PHYS512_HW6

November 11, 2022

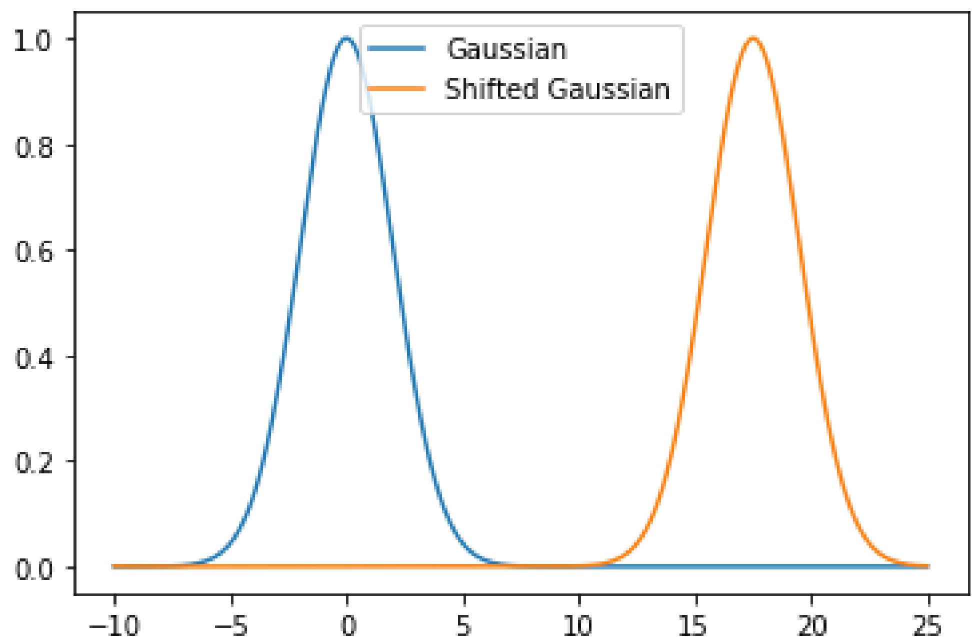
[1]:
```python
import math
import matplotlib.pyplot as plt
import numpy as np
from scipy.stats import chisquare
```

1 Problem 1

[149]:
```python
def func(ar, shift):
    J = np.complex(0, 1)

    F=np.fft.fft(ar)
    k = np.arange(len(F))
    G = np.exp(2 * np.pi * J * shift * k/ 1000)
    return np.real(np.fft.ifft(F*G))

N = 1000
x = np.linspace(-10, 25, N)
  = 2
y = np.exp(-0.5 * x**2/(  **2 ))
print(len(y))

plt.plot(x, y, label = 'Gaussian')
plt.plot(x, func(y, len(y)/2), label = 'Shifted Gaussian')
plt.legend()
```

```
1000
```

[149]:
```
<matplotlib.legend.Legend at 0x7fa9f7c7f250>
```



2 Problem 2

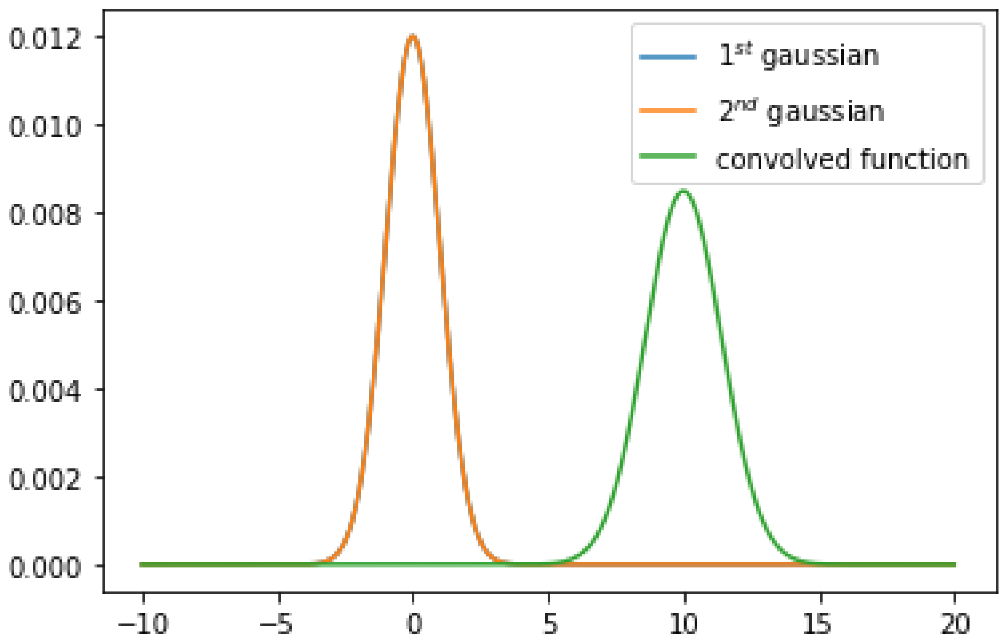
2.1 a)

[4]:
```
def func1(ar1 ,ar2):
    ar1 = ar1/ar1.sum()
    ar2 = ar2/ar2.sum()

    F = np.fft.fft(ar1)
    G = np.fft.fft(ar2)

    return (np.fft.ifft(F * G))

def gaussian(x,  ,  ):
    return np.exp(-0.5 * (x -  )**2/ **2)

N = 1000
x = np.linspace(-10, 20, N)
y1 = gaussian(x, 1, 0)
y2 = gaussian(-x,1 , 0) # since conj(F(k)) = DFT(f(-x)), we'are taking -x␣
↪everywhere, doesnt matter with

plt.plot(x, y1/y1.sum(), label =  '1$^{st}$ gaussian' )
plt.plot(x, y2/y2.sum(), label = '2$^{nd}$ gaussian')
plt.plot(x, func1(y1, y2), label = 'convolved function')
```

```
plt.legend()
```

```
/Users/junalexsugiyama/opt/anaconda3/lib/python3.7/site-
packages/numpy/core/_asarray.py:85: ComplexWarning: Casting complex values to
real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
```

