Foundations DM. hw 1.

(1). Determine if a relation is a function.

Sol-n:

no b/c (yolid) maps to two outputs which contradicts the defn of a f-n

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(2). Determine if a f-n:

a). $y = x^2 + 1$

b) $y^2 = x + 1$.

Sol-n:

a). ~~depends on the domain.~~

suppose $f: \mathbb{R} \to \mathbb{R}$.

given $x \in \mathbb{R}$.

Then $y = x^2 + 1 \in \mathbb{R}$.

Suppose $\exists\ z = x^2 + 1 \in \mathbb{R}$ s.t. $y \neq z$.

but $x^2 + 1 = x^2 + 1$ contr-n.

hence $y = z$.

hence f - f-n.

b). Suppose $f: \mathbb{R} \to \mathbb{R}$.

suppose $y^2 = x + 1$ represents a function.

let $x = 3$.

Then, $y = 2$ and $y = -2$ satisfies $y^2 = x + 1$.

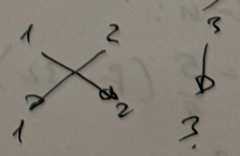
$\Rightarrow$ not a f-n.

(3) Which functions - onto?

1) $f: \mathbb{Z} \to \mathbb{Z}$, $f(n) = 3n$.

2) $g: \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$

3) $h: \{1, 2, 3\} \to \{1, 2, 3\}$ defined as



Sol-n:

1). let $y = 2$.

suppose $\exists\ x \in \mathbb{Z}$ s.t. $2 = 3x \Rightarrow x = \frac{2}{3} \notin \mathbb{Z} \Rightarrow \nexists x$ s.t. $f(x) = y$ for $y = 2$. $\Rightarrow$ not onto

2). not onto b/c $\nexists x \in \{1, 2, 3\}$ s.t. $f(x) = b$ for $b \in \{a, b, c\}$ $\Rightarrow$ not onto

3) onto.

4. $f(x) = \frac{1}{x+2}$, $g(x) = \frac{1}{x} - 2$.

$g = f^{-1}$?

Sol-n:

$f(x) = \frac{1}{x+2}$

$x f(x) + 2f(x) = 1$

$x = \frac{1 - 2f(x)}{f(x)} = \frac{1}{f(x)} - 2.$

$f^{-1}(x) = \frac{1}{x} - 2.$

yes.

5. $f(x) = 2 + \sqrt{x-4}$

f^{-1} - ?

Sol-n:

$f: D \to R$ $\quad x - 4 \geq 0, x \geq 4$

$D = [4, +\infty)$

$R = [2, +\infty)$

$f(x) = 2 + \sqrt{x-4}$

$f(x) - 2 = \sqrt{x-4}$

$x - 4 = (f(x) - 2)^2$

$x = (f(x) - 2)^2 + 4$

$f^{-1}(x) = (x-2)^2 + 4.$

domain of inverse: $[2, +\infty)$

6. $C = \frac{5}{9}(F - 32)$

find inv.?

Sol-n:

$C = \frac{5}{9}(F - 32)$

$\frac{9C}{5} + 32 = F.$

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7. $g(x) = 2\sqrt{x-4}$

D, R - ?

Sol-n:

$D: \{x \in R \mid x-4 \geq 0\} = \{x \in R \mid x \geq 4\} = [4, +\infty)$

$R = [0, +\infty)$

8. $h(x) = -2x^2 + 4x - 9$.

D, R - ?

Sol-n:

$h(x) = -2x^2 + 4x - 9 = -2\left(x^2 - 2x + \frac{9}{2}\right)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4 \cdot \frac{9}{2}}}{2} = \cancel{\frac{2 \pm \sqrt{-14}}{2}}$

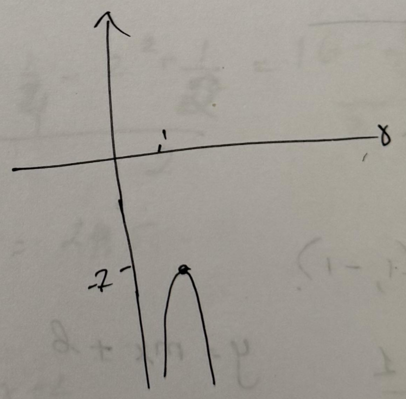
$D: x \in R$.

$-2x^2 + 4x - 9 = -2(x-1)^2 - 7$.

$x^2 - 2x + 1$

vertex $= (1, -7)$

$R: y \leq -7$.



9. $f(x) = \frac{x-4}{x^2 - 2x - 15}$

domain - ?

