Reall Dr= Vo Va. So (AdV= StdV Ky Du. Thin Skidv= (Volva) dV = ((Thon) ds or (duds So Son do = Son foll By Newan Conditions,

Sold S = Spall must be line for any silh in to the non-terrigonees POET. b. For harageneeus case, he get (gds = C. Here the average value of gxy, 73 a SIS must be Zev.

 $\partial_{t}a_{t}$ in harmonic \Rightarrow dues an disk (2).

By max (min) principle, wax (min) occurs
on the bandary $u(x,y) = u(\partial_{t}a_{t}) = 3 (\partial_{t}a_{t}a_{t})(\partial_{t}a_{t}a_{t}) + 1$ $x^{2}+y^{2}=4$ UCD, 61 = 6 COS6SNE + 1 = 3 sn 2 + 1 CICI: U'= 6 cost =0 ch do= II+ATT $G = \Pi + M\Pi$ 4 cihal # in paramplizh 0=62 dr 6,= # 1 6,= 31, 63=57, 64=20 4 u(2, T/4) = u(1, 5T/4) = 3sin (T/1)+1 = 4 MAX u(2,311/4) = L(2,711/4) = 35n(31/4)+1 =-1, mn By the Men Khe Papuly,

UIC, 0 = I (h(x, y) ds

IT Sx2+y2=4

= I (2) h(2) D) ds

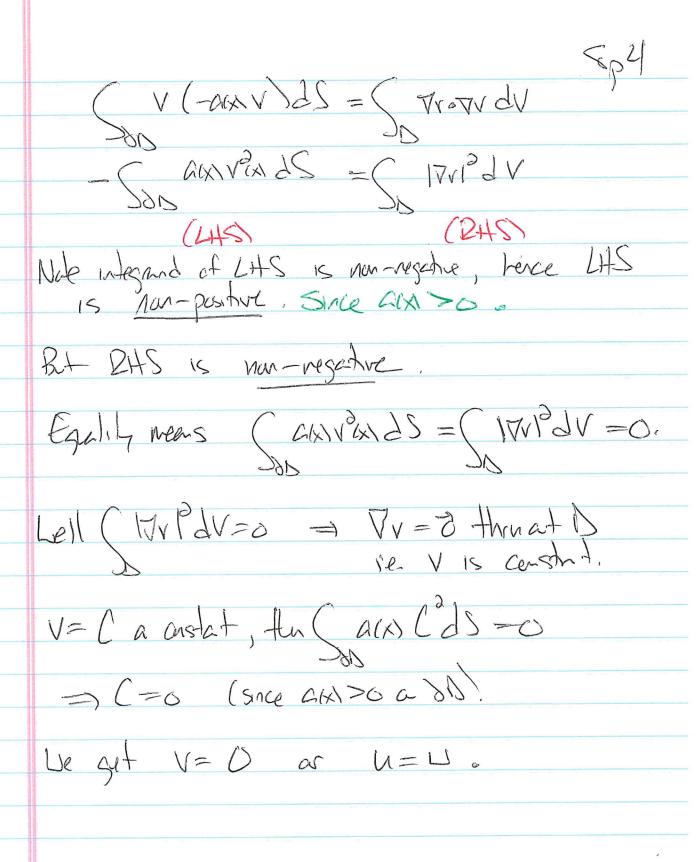
= I (35 ndz +1) dc

= IT)6

-1 (-3 cosdo + 6) PT (Not a surprise, men of the men + min. 3. Let u and us solve 5 kn=0 in D

an +ackness=0 an 81)

on w/acxl > 0 an d1 Dehry V=M-U Note V solve the exact same BVD. O AV = AU - AU = O a du = VION = (Vu - Vis) O 1 = Vuon -Vuon $\frac{ds}{ds} - \frac{ds}{ds} = \frac{ds}{ds}$ = - G(x) W - (-ax) W = -a(x) (u-v) = - GIXXV = O. tor unqueress, reed to show V=0 Cansol VAW = 0 => SNAWGN = O Son and S - (Tropred = 0 by freen's 1st)



4. At a minimum, the directional derivative in the direction of v needs to the zero for any v.

That is, lim Elinter 3-Elin = 0.

GO E let VEC'(D). E [u+Ev] = 1 (17(u+Ev)) - (M(u+Ev) = 15/12/13 + 6 2001 + 6 (12/2 - (1/2 - 6) hr = ELta) + E (Vn-Vv - E (hv + E (IV)) So lim E [U+EV]-ELh] = STUOTV - ShV = StroTu - Shr (mac derivs off r) = SNO on -SNAW-Shr Green's !" = SON - SIVAW - Shy parided = - (VAW dV.

Atamin, Svau=0 trany adall vec! :- shoo thruant Duly du -h. i.e. u solves Laplace's Egn.