Exam 3-Spring 2023-MTH 427

Conceptuals problems

(Make sure to clearly show your work in order to earn full credit)

1 Exercise 1 (18 points)

For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are ARMA*(p,q)* (watch out for parameter redundancy), determine whether they are causal, and determine whether they are invertible.

(a) $x_t = -0.3x_{t-1} + w_t - 0.4w_{t-1} - 0.21w_{t-2}$

(b) $x_t = 3x_{t-1} + w_t + w_{t-1} - 2w_{t-2}$

(c) $x_t = 0.5x_{t-1} + 0.5x_{t-2} + w_t - w_{t-1}$

2 Exercise 2 (8 points)

Suppose $x_t = w_t - w_{t-4}$, where w_t is a Gaussian white noise $WN(0, \sigma_w^2)$. Find the the mean and the autocovariance function (ACVF) of this series. Show that its autocorrelation function is nonzero only for lag $h = 4$. $(h \geq 1)$.

3 Exercise 3 (11 points)

For those models of "Exercise 1" that are causal and invertible, compute the first four coefficients $\pi_0, \pi_1, \pi_2, \pi_3$ in the invertible linear process representation $\pi(B)x_t = \sum_{j=0}^{\infty} \pi_j x_{t-j}$. (hint:set $\sum_{j=0}^{\infty} \pi_j z^j = \frac{\phi(z)}{\theta(z)}$)

4 Exercise 4 (14 points)

Exhibit an equation of the following models. (hint: use you may first use backshift operators, then expand both sides to obtain an equation in difference form.)

(a) $ARIMA(2, 1, 1)$.

(b) $ARIMA(1, 1, 1) \times (0, 1, 1)_{12}$ or $SARIMA(1, 1, 1) \times (0, 1, 1)_{12}$ (notation from other text books).

R Project

5 Exercise 5 (8 points)

Generate $n = 100$ observations from $ARMA(1,1), AR(1)$ and $MA(1)$ processes respectively and plot the sample ACF and PACF of each series for $\phi = 0.6$, $\theta = 0.9$. Compare the sample ACFs and PACFs of the generated series with the results given in Table 3.1. (Are these series consistent with the table?)

6 Exercise 6 (13 points)

Consider the Lake Huron data set("LakeHuron") in R or in the package "itsmr" as ("lake"). It consists of $n = 98$ observations of annual levels from 1875-1972. Let's investigate if there is evidence of decline in the level of Lake Huron

(a) Plot this series. Plot a sample **acf** and the sample **pacf** for this series.

(b) Based on the **acf** and **pacf** in part (a), fit an appropriate ARMA model using the ***sarima(p,d,q)*** function in R, performing all necessary diagnostics. Comment.

(c) After deciding on an appropriate model, forecast the data into the future 12 time periods ahead. Make sure to display the plot.

7 Exercise 7 (28 points)

Consider the Monthly Airlines Passengers numbers 1949-1960 (airpass) from the package "itsmr", or (AirPassengers) from package "astsa" in R.

(a) Display the plot of this series and its **acf** and **pacf**. Comment on the behavior of this series.

(b) Is any transformation necessary? Why?

(c) Detrend the series by fitting a linear regression of the log-transformed of the series on time t. Based on the R^2 of this regression fit, can we say that there a significant trend? Why? (Make sure to display your code and results)

One can also remove the trend by differencing

(d) Compute the first difference of the log-transformed of the original series. Examine

its sample **acf** plot and compare it with the **acf** plot of the residuals in part (c).Make-sure to display your plots.

(e) Is there a strong seasonality in the series? Explain.

(f) Display a sample **acf** and **pacf** of the seasonal difference of the differenced series in part(d) *(hint: this line of code is given in the text file "Rcode times series-hwk8..." posted in Canvas for* ***"unemp****" series).*

(g) Based on the **acf** and **pacf** in (f), fit an appropriate seasonal ARIMA model to the log-transformed Airlines Passengers series. Make sure to explain why you select that model (model diagnostics).

(h) Use the estimated model to forecast the next 12 values.