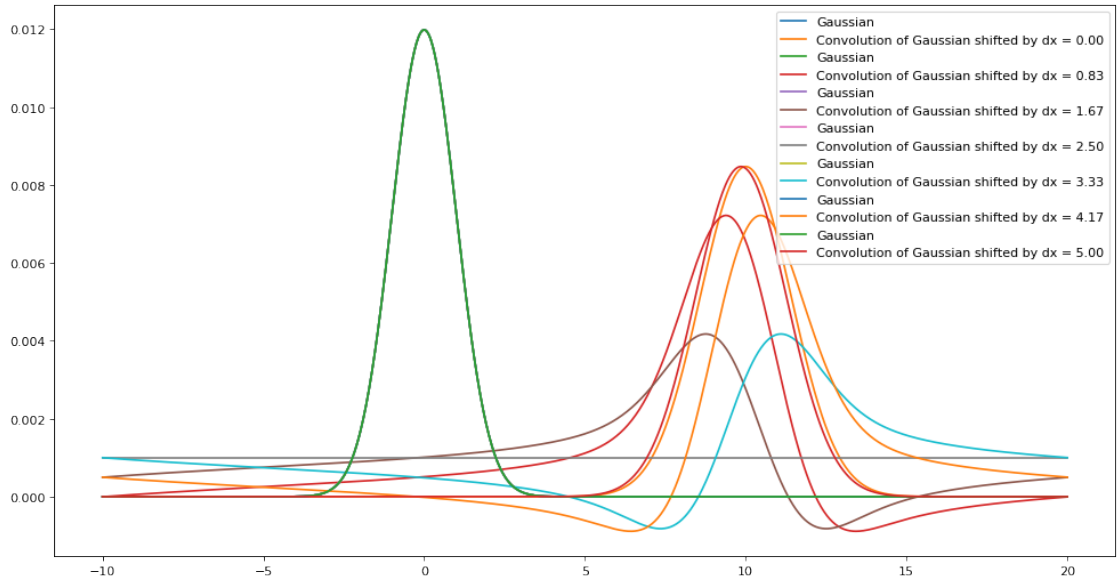
[4]: <matplotlib.legend.Legend at 0x7faa09a44550>



2.2 b)

[5]:
```
N = 1000
x = np.linspace(-10, 20, N)
y1 = gaussian(x, 1, 0)
y2 = gaussian(-x,1 , 0)

dx = np.linspace(0, 5, 7)

plt.figure(figsize=(15, 8), dpi=80)
for i in dx:
    y_shift = func(y1, i)

    plt.plot(x, y1/y1.sum(), label =  'Gaussian' )
    plt.plot(x, func1(y_shift, y2), label = f'Convolution of Gaussian shifted␣
 ↪by dx = {i:.2f}')
```

```
    plt.legend()
```

