Math 345 – Homework 5 **Due Friday, October 7, 2022**

1. Find the general solution of each of the differential equations.

   (a) $y''' - 3y'' + 3y' - y = 4e^t$

   (b) $y^{(4)} + 2y'' + y = 3\sin t - 5\cos t$

   (c) $y''' + 4y' = t + \cos t + e^{-2t}$

2. Find the solution of the given IVP

$$y^{(4)} + 2y'' + y = 3\sin t - 5\cos t,\ y(0) = y'(0) = 0,\ y''(0) = y'''(0) = 1.$$

3. Convert the given IVP to its equivalent first-order linear initial value system.

$$y^{(4)} + 2y'' + y = 3\sin t - 5\cos t,\ y(0) = y'(0) = 0,\ y''(0) = y'''(0) = 1.$$

Definition: A function $f(t, x)$ on a domain $D \subset \mathbb{R} \times \mathbb{R}^n$ is said to satisfy a **Lipschitz condition** with respect to x on D if there exists a constant $K > 0$ such that

$$|f(t, x_a) - f(t, x_b)| \leq K|x_a - x_b|,\ \text{for any } (t, x_a), (t, x_b) \in D.$$

(Colloquially, we just say that "f is Lipschitz". K is called the *Lipschitz condition.* (And, yes, the 8 year old in all of us loves this name.))

4. Show that the function $f(t, x) = 5\sin t\cos x$ is Lipschitz on its domain. Be sure to clearly justify your answer.