The concavity of price impact

Master Project - MATH 594

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SUMMARY

The concept of price impact concavity refers to the idea that the relationship between the quantity of an asset traded and the resulting price impact is not linear, but rather concave. In other words, as the quantity of an asset traded increases, the price impact per unit of trade decreases at an increasing rate.

This means that a small trade in a relatively illiquid market may have a larger price impact than a much larger trade in a more liquid market. As more of an asset is traded, market participants may start to adjust their expectations about future prices, reducing the impact of subsequent trades on the market price.

Understanding the concavity of price impact is important for market participants, particularly those engaged in large trades or those trading in relatively illiquid markets. By taking into account the non-linear relationship between trade size and price impact, market participants can better estimate the costs and benefits of their trades, and adjust their strategies accordingly.

INTRODUCTION

Let’s use a theoretical model that provides a microstructural explanation for the concavity of the price impact function observed in financial markets.

Empirical evidences of the concavity of the price impact function, shows that larger trades have a smaller impact on market prices than smaller trades of the same size relative to the market's trading volume.

A simple model can be based on a limit order book that captures the key features of price impact in financial markets. The model assumes that the limit order book is composed of two groups of traders: liquidity traders who submit market orders and informed traders who submit limit orders. The informed traders have superior information about the true value of the asset and adjust their orders based on this information.

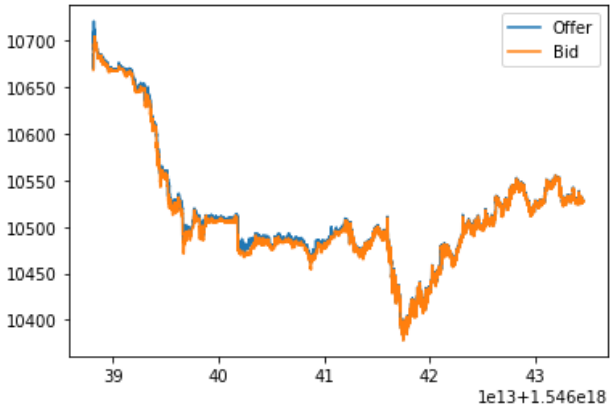
Concavity of the price impact function can be explained by the strategic interaction between the informed traders and the liquidity traders. The informed traders submit limit orders that reflect their private information, which leads to a widening of the bid-ask spread. This, in turn, reduces the price impact of large trades because they need to trade against a larger number of limit orders that are located farther from the current market price.

The concavity of the price impact function is sensitive to the market's microstructure, such as the order flow imbalance, the liquidity supply, and the trading frequency. The model's predictions are consistent with empirical observations in financial markets.

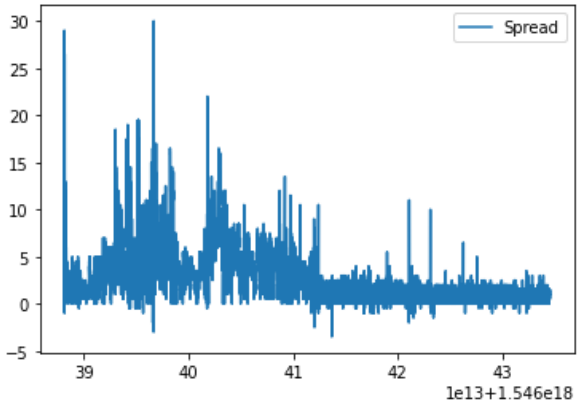
ANALYSIS

To illustrate and verify the concavity of the price impact, we need to visualize the distribution of traded volume across imbalance levels.

We gather informations on the ask price P_A and volume V_A, and the bid price P_B and volume V_B according to a given time t. After matching the ask and bid to the same time, those informations constitute the Limit Order Book. Here is a graph of the bid and ask and a graph of the spread $P_A - P_B$ depending on the time. Few values of the spread where negatives, certainly due to the original file of data provided. Consequently the price and volume corresponding to the negative spread have been ignored.



