Math 414 – Homework 4 **Due Friday, February 24, 2023**

Heat: Section 2.3, mostly

1. The damped string equation is given by

$$u_{tt} - c^2 u_{xx} + \gamma u_t = 0, x \in \mathbb{R}, t \in R.$$

   Using the energy function $E(t)$ from lecture, show that energy is decreasing. As usual, assume the data is of compact support.

2. Let $u(x,t) = 1 - x^2 - 2kt$.

   (a) Show that u solves the heat equation $u_t = ku_{xx}$ on $-1 \le x \le 2$, $t > 0$ and determine the initial temperature distribution.

   (b) Determine the maximum and minimum of $u(x,t)$ on the rectangle $[-1,1] \times [0,T]$.

3. Consider the diffusion equation $u_t = u_{xx}$ in $[0,1] \times [0,\infty)$ with $u(0,t) = u(1,t) = 0$ and $u(x,0) = 4x(1-x)$.

   (a) Show that $0 < u(x,t) < 1$ for all $t > 0$ and $0 < x < 1$.

   (b) Show that $u(x,t) = u(1-x,t)$ for all $t \ge 0$ and $0 \le x \le 1$.

   (c) Use the energy method to show that $\int_0^1 u^2(x,t)\,dx$ is a strictly decreasing function of t.

4. Prove the comparison principle for the heat equation on $0 \le x \le L$: If u and v are two solutions, and $u \le v$ for $t = 0$, $x = 0$ and $x = L$, then $u \le v$ for all time.