306 Homework 3

1. a) $P(Y<y) = \int_0^y \frac{2(\theta-y)}{\theta^2}\,dt = \frac{2\theta y - y^2}{\theta^2}$

which is dependent on θ.

b) $P(U<u) = P(Y/\theta < u) = P(Y < \theta u)$

$= \int_0^{\theta u} \frac{2(\theta-t)}{\theta^2}\,dt$

$= 2u - u^2$ which is indep of θ.

c) $P(U<u) = 0.90 \Rightarrow 2u - u^2 = 0.90$

$\Rightarrow u = 1 - \frac{1}{\sqrt{10}} = \frac{\sqrt{10}-1}{\sqrt{10}}$

≈ 0.683772

Then $P(U<u) = 0.90$

$\Rightarrow P(Y/\theta < u) = 0.90$

$\Rightarrow P(Y/u < \theta) = 0.90$

C.I is $\theta > Y/0.683772$.

2. $\hat{p} = \dfrac{171}{880}$, $n = 880$ is a large sample.

90% C.I $\Rightarrow z_{0.05} = 1.645$

Using $\hat{q} = 1 - \hat{p} = \dfrac{708}{880}$

yields $\dfrac{171}{880} \pm (1.645)\left(\sqrt{\dfrac{\left(\frac{171}{880}\right)\left(\frac{708}{880}\right)}{880}}\right)$

or 0.194 ± 0.023

3. $n = 9 \Rightarrow$ small sample ∴ t-dist'n.
df = 8

99% C.I $\Rightarrow t_{0.005}(8) = 3.355$

a) $\sigma_{\bar{x}}^2 = \dfrac{\sigma_x^2}{n} \Rightarrow \sigma_{\bar{x}} = \dfrac{11.1}{\sqrt{9}} = 3.7$

b) C.I: $218 \pm (3.355)(3.7)$
or 218 ± 12.414 ppm

4. $\sigma = \sqrt{\frac{p_m q_m}{100} + \frac{p_w q_w}{100}} \le \sqrt{\frac{1/4}{100} + \frac{1/4}{100}} = \frac{1}{20\sqrt{10}}$.

a) For 2σ, $E = \frac{1}{10\sqrt{10}} = 0.316$.

b) For C.I 90%, $\alpha = 0.10$ and $z_{0.05} = 1.645$.

$$z_{\alpha/2}\,\sigma = 0.02 \Rightarrow \sigma^2 = \left(\frac{0.02}{1.645}\right)^2$$

$$\Rightarrow \frac{p_m(1-p_m)}{n} + \frac{p_w(1-p_w)}{n} = \left(\frac{0.02}{1.645}\right)^2$$

$$\Rightarrow n \ge \frac{p_m(1-p_m) + p_w(1-p_w)}{\left(\frac{0.02}{1.645}\right)^2}$$

$$n \ge \frac{1}{2} \cdot \left(\frac{1.645}{0.02}\right)^2$$

$$n \ge 3383$$

5. $\bar{X} = 8$, $S^2 = 13$, $S \approx 3.606$

$n = 7 \Rightarrow df = 6$ on t-distn

a) For 98% C.I, $t_{0.01}(6) = 3.143$

$$\bar{X} \pm t_{0.01}(6)\frac{S}{\sqrt{n}} \Rightarrow 8 \pm (3.143)\sqrt{\frac{13}{7}}$$

$$\Rightarrow 8 \pm 4.283$$

or $(3.717, 12.283)$ hrs

b) C.I. 95% for S^2 Again $df = 6$.

$$\alpha = 0.05 \Rightarrow \chi^2_L = \chi^2_L(6) = \chi^2_{0.975} = 1.237$$

$$\Rightarrow \chi^2_u = \chi^2_{0.025}(6) = 14.449.$$

Then $\dfrac{(n-1)S^2}{\chi^2_u} \le \sigma^2 \le \dfrac{(n-1)S^2}{\chi^2_L}$

$$\frac{6 \cdot 13}{14.449} \le \sigma^2 \le \frac{6 \cdot 13}{1.237}$$

$$5.3983 \le \sigma^2 \le 63.0558.$$