Math 326 – Homework 05 (9.6 – 9.7)
Due (via upload to Canvas) Friday, February 25, 2022 at 11:59 PM

1. Suppose $X_1, X_2, \ldots, X_n$ are iid with the common density function

$$f(x) = \frac{2\theta^2}{x^3}, \quad x > \theta$$

where θ is unknown.

   (a) Find the method of moments estimator of θ.

   (b) Find the bias and variance of your estimator.

2. Suppose that $X_1, X_2, \ldots, X_n$ are a random sample from an exponentially distributed population with unknown mean θ. Find the maximum likelihood estimator of the population variance θ^2.

3. Let $X_1, X_2, \ldots, X_n$ denote a random sample from the density function given by

$$f(x) = \left(\frac{1}{\theta}\right) r x^{r-1} e^{-x^r/\theta}, \quad x > 0, \quad \theta > 0$$

where r is a known positive constant. Consider the statistic defined by $U = \frac{1}{n}\sum X_i^r$.

   (a) Find and simplify the likelihood function $L(x_1, x_2, \ldots, x_n|\theta)$ and complete the factorization.

   (b) Show that U is a sufficient statistic for θ.

   (c) Show that U is the MLE for θ.

   (d) Show that U is an unbiased estimator of θ.

   (e) What can we now conclude about the estimator U.

   (f) Explain why $V(U)$ is finite. (You do not have to compute it, but clearly explain why it is finite.)

   (g) Show that U is a consistent estimator of θ.