Matthew Wilder

1 a $(A \cup B) \cap (A \cup C) \overset{?}{=} A \cup (B \cap C)$

$= (A \cup B) \cap (A \cup C)$ True

by distribution

1 b $(A \cup B) = ((A \cap \bar{B}) \cup B)$

$= B \cup (A \cap \bar{B})$ commu~~Associ~~ative

$= (B \cup A) \cap (B \cup \bar{B})$ Distributive

$= (B \cup A) \cap U$

$= B \cup A$

$= A \cup B$ True

1 c $(\bar{A} \cup B) = (A \cup B)$

Implies $\bar{A} = A$ which is false and contradicts the original statement

1 d $(\overline{A \cup B}) \cap C = \bar{A} \cap \bar{B} \cap \bar{C}$ False

$C \cap (\overline{A \cup B}) =$

$C \cap \bar{A} \cap \bar{B}$ = by de morgan's law

$\bar{A} \cap \bar{B} \cap C$ $\bar{A} \cap \bar{B} \cap \bar{C}$

$C \neq \bar{C}$ false False

1 e $(A \cap B) \cap (\bar{B} \cap C) = \emptyset$

$((A \cap B) \cap \bar{B}) \cap ((A \cap B) \cap C)$ distributive

$(A \cap (B \cap \bar{B})) \cap ((\cdot \cap \ ) \cap C)$ associative

$(A \cap \emptyset \ ) \cap (\quad)$ def of $\emptyset$

$\emptyset \cap (\ )$

$\emptyset = \emptyset$ simplify True

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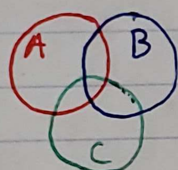
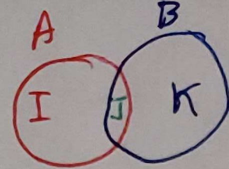
2 a.) $A \cup B \cup C$

b.) $(A \cup B \cup C) \cap \overline{((A \cap B) \cup (A \cap C) \cup (B \cap C))}$

c.) $(A \cap \overline{B} \cap C)$

d.) $(A \cup B \cup C) \cap \overline{(A \cap B \cap C)}$





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3 a. Let I denote $A \cap \bar{B}$
Let J denote $A \cap B$
Let K denote $\bar{A} \cap B$

Then I, J, K are mutually exclusive, that is
$I \cap J = \emptyset$, $I \cap K = \emptyset$, and $J \cap K = \emptyset$

$$
\begin{aligned}
I \cap J &= (A \cap \bar{B}) \cap (A \cap B) && \text{substitute} \\
&= (A \cap \bar{B} \cap A) \cap (A \cap (\bar{B} \cap B)) && \text{distribute} \\
&= (A \cap \bar{B} \cap A) \cap (A \cap \emptyset) && S \cap \bar{S} = \emptyset \text{ definition} \\
&= \emptyset && \text{simplify}
\end{aligned}
$$

$$
\begin{aligned}
I \cap K &= (A \cap \bar{B}) \cap (\bar{A} \cap B) && \text{substitute} \\
&= (A \cap \bar{B} \cap \bar{A}) \cap (A \cap (\bar{B} \cap B)) && \text{distribute} \\
&= (A \cap \bar{B} \cap \bar{A}) \cap (A \cap \emptyset) && \text{def of } \emptyset \\
&= \emptyset && \text{simplify}
\end{aligned}
$$

$$
\begin{aligned}
J \cap K &= (A \cap B) \cap (\bar{A} \cap B) && \text{substitute} \\
&= ((A \cap B) \cap \bar{A}) \cap ((A \cap B) \cap B) && \text{distribute} \\
&= ((A \cap \bar{A}) \cap B) \cap ((A \cap B) \cap B) && \text{associative and commutative laws} \\
&= (\emptyset \cap B) \cap ((A \cap B) \cap B) && \text{def of } \emptyset \\
&= \emptyset && \text{simplify}
\end{aligned}
$$

