motivating Parabola Example in the notes

In[442]:= `X = {{1, -1, 1}, {1, 1, 1}, {1, 2, 4}, {1, 3, 9}}`

Out[442]= `{{1, -1, 1}, {1, 1, 1}, {1, 2, 4}, {1, 3, 9}}`

In[443]:= `MatrixForm[X]`

Out[443]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$$

In[444]:= $Y = \left\{\left\{\frac{1}{2}\right\}, \{-1\}, \left\{-\frac{1}{2}\right\}, \{2\}\right\}$

Out[444]= $\left\{\left\{\frac{1}{2}\right\}, \{-1\}, \left\{-\frac{1}{2}\right\}, \{2\}\right\}$

In[445]:= `MatrixForm[Y]`

Out[445]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} \\ -1 \\ -\frac{1}{2} \\ 2 \end{pmatrix}$$

In[446]:= `MatrixForm[Transpose[X].X]`

Out[446]//MatrixForm=

$$\begin{pmatrix} 4 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{pmatrix}$$

In[447]:= `Det[%]`

Out[447]= 440

In[448]:= `betaparb = Inverse[Transpose[X].X].Transpose[X].Y`

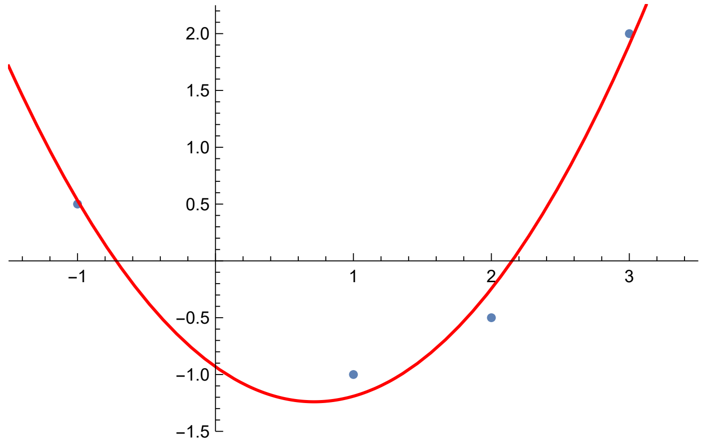
Out[448]= $\left\{\left\{-\frac{41}{44}\right\}, \left\{-\frac{379}{440}\right\}, \left\{\frac{53}{88}\right\}\right\}$

In[449]:= $N\left[\left\{\left\{-\frac{41}{44}\right\}, \left\{-\frac{379}{440}\right\}, \left\{\frac{53}{88}\right\}\right\}\right]$

Out[449]= `{{-0.931818}, {-0.861364}, {0.602273}}`

In[450]:=
```
Show[ListPlot[{{-1, 1/2}, {1, -1}, {2, -1/2}, {3, 2}},
  PlotRange → {{-1.5, 3.5}, {-1.5, 2.25}}], Plot[-41/44 - 379 x/440 + 53 x^2/88,
  {x, -1.5, 3.5}, PlotRange → {{-1.5, 3.5}, {-2, 2.25}}, PlotStyle → Red]]
```

Out[450]=



testing b2 is non-zero

In[451]:=
```
Inverse[Transpose[X].X]
MatrixForm[%]
440 * Inverse[Transpose[X].X]
MatrixForm[%]
```

Out[451]= $\left\{\left\{\frac{13}{22}, \frac{3}{44}, -\frac{5}{44}\right\}, \left\{\frac{3}{44}, \frac{171}{440}, -\frac{13}{88}\right\}, \left\{-\frac{5}{44}, -\frac{13}{88}, \frac{7}{88}\right\}\right\}$

Out[452]//MatrixForm=

$$\begin{pmatrix} \frac{13}{22} & \frac{3}{44} & -\frac{5}{44} \\ \frac{3}{44} & \frac{171}{440} & -\frac{13}{88} \\ -\frac{5}{44} & -\frac{13}{88} & \frac{7}{88} \end{pmatrix}$$