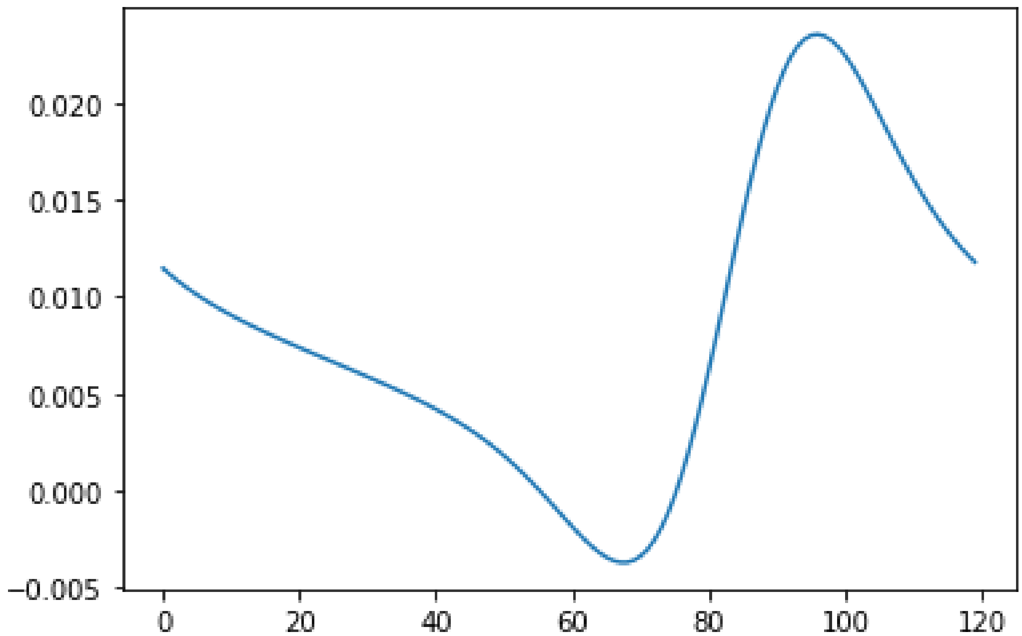


3 Problem 3

[165]:
```
N = 100
x = np.linspace(-10, 10, N)
y1 = gaussian(x, 1, 0)
y2 = gaussian(-x, 2, 4)

def window(ar1, ar2):
    zrs = np.zeros(10)

    #dar1 = np.insert(ar1,len(ar1),zrs)
    #ar1 = np.insert(ar1, 0,zrs)
    #print(ar1)
    #ar2 = np.insert(ar2,len(ar2), zrs)
    #ar2 = np.insert(ar2, 0, zrs)

    #ar1_shift = func(ar1, len(ar1)/2)
    #ar2_shift = func(ar2, len(ar2)/2)
    #plt.plot(ar1)
    #plt.plot(ar1_shift)
    y1_w = ar1 #* window
    y2_w = ar2 #* window
    return func1(ar1_shift, ar2_shift)
```

```python
#plt.plot(x, y1/y1.sum(), label =  'Gaussian' )
#plt.plot(x, y2/y2.sum())
plt.plot(window(y1, y2))
```

```
[165]: [<matplotlib.lines.Line2D at 0x7fa9f8e997d0>]
```



4 Problem 4

4.1 a)

$$\sum_{x=0}^{N-1} e^{-\frac{2\pi ikx}{N}}$$

Let $e^{-\frac{-2\pi ik}{N}=\alpha}$, then:

$$\sum_{x=0}^{N-1} e^{-\frac{2\pi ikx}{N}} = \sum_{x=0}^{N-1} \alpha^x$$

$$S_n = \alpha^0 + \alpha^1 + ... + \alpha^{N-1},$$

$$\alpha S_n = \alpha(\alpha^0 + \alpha^1 + ... + \alpha^{N-1}) = \alpha^1 + ... + \alpha^N$$

Subsctrating $S_n - S_{n-1}$:

$$S_n - S_{n-1} = S_n - \alpha S_n =$$
$$S_n(1-\alpha) = (\alpha^0 + \alpha^1 + ... + \alpha^{N-1}) - (\alpha^1 + ... + \alpha^N) = \alpha^0 - \alpha^N$$

So,

$$S_n = \frac{1-\alpha^N}{1-\alpha} = \frac{1-e^{-2\pi ik}}{1-e^{-\frac{2\pi ik}{N}}}$$

5 b)

For k $\to$ 0, the value in the exponent is small. Using Taylor expansion:

$$\sum_{x=0}^{N-1} e^{-\frac{2\pi ikx}{N}} = \frac{1-\alpha^N}{1-\alpha} = \frac{1-e^{-2\pi ik}}{1-e^{-\frac{2\pi ik}{N}}} =$$

$$\frac{1-(1-2\pi ik)}{1-(1-\frac{2\pi ik}{N})}) =$$

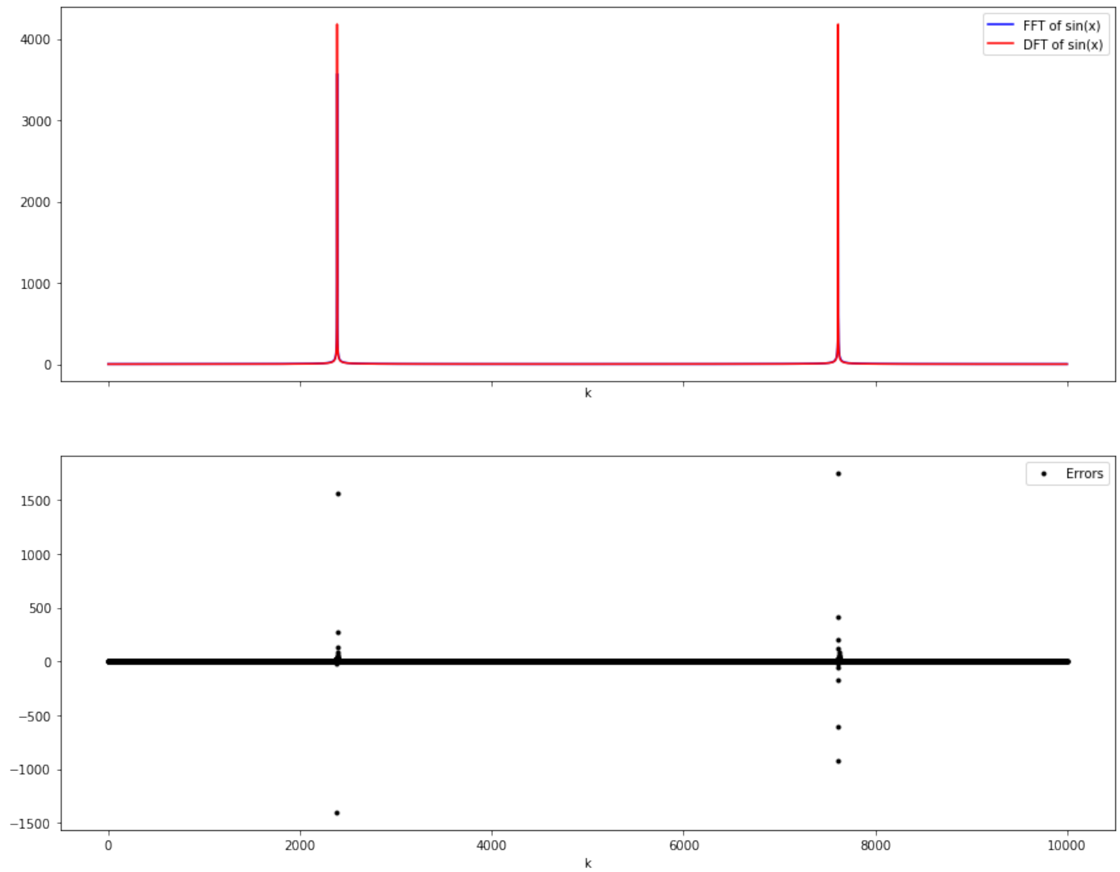
$$\frac{2\pi ik}{\frac{2\pi ik}{N}} \to N$$

