Math 325_01   **Name:**_____
November 12, 2018
5 questions for 75 points possible

1. No hats or dark sunglasses. All hats are to be removed.

2. All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.

3. Use of the battery or solarpowered Texas Instruments BA35 model calculator, the BA II Plus, the BA II Plus Professional, the TI30Xa or TI30X II (IIS solar or IIB battery), or TI-30X MultiView (XS Solar or XB Battery) allowed.

4. No cell phones or music devices. Turn them off now. If you are seen with a cell phone in hand during the exam, it will be construed as cheating and you will be asked to leave. This includes using it as a time-piece.

5. If you have a question, raise your hand and I will come to you. Once you stand up, you are done with the exam. If you have to use the facilities, do so now. You will not be permitted to leave the room and return during the exam.

6. Every exam is worth a total of **75 points**. Check to see that you have all of the pages. Including the cover sheet, each exam has 5 pages.

7. Be sure to print your proper name clearly.

8. If you finish early, quietly and respectfully get up and hand in your exam.

9. When time is up, you will be instructed to put down your writing utensil, close the exam and remain seated. Anyone seen continuing to write after this announcement will have their exam marked and lose all points on the page they are writing on. I will come and collect the exams from you.

10. You have fifty minutes to complete the exam. Good luck.

1. [25 points] Suppose X and Y have the joint density function given by

$$f(x, y) = \begin{cases} 6x, & 0 \leq x, 0 \leq y, x + y \leq 1 \\ 0, & \text{else} \end{cases}$$

(a) Find $P(X > Y)$.

(b) Find the marginal density function of X.

(c) Find $E[X]$.

(d) Find the conditional density of X given Y.

(e) Find $P(X > 2/3 \,|\, Y = 1/6)$

(f) Find $E(X \,|\, Y = 1/6)$

2. [20 points] Consider the discrete random variable X and Y with joint probability function given in the following table:

$p(x,y)$	$y = 0$	$y = 1$	$y = 2$
$x = 0$	0.21	0.05	0
$x = 1$	0.12	0.20	0.13
$x = 2$	0.04	0	0.25

(a) Find $P(X \leq 1 \mid Y \leq 1)$.

(b) Find $P(X = 1 \mid Y = 1)$.

(c) Find the marginal density function of X.

(d) Are X and Y independent? Justify your answer.

(e) Find $\mathrm{Cov}(X, Y)$.

3. [10 points] Given that X, Y and Z are random variable with $E(X) = 7$, $\text{Var}(X) = 8$, $E(Y) = 3$, $\text{Var}(Y) = 6$, $E(Z) = 5$, $\text{Var}(Z) = 12$, $\text{Cov}(X, Y) = -2.5$, $\text{Cov}(X, Z) = -4$, and $\text{Cov}(Y, Z) = 1$. Find the mean and variance of $U = X + 3Y - 2Z$.

4. [10 points] Suppose that X and Y are independent random variables each with an exponential distribution with parameter $\beta = 1$.

   (a) Find the joint density function $f(x, y)$.

   (b) Find $P(X + Y < 2)$.

5. [10 points] Let X be a random variable with density function given by

$$f(x) = \begin{cases} x/2, & 0 \le x \le 2 \\ 0, & \text{else} \end{cases}$$

Let $U = X^2$.

(a) Find the density function of U. Be sure to indicate the range of U.

(b) Determine $\mathrm{E}(U)$.