Math 345 – Homework 7 **Due Monday, October 31, 2022**

1. For the following matrices A:

   (a) do the eigenvalue decomposition and determine the associated eigenvectors,

   (b) determine the transition matrix P who is columns space the (generalized) eigenspaces of A,

   (c) compute e^{At}, and

   (d) show use P to compute the (up to sign) Jordon Canonical form of each.

   A. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$ B. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}$ C. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ D. $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

2. For the following, find the stable unstable and center subspaces for $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$. Then sketch the phase portrait in each of these cases.

   A. $\begin{pmatrix} 2 & 4 \\ 0 & -2 \end{pmatrix}$ B. $\begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ C. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 2 & 2 \end{pmatrix}$

3. Find the stable, unstable and center subspaces for the systems $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ as they are defined in Problem 1 C and D.

4. Consider the autonomous non-linear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ where

$$\mathbf{f}(x_1, x_2, x_3) = \langle x_1 + x_1 x_2^2 + x_1 x_3^2, -x_1 + x_2 - x_2 x_3 + x_1 x_2 x_3, x_2 + x_3 - x_1^2 \rangle.$$

   (a) Find the rest points of the non-linear system.

   (b) Find the derivative $D\mathbf{f}(\mathbf{x})$.

   (c) Find the linearization of $\mathbf{f}(\mathbf{x})$ at each of the rest points.

5. TBD