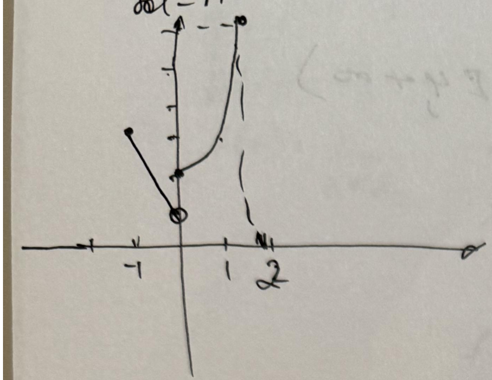
Sol-n:

$f(x) = \frac{x-4}{(x-5)(x+3)}$

domain: $x \in (-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

(10) $f(x)=\begin{cases} -2x+1 & -1 \le x < 0 \\ x^2+2 & 0 \le x \le 2. \end{cases}$

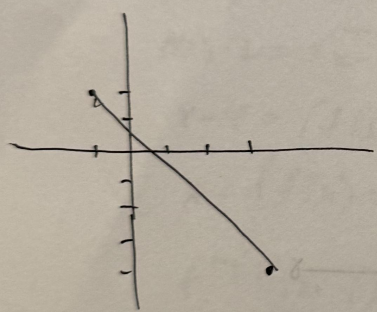
Sol-n:



(11). $(-1,2),\ (3,-4)$. slope?

Sol-n:

$m = \dfrac{-4-2}{3-(-1)} = \dfrac{-6}{4} = -\dfrac{3}{2}.$



(12) $m = \dfrac{3}{4}$. $(1,-1)$.

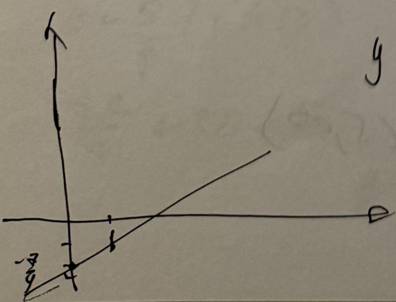
$\dfrac{3}{4} = \dfrac{y_2+1}{x_1-1}$

$y = mx + b$

$y = \dfrac{3}{4}x + b.$

$-1 = \dfrac{3}{4} + b \Rightarrow b = -1 - \dfrac{3}{4} = \dfrac{-7}{4}$

$y = -\dfrac{7}{4} + \dfrac{3}{4}x.$



(13). $g(t)$.
average rate of change on $[-1, 2]$.
sol-n:
parabola. vertex: $(1, 0)$.

$y = (x-h)^2 + k$.

$y = (x-1)^2$

$y' = 2(x-1) = 2x-2$.

average rate of change.

$\Delta x = 3$

$\Delta y = 1 - (4) = -3$.

$m = \frac{-3}{3} = -1$.

(14). $f(x) = x^2 - \frac{1}{x}$ on $[2, 4]$.

$m = \frac{f(4) - f(2)}{4-2} = \frac{4^2 - \frac{1}{4} - 2^2 + \frac{1}{2}}{2} = \frac{16 - \frac{1}{4} - 4 + \frac{1}{2}}{2} = \frac{12 + \frac{1}{4}}{2} =$

$= \frac{49 \cdot 2}{4} = \frac{49}{2} = 24,5$

(15). $f(t) = t^2 - t$, $h(x) = 3x + 2$
$f(h(1))$.
$h(1) = 5$
$f(5) = 20$.

(16). $(f \circ g)(x)$, $f(x) = \frac{5}{x-1}$, $g(x) = \frac{4}{3x-2}$.

domain -?

$f \circ g(x) = f(g(x)) = \dfrac{5}{\frac{4}{3x-2} - 1} = \dfrac{5(3x-2)}{4-3x+2} = \dfrac{5(3x-2)}{6-3x} = \dfrac{5(3x-2)}{-3(x-2)}$

domain: $(-\infty, 2) \cup (2, \infty)$

$g(x) = \dfrac{4}{3x-2}$, $x \neq \dfrac{2}{3}$.

~~$f: x \to y$, $g: y \to z$~~

~~$f: x \to z$.~~ ?

$(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, 2) \cup (2, \infty)$.

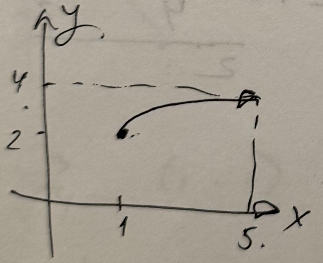
(17). $f(x) = x-1$, $g(x) = x^2-1$

$(g-f)(x) = x^2-1-(x-1) = \cancel{(x-1)^2-(x-1) = (x-1)(x-2)}$.

$= x^2 - x = x(x-1)$.

$\dfrac{g}{f}(x) = \dfrac{x^2-1}{x-1} = x+1$, $x \neq 1$.

(18).



