Math 325_01 **Name:______________________________**

October 19, 2018

6 questions for 75 points possible

1. No hats or dark sunglasses. All hats are to be removed.
2. All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.
3. Use of the battery or solarpowered Texas Instruments BA35 model calculator, the BA II Plus, the BA II Plus Professional, the TI30Xa or TI30X II (IIS solar or IIB battery), or TI-30X MultiView (XS Solar or XB Battery) allowed.
4. No cell phones or music devices. Turn them off now. If you are seen with a cell phone in hand during the exam, it will be construed as cheating and you will be asked to leave. This includes using it as a time-piece.
5. If you have a question, raise your hand and I will come to you. Once you stand up, you are done with the exam. If you have to use the facilities, do so now. You will not be permitted to leave the room and return during the exam.
6. Every exam is worth a total of **75 points**. Check to see that you have all of the pages. Including the cover sheet, each exam has 6 pages.
7. Be sure to print your proper name clearly.
8. If you finish early, quietly and respectfully get up and hand in your exam.
9. When time is up, you will be instructed to put down your writing utensil, close the exam and remain seated. Anyone seen continuing to write after this announcement will have their exam marked and lose all points on the page they are writing on. I will come and collect the exams from you.
10. You have fifty minutes to complete the exam. Good luck.

1. [20 points] Suppose c is a constant and the density function for a random variable Y is given by:

$$f(y) = \begin{cases} cy^3 + y, & 0 \le y \le 2 \\ 0, & \text{else} \end{cases}.$$

(a) Find the value of c that makes f a probability density function.

(b) Determine the mean of Y.

(c) Determine the cumulative distribution function for Y.

(d) Find the quantile corresponding to 75%.

2. [15 points] Customers arrive at a mortgage loan office at a mean rate of 15 per hour. Assume that the number of arrivals per hour has a Poisson distribution.

   (a) Give the probability that no more than 5 customers arrive in a given hour.

   (b) Determine, exactly, the probability that the office waits 10 minutes for the first customer to arrive.

   (c) Express, as an integral, the probability that it takes less than 30 minutes for the first 7 customers to arrive.

3. [10 points] A r.v. Y has the following cumulative distribution function:

$$F(y) = P(Y \leq y) = \begin{cases} 0, & y < 2 \\ 1/8, & 2 \leq y < 2.5 \\ 3/16, & 2.5 \leq y < 4 \\ 1/2, & 4 \leq y < 5.5 \\ 5/8, & 5.5 \leq y < 6 \\ 1, & y \geq 6 \end{cases}$$

(a) Is Y a continuous or discrete random variable? Why?

(b) What values of Y are assigned positive probabilities?

(c) Find the probability function for Y.

(d) What is the median, $q_{0.5}$, of Y?

4. [15 points] Scores on an exam are assumed to be Normally distributed with mean 78 and variance 36.

   (a) What is the probability that a person taking the exam scores higher than 72?

   (b) The top 15% of students are to receive an A grade. What is the minimum score a student must achieve to earn an A?

   (c) If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

5. [8 points] Let $m(t) = (2/6) + (1/6)e^t + (2/6)e^{2t} + (1/6)e^{3t}$ be the moment generating function for a random variable Y.

   (a) Find $E[y]$.

   (b) Determine the distribution of Y.

6. [7 points] Using the definition, derive the moment-generating function $m(t)$ for the exponential distribution.