

Name: _____

1. No hats or dark sunglasses. All hats are to be removed.
2. All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.
3. Be sure to print your proper name clearly.
4. Use of the battery or solarpowered Texas Instruments BA35 model calculator, the BA II Plus, the BA II Plus Professional, the TI30Xa or TI30X II (IIS solar or IIB battery), or TI-30X MultiView (XS Solar or XB Battery) allowed.
5. Watches with recording, internet, communication or calculator capabilities (e.g., a smart watch or fitness band) are prohibited.
6. All electronic devices, including cell phones and other wearable devices, must be powered off and stored out of sight for the entirety of the exam.
7. If you have a question, raise your hand and I will come to you. Once you stand up, you are done with the exam. If you have to use the facilities, do so now. You will not be permitted to leave the room and return during the exam.
8. Every exam is worth a total of **45 points**. Including the cover sheet, each exam has 6 pages.
9. If you finish early, feel free to quietly and respectfully get up and hand in your exam.
10. When time is up, you will be instructed to put down your writing utensil, close the exam and remain seated. Anyone seen continuing to write after this announcement will have their exam marked and lose all points on the page they are writing on. I will come and collect the exams from you.
11. You have fifty minutes to complete the exam. Good luck.

1. (10 points) Suppose that the two-dimensional discrete random variable (X, Y) has joint pdf

$p(x, y)$	$y = 1$	$y = 2$	$y = 3$
$x = 1$	$1/12$	$1/6$	0
$x = 2$	0	$1/9$	$1/5$
$x = 3$	$1/18$	$1/4$	$2/15$

- (a) Determine all the marginal distributions.
- (b) Determine all the conditional distributions of Y given X .
- (c) Determine the expected value of Y given that $x = 3$.
- (d) Find the $\text{Cov}(X, Y)$

2. (10 points) Let X and Y have the joint p.d.f. $f(x, y) = x + y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$.
- (a) Find the marginal p.d.f.'s $f_1(x)$ and $f_2(y)$ and show that X and Y are dependent.

- (b) Show that X and Y are dependent via computing the covariance.

3. (12 points) Suppose that the two-dimensional random variable (X, Y) has joint pdf

$$f(x, y) = x^2 + \frac{xy}{3} \text{ on the support } 0 < x < 1, 0 < y < 2.$$

(a) Compute $P(X < \sqrt{Y})$.

(b) Compute $P(Y < \frac{1}{2} | X < \frac{1}{2})$.

(c) Determine if X and Y are independent.

4. (9 points) If X , Y , and Z are uncorrelated random variables with standard deviations of 5, 12, and 9, respectively.
- (a) If $W = 2X - 3Y + Z$, determine the variance of W .

- (b) If $U = X + Y$ and $V = Y + Z$, evaluate the correlation coefficient between U and V .

5. (4 points) Suppose that the number of eggs laid by a certain bird has a Poisson distribution with mean λ . The probability that any one egg hatches is p . Assume that the eggs hatch independently of one another. Find the

(a) expected value of Y , the total number of eggs that hatch.

(b) variance of Y .