# Math 325 – Homework 01 (MTH 325 review topics)
# Due (via upload to Canvas) Wednesday, January 26, 2022 at 6 PM

1. Benford's Law states that in a legitimate financial record, 30.1% of all randomly selected first digits will be "one".
   (a) What is the probability that exactly two out of 10 records begin with a "1"?
   (b) What is the probability that at least 20 out of 100 records begin with a "1"?
   (c) What is the probability that we have not seen a "1" in our first eight records?

2. Consider a continuous random variable $Y$ with density function

$$f(y) = \frac{k}{y} \text{ with support } 1 \leq y \leq 3.$$

   (a) Find the value of $k$ that will make $f(y)$ a legitimate density function.
   (b) Find $P(Y \leq 2.5)$.
   (c) Find the mean and standard deviation of $Y$.
   (d) Suppose random variables $Y_1$, $Y_2$, $Y_3$, and $Y_4$ are independent random event from the above distribution $f(y)$. Let $M = \max(Y_1, Y_2, Y_3, Y_4)$. Find $P(M \leq 2.5)$.

3. Consider the random variable $X$ and $Y$ whose joint probability distribution $p(x, y)$ is given in the following table.

| $X$ \ $Y$ | 0 | 1 | 2 |
|---|---|---|---|
| 1 | 0.15 | 0.10 | 0.05 |
| 2 | 0.05 | 0.20 | 0.10 |
| 3 | 0.05 | 0.05 | 0.25 |

   Find each of the following:
   (a) $p_x(1) = P(X = 1)$
   (b) $E[X]$
   (c) $P(X = 1|Y = 2)$
   (d) $E(X|Y = 2)$
   (e) Find the mean and variance of each random variable.
   (f) Find $\text{Cov}(X, Y)$.
   (g) Suppose that $U = 3X - 2Y$. Use Theorem 5.12 to find the mean and variance of U.

4. The time needed to complete a certain factory job is a normal random variable with mean $\mu = 50$ minutes and standard deviation $\sigma = 5$ minutes.

   (a) What is the probability that a (randomly selected) job will be completed in 53 minutes or less?

   (b) What is the probability that the average time of ten randomly selected jobs will be less than 53 minutes or less?

   (c) Only 5% of the time will a single job be completed in M minutes or less. Determine $M$.