Homework 5

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1. To prove φ_α is closed under addition, first let f, g be any elements of $S[x]$. We know that $f+g$ is also an element of $S[x]$ because the coefficients of f and g are elements of S, which is a ring. This implies that the coefficients of f and g are closed under addition, so the coefficients of $f+g$ are also in S.

   With this having be said, we can prove φ_α is closed under addition as such:

   $$\varphi_\alpha(f+g) = (f+g)(\alpha) = f(\alpha) + g(\alpha) = \varphi_\alpha(f) + \varphi_\alpha(g)$$

   We know $f \cdot g$ is an element of $S[x]$ since S is closed under multiplication, so the coefficients of $f \cdot g$ are elements of S.

   We can prove φ_α is closed under multiplication as such:

   $$\varphi_\alpha(f \cdot g) = (f \cdot g)(\alpha) = f(\alpha) \cdot g(\alpha) = \varphi_\alpha(f) \cdot \varphi_\alpha(g)$$

2. TODO

3. TODO