It follows that $A \cup B = I \cup J \cup K$
so $P(A \cup B) = P(I) + P(J) + P(K) = 1$
and $P(A) = P(I) + P(J)$
and $P(B) = P(J) + P(K)$
$P(I) = P(A) - P(J)$
$P(K) = P(B) - P(J)$

So by substitution, $P(A \cup B) = P(A) - P(J) + P(J) + P(B) - P(J)$
$= P(A) + P(B) - P(J)$ by simplification
$= P(A) + P(B) - P(A \cap B)$ by substitution

QED

3c $P(A\cup(B\cup C)) = P(A) + \underbrace{P(B\cup C)}_{= P(B)+P(C)-P(B\cap C)} - P(A\cap(B\cup C))$

$= P(A)+P(B)+P(C)-P(B\cap C) \underbrace{- P(A\cap(B\cup C))}_{\text{Distribute}}$

$= -P((A\cap B)\cup(A\cap C))$

$= -P(A\cap B) + P(A\cap C) - \underbrace{P((A\cap B)\cap(A\cap C))}_{= +P(A\cap B\cap C)}$

$A\cap A = A \longrightarrow$

so

$P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

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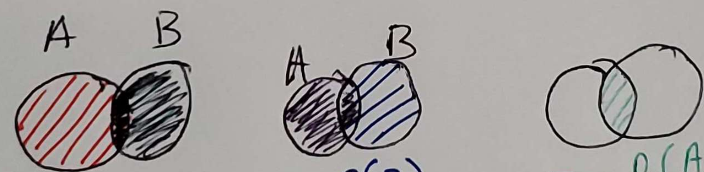

$X = P(A)$ $P(\bar{A})$

$y = P(B)$ $P(\bar{B})$

$Z = P(A\cap B)$

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4. a. $P(\bar{A}\cap\bar{B})$ ~~$= P(\emptyset) = 0$~~ $= P(Z)$

b. $P(A\cup\bar{B})$ $= P(X+Z)$

c. No because if Z is the ~~union~~ intersection of two sets, then the largest it could possibly be is $\text{Min}\{S_1, S_2\}$ (S_1, S_2). In this case, $\text{Min}\{0.17, 0.13\}$ the largest Z can be is 0.13, aka $|Z| \leq 0.13$

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5. a 8 dials, 5 combs, $\boxed{5^8 = 390,625}$

b.) 5 choices on first, 4 on remaining 7
$5 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 5 \cdot 4^7 = \boxed{81,920}$

c.) $a^4 b^4$ $\quad C^8_4 = \frac{8!}{4!4!} = \boxed{80}$

d.) 4a, 3b, 1c $\quad \binom{8}{4,3,1} = \frac{8!}{4!3!1!} = \boxed{280}$

e. 4x, ~~3z~~ 3y, 1z $\quad P(5,3) \cdot \binom{8}{4,3,1}$

$\frac{5!}{2!} \cdot \frac{8!}{4!3!} = \boxed{16,800}$

6. a. S = Set of 1500 washers

$G \subset S \mid G$ is 1100 good ones ~~3/5 chan~~ 11/15 chance

$B \subset S \mid B$ is 400 bad ones 4/15 chance

$B \cap G = \emptyset$ and $B \cup G = S$

(Choosing 200 washers

$$= {}_{200}C_{75} \cdot \left(\frac{4}{15}\right)^{75} \cdot \left(\frac{11}{15}\right)^{\cancel{200}\ 125}$$

$= 0.00021768787$
or 0.02%

b.

$$\boxed{\sum_{k=3}^{200} \binom{200}{k} \cdot \left(\frac{4}{15}\right)^{k} \cdot \left(\frac{11}{15}\right)^{200-k}}$$

$\frac{P^{200}_{75}}{75!}$ $\frac{200!}{75!\,125!}$