5.1 c)

[100]:
```
  = 1.5 #non-integer values of frequency of sin function
def prod(k, N):
    J = np.complex(0, 1)
    x = np.arange(0.0, N) # x interval for every k
    #here i used the sin(x) in exponential form: sin(x) = (e^(ix) - e^(-ix))/2i
    return np.sum(np.exp(- 1 * x * k * 2 * J * np.pi/N) * np.sin(1.5 *x))
                  #(np.exp(J *  * x) - np.exp(-J *   * x) )/(2*J ))

N = 10000
k = np.linspace(0, N , N +1)

dftar = []
for i in k:
    dftar.append(prod(i, N))


x = np.linspace(0, N , N+1)
y = np.sin(  * x)
yft = np.fft.fft(y)
```

```python
fig0, (ax1, ax2) = plt.subplots(nrows=2, ncols=1, sharex = True, figsize = (15,␣
↪12) ) # two axes on figure

ax1.plot(np.abs(yft), color = 'b', label = 'FFT of sin(x)')
ax1.plot(np.abs(dftar), color = 'r', label = 'DFT of sin(x)')
ax1.set_xlabel('k')
ax1.legend()

ax2.plot(np.abs(yft) - np.abs(dftar), '.k', label = 'Errors')
ax2.legend()
ax2.set_xlabel('k')
```

[100]: Text(0.5, 0, 'k')



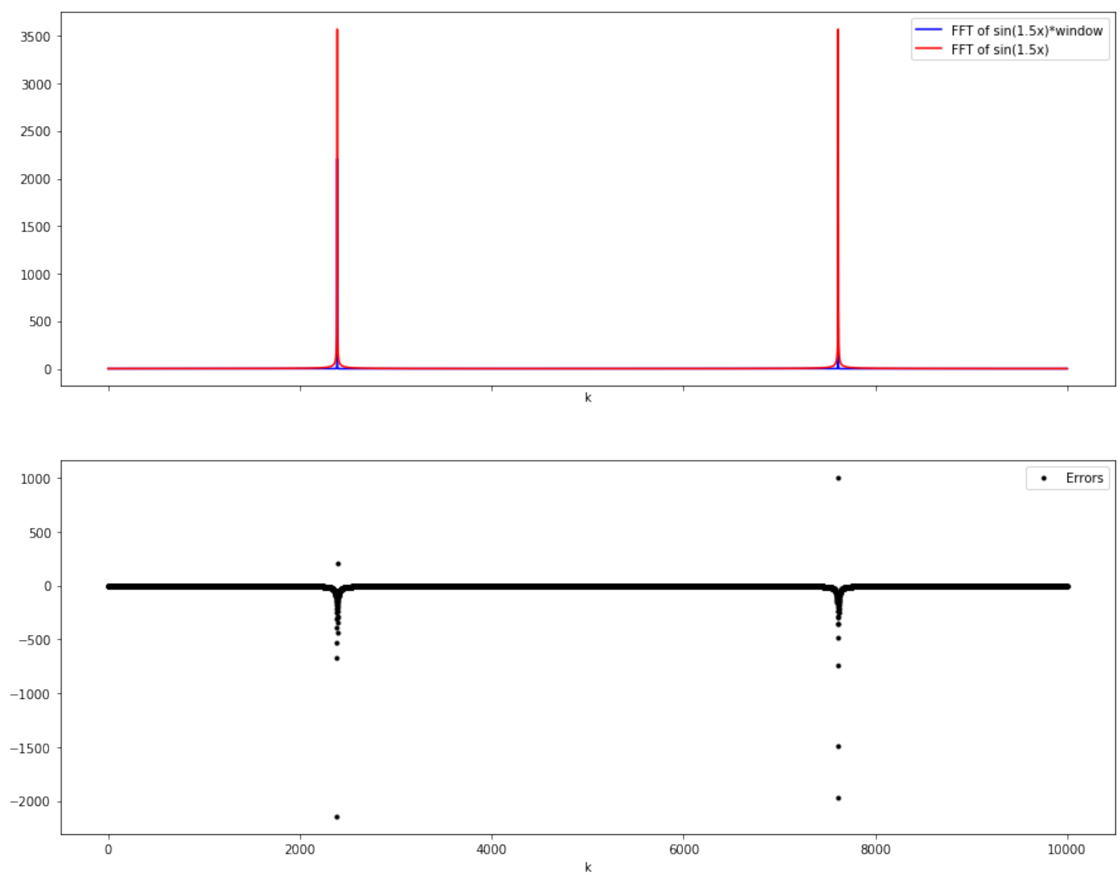
[101]:
```python
# chi-square: sum((observed - expected)**2/expected
print(f'{(np.sum((np.abs(yft) - np.abs(dftar))**2/np.abs(yft))):.2f}')
```

7030.68

5.2 d)

[93]:
```
window = 0.5 - 0.5*(np.cos(2*np.pi*x/N))
y_window = y * window

yft_window = np.fft.fft(y_window)
fig0, (ax1, ax2) = plt.subplots(nrows=2, ncols=1, sharex = True, figsize = (15,␣
↪12) ) # two axes on figure

ax1.plot(np.abs(yft_window), color = 'b', label = 'FFT of sin(1.5x)*window')
ax1.plot(np.abs(yft), color = 'r', label = 'FFT of sin(1.5x)')
ax1.set_xlabel('k')
ax1.legend()

ax2.plot(np.abs(yft_window) - np.abs(dftar), '.k', label = 'Errors')
ax2.legend()
ax2.set_xlabel('k')
```

