

Math 326 – Homework 07 (10.10 – 11.5)

Due (via upload to Canvas) Friday, April 01, 2022 at 11:59 PM

1. Suppose that we have a random sample of n observations from the density function

$$f(y|\theta) = \frac{y^2 e^{-y/\theta}}{2\theta^3} \quad \text{on support } y > 0$$

- (a) Determine the rejection region for the most powerful test of $H_0 : \theta = \theta_0$ versus $H_a : \theta = \theta_a$, assuming $\theta_a > \theta_0$.
- (b) Is the test you defined in part (a) uniformly most powerful for the alternative $\theta > \theta_0$? Briefly explain your answer.

The following table contains dietary data (calories and the content of fat, sodium, carbohydrate, and protein) in some standard hamburgers that can be found at local fast food restaurants.

	cal	fat (g)	sodium (mg)	carbs (g)	protein (g)
BK Jr.	310	18	390	27	13
Wendy's Jr.	250	11	420	25	13
McDonald's	250	9	480	31	12
Culvers	390	17	480	38	20
Steak-n-Shake	320	14	830	32	15
Sonic Jr.	330	16	610	32	15

2. We wish to explore if there is a relationship between fat and sodium. The conjecture is that leaner meat need more salt to enhance flavor.
- (a) Compute the least squares regression line with response variable sodium content and input variable fat content. Clearly state sums of the intermediate calculations: \bar{x} , \bar{y} , S_{xy} , S_{xx} .
- (b) Calculate S^2 . Again, state any necessary intermediary sums.
- (c) Calculate the correlation coefficient ρ^2 ,
- (d) A good rule of thumb for the correlation coefficient in regards to best-fit lines is that if ρ^2 is greater than 0.70, then the line is a good model for the spread of the data. Is using a line a good model for this data?
- (e) Sketch a scatterplot of the data and draw the best-fit line and interpret the picture in context of your answer in (d).
3. American culture is focused on fat intake as corresponding to a high-calorie diet.
- (a) Compute the least squares regression line with response variable calorie count and input variable fat content. Clearly state sums of the intermediate calculations: \bar{x} , \bar{y} , S_{xy} , S_{xx} .

- (b) Suppose a new burger on the market is known to have 650 calories. What is a good estimate for how much fat is in the burger?
- (c) Find an 80% confidence interval for the slope of the regression line.
- (d) Is there statistical evidence that the slope of the regression line is greater than 10? Run a hypotheses test at $\alpha = 0.05$.