Homework 6

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1. To prove that ”is an associate of” is an equivalence relation on R, we must show that it is reflexive, symmetric, and transitive.

   First, since R is a commutative ring, there exists some identity element in R, 1_R, such that $a1_R = a$. We can say that 1_R is a unit since $1_R\dot{1}_R = 1_R$. Since $a1_R = a$, a is an associate of a and ”is an associate of” is reflexive.

   Next, let a, $b \in R$ such that a is an associate of b. Therefore, there exist some unit, u, in R sucht that $au = b$. Since u is a unit, there also exists some unit $v \in R$ such that $uv = 1_R$.

   If $a = bu$, then
   $$av = (bu)v$$
   $$av = buv$$
   $$av = b(uv)$$
   $$av = b1_R$$
   $$av = b$$
   Therefore, b is an associate of a and ”is an associate of” is reflexive.

   Finally, let a be an associate of b and let b be an associate of c for some elements a, b, $c \in R$. Therefore, $a = bu$ and $b = cv$ for some units u, $v \in R$.

   We notice that
   $$a = bu$$
   $$a = (cv)u$$
   $$a = c(vu)$$
   We know that vu is a unit because there exists some v^{-1}, $u^{-1} \in R$ such that $vv^{-1} = 1_R$ and $uu^{-1} = 1_R$ since v and u are units. Thus, $(vu)(v^{-1}u^{-1}) = (vv^{-1})(uu^{-1}) = 1_R$ and so vu is a unit. Since vu is a unit, a is an associate of c and ”is an associate of” is transitive.