[93]:
```
Text(0.5, 0, 'k')
```



[94]:
```
print(f'{(np.sum((np.abs(yft_window) - np.abs(yft))**2/np.abs(yft))):.2f}')
```

```
47844.69
```

[96]:
```

```

[96]:
```
array([-0.89342866, -0.89342858, -0.89342835, …, -0.89342948,
       -0.89342926, -0.89342888])
```

5.3 e)

[61]:
```
def prod1(k, N):
    J = np.complex(0, 1)
    x = np.arange(0.0, N) # x interval for every k
    #here i used the cos(x) in exponential form: cos(x) = (e^(ix) - e^(-ix))/2i
    return np.sum(np.exp(- 1 * x * k * 2 * J * np.pi/N) * (0.5 + 0.5 * 0.5 *␣
 ↪(np.exp(J * 2* np.pi * x/N) + np.exp(-J * 2* np.pi * x/N))))

dftwindow = []
for i in k:
    dftwindow.append(prod1(i, N))
print('Fourier Transform of the window function: ', np.abs(dftwindow), ", where␣
 ↪N = 10000. As it's seen the list is [N/2, N/4, ..., N/2, N/4]")
```

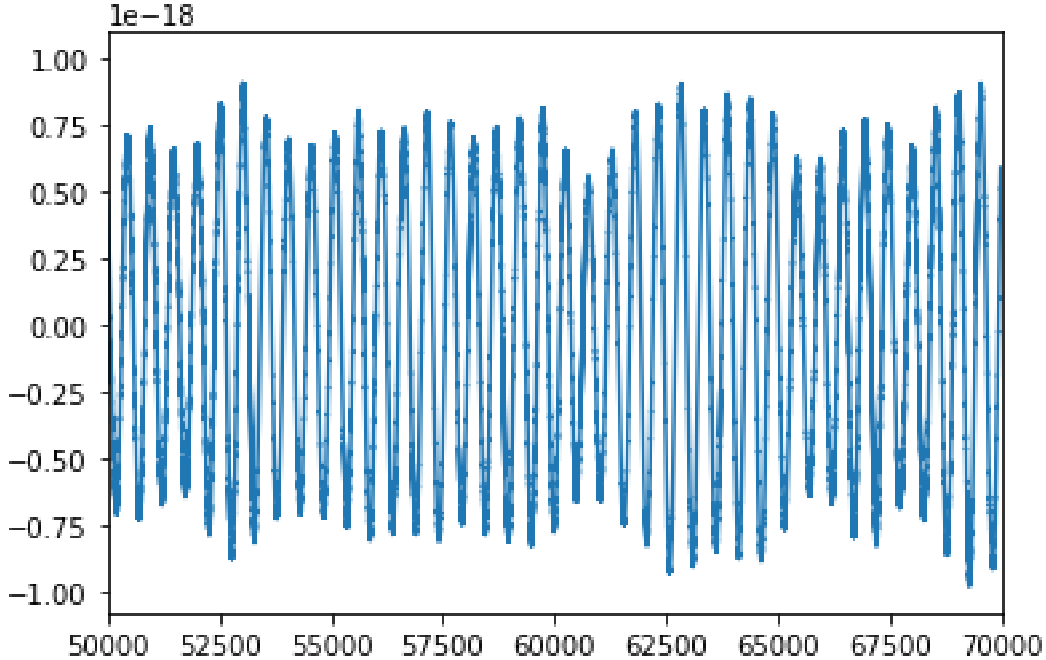
```
Fourier Transform of the window function:  [5.00000000e+03 2.50124974e+03
2.33292178e+00 … 2.50249869e+03
 4.99999947e+03 2.50000000e+03] , where N = 10000. As it's seen the list is
[N/2, N/4, …, N/2, N/4]
```