Bid and Ask as a function of time

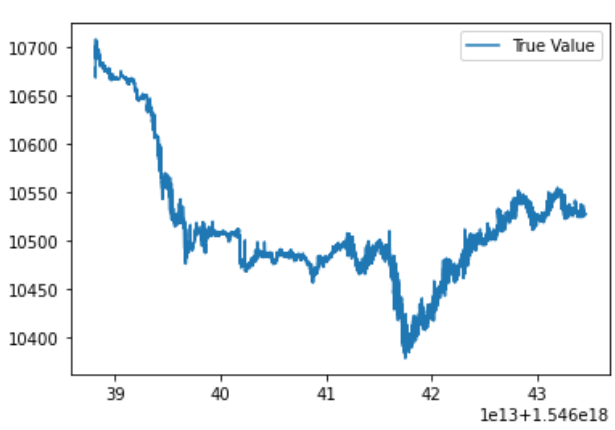


Spread as a function of time

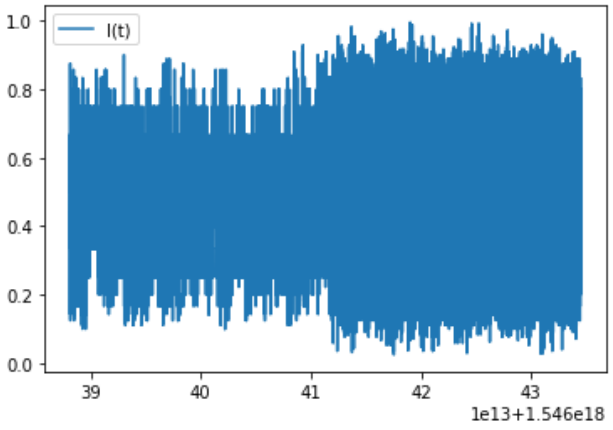
What's more we want to use small spreads to trade because they allow traders to buy and sell assets with minimal transaction costs. The spread refers to the difference between the bid price (the price at which a trader can sell an asset) and the ask price (the price at which a trader can buy an asset). When the spread is small, it means that the bid and ask prices are close to each other, which makes it easier and cheaper for traders to enter and exit positions in the market.

In addition to minimizing transaction costs, trading with small spreads can also provide traders with more liquidity and faster execution. When the spread is small, it means that there are many buyers and sellers in the market, which makes it easier and faster for traders to find a counterparty for their trades.

In a second part, we graph the True price $X_t = B_t + (A_t - B_t)I_t$ at time t with the imbalance $I_t = \frac{V_B}{V_A + V_B}$. Order imbalance act as a predictive factor. If the current LO imbalance is high, the next MO is more likely to be a buy an the next price move is more likely to be upward. If the current LO imbalance is low, the next MO is more likely to be a sell an the next price move is more likely to be downward.

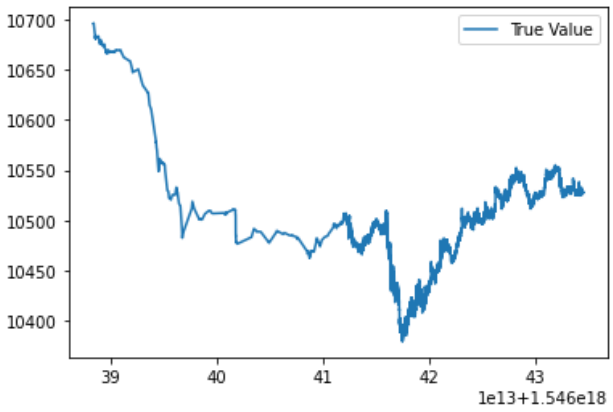


True Price as function of time



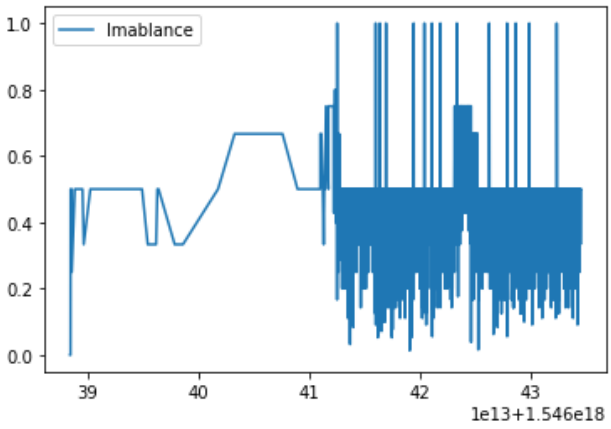
Imbalance as function of time

We will focus on certain values of the spread between ask and bid prices, especially when the spread is equal to 1.



