Homework 8

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1. (a) Let I be a nonempty ideal in F such that I does not equal F. Let $x \in F$ and $x \notin I$, let $a \in I$, and let a^{-1} be the multiplicative inverse of a. If $a \in I$, then $ax \in I$ and $a^{-1}ax \in I$. However, $a^{-1}ax = 1_F x = x$ and $x \notin I$ so this is a contradiction.

   (b) Since f is a homomorphism of rings, the kernel of f is an ideal of F. The only ideals of F are F and (0). If the kernel of f is (0), then f is injective. If the kernel of f is F, then f is the zero function.