HW 1 PDE Sp'23

1. $f(x,y) = \sin(xy) - x^3y + xy^4 - 12$

a) $f_x = y\cos(xy) - 3x^2y + y^4$

$f_y = x\cos(xy) - x^3 + 4xy^3$

b) $f_{xx} = -y^2\sin(xy) - 3x^2$

$f_{xy} = \cos(xy) - xy\sin(xy) - 3x^2 + 4y^3$
$= f_{yx}$

$f_{yy} = -x^2\sin(xy) + 12xy^2$

2 a) $\partial_x f(u(x,y)) = f'(u)\,u_x$
$= (-\sin u + 2u)(2xy + 2)$

b) $\nabla(f \circ u) = \langle f'(u)u_x, f'(u)u_y \rangle$
$= f'(u)\langle u_x, u_y \rangle$
$= (-\sin u + 2u)\langle 2xy+2, x^2 + 3y^2 \rangle$

3. $\mathrm{dom}(f) = \{\mathbb{R}^2 \setminus (0,0)\}$

$f_x = \frac{2x}{x^2+y^2}$, $f_y = \frac{2y}{x^2+y^2}$ $\boxed{f_{xx} + f_{yy} = 0}$

$f_{xx} = \frac{2(x^2+y^2) - 2x(2x)}{(x^2+y^2)^2}$, $f_{yy} = \frac{2(x^2+y^2) - 2y(2y)}{(x^2+y^2)^2}$

HW1 p2

(4) $y' + \frac{2}{r} y = \frac{e^{t^2}}{r}$ 1st order linear standard form.

$\mu(t) = \exp\left[\int \frac{2}{r} dt\right] = e^{2t/r}$

$\mu(t) y' + \frac{2}{r}\mu(t) y = \frac{1}{r}\exp\left[t^2 + \frac{2t}{r}\right].$

$(\mu y)' = \quad ''$

$\mu(t) y = \int \frac{1}{r}\exp\left[t^2 + \frac{2t}{r}\right] dt$

$y(t) = e^{-2t/r} \int \frac{1}{r}\exp\left[t^2 + \frac{2t}{r}\right] dt.$

5 a) $\frac{dy}{y^2} = \cos t \, dt$, $\frac{-1}{y} = \sin t + C$

$y(0) = \alpha \Rightarrow \frac{-1}{\alpha} = 0 + C \Rightarrow C = \frac{-1}{\alpha}$

$\frac{-1}{y} = \frac{\alpha \sin t - 1}{\alpha}$, $y(t) = \frac{-\alpha}{\alpha \sin t - 1}$

b) Note $y = 0$ is an equilibrium sol'n.
So if $\alpha = 0$, the only sol'n is $y = 0. \ \forall t \in \mathbb{R}$.

$\alpha \neq 0$, $\alpha \sin t - 1 = 0$ has sol'ns is $|\alpha| \geq 1$.
So $y(t) = \frac{-\alpha}{\alpha \sin t - 1}$ defined for all t when $|\alpha| < 1$

HW p3

(6) $(D^2+2D+5)y=0$

$$\frac{-2\pm\sqrt{4-20}}{2} = \frac{-2\pm\sqrt{-16}}{2} = \frac{-2\pm 4i}{2} = -1\pm 2i.$$

$$y(x) = C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t$$

(7) a) trivial

b) superposition.

$$(\alpha y_1'' + \beta y_2'') + P(\alpha y_1' + \beta y_2') + Q(\alpha y_1 + \beta y_2) -$$

$$= \alpha\left[y_1'' + Py_1' + Qy_1\right] + \beta\left[y_2'' + Py_2' + Qy_2\right]$$

$$= \alpha\cdot 0 + \beta\cdot 0$$

$$= 0.$$