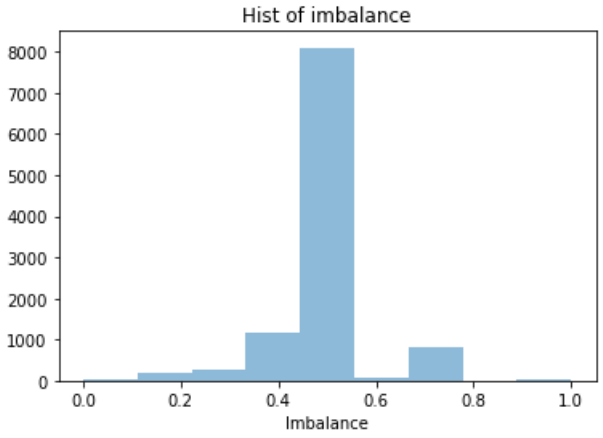
True Price as function of time

when $P_A - P_B = 1$



Imbalance as function of time

when $P_A - P_B = 1$

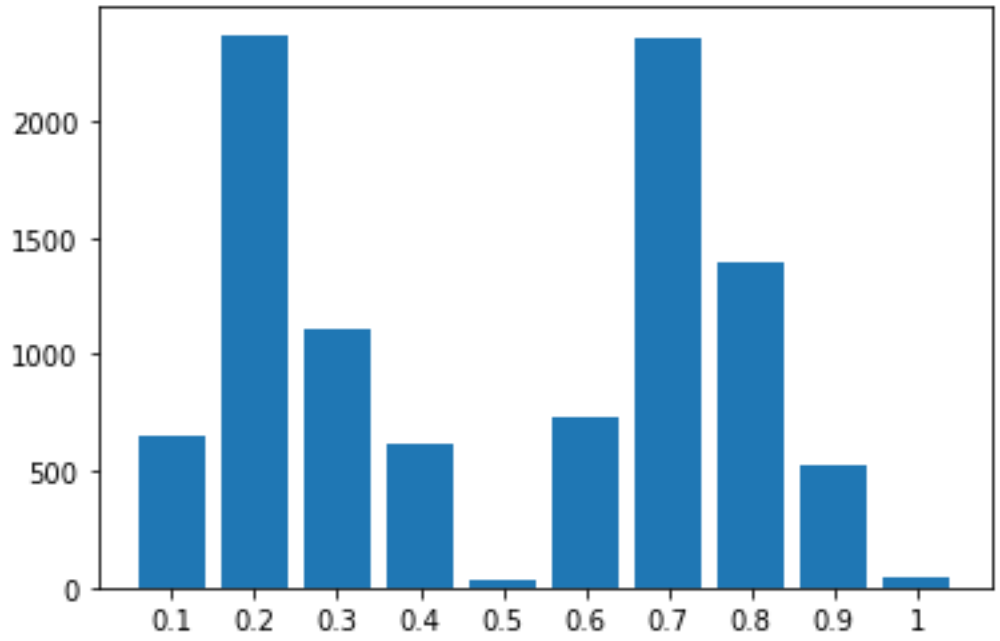


Histogram of Imbalance,

when $P_A - P_B = 1$

Finally to compute the Distribution of traded volume across imbalance levels we need to consider Market Orders and their matching Limited Orders with a certain spread. In that part we chose a spread equals to 1±0.5.

We define traded volume as the some sum of the volumes of all Market Orders submitted at or before time t.



Absolute distribution of traded volume across imbalance levels

In our case all the Market Order's times matched one Limit Order time. However it could have not be the case. If there was a rounding errors (< 5%) we can match the times together. If the error was greater than (5%), the times aren't the same and we cannot consider the corresponding values.

CONCLUSION

The U-shape of traded volume refers to the empirical observation that large trades have a smaller price impact relative to smaller trades of the same size relative to the market's trading volume. This phenomenon is also known as the concavity of the price impact function.

The connection between the U-shape of traded volume and concavity can be explained by the microstructure of financial markets. In particular, the concavity of the price impact function arises from the strategic interaction between informed traders and liquidity traders in the market.

Informed traders submit limit orders that reflect their private information, which leads to a widening of the bid-ask spread. This widening reduces the price impact of large trades because they need to trade against a larger number of limit orders that are located farther from the current market price. As a result, the price impact function becomes concave, with larger trades having a smaller impact on market prices than smaller trades of the same size relative to the market's trading volume.

Therefore, the U-shape of traded volume and the concavity of the price impact function are interconnected, with the former being a manifestation of the latter due to the market's microstructure and the strategic behavior of traders.

REFERENCES

S. Nadtochiy (2020). *A simple microstructural explanation of the concavity of price impact*

Git repository of the code : https://github.com/madidina/PriceImpact