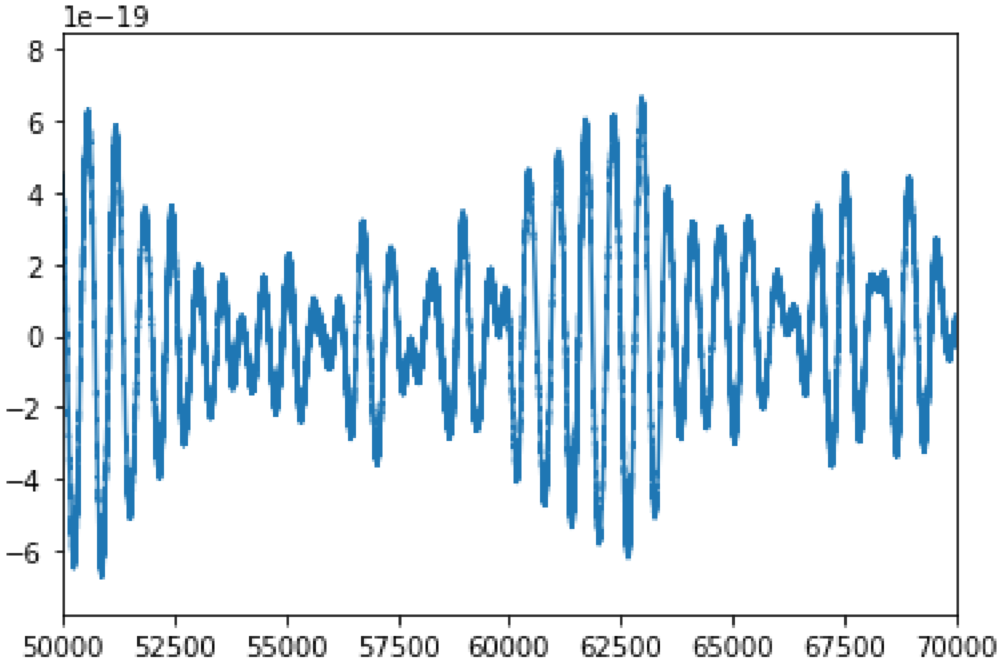
6 Problem 5

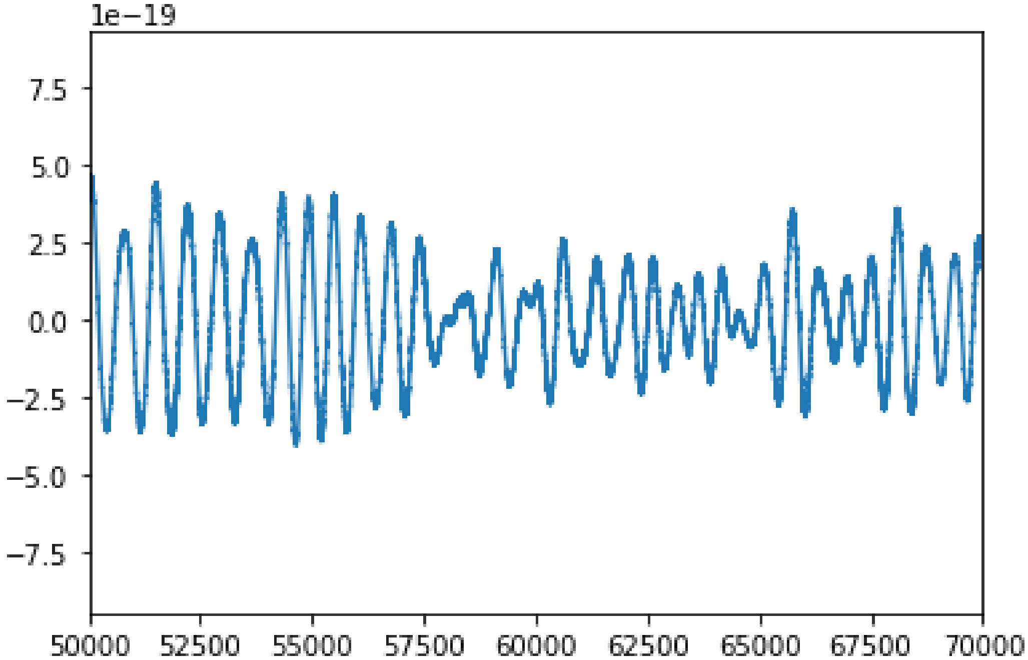
[17]:
```
import h5py
import glob
plt.ion()

def read_template(filename):
    dataFile=h5py.File(filename,'r')
    template=dataFile['template']
    tp=template[0]
    tx=template[1]
    return tp,tx
def read_file(filename):
    dataFile=h5py.File(filename,'r')
    dqInfo = dataFile['quality']['simple']
    qmask=dqInfo['DQmask'][...]
```

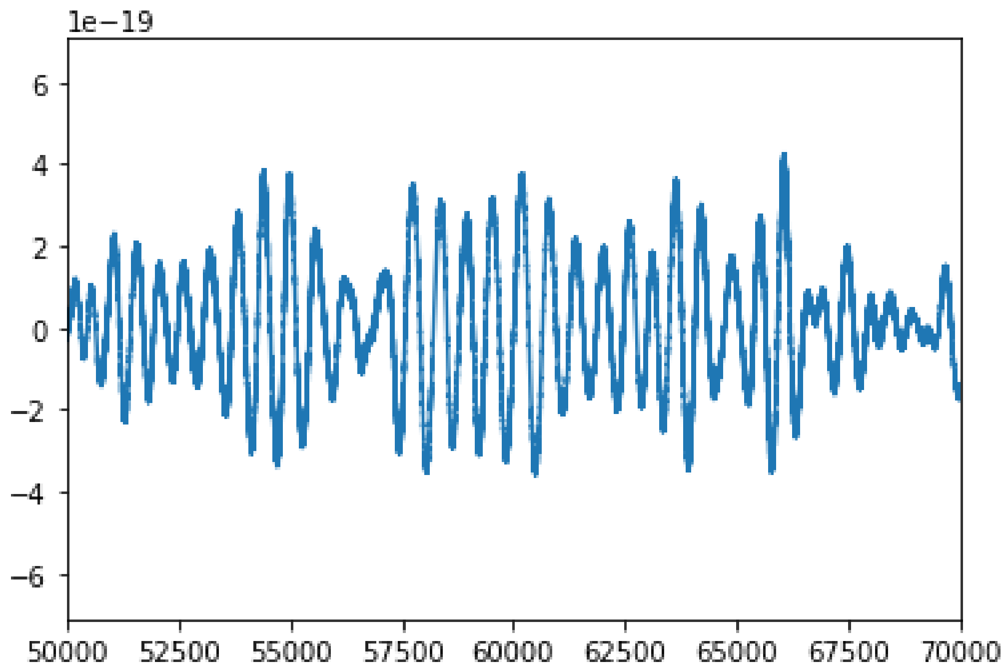
```python
    meta=dataFile['meta']
    #gpsStart=meta['GPSstart'].value
    gpsStart=meta['GPSstart'][()]
    #print meta.keys()
    #utc=meta['UTCstart'].value
    utc=meta['UTCstart'][()]
    #duration=meta['Duration'].value
    duration=meta['Duration'][()]
    #strain=dataFile['strain']['Strain'].value
    strain=dataFile['strain']['Strain'][()]
    dt=(1.0*duration)/len(strain)

    dataFile.close()
    return strain,dt,utc



#fnames=glob.glob("[HL]-*.hdf5")
#fname=fnames[0]
name = '/phys512-fall2022/LOSC_Event_tutorial-master/'
fname=['H-H1_LOSC_4_V1-1167559920-32.hdf5',
      'H-H1_LOSC_4_V2-1126259446-32.hdf5',
      'H-H1_LOSC_4_V2-1128678884-32.hdf5',
      'H-H1_LOSC_4_V2-1135136334-32.hdf5',
      'L-L1_LOSC_4_V1-1167559920-32.hdf5',
      'L-L1_LOSC_4_V2-1126259446-32.hdf5',
      'L-L1_LOSC_4_V2-1128678884-32.hdf5',
      'L-L1_LOSC_4_V2-1135136334-32.hdf5']
#print('reading file ',fname)
A = np.zeros((8))
print(np.shape(A))
for i in range(len(fname)):
    strain,dt,utc=read_file(name + fname[i] )
    plt.figure()
    plt.plot(strain)
    plt.xlim([50000,70000])

print(dt)
#th,tl=read_template('GW150914_4_template.hdf5')
#template_name='GW150914_4_template.hdf5'
#tp,tx=read_template(template_name)

#plt.xlim([20000,30000])
#plt.xlim([21600,21800]);plt.ylim([-2e-19,2e-19])
```

