1. (6 points) In the greater Peoria area, 35% of people are St. Louis Cardinal (S) fans, 30% of people are Chicago Cubs (C) fans, and a deranged 5% are a fan of both teams.

0.35	Cards (S)
0.30	Cubs (C)
0.05	Both

(a) For the following, determine the event in terms of set notation using S and C, and then the probability of the event.

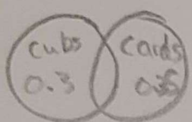
i. That a person chosen at random is not a fan of either team.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.35 + 0.3 - 0.05 = 0.6$ fan of something

$1 - P(A \cup B) = 0.4$ that they arent a fan of any

ii. That a person chosen at random is only a Cubs fan.



$= P(\text{Cubs}) - P(\text{deranged})$

$0.3 - 0.05$

0.25

only cubs	0.25
only cards	0.3
both	0.05
neither	0.4

(b) Determine the probability that a person who is a known Cubs fan is also deranged. (That is, they are a Cardinals fan as well.)

$P(\text{deranged} \mid \text{cubs}) = \dfrac{P(D \cap C)}{P(C)} \quad \dfrac{0.05}{0.03} = \dfrac{1}{6}$

2. (6 points) Bowl A contains three red chips, Bowl B contains two blue chips and one red chip, and bowl C contains one red, one blue chip, and one black chip. A bowl is selected at random and one chip is taken from that bowl.

(a) Compute the probability of selecting a blue chip.

	A	B	C
R	3	1	1
Blue		2	1
Bk			1

$\overset{A}{\left(\frac{1}{3}\right)\cdot(0)} + \overset{B}{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} + \overset{C}{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)} \qquad \frac{1}{3}$

$0 + \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$

(b) If the selected chip is blue, compute the probability that the ~~other~~ next chip in the bowl is red.

	B	C
R	1	1
Be	1	0
Bk		1
	$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{2}$

3. (6 points) A weighted six-sided die has the following probabilities

y: # of pips	1	2	3	4	5	6
$p(y)$: probability	1/12	1/12	1/4	1/4	1/12	1/4

(a) Show that the above is a proper probability distribution.

$$\sum_{y=1}^{6} p(y) = \frac{1}{12}+\frac{1}{12}+\frac{1}{4}+\frac{1}{4}+\frac{1}{12}+\frac{1}{4} = \frac{3}{12}+\frac{3}{4} = 1 \quad \text{true}$$

(b) Determine the average value of a single die roll.

$$\sum_{y=1}^{6} p(y)\cdot y = \frac{1}{12}+\frac{2}{12}+\frac{3}{4}+\frac{4}{4}+\frac{5}{12}+\frac{6}{4} = \frac{1+2+9+12+5+18}{12} = \frac{47}{12}$$

$\approx 3.91\overline{6}$

(c) Determine $E\left[\frac{1}{y}\right]$.

$$E\left[\frac{1}{y}\right] = \sum_{y=1}^{6} p(y)\cdot\frac{1}{y} = \frac{1}{12}+\frac{1}{24}+\frac{1}{12}+\frac{1}{16}+\frac{1}{60}+\frac{1}{24}$$

lcm = 240

$$= \frac{20+10+20+15+4+10}{240} = \frac{79}{240} \approx 0.3291\overline{6}$$

4. (6 points) Let Y have a Poisson distribution with a variance of 3. Find $P(Y = 2)$.

Let $X = Y$ (λ and Y are reflections about $y=0$ and I miswrite them visually)

$$\frac{\lambda^x e^{-\lambda}}{x!} \qquad P(X=2) = \frac{3^2 e^{-3}}{2!} \approx 0.224$$

$V[X] = 3 = \sigma^2 = \lambda$

$\lambda = 3$

5. (6 points) It is claimed that for a particular scratch-off lottery, 1/20 of the 50,000,000 tickets produced will win a prize.

(a) You decide to purchase 15 tickets. What probability distribution should you use to determine the probability that you win and why? What is the probability that you win?

$n = 15$
$p = 1/20$
$q = 19/20$
$P(X = 1)$

$\frac{1}{20}$ of $5 \cdot 10^7 = 2.5 \cdot 10^6$ (2.5 mil)

$\frac{1}{2.5 \cdot 10^6}$ chance of 1 ticket winning

Binomial bc we're dealing w/ success or failure

$P(\text{winning}) = 1 - P(X=0) = \binom{15}{0}\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{15} \approx 0.5367$

(b) How many tickets should you purchase if you want a 90% chance of winning?

$1 - \left(\frac{19}{20}\right)^n \geq 90$

$n = 45$ tickets for 90% chance

6. (6 points) Suppose that in manufacturing O-rings, it is known that approximately 1% are defective. Suppose further that the process of producing an O-ring is an independent process. An inspector from NASA comes to inspect a lot and randomly selects 20 O-rings for testing. It is known that the inspector will reject the order if more than one O-ring in the sample is deemed defective. What is the probability that the order is rejected.

$p(y > 1) = 1 - (p(y=1) + P(Y=0))$

~1% defective

$n = 20$

hyper

rejected if $y > 1$

$y = 1$ $\quad pq^{y-1}$ $\quad 0.01 \cdot 0.99^1$
$= 0.01$

2%

$P\ y = 0$ $\quad pq$

7. (4 points) Using the definition, derive the moment generating function for a geometric probability distribution.

pq^{y-1} Idk