Homework 8

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1. (a) Since $(x^7+25x^6-25x+5)$ is a nonconstant polynomial in $\mathbb{Q}$, $\mathbb{Q}/(x^7+25x^6-25x+5)$ is a field if and only if $(x^7+25x^6-25x+5)$ is irreducible in $\mathbb{Q}$. We know that $(x^7+25x^6-25x+5)$ is irreducible in $\mathbb{Q}$ by Eisenstein's Criterion for prime 5. Thus, $\mathbb{Q}/(x^7+25x^6-25x+5)$ is a field.

   (b) Consider $\mathbb{Z}/2\mathbb{Z}$. If $f(x) = x^3+2x^2-x+1$, then $\overline{f}(x) = x^3-x+1$. We know that $\overline{f}(x)$ is irreducible in $\mathbb{Z}/2\mathbb{Z}$ because $\overline{f}(0) = \overline{f}(1) = 1$. Since $\overline{f}(x)$ is irreducible in $\mathbb{Z}/2\mathbb{Z}$, $f(x)$ is irreducible in $\mathbb{Q}$, which means that $\mathbb{Q}[x]/(x^3+2x^2-x+1)$ is a field.

   (c) Only first-degree and second-degree polynomials can be irreducible in $\mathbb{R}[x]$. Since $(x^5+42x^4+\pi x^3-1729x^2+ln(2)x-2019)$ is a fifth-degree polynomial, it is reducible in $\mathbb{R}[x]$. Therefore $\mathbb{R}[x]/(x^5+42x^4+\pi x^3-1729x^2+ln(2)x-2019)$ is NOT a field.

2. (a) Let $f(x) = x^3+2x+1$. We know that $f(x)$ is irreducible in $\mathbb{Z}/3\mathbb{Z}$ because $f(0) = f(1) = f(2) = 1$. Therefore, K is a field.

   (b) TODO