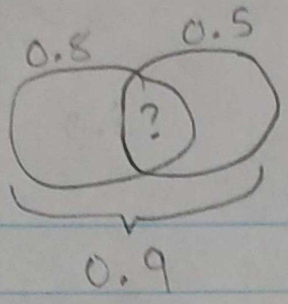
MTH 325
HW 2

Matt Wilder



1. a. $A \cup B = A + B - A \cap B$

$$0.9 = 0.8 + 0.5 - A \cap B$$
$$-0.4 = -A \cap B$$
$$A \cap B = 0.4$$

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if (P(A∩B) == P(A)·P(B))
    return true;
return false;
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$$0.4 = 0.8 \cdot 0.5$$
$$0.4 = 0.4, \quad \text{true}$$

$\boxed{\text{A and B are independent}}$

because "two events A and B are said to be independent if any of the following holds:

1.) $P(A|B) = P(A)$
2.) $P(B|A) = P(B)$
3.) $P(A \cap B) = P(A) \cdot P(B)$

and the third condition holds, all we needed was to compute $A \cap B$.

MTH 325
HW 2

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Wilder

1. b. Two events are mutually exclusive if $A \cap B = 0$, that is, both cannot happen simultaneously.
Using the work from part a.) $A \cap B = 0.4$ and $0.4 != 0.0$ $\therefore$ A and B are NOT mutually exclusive

MTH 325 HW2 Matt Zittdel

2. A: 20% $A \cap B$: 8%
B: 16% $A \cap C$: 5%
C: 14% $B \cap C$: 4%
$A \cap B \cap C$: 2%

a.) None of the papers

$1-(A \cup B \cup C)$

$A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$

$0.5 - 0.08 - 0.05 - 0.04 + 0.02$

$A \cup B \cup C = 0.35$

$P(\overline{A \cup B \cup C}) = 1 - 0.35 = \boxed{0.65}$ no papers

b.) Exactly 1 paper

from #a, at least 1 paper is 0.35

$0.35 - A \cap B - A \cap C - B \cap C + A \cap B \cap C$

$0.35 - 0.08 - 0.05 - 0.04 + 0.02$

0.20 0.20 exactly 1

c.) Reads at least $A \cap B$ given $A \cup B \cup C$

$P(A \cup B \cup C) = 0.35$

$P((A \cap B) \cap (A \cap B \cap C) \mid A \cup B \cup C)$

$P(A \cap B \mid A \cup B \cup C)$

$A \cap B \cap C \subset A \cap B$

$= \frac{P((A \cap B) \cap (A \cup B \cup C))}{P(A \cup B \cup C)} = \frac{P(A \cap B)}{P(A \cup B \cup C)}$

$\boxed{= \frac{0.08}{0.35} \approx 0.2289}$

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3. 10 urns

9/10 contain 3W ∧ 3R = Urn ~B

1/10 contain 5W ∧ 1R = Urn B

Randomly pick 1 urn $\left(\frac{1}{10}\right)$, then 3 chips w/o replacement

~~P(B) 3W)~~

$$P(3 \text{ white} \mid \sim B) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{20}$$

$$P(3 \text{ white} \mid B) = \left(\frac{5}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right) = \frac{1}{2}$$

let A = prob of 3 whites

$$P(B) = \frac{1}{10}, \quad P(A \mid B) = \frac{1}{2}$$

Group the 9 as 1 event →

$$P(\sim B) = \frac{9}{10}, \quad P(A \mid \sim B) = \frac{1}{20}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \mid B) \cdot P(B)}{P(A)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{10}}{\sum_{1}^{10} P(A \mid B_i) \cdot P(B_i)} = \frac{\frac{1}{20}}{1\left(\frac{1}{20} \cdot \frac{9}{10}\right) + 1\left(\frac{1}{2} \cdot \frac{1}{10}\right)}$$

still 1 event

$$= \frac{\frac{1}{20}}{\left(\frac{9}{200}\right) + \frac{1}{20}} = \frac{\frac{10}{200}}{\frac{19}{200}} = \frac{10}{19}$$

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4. X, 4 sided die

$P(1) = P(2) = P(4) = 0.3$

$P(3) = 0.1$

$\mu = E[X] = 1 \cdot \frac{3}{10} + 2 \cdot \frac{3}{10} + 1 \cdot \frac{1}{10} + 4 \cdot \frac{3}{10}$ mean

$= \frac{3}{10} + \frac{6}{10} + \frac{1}{10} + \frac{12}{10} = \frac{22}{10}$ $\boxed{2.2 = \mu = \text{avg}}$

Variance $= \sigma^2 = E\left[(X-\mu)^2\right]$

$\frac{1}{10}\left[3(1-2.2)^2 + 3(2-2.2)^2 + (3-2.2)^2 + 3(4-2.2)^2\right]$

Varience $= 1.48$

Standard distribation $= \sqrt{1.48} \approx 1.21655$

MTH 325 Matt Wilder

5. $E(X+4)=10$

$E(X)+E(4)=10$

$E(X)+4=10$

$E(X)=6=\mu$

$\sigma^2=E[(X-6)^2]$

$=E(X^2)-12E[\boxed{X}]+36$

$=52-12\cdot 6+36$

$\sigma^2=16$

$\mu=6$

$E((X+4)^2)=116$

$=E(x^2+8x+16)=116$

$=E(x^2)+E(8x)+E(16)=116$

$E(X^2)+8E(X)=100$

substitute $E(X)=6$

$E(X^2)+8\cdot 6=100$

$E(X^2)=52$

$V[X-4]=E[(X-4-\mu)^2]$

$=E[(X-4-6)^2]$

$E[(X-10)^2]$

$E(X^2)-20E(X)+100$

$52-20\cdot 6+100$

$=\cancel{32}$

$V[X-4]=32$