Math 345 – Homework 2 **Due Monday, September 13, 2022**

1. Express the solution of the initial value problem

$$2x\frac{dy}{dx} = y + 2x\cos x, \;\; y(1) = 0$$

as an integral.

2. Find the general solutions of the differential equations.

   (a) $x^3 + 3y - xy' = 0$

   (b) $xy^2 + 3y^2 - x^2y' = 0$

   (c) $6xy^3 + 2y^4 + (9x^2y^2 + 8xy^3)y' = 0$

3. Solve the differential equation

$$(x + ye^y)\frac{dy}{dx} = 1$$

by regarding y as the independent variable rather than x.

4. (a) Consider the ODE $y(1 + x^3)y' = x^2$. Determine where in the xy-plane existence and uniqueness issues to an associated initial value problem may occur.

   (b) Solve the IVP $y(1 + x^3)y' = x^2$, $y(0) = y_0$ and determine how the interval in which the solution exists depends on the initial value y_0.

5. Consider the IVP $y' = ty^2$, $y(0) = 1$.

   (a) Explain why this IVP has a unique solution.

   (b) Covert the IVP into an equivalent integral equation.

   (c) Set up the approximate integral equation used in Picard's method and carry out the iteration for three steps.

   (d) Solve the IVP by separation of variables.

   (e) Determine the series representation of the solution to the IVP and compare it to the successive approximations computed above.

Reduction of Order

The general form of a second-order differential equation has the form

$$F(x, y, y', y'') = 0.$$

Reduction of order is the idea of using a change of variable to produce an equivalent, lower-order differential equation.

1. *dependent variable missing*

   When y is not explicitly present in the ODE, the ODE has the general form $F(x, y', y'') = 0$.

   (a) Explain how the introduction of the new dependent variable $p(x) = y'$ converts the second-order ODE into a first-order system in p.

   (b) Solve the equation $xy'' - y' = 3x^2$. (Note that in the end your solution for y will have two arbitrary constants; as it should.)

2. *independent variable missing*

   When x is not explicitly present in the ODE, the ODE has the general form F(y,y',y")=0. (Equations of this form are know as **autonomous**.)

   (a) Explain how the introduction of the new dependent variable $p(y) = y'$ converts the second-order ODE into a first-order system where y can be momentarily interpreted as the independent variable. (Hint: Take care with the derivative computed to replace the y'' term.)

   (b) Solve the equation $y'' + k^2 y = 0$ (where k a positive constant). (This is actually ill-posed. At some point you are going to have to choose a sign convention. This will yield a solution. The other sign convention would yield a separate solution. We will learn a much easier way to do this problem in the near future.)