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MTH 325 WH 5 Matt Wilder

1.)

a.) $\int_0^\infty \frac{e^{-x/1000}}{1000} = \frac{-1}{e^{x/1000}}\Big|_0^\infty$

$= \lim_{x\to\infty}\left(\frac{-1}{e^{x/1000}}\right) - \lim_{x\to 0}\left(\frac{-1}{e^{x/1000}}\right)$

$= 0 - \left(\frac{-1}{1}\right) = 1$

Proper dist

b.) $P(Y > 1500)$

$= \int_{1500}^\infty f(x)dx = \frac{-1}{e^{x/1000}}\Big|_{1500}^\infty$

$= 0 - \left(\frac{-1}{e^{3/2}}\right) = 0.22313$

says $e^{3/2}$

$\mu = \int_D x f(x)\, dx$

c.) $\mu = \int_0^\infty x f(x) dx = \cancel{\frac{x}{1000}}$

$= \frac{-1}{e^{x/1000}}(x+1000)\Big|_0^\infty$

$\lim_{x\to\infty}\left(\frac{-x}{e^{x/1000}}\right) \overset{H}{=} \lim_{x\to\infty}\left(\frac{-1}{\frac{e^{x/1000}}{1000}}\right) = -1000\lim_{x\to\infty}\left(\frac{1}{e^{x/1000}}\right) = -1000(0) = 0$

$\lim_{x\to 0}\left(\frac{-1000}{e^{x/1000}}\right) = \frac{-1000}{e^{0/1000}} = \frac{-1000}{1}$ (direct sub cont. f(x))

$0-(-1000) = 1000 = \mu$

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2\. a.) The graph of $F(y)$ is a discontinuous step function, so it is discrete.

b.) $f(y) = \begin{cases} 1/6 & y=1 \\ 1/2 & y=3 \\ 1/3 & y=4 \end{cases}$

c.) $\boxed{P(Y=2) = 0}$

d.) $\frac{e^{t}}{6} + \frac{e^{3t}}{2} + \frac{e^{4t}}{3} = m(t)$

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3 a.)

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 -ax+3a\,dx$$

$$\frac{ax^2}{2}\Big|_0^1 + ax\Big|_1^2 + \left(\frac{-ax^2}{2}+3ax\right)\Big|_2^3$$

$$\frac{a}{2} + (2a-1a) + \left[\left(\frac{-9a}{2}+9a\right)-\left(\frac{-4a}{2}+6a\right)\right]$$

$= 2a = 1$ (by def of proper dist)

$$\boxed{a=\frac{1}{2}}$$

3b

$$F(y)=\begin{cases} 0 & y<0 \\ \frac{y^2}{4} & 0\le y<1 \\ \frac{y}{2}-\frac{1}{4} & 1\le y<2 \\ \frac{-y^2}{4}+\frac{3}{2}y-\frac{5}{4} & 2\le y<3 \\ 1 & y>3 \end{cases}$$

r clarity $\frac{-y^2}{4}+\frac{3y}{2}-\frac{5}{4}$

$\int \frac{1}{2}x\,dx = \frac{1}{4}x^2$, $\frac{y^2}{4}$ $0\le y<1$

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$\frac{1}{2}y + C = F(1) = \frac{1}{4}$, let $y=1$ $\frac{1}{2}+c=\frac{1}{4}$, $c=\frac{-1}{4}$

anti 3

Let $y=2$ $\frac{-y^2}{4} + \frac{3y}{2} + C = F(2) = 3/4$

$\frac{-4}{4}+\frac{12}{2}+C = 3/4 \Rightarrow C = -5/4$

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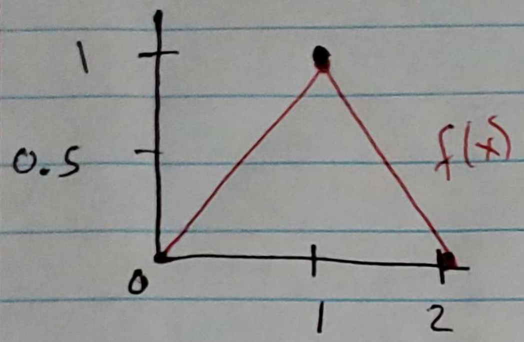
4. $f(x) = 1-|x-1|$ $[0,2]$

a.)



$y = x$ $(0,1)$ $\frac{x^2}{2} = y'$

$y = 2-x$ $(1,2)$ $\frac{-x^2}{2} = y'$

b.)

$$F(y) = \begin{cases} 0 & y<0 \\ \frac{y^2}{2} & 0 \le y < 1 \\ 2y - \frac{y^2}{2} - 1 & 1 \le y < 2 \\ 1 & y \geq 2 \end{cases}$$

$\int 2-x = 2y + -\frac{y^2}{2} + C = \frac{1}{2}$ when $y = 1$

$y=1 \Rightarrow C = -1$

$2y - \frac{y^2}{2} - 1$

c.) $F(x) = 0.75$ $2y - \frac{y^2}{2} - 1 = 0.75$

$2y - \frac{y^2}{2} - 1.75 = 0$

$\frac{-2 \pm \sqrt{4-3.5}}{-1} = 2 \pm \sqrt{0.5}$

$2 \pm \frac{1}{\sqrt{2}}$

$2 + \frac{1}{\sqrt{2}}$ out of bounds

$2 - \frac{1}{\sqrt{2}}$

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5. a.

$$f(y) = \begin{cases} \frac{c}{x^2} & (1, \infty) \\ 0 & \text{else} \end{cases}$$

$$\int_1^{\infty} cx^{-2} = 1$$

$$\left.\frac{-c}{x}\right|_1^{\infty} = 1$$

$$\lim_{x \to \infty}\left(\frac{-c}{x}\right) = 0 \quad , \quad -\lim_{x \to 1}\left(\frac{-c}{x}\right) = 1$$

$$\lim_{x \to 1}\left(\frac{c}{x}\right) = 1 \quad \frac{c}{1} = 1$$

$$c = 1$$

b.) $\frac{1}{x^2}$ $\int_1^{\infty} x f(x) = \int_1^{\infty} \frac{1}{x} \xrightarrow{\text{Diverges}}$ DNE

harmonic series

$$\mu = \int_0 x f(x)$$

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6. $A = \pi r^2$ ~~$f \in (0$~~ $r \in (0,2)$ uniformly

$E[A] = E[\pi r^2] = \pi \cdot E[r^2]$

$= \pi \cdot \int_0^2 r^2 \cdot f(r)\, dr$

$f(r) = \frac{1}{(2-0)} = \frac{1}{2}$ $= \pi \cdot \frac{1}{2} \int_0^2 r^2 dr = \frac{\pi}{2} \cdot \left(\frac{r^3}{3} \Big|_0^2 \right)$

$= \frac{\pi}{2} \cdot \frac{2^3}{3} = \frac{\pi}{3} \cdot 2^2 = \frac{4\pi}{3} = \mu$

$E[A^2] = E[\pi^2 r^4] = \frac{\pi^2}{2} \cdot \int_0^2 r^4 dr = \frac{\pi^2}{2} \cdot \frac{r^5}{5} \Big|_0^2$

$\frac{16\pi^2}{5}$

$V[A] = \frac{16\pi^2}{5} - \left(\frac{4\pi}{3}\right)^2 = \frac{64\pi^2}{45} = \sigma$

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3c

$$1 - F(1.5) = 1 - \frac{1.5}{2} - \frac{1}{4} = \frac{1}{2}$$

$$\binom{3}{1} \cdot \left(\frac{1}{2}\right)^3 = \frac{3}{8}$$