Math 414 – Homework 3 Due Wednesday, February 8, 2023?

Waves: Section 2.1 and 2.2

1. Solve $u_{tt} = c^2 u_{xx}$, $u(x, 0) = \ln(1 + x^2)$, $u_t(x, 0) = 4 + x$.

2. The midpoint of a piano string of tension T, density ρ, and length l is hit by a hammer whose head diameter is $2a$. A flea is sitting at a distance $l/4$ from one end. (Choose a so that know fleas are killed in this thought experiment.) How long does it take for the disturbance to reach the flea?

3. If both initial data functions ϕ and ψ are odd functions of x, show that D'Alembert's solution is also odd in x for all t.

4. Solve $-4u_{tt} - 3u_{xt} + u_{xx} = 0$, $u(x, 0) = x^2$, $u_t(x, o) = e^x$. (*Hint*: Factor the operator as we did in deriving the traveling wave solution.)

5. Use the energy conservation of the wave equation to prove that the only solution with $\phi \equiv 0$ and $\psi \equiv 0$ is $u \equiv 0$.

6. Show that the wave equation has the following invariance properties.

   (a) Any translate $u(x - y, t)$, where y is fixed, is also a solution.

   (b) Any derivative, say u_x, of a solution is also a solution.

   (c) The dilated function $u(ax, at)$ is also a solution, for any constant a.