MATH 307
Assignment #1
Due Friday, January 21, 2022

For each problem, include the statement of the problem. Leave a blank line. At the beginning of the next line, write **Solution** or **Proof** – as appropriate.

1. Label the following statements as being true or false. Provide some justification from the text for your label.
   (a) Every vector space contains a zero vector.
   (b) A vector space may have more than one zero vector.
   (c) In any vector space $au = bu$ implies that $a = b$.
   (d) In any vector space $au = av$ implies that $u = v$.
   (e) In $\mathcal{P}(\mathbf{F})$ only polynomials of the same degree may be added.
   (f) If f and g are polynomials of degree n, then $f + g$ is a polynomial of degree n.
   (g) If f is a polynomial of degree n and c is a nonzero scalar, then cf is a polynomial of degree n.
   (h) A nonzero element of $\mathbf{F}$ may be considered to be an element of $\mathcal{P}(\mathbf{F})$ having degree zero.
   (i) Two functions in $\mathbf{F}^S$ are equal if and only if they have the same values at each element of S.

2. Let $v_1, \ldots, v_4$ be four vectors in a vector space V. Verify $(v_1 + v_2) + (v_3 + v_4) = [v_2 + (v_3 + v_1)] + v_4$. Use the definition, properties, and theorems on pp.12-15 to justify each step in the transitions from the LHS to the RHS.

3. Which vectors in $\mathbf{R}^3$ are linear combinations of $(1, 0, -1), (0, 1, 1), (1, 1, 1)$?

4. Let $V = \mathbf{R}^2$ with *new* operations

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$$
$$c(x, y) = (cx, y)$$

   Is V a vector space? Justify.

5. Let $V = \mathbf{R}^2$ with *new* operations

$$(x, y) + (x_1, y_1) = (x + x_1, 0)$$
$$a(x, y) = (ax, 0)$$

   Is V a vector space? Justify.

6. Consider $\mathbf{R}^n$ with new operations

$$v \boxplus w = v - w$$
$$a \cdot v = -av$$

   Which of the parts of the definition of vector space are satisfied with these new operations?

7. Which subsets of $\mathcal{P}(\mathbf{R})$ form a vector space? Justify.

   (a) All $p(x)$ such that $p(0) = 1$.

   (b) All $p(x)$ such that $p(0) = 0$.

   (c) All $p(x)$ such that $2p(0) - 3p(1) = 0$.