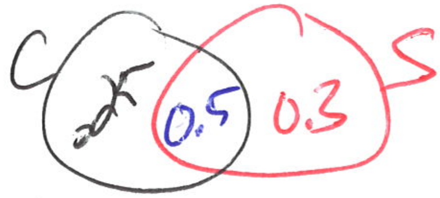
Math 325 Section 01
Professor Carty
40 points possible

Exam 1
October 1, 2021

Name: ____

1. No hats or dark sunglasses. All hats are to be removed.
2. All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.
3. Be sure to print your proper name clearly.
4. Calculators are optional, but if you choose to use one you may ONLY use the battery or solarpowered Texas Instruments BA35 model calculator, the BA II Plus, the BA II Plus Professional, the TI30Xa or TI30X II (IIS solar or IIB battery), or TI-30X MultiView (XS Solar or XB Battery).
5. All electronic devices, including cell phones and other wearable devices, must be silenced or powered off and stored out of sight for the entirety of the exam.
6. If you have a question, raise your hand and I will come to you. Once you stand up, you are done with the exam. If you have to use the facilities, do so now. You will not be permitted to leave the room and return during the exam.
7. Every exam is worth a total of **40 points**. Including the cover sheet, each exam has 5 pages.
8. At 11:45, you will be instructed to put down your writing utensil. Anyone seen continuing to write after this announcement will have their exam marked and lose all points on the page they are writing on. At this time, you will use your phone to take PDFs of the pages and upload them into Canvas as a single PDF document (the same way you do with homework.) Solutions must be uploaded by 11:55.
9. If you finish early, quietly and respectfully perform the preceding tasks. You may leave early.
10. You will hand in the paper copy of the exam on your way out of the classroom.
11. You have forty-five minutes to complete the exam. I hope you do well.



1. (6 points) In the greater Peoria area, 35% of people are St. Louis Cardinal (S) fans, 30% of people are Chicago Cubs (C) fans, and a deranged 5% are a fan of both teams.

   (a) For the following, determine the event in terms of set notation using S and C, and then the probability of the event.

      i. That a person chosen at random is not a fan of either team.

      $$\overline{C \cup S}, \quad P(\overline{C \cup S}) = 1 - P(S \cup C) = 0.4$$

      ii. That a person chosen at random is only a Cubs fan.

      $$C \cap \bar{S}, \quad P(C \cap \bar{S}) = P(C) - P(C \cap S) = 0.25$$

   (b) Determine the probability that a person who is a known Cubs fan is also deranged. (That is, they are a Cardinals fan as well.)

   $$P(S \mid C) = \frac{P(S \cap C)}{P(C)} = \frac{0.05}{0.30} = \frac{1}{6}$$

2. (6 points) Bowl A contains three red chips, Bowl B contains two blue chips and one red chip, and bowl C contains one red, one blue chip, and one black chip. A bowl is selected at random and one chip is taken from that bowl.

   (a) Compute the probability of selecting a blue chip.

   $$P(A)P(\text{blue} \mid A) + P(B)P(b \mid B) + P(C)P(b \mid C)$$
   $$= \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3}$$

   (b) If the selected chip is blue, compute the probability that the ~~other~~ next chip ~~in the bowl~~ chosen fr is red.

   Could use Bayes or given blue, 50-50 chance picked bowl B or C:

   $$\frac{1}{2} P(\text{blue remain } B) + \frac{1}{2} P(\text{blue remains in } C)$$
   $$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

3. (6 points) A weighted six-sided die has the following probabilities

y: # of pips	1	2	3	4	5	6
$p(y)$: probability	1/12	1/12	1/4	1/4	1/12	1/4

(a) Show that the above is a proper probability distribution.

$p(y) \geq 0 \;\; \forall y$

$\sum p(y) = 3\left(\frac{1}{12}\right) + 3\left(\frac{1}{4}\right) = 1.$

(b) Determine the average value of a single die roll.

$E[Y] = 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{4} + 5 \cdot \frac{1}{12} + 6 \cdot \frac{1}{4} = \frac{47}{12}$

(c) Determine $E\left[\frac{1}{y}\right]$.

$= \frac{1}{1} \cdot \frac{1}{12} + \frac{1}{2} \cdot \frac{1}{12} + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{12} + \frac{1}{6} \cdot \frac{1}{4} = \frac{79}{240}$

4. (6 points) Let Y have a Poisson distribution with a variance of 3. Find $P(Y = 2)$.

Since $\sigma^2 = \lambda$, $\lambda = 3$

$p(y) = \frac{3^y e^{-3}}{y!}$

$P(Y=2) = \frac{3^2 e^{-3}}{2!} = \frac{9}{2e^3} \approx 0.224$

5. (6 points) It is claimed that for a particular scratch-off lottery, 1/20 of the 50,000,000 tickets produced will win a prize.

   (a) You decide to purchase 15 tickets. What probability distribution should you use to determine the probability that you win and why? What is the probability that you win?

While actually a hypergeo distn $n=15$ is so small $P(\text{win})$ is essentially constant at $1/20$. Use binomial.

$n=15,\ p=1/20,\ q=19/20,\ y=\#\text{ wins},\ p(y)=\binom{15}{y}p^y q^{15-y}$.

$P(Y>0)=1-P(Y=0)=1-\binom{15}{0}p^0q^{15}=1-q^{15}\approx 0.536$.

   (b) How many tickets should you purchase if you want a 90% chance of winning?

Want $1-q^n=0.9$ $\quad n=\dfrac{\ln(0.1)}{\ln(19/20)}$

(if have calculator $44<\dfrac{\ln(0.1)}{\ln(19/20)}<45$..

Choose $n=45$.)

6. (6 points) Suppose that in manufacturing O-rings, it is known that approximately 1% are defective. Suppose further that the process of producing an O-ring is an independent process. An inspector from NASA comes to inspect a lot and randomly selects 20 O-rings for testing. It is known that the inspector will reject the order if more than one O-ring in the sample is deemed defective. What is the probability that the order is rejected.

binomial w/ $p=0.01,\ q=0.99,\ y=\#$ of defective.

$P(\text{reject})=P(Y>1)=1-(P(Y=0)+P(Y=1))$

$=1-(0.99)^{20}-20(0.01)(0.99)^{19}$.

7. (4 points) Using the definition, derive the moment generating function for a geometric probability distribution.

$$m(t) = \sum_{1}^{\infty} e^{ty} p q^{y-1}$$

$$= \sum_{1}^{\infty} e^{ty-\cancel{t}+t} p q^{y-1}$$

$$= pe^{t} \sum_{1}^{\infty} e^{t(y-1)} q^{y-1}$$

$$= pe^{t} \sum_{1}^{\infty} (qe^{t})^{y-1}$$

$$= \frac{pe^{t}}{1-qe^{t}} \text{ provided } qe^{t} < 1$$

$$\Rightarrow t < \ln(1/q).$$