# Math 325, Distribution Formula Sheet

## Discrete Distributions

**Binomial**

$f(y) = \binom{n}{y} p^y q^{n-y}$, $y = 0, 1, \ldots, n$;

$\mu = np$, $\sigma^2 = npq$,

$m(t) = [pe^t + q]^n$

**Geometric**

$f(y) = pq^{y-1}$, $y = 1, 2, \ldots$;

$\mu = 1/p$, $\sigma^2 = q/p^2$,

$$m(t) = \frac{pe^t}{1 - qe^t}$$

**Hypergeometric**

$$f(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}},\ y \le n,\ y \le r,\ n - y \le N;$$

$$\mu = nr/N,\ \sigma^2 = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

**Negative Binomial**

$f(y) = \binom{y-1}{r-1} p^r q^{y-r}$, $y = r, r+1, \ldots, n$;

$\mu = r/p$, $\sigma^2 = rq/p^2$,

$$m(t) = \left[\frac{pe^t}{1 - qe^t}\right]^r$$

**Poisson**

$$f(y) = \frac{\lambda^y e^{-\lambda}}{y!},\ y = 0, 1, 2, \ldots;$$

$\mu = \lambda$, $\sigma^2 = \lambda$,

$m(t) = \exp[\lambda(e^t - 1)]$

**Multinomial**

$$f(y_1, y_2, \ldots, y_n) = \frac{n!}{y_1! y_2! \ldots y_n!}\ p_1^{y_1} \ldots p_n^{y_n},\ y_i = 0, 1, 2, \ldots, n;$$

$E(Y_i) = np_i$, $V(Y_i) = np_iq_i$

## Continuous Distributions

**Uniform**

$$f(y) = \frac{1}{b-a},\ a \le y \le b;$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12},$$

$$m(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$$

**Normal**

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right],\ y \in \mathbb{R};$$

mean $\mu$, variance $\sigma^2$,

$$m(t) = \exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$$

**Gamma**

$$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta},\ 0 \le y \le \infty,$$

where $\Gamma(t) = \int_0^\infty y^{t-1} e^{-y} dy$;

$$\mu = \alpha\beta,\ \sigma^2 = \alpha\beta^2,$$

$$m(t) = (1 - \beta t)^{-\alpha}$$

**Beta**

$$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1},\ 0 \le y \le 1;$$

$$\mu = \frac{\alpha}{\alpha+\beta},\ \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)},$$