Math 325 Section 01 **Exam 1**
Professor Carty **April 3, 2019**
45 points possible

Name: ____

1. No hats or dark sunglasses. All hats are to be removed.
2. All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.
3. Be sure to print your proper name clearly.
4. Use of the battery or solarpowered Texas Instruments BA35 model calculator, the BA II Plus, the BA II Plus Professional, the TI30Xa or TI30X II (IIS solar or IIB battery), or TI-30X MultiView (XS Solar or XB Battery) allowed.
5. Watches with recording, internet, communication or calculator capabilities (e.g., a smart watch or fitness band) are prohibited.
6. All electronic devices, including cell phones and other wearable devices, must be powered off and stored out of sight for the entirety of the exam.
7. If you have a question, raise your hand and I will come to you. Once you stand up, you are done with the exam. If you have to use the facilities, do so now. You will not be permitted to leave the room and return during the exam.
8. Every exam is worth a total of **45 points**. Including the cover sheet, each exam has 7 pages.
9. If you finish early, feel free to quietly and respectfully get up and hand in your exam.
10. When time is up, you will be instructed to put down your writing utensil, close the exam and remain seated. Anyone seen continuing to write after this announcement will have their exam marked and lose all points on the page they are writing on. I will come and collect the exams from you.
11. You have fifty minutes to complete the exam. Good luck.

1. (5 points) Let X have the pdf

$$f(x) = \frac{1}{2}(x+1), \quad -1 \le x \le 1.$$

Find the moment generating function for the random variable X.

2. (5 points) Let Y be a random variable distributed by the general hyperbolic secant distribution

$$f(y) = \frac{1}{2\sigma}\text{sech}\left[\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)\right].$$

The moment generating function $m(t)$ of Y is given by

$$m(t) = e^{\mu t}\sec(\sigma t), \quad -\pi/2\sigma < t < \pi/2\sigma.$$

Determine the mean and variance of the distribution of Y.

3. (10 points) The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function given by

$$F(y) = \begin{cases} 0, & y \leq 0 \\ 1 - e^{-y^2}, & 0 \leq y \end{cases}$$

(a) Show that F has the properties of a cumulative distribution function for a continuous random variable.

(b) Find the 0.30-quantile of Y.

(c) Find the probability distribution $f(y)$.

(d) Find the probability that a transistor lasted longer than 100 hours given that you know the transistor failed by 200 hours.

4. (5 points) Suppose that the distribution of scores on an IQ test has mean 100 and standard deviation of 16. Estimate the probability of a student having an IQ above 136 or below 64.

5. (8 points) At a certain college, the average score of a first-year student on the verbal part of the SAT is 565, with a standard deviation of 75. The admissions office assumes that the distribution of scores is Normal.

   (a) What proportion of the school's first-year students have verbal SAT scores under 500?

   (b) Find the 60th percentile for the verbal SAT score of a first-year student.

6. (6 points) (a) Show that for the gamma function, $\Gamma(r) = (r-1)\Gamma(r-1)$ for all $r \in \mathbb{R}$.

(b) Evaluate $\Gamma(7/2)$.

7. (6 points) Telephone calls enter a college switchboard according to a Poisson process on the average of two every 4 minutes. Let X denote the waiting time until the tenth call that arrives.

   (a) What is the pdf of X?

   (b) What is the moment-generating function for the random variable X?