326 Homework 1.

1. Let $p = 0.301$ and $q = 0.699$

a) binom(10, 0.301)

$p(2) = \binom{10}{2} (0.301)^2 (0.699)^8 = 0.232361$

b) $P(Y \geq 20) = 1 - P(Y \leq 19)$

$= 1 - \sum_{n=0}^{19} \binom{100}{n} (0.301)^n (0.699)^{100-n}$

$= 1 -$ pbinom(100, 19)

$= 0.9843832$

Or, using the fact that binom dist'ns can be approximated by Normal dist'n $N(\mu, \sigma^2)$ where $\mu = np = 30.1$, $\sigma^2 = npq = 21.04$

$P(Y \geq 20) \approx P(X \geq 19.5)$

$= 1 - P(X \leq 19.5)$

$= 1 - P\left(Z \leq \frac{19.5 - 30.1}{\sqrt{21.04}}\right)$

$= 1 - P(Z \leq -2.31)$

$= 1 - P(Z \geq 2.31)$

$= 1 - 0.0104$ by Table 4

$= 0.9896$

c) geometric dist'n w/ $p = 0.699$

$p(8) = (0.699)^8 = 0.0569925$

2a) $\int_1^3 \frac{k}{y}\,dy = 1 \Rightarrow k\ln 3 = 1$ or $k = \frac{1}{\ln 3}$

b) $P(Y \le 2.5) = \int_1^{2.5} \frac{1}{y\ln 3} = \frac{\ln(2.5)}{\ln 3} \approx 0.834\ldots$

c) $E[Y] = \int_1^3 y \cdot \frac{1}{y\ln 3} = \frac{2}{\ln 3} = \mu$

$V[Y] = E[Y^2] - (E[Y])^2$

$E[Y^2] = \int_1^3 y^2 \cdot \frac{1}{y\ln 3} = \frac{1}{\ln 3}\int_1^3 y = \frac{4}{\ln 3}$

$\sigma^2 = \frac{4}{\ln 3} - \left(\frac{2}{\ln 3}\right)^2 \approx 0.3026815$

and $\sigma \approx 0.5716\,77$

d) Have $P(Y \le 2.5) = \frac{2}{\ln 3}$,

max order stats $P(Y_{(4)} \le 2.5) = (P(Y \le 2.5))^4$

$= \left(\frac{2}{\ln 3}\right)^4 \approx 0.48390\ldots$

3.

		$Y = 0$	1	2	
	1				0.30
X	2				0.35
	3				0.35
		0.25	0.35	0.40	

marginals

a) $p_X(1) = 0.30$

b) $E[X] = \sum_x x\, p_X(x) = 1 \cdot 0.3 + 2 \cdot 0.35 + 3 \cdot 0.35$
$= 2.05$

c) $P(X=1 \mid Y=2) = \dfrac{p(1,2)}{p_Y(2)} = \dfrac{0.05}{0.40} = \dfrac{1}{8} = 0.125$

d) $E(X \mid Y=2) = \sum x\, p(x \mid y=2)$
$= 1 \cdot \dfrac{p(1,2)}{p_Y(2)} + 2 \cdot \dfrac{p(2,2)}{p_Y(2)} + 3\, \dfrac{p(3,2)}{p_Y(2)} = 2.5$

e) $\mu_X = 2.05$ by above.
$\sigma_X^2 = 0.30 + 4 \cdot 0.35 + 9 \cdot 0.35 - (2.05)^2$

$\mu_Y = \sum_y y\, p(y) = 0 + 1 \cdot 0.35 + 2 \cdot 0.40 = 1.15$

$\sigma_Y^2 = 0 + 0.35 + 2^2 \cdot 0.40 - (1.15)^2$

f) $Cov(X,Y) = E(XY) - \mu_x \mu_y$

$E(XY) = 0 + 0 + 0 + 1\cdot 1(0.10) + 2\cdot 1(0.20) + 3\cdot 1(0.05)$
$+ 1\cdot 2(0.05) + 2\cdot 2(0.10) + 3\cdot 2(0.25)$
$= 2.65$

$Cov(X,Y) = 2.65 - (2.05)(1.15)$
$= 0.2925$

g) $E(U) = 3E(X) - 2E(Y) = 3(2.05) - 2(1.15)$

$V(U) = 3^2 V(X) + (-2)^2 V(Y) + 2\cdot 3\cdot(-2)\, Cov(X,Y)$

4. $N(50, 25)$

a) $P(X \leq 53) = P\left(Z \leq \frac{53-50}{5}\right) = P(Z \leq 0.60)$

$= 1 - P(Z > 0.60) = 0.7257$

b) $\bar{X} \leq 53$ sample means dist'n $N(50, 25/10)$

$\sigma_{\bar{x}} = \frac{5}{\sqrt{10}} = 1.58$

$P\left(\bar{X} \leq \frac{53-50}{1.58}\right) = P(Z \leq 1.90)$

$= 1 - P(Z \geq 1.90)$

$= 0.9713$

c) $P(X \leq m) = 0.05 \leftrightarrow P(X \geq m) = 0.05$

$Z = 1.645$

$Z \leq 1.645 \leftrightarrow \frac{m-50}{5} = -1.645$

$m = 41.775$