Homework 8

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1. (a) Let I be a nonempty ideal in F such that I does not equal F. Let $x \in F$ and $x \notin I$, let $a \in I$, and let a^{-1} be the multiplicative inverse of a. If $a \in I$, then $ax \in I$ and $a^{-1}ax \in I$. However, $a^{-1}ax = 1_F x = x$ and $x \notin I$ so this is a contradiction.

   (b) Since f is a homomorphism of rings, the kernel of f is an ideal of F. The only ideals of F are F and (0). If the kernel of f is (0), then f is injective. If the kernel of f is F, then f is the zero function.

2. First, assume $(p(x))$ is maximal. Let $p(x) = q_1(x)q_2(x)...q_n(x)$ be the prime factorization of $p(x)$. Therefore $(p(x)) \subset (q_1(x))$ since $q_1(x)$ divides $p(x)$. Since $(p(x))$ is maximal, either $(p(x)) = (q_1(x))$ or $F[x] = (q_1(x))$. Assume $(p(x)) = (q_1(x))$. Therefore $p(x)$ must be some divisor of $q_1(x)$ such that $q_1(x) = p(x)q_2^{-1}(x)...q_n^{-1}(x)$.