MATH 307
Assignment #8
Due Friday, March 11, 2022; CORRECTION: Problems 3 & 5: notation for inner product. Fixed $\LaTeX$ typo in #7. Deleted 3 probllems: 9Feb2022.

For each problem, include the statement of the problem. Leave a blank line. At the beginning of the next line, write **Solution** or **Proof** – as appropriate.

1. Prove that
$$16 \leq (a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$
for all positive numbers a, b, c, d.
Hint: find two vectors having lots of square roots; compute an inner product and also use Cauchy-Schwarz.

2. Prove or disprove: there is an inner product on $\mathbf{R}^2$ such that the associated norm is given by
$$\|(x,y)\| = \max\{|x|,|y|\}$$
for all $(x,y) \in \mathbf{R}^2$.

3. Suppose V is a real inner product space. Prove that
$$\langle u, v\rangle = \frac{\|u+v\|^2 - \|u-v\|^2}{4}$$
for all $u, v \in V$.

4. Show that if $a_1, \ldots a_n \in \mathbf{R}$, then the square of the average of $a_1, \ldots, a_n$ is less than or equal to the average of $a_1^2, \ldots, a_n^2$.

5. Convert $\mathcal{P}_2([0,1])$ into an inner product space by writing $\langle p, q\rangle = \int_0^1 p(x)\overline{q(x)}\,dx$ for $p, q \in \mathcal{P}_2([0,1])$. Find a complete orthonormal set in that space.

6. Suppose U is the subspace of $\mathbf{R}^4$ defined by
$$U = \text{span}\,((1,2,3,-4),(-5,4,3,2)).$$
Find an orthonormal basis of U and an orthonormal basis of $U^\perp$.