Note: The quiz is open book but has one limitation: You are not allowed to get help from another human including your classmates.

Question 1. Given $R = Rot(\hat{x}, \frac{\pi}{2})Rot(\hat{z}, \pi)$, find the unit vector $\hat{\omega}$ and angle θ such that $R = e^{[\hat{\omega}]\theta}$. Explain the physical meaning of this.

Question 2. Find the exponential coordinates $\hat{\omega}\theta \in \mathbb{R}^3$ for the SO(3) matrix:

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

- Why is this matrix a member of the Special Orthogonal Group (SO(3))?
- What is the physical interpretation of this problem?

Question 3. Answer the following questions related to ZXZ Euler angles (successive rotations about the body frame).

(a) Derive a procedure for finding the ZXZ Euler angles of a rotation matrix.

(b) Using the results of (a), find the ZXZ Euler angles for the following rotation matrix (check to see if it is really a rotation matrix):

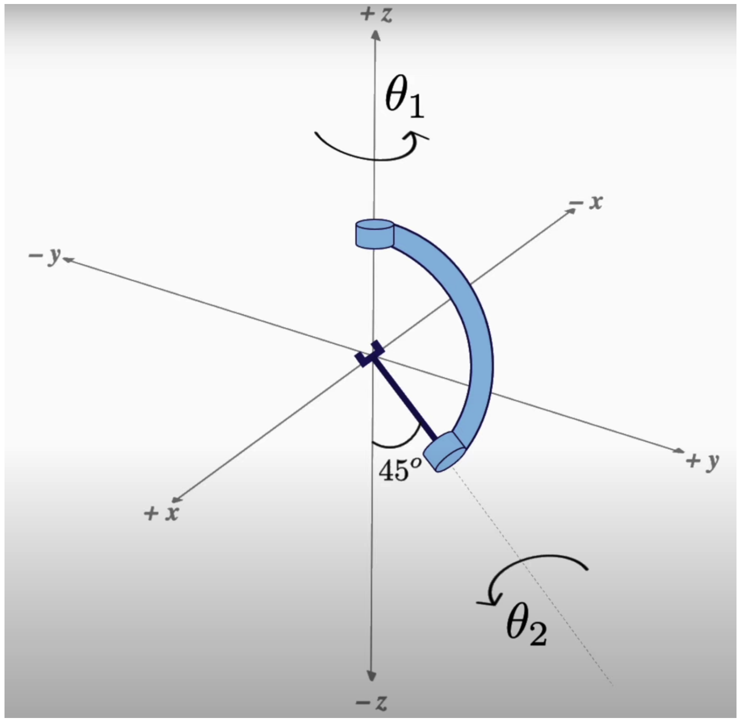
$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(c) Visualize this rotation using the software RoboDK (Catia/Solidworks uses this convention). If you calculated different sets of Euler angles that represent this rotation matrix, show that the final orientations are the same for all of them.

Question 4. Consider a wrist mechanism with two revolute joints θ_1 and θ_2, in which the end-effector frame orientation $R \in SO(3)$ is given by:

$$R = e^{[\hat{\omega}_1]\theta_1} e^{[\hat{\omega}_2]\theta_2} I$$

with $\hat{\omega}_1 = (0, 0, 1)$ and $\hat{\omega}_2 = (0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.



Determine whether the following orientation is reachable (that is, find, if it exists, a solution (θ_1, θ_2)) for the following R):

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$