Out[453]= {{260, 30, -50}, {30, 171, -65}, {-50, -65, 35}}

Out[454]//MatrixForm=

$$\begin{pmatrix} 260 & 30 & -50 \\ 30 & 171 & -65 \\ -50 & -65 & 35 \end{pmatrix}$$

In[455]:=
```
SSEparb = Transpose[Y].Y + Transpose[betaparb].Transpose[X].Y
S2parb = SSEparb / 3
N[%]
```

Out[455]= $\left\{\left\{\frac{4791}{440}\right\}\right\}$

Out[456]= $\left\{\left\{\frac{1597}{440}\right\}\right\}$

Out[457]= {{3.62955}}

In[458]:=
```
varb2parb = S2parb * (35 / 400)
N[%]
Sb2parb = Sqrt[varb2parb]
N[%]
```

Out[458]= $\left\{\left\{\frac{11179}{35200}\right\}\right\}$

Out[459]= {{0.317585}}

Out[460]= $\left\{\left\{\frac{\sqrt{\frac{11179}{22}}}{40}\right\}\right\}$

Out[461]= {{0.563547}}

In[462]:=
```
T = (53 / 88 - 0) / Sb2parb
N[%]
```

Out[462]= $\left\{\left\{265\sqrt{\frac{2}{122969}}\right\}\right\}$

Out[463]= {{1.06872}}

Potency Example from the notes

In[464]:=
```
X := {{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, {30, 30, 30, 50, 50, 50, 70, 70, 70, 90, 90, 90}}
```

In[465]:=
```
MatrixForm[Transpose[X]]
```

Out[465]//MatrixForm=

$$\begin{pmatrix} 1 & 30 \\ 1 & 30 \\ 1 & 30 \\ 1 & 50 \\ 1 & 50 \\ 1 & 50 \\ 1 & 70 \\ 1 & 70 \\ 1 & 70 \\ 1 & 90 \\ 1 & 90 \\ 1 & 90 \end{pmatrix}$$

In[466]:=
```
X.Transpose[X]
```

Out[466]= {{12, 720}, {720, 49200}}

In[467]:=
```
Inverse[X.Transpose[X]]
```

Out[467]= $\left\{\left\{\frac{41}{60}, -\frac{1}{100}\right\}, \left\{-\frac{1}{100}, \frac{1}{6000}\right\}\right\}$

In[468]:=
```
Y := {{38, 43, 29, 32, 26, 33, 19, 27, 23, 14, 19, 21}}
```

In[469]:=
```
MatrixForm[Y]
```

Out[469]//MatrixForm=

$$\begin{pmatrix} 38 & 43 & 29 & 32 & 26 & 33 & 19 & 27 & 23 & 14 & 19 & 21 \end{pmatrix}$$

In[470]:=
```
X.Transpose[Y]
```

Out[470]= {{324}, {17 540}}

In[471]:=
```
Blinear = Inverse[X.Transpose[X]].X.Transpose[Y]
MatrixForm[Blinear]
N[Blinear]
```

Out[471]= $\left\{\{46\}, \left\{-\frac{19}{60}\right\}\right\}$

Out[472]//MatrixForm=

$$\begin{pmatrix} 46 \\ -\frac{19}{60} \end{pmatrix}$$

Out[473]= {{46.}, {-0.316667}}

In[474]:=
```
Transpose[Blinear].X
```

Out[474]= $\left\{\left\{\frac{73}{2}, \frac{73}{2}, \frac{73}{2}, \frac{181}{6}, \frac{181}{6}, \frac{181}{6}, \frac{143}{6}, \frac{143}{6}, \frac{143}{6}, \frac{35}{2}, \frac{35}{2}, \frac{35}{2}\right\}\right\}$

In[475]:=
```
Y.Transpose[Y]
```

Out[475]= {{9540}}

In[476]:=
```
Transpose[Blinear].X.Transpose[Y]
```

Out[476]= $\left\{\left\{\frac{28\,049}{3}\right\}\right\}$

In[477]:=
```
SSE = Y.Transpose[Y] - Transpose[Blinear].X.Transpose[Y]
N[SSE]
```

