Math 414 – Homework 2 **Due Monday, January 30, 2023**

Chapter 1

1. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous; provide reasons.

   (a) $u_t - u_{xx} + 1 = 0$

   (b) $u_t - u_{xx} + xu = 0$

   (c) $u_t - u_{xxt} + uu_x = 0$

   (d) $u_{tt} - u_{xx} + u/x = 0$

   (e) $u_x(1 + u_x^2)^{-1/2} - u_{xy} = 0$

   (f) $u_t + u_{xxxx} + \sqrt{1 + u} = 0$

   (g) $u_x - e^y u_y = 0$

2. Solve the equation $2u_t + 3u_x = 0$ with the auxiliary condition $u = \sin x$ when $t = 0$.

3. (a) Solve the equation $yu_x + xu_y = 0$ with the condition $u(0, y) = e^{-y^2}$.

   (b) In which region of the xy-plane is the solution the IVP in (a) uniquely determined? (Hint: Recall that data is transported along characteristic curves. Look at the characteristic curves and think about what happens to the data of our initial condition.)

4. Solve $au_x + bu_y + cu = 0$. (Assume $a, b \in (-\infty, 0) \cup (0, \infty)$.)

5. Solve the equation

$$u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2.$$

6. Consider the equation

$$u_x + yu_y = 0$$

   with the boundary condition $u(x, 0) = \phi(x)$.

   (a) Find the general solution to the PDE.

   (b) (BVP without a solution) For $\phi(x) \equiv x$, show that no solution exists.

   (c) (BVP without uniqueness) For $\phi(x) \equiv 1$, show that there are many solutions.