MATH 307
Assignment #14
Due any time, any year

For each problem, include the statement of the problem. Leave a blank line. At the beginning of the next line, write **Solution** or **Proof** – as appropriate.

1. Suppose $T \in \mathcal{L}(\mathbf{C}^4)$ is such that the eigenvalues of T are 3, 5 8. Prove that $(T-3I)^2(T-5I)^2(T-8I)^2 = 0$.

2. Give an example of an operator on $\mathbf{C}^4$ whose characteristic polynomial equals $(z-1)(z-5)^3$ and whose minimal polynomial equals $(z-1)(z-5)^2$.

3. Suppose V is a complex vector space. Suppose $P \in \mathcal{L}(V)$ is such that $P^2 = P$. Prove that the characteristic polynomial of P is $z^m(1-z)^n$, where $m = \dim P$ and $n = \dim \operatorname{range} P$.

4. Suppose $T \in \mathcal{L}(V)$ has minimal polynomial $2 - 3z + z^2 - 5z^3 + 7z^4 + 6z^5$. Find the minimal polynomial of T^{-1}.

5. Suppose $T \in \mathcal{L}(V)$ is invertible. Prove that there exists a polynomial $p \in \mathcal{P}(\mathbf{F})$ such that $T^{-1} = p(T)$.

6. Suppose V is an inner product space and $T \in \mathcal{L}(V)$ is normal. Prove that the minimal polynomial of T has no repeated zeros.

7. Suppose $N \in \mathcal{L}(V)$ is nilpotent. Prove that the minimal polynomial of N is z^{m+1}, where m is the length of the longest consecutive string of 1's that appears on the line directly above the diagonal in the matrix of N with respect to any Jordan basis for N.

8. Suppose $T \in \mathcal{L}(V)$ and $v_1, \ldots, v_n$ is a basis of V that is a Jordan basis for T. Describe the matrix of T^2 with respect to this basis.

9. Suppose $p, q \in \mathcal{P}(\mathbf{C})$ are monic polynomials with the same zeros and q is a polynomial multiple of p. Prove that there exists $T \in \mathcal{L}(\mathbf{C}^{\deg q})$ such that the characteristic polynomial of T is q and the minimal polynomial of T is p.