Math 326 – Homework 08 (11.6 – 11.7 and 11.10 – 11.12)
Due (via upload to Canvas) Wednesday, April 13, 2022 at 11:59 PM

1. Suppose the following represents a random sample of points (x, y):

x	-2.0	-1.0	0.0	1.0	2.0
y	3.0	2.0	1.0	1.0	0.5

   (a) Find the 90% confidence interval for $E(Y)$ when $x^* = 0$ and again when $x^* = 2$.

   (b) Fine the 90% confidence interval for Y^* when $x^* = 0$ and again when $x^* = 2$.

   (c) Are the intervals foe $E(Y)$ or the intervals for Y^* wider? How can this be explained?

The following table contains dietary data (calories and the content of fat, sodium, carbohydrate, and protein) in some standard hamburgers that can be found at local fast food restaurants.

	cal	fat (g)	sodium (mg)	carbs (g)	protein (g)
BK Jr.	310	18	390	27	13
Wendy's Jr.	250	11	420	25	13
McDonald's	250	9	480	31	12
Culvers	390	17	480	38	20
Steak-n-Shake	320	14	830	32	15
Sonic Jr.	330	16	610	32	15

2. Assume the relationship between defining calories is a linear one. That is,

$$\text{cal} = A \cdot (\text{fat}) + B \cdot (\text{carbs}) + C \cdot (\text{protein}).$$

   (a) Determine the best-fit hyperplane that predicts calories of a burger based upon the fat, carbohydrate, and protein content of the burger. Explicitly state the terms in of the normal forms: X, X^TX, $(X^TX)^{-1}$, and X^TY.

   (b) Based upon your least-squares regression analysis, how many calories are in expected in a burger made with 10 grams of fat, 20 grams of carbohydrates, and 15 grams of protein.

   (c) Find SSE and S.

   (d) Find a 95% confidence interval for the amount of calories in a burger made with 10 grams of fat, 20 grams of carbohydrates, and 15 grams of protein.

3. A family kept track of its natural gas usage for two heating seasons and the accompanying outdoor temperatures. Gas usage is measured in hundreds of cubic feet, and temperature is average temperature in degrees Fahrenheit.

month:	Oct 1	Nov 1	Dec 1	Jan 1	Feb 1	Mar 1
temp:	53	41	14	22	32	39
gas:	3.0	6.2	12.6	9.2	7.5	5.5
month:	Oct 2	Nov 2	Dec 2	Jan 2	Feb 2	Mar 2
temp:	56	36	33	13	35	43
gas:	2.0	6.5	7.3	12.5	6.9	5.3

(a) Fit he model $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon$ where X is the temperature and Y is the gas usage.

(b) Test if the quadratic term is really necessary. That is, test $H_0 := \beta_2 = 0$.