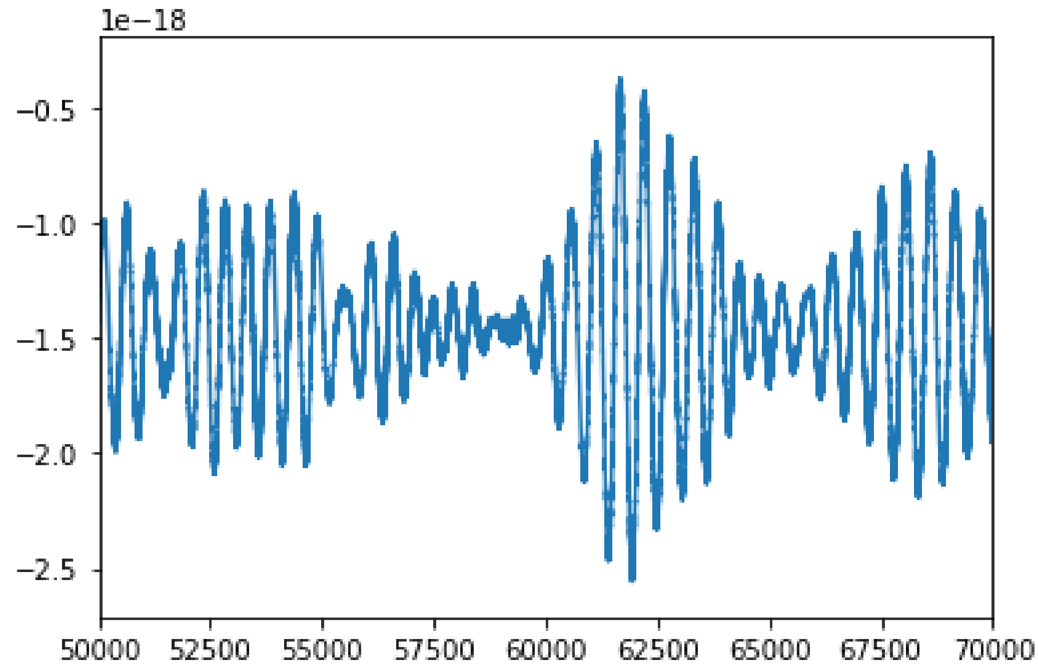
```
(8, 33)
0.000244140625
```

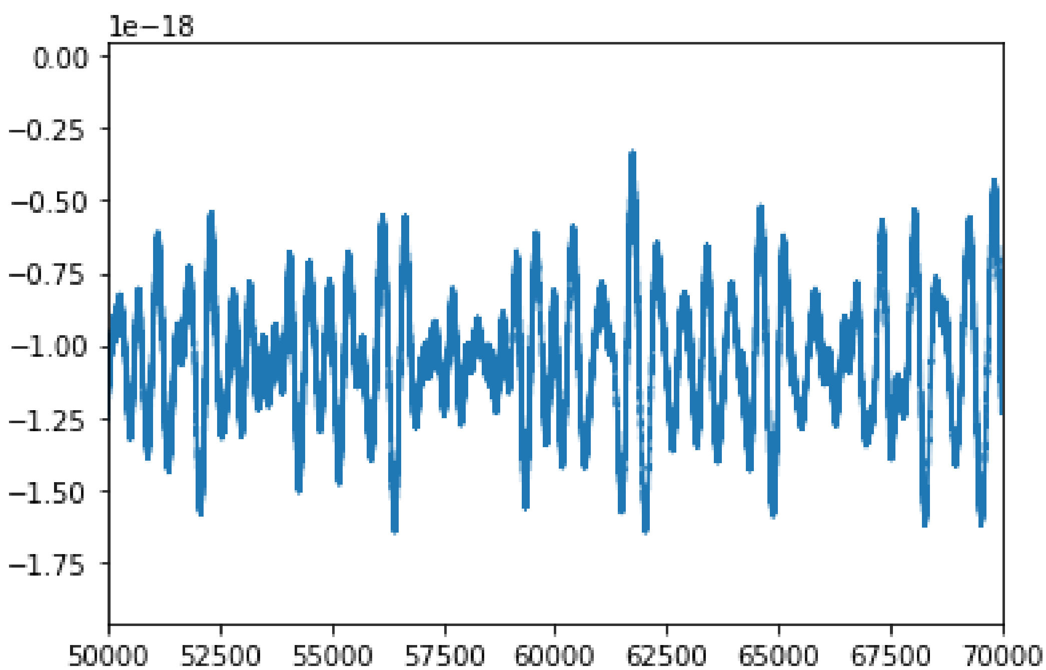

1e−18
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
50000
52500
55000
57500
60000
62500
65000
67500
70000