Out[477]= $\left\{\left\{\frac{571}{3}\right\}\right\}$

Out[478]= {{190.333}}

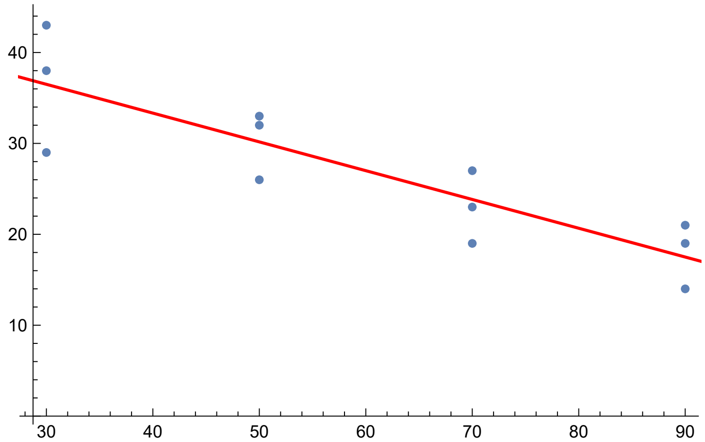
In[479]:=
```
x = {{30, 30, 30, 50, 50, 50, 70, 70, 70, 90, 90, 90}}
potencydata = ArrayFlatten[{{Transpose[x], Transpose[Y]}}]
```

Out[479]= {{30, 30, 30, 50, 50, 50, 70, 70, 70, 90, 90, 90}}

Out[480]= {{30, 38}, {30, 43}, {30, 29}, {50, 32}, {50, 26},
{50, 33}, {70, 19}, {70, 27}, {70, 23}, {90, 14}, {90, 19}, {90, 21}}

In[481]:=
```
Show[ListPlot[potencydata],
 Plot[Transpose[Blinear].{{1}, {t}}, {t, 20, 100}, PlotStyle → Red]]
```

Out[481]=



best fit quadratic for potency data?

In[482]:=
```
One = {{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}}
x2 = {{30^2, 30^2, 30^2, 50^2, 50^2, 50^2, 70^2, 70^2, 70^2, 90^2, 90^2, 90^2}}
```

Out[482]= {{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}}

Out[483]= {{900, 900, 900, 2500, 2500, 2500, 4900, 4900, 4900, 8100, 8100, 8100}}

In[484]:=
```
X2 = ArrayFlatten[{{Transpose[One], Transpose[x], Transpose[x2]}}]
```

Out[484]= {{1, 30, 900}, {1, 30, 900}, {1, 30, 900}, {1, 50, 2500}, {1, 50, 2500}, {1, 50, 2500}, {1, 70, 4900}, {1, 70, 4900}, {1, 70, 4900}, {1, 90, 8100}, {1, 90, 8100}, {1, 90, 8100}}

In[485]:=
```
MatrixForm[X2]
```

Out[485]//MatrixForm=

$$\begin{pmatrix} 1 & 30 & 900 \\ 1 & 30 & 900 \\ 1 & 30 & 900 \\ 1 & 50 & 2500 \\ 1 & 50 & 2500 \\ 1 & 50 & 2500 \\ 1 & 70 & 4900 \\ 1 & 70 & 4900 \\ 1 & 70 & 4900 \\ 1 & 90 & 8100 \\ 1 & 90 & 8100 \\ 1 & 90 & 8100 \end{pmatrix}$$

In[486]:=
```
Transpose[X2].X2
```

Out[486]= {{12, 720, 49 200}, {720, 49 200, 3 672 000}, {49 200, 3 672 000, 290 040 000}}

In[487]:=
```
Inverse[Transpose[X2].X2]
MatrixForm[%]
```

Out[487]= $\left\{\left\{\frac{5461}{960}, -\frac{163}{800}, \frac{31}{19200}\right\}, \left\{-\frac{163}{800}, \frac{23}{3000}, -\frac{1}{16000}\right\}, \left\{\frac{31}{19200}, -\frac{1}{16000}, \frac{1}{1920000}\right\}\right\}$

