

1. Find the general solution of each of the differential equations.

(a)  $y''' - y = 0$

(b)  $y''' + 4y'' + 4y' = 0$

(c)  $y^{(4)} + 2y'' + y = 0$

(d)  $y^{(4)} + 4y''' + 6y'' + 4y' + y = 0$

2. If  $m$  is a positive constant, find the solution of the initial value problem

$$y''' - my'' + m^2y' - m^3y = 0$$

where  $f(0) = f'(0) = 0$ ,  $f''(0) = 1$ .

3. Find a linear differential equation  $Ly = 0$  with constant coefficients where  $y_1(x) = e^{-2x} \cos 3x$ ,  $y_2(x) = x^2$ , and  $y_3(x) = x \sin x$  are solutions.

4. Find the general solution (by hand) of the differential equation

$$y'' - y = \frac{2}{1 + e^x}.$$

5. Consider the non-homogenous ODE

$$y''' + \frac{1}{x}y'' - \frac{2}{x^2}y' + \frac{2}{x^3}y = 2x.$$

- (a) Verify that  $\{x, x^2, 1/x\}$  form a fundamental set to the corresponding complementary ODE.
- (b) Determine a particular solution of the non-homogenous ODE.