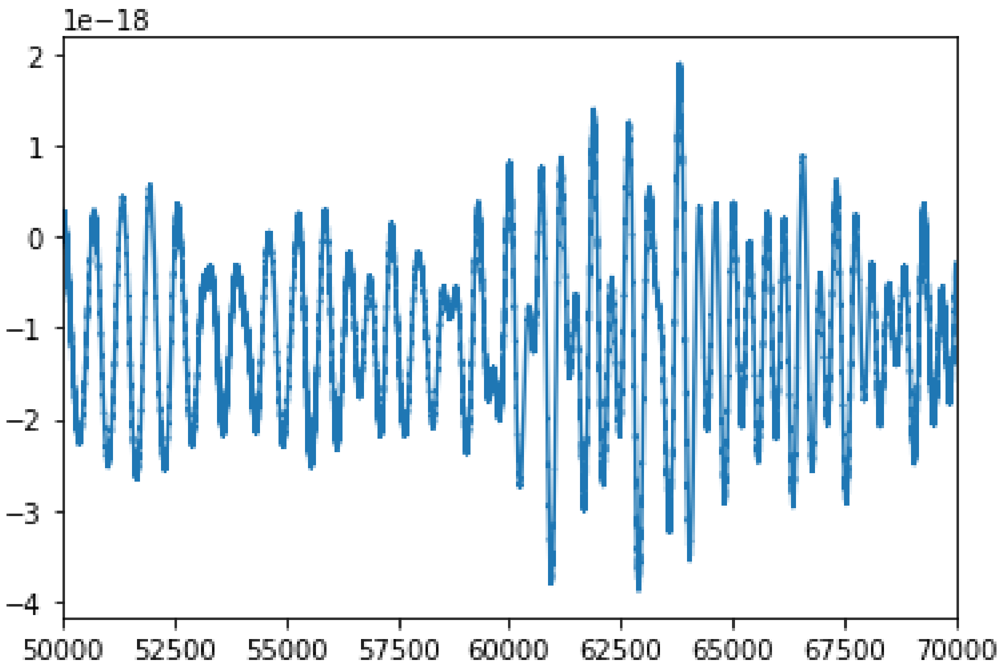

1e−19
8
6
4
2
0
−2
−4
−6
50000
52500
55000
57500
60000
62500
65000
67500
70000

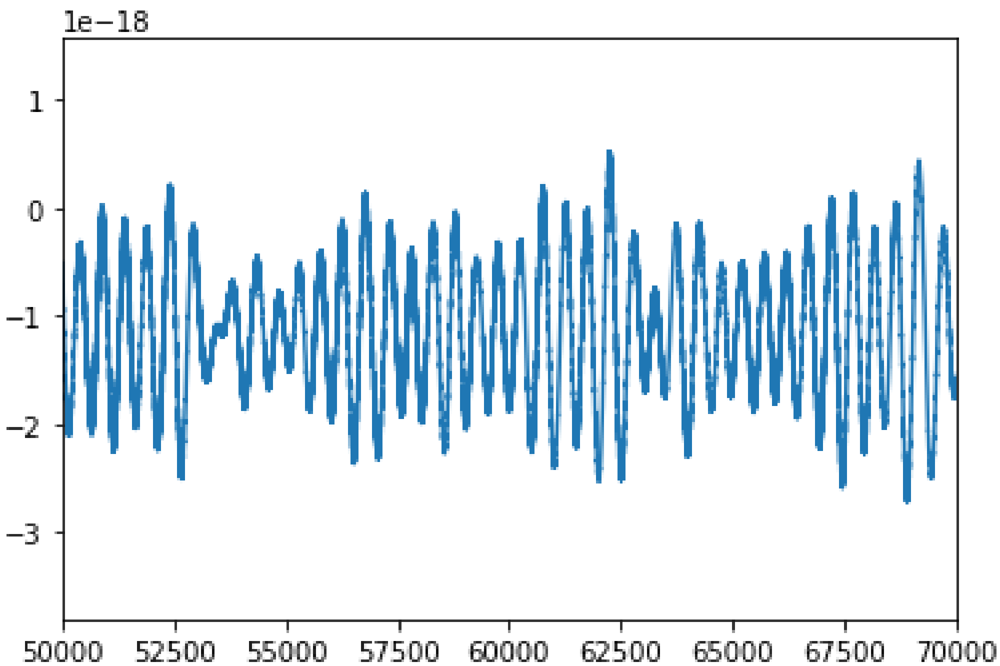

1e−19
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
50000
52500
55000
57500
60000
62500
65000
67500
70000


1e−19
6
4
2
0
−2
−4
−6
50000
52500
55000
57500
60000
62500
65000
67500
70000


1e−18
−0.5
−1.0
−1.5
−2.0
−2.5
50000
52500
55000
57500
60000
62500
65000
67500
70000


1e−18
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
50000
52500
55000
57500
60000
62500
65000
67500
70000


1e−18
2
1
0
−1
−2
−3
−4
50000
52500
55000
57500
60000
62500
65000
67500
70000


1e−18
1
0
−1
−2
−3
50000
52500
55000
57500
60000
62500
65000
67500
70000

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