Out[488]//MatrixForm=

$$\begin{pmatrix} \frac{5461}{960} & -\frac{163}{800} & \frac{31}{19200} \\ -\frac{163}{800} & \frac{23}{3000} & -\frac{1}{16000} \\ \frac{31}{19200} & -\frac{1}{16000} & \frac{1}{1920000} \end{pmatrix}$$

In[489]:=
```
1920000 * Inverse[Transpose[X2].X2]
MatrixForm[%]
```

Out[489]= {{10 922 000, -391 200, 3100}, {-391 200, 14 720, -120}, {3100, -120, 1}}

Out[490]//MatrixForm=

$$\begin{pmatrix} 10922000 & -391200 & 3100 \\ -391200 & 14720 & -120 \\ 3100 & -120 & 1 \end{pmatrix}$$

In[491]:=
```
Bquad = Inverse[Transpose[X2].X2].Transpose[X2].Transpose[Y]
MatrixForm[Bquad]
N[Bquad]
```

Out[491]= $\left\{\left\{\frac{583}{12}\right\}, \left\{-\frac{5}{12}\right\}, \left\{\frac{1}{1200}\right\}\right\}$

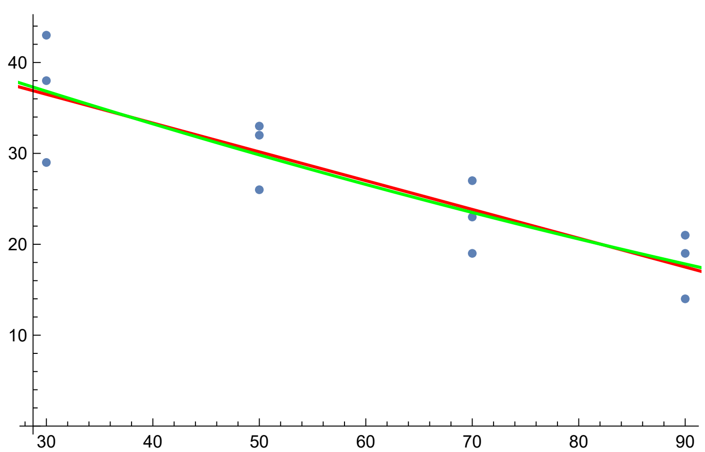
Out[492]//MatrixForm=

$$\begin{pmatrix} \frac{583}{12} \\ -\frac{5}{12} \\ \frac{1}{1200} \end{pmatrix}$$

Out[493]= {{48.5833}, {-0.416667}, {0.000833333}}

In[494]:=
```
Show[ListPlot[potencydata],
 Plot[Transpose[Blinear].{{1}, {t}}, {t, 20, 100}, PlotStyle → Red],
 Plot[Transpose[Bquad].{{1}, {t}, {t^2}}, {t, 20, 100}, PlotStyle → Green]]
```

Out[494]=



testing b2 is non-zero

```
In[495]:= SSE2 = Y.Transpose[Y] - Transpose[Bquad].Transpose[X2].Transpose[Y]
          N[SSE2]

Out[495]= {{189}}

Out[496]= {{189.}}

In[507]:= S2quad = SSE2 / 9
          N[%]
          Squad = Sqrt[SSE2 / 9]
          N[%]
```

Out[507]= $\{\{21\}\}$

Out[508]= $\{\{21.\}\}$

Out[509]= $\{\{\sqrt{21}\}\}$

Out[510]= $\{\{4.58258\}\}$

```
In[515]:= varb2 = (SSE2 / 9) / 1920000
          N[%]
          Sofb2 = Sqrt[varb2]
          N[%]
```

Out[515]= $\left\{\left\{\frac{7}{640000}\right\}\right\}$

Out[516]= $\{\{0.0000109375\}\}$

Out[517]= $\left\{\left\{\frac{\sqrt{7}}{800}\right\}\right\}$

Out[518]= $\{\{0.00330719\}\}$

In[519]:= $T = \left(\frac{1}{1200} - 0\right) \Big/ \text{Sofb2}$

N[T]

Out[519]= $\left\{\left\{\frac{2}{3\sqrt{7}}\right\}\right\}$

Out[520]= $\{\{0.251976\}\}$