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1a Y is a binomial distribution
$n = 15, \quad p = 0.25$
Y = # of correct guesses

1b. $E[Y] = n \cdot p = (15 \cdot 0.25) = 3.75 = E[Y]$

$V[Y] = npq = (15 \cdot 0.25 \cdot 0.75) = 2.8125 = V[Y]$

1c. $P(Y \leq 5) = \text{pbinom}(5, 15, 0.25)$

≈ 0.8516

1d. $1 - P(Y \leq 7)$ $\quad$ pbinom(7, 15, 0.25)
$1 - 0.9827$
≈ 0.0173

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2. $n \cdot p = \mu = 84$

$n \cdot p \cdot q = V = 36$

$n \cdot p \cdot q = 36 \xrightarrow{\text{substitute}} 84 \cdot q = 36$

$q = \frac{3}{7} \approx 0.42857$

$p = \frac{4}{7} \approx 0.57143$

$n \cdot \frac{4}{7} = 84$

$n = 84 \cdot \frac{7}{4}$

$n = 147$

$P(Y \geq 50) = 1 - \text{pbinom}(49, 147, 4/7)$

≈ 1

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3a. Y = # of trials until first error

geo

$p = 0.04$

$q = 0.96$

$P(Y=y) = q^{y-1} p$

$P(Y==5) = 0.96^{4} 0.04$

≈ 0.03397

3b. $P(Y \leq 5) = 1 - q^{k}$

$= 1-(0.96)^{5}$

≈ 0.18463

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4a. $p = \frac{1}{5000}$ $q = \frac{4999}{5000}$

$E[Y] = \frac{1}{(1/5000)} = 5000$

4b. $P(Y=2000) = q^{y-1}p$

$= \left(\frac{4999}{5000}\right)^{1999} \cdot \frac{1}{5000}$

≈ 0.00013408546

4c. $P(Y \leq 2000) = 1 - q^k = 1 - \left(\frac{4999}{5000}\right)^{2000}$

≈ 0.3297067698

4d. $1 - P(P \leq 2999) = 1 - \left(1 - \left(\frac{4999}{5000}\right)^{2999}\right)$

≈ 0.5488848168

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5a best of 7 (first to 4)

$p = \text{prob}(\text{A wins a game}) = 0.54$

$P(\text{A wins in } 6) = \binom{5}{3}(0.54)^4(0.46)^{(6-4)}$

≈ 0.1799246496

5b. $P(\text{A wins}) = P(4) + P(5) + P(6) + P(7)$

$P(4) = \binom{3}{3}(0.54)^4(0.46)^0 \approx 0.08503$

$P(5) = \binom{4}{3}(0.54)^4(0.46)^1 \approx 0.156456$

$P(6) =$ 5a

$P(7) = \binom{6}{3}(0.54)^4(0.46)^3 \approx 0.1653069$

$\sum_{i=4}^{7} P(i) \approx 0.5869421471 2$

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6. Neg Bin

$p(y) = \binom{y-1}{r-1} p^r q^{y-r}$

$p = 0.20$
$q = 0.8$

$n = 15$
$r = 5$

$p(y=15) = \binom{14}{4} (0.2)^5 (0.8)^{10}$

≈ 0.0343940981
that exactly 15 couples need to be asked to find 5