$x = (y-2)^2 + 1$

$x = (y-2)^2 + 1$, $x \in [1, \infty)$
$y \in [2, \infty)$

$x - 1 = (y-2)^2$.

$\sqrt{x-1} = y-2$, $x \in [1, \infty)$

$y = \sqrt{x-1} + 2$, $x \in [1, \infty)$.

$y = \sqrt{x-1} + 2$, $x \geq 1$.

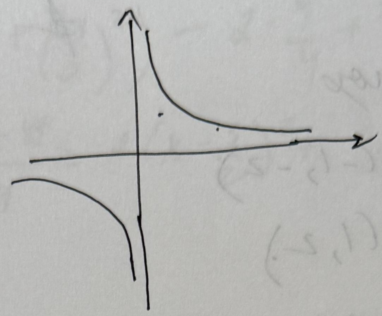
19. $f(x) = \frac{1}{x}$

one unit to the right, one unit up.

function - ?

Sol-n:

$g(x) = 1 + \frac{1}{x-1}$



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20. $f(x) = x^3 + 2x$.

$f(-x) = -x^3 - 2x = -(x^3 + 2x) = -f(x)$

$\Rightarrow$ odd.

21. $f(s) = s^4 + 3s^2 + 7$.

$f(-s) = f(s) \Rightarrow$ even.

22. point-slope form.

$(5,1), (8,7)$.

$m = \frac{7-1}{8-5} = \frac{6}{3} = 2$

$y - 1 = m(x-5)$

$y - 7 = m(x-8)$.

$y = mx + b = 2x + b$.

$1 = 10 + b \Rightarrow b = -9$

$7 = 16 + b$

$y = 2x - 9$.

#

23. $f(x)$ - linear.

$(3,-2), (8,1)$.

$m = \frac{1+2}{8-3} = \frac{3}{5} = 0{,}6$

$-2 = \frac{3}{5} \cdot 3 + b$

$b = -2 - \frac{9}{5} = \frac{-19}{5} = -\frac{38}{10} = -3{,}8$.

$y = 0{,}6x + (-3{,}8)$.

$y' = 0{,}6 > 0 \Rightarrow$ increasing

(24). abs. min : $(3, -10)$

abs. max: $(-2, 16)$, $(2, 16)$

(25). local min, max

local min: $(-1, -2)$

local max: $(1, 2)$

(26). $f(x) = 2x+3$, $h(x) = -2x+2$, $g(x) = \frac{1}{2}x - 4$, $j(x) = 2x - 6$.

Sol-n:

f, j - parallel

h, g - perp.

(27). $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

$y = 7-2x$

$x - 2(7-2x) = 6$

$5x - 14 = 6$

$5x = 20 \Rightarrow x = 4$

$y = 7 - 2x = -1$.

$(4, -1)$

(28). $\begin{cases} 4x+2y=4 \\ 6x-y=8. \end{cases}$

$\begin{cases} 4x+2y=4 \\ 12x-2y=16 \end{cases}$

$16x = 20$

$x = \frac{5}{4}$ $y = 6x - 8 = 6 \cdot \frac{5}{4} - 8 = \frac{15}{2} - 8 = -\frac{1}{2}$

$\left(\frac{5}{4}, -\frac{1}{2}\right)$

o mouse keyboard.

~~o apple core?~~

(29). $f(x) = 2x^2 - 6x + 7$.

$f(x) = \cancel{2(x^2 - 3x + 2) + 3 = 2(x-2)}$

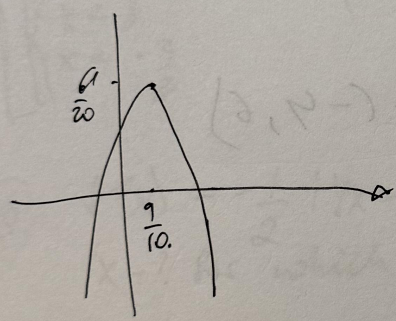
$= 2\left(x^2 - 2\cdot\frac{1}{2}\cdot 3x + \left(\frac{3}{2}\right)^2\right) - 2\cdot\frac{9}{4} + 7 =$

$= 2\left(x - \frac{3}{2}\right)^2 + \frac{28-18}{4} = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$. #

(30) $f(x) = -5x^2 + 9x - 1$.

$f(x) = -5\left(x^2 - 2\cdot\frac{1}{5}\cdot\frac{9}{2}x + \left(\frac{9}{10}\right)^2\right) + 5\left(\frac{9}{10}\right)^2 - 1 =$

$= -5\left(x - \frac{9}{10}\right)^2 + \frac{81}{20} - 1 = -5\left(x - \frac{9}{10}\right)^2 + \frac{61}{20}$.



domain: $x \in \mathbb{R}$

range: $x \leq \frac{61}{20}$

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(31). $f(x) = 3x^2 + 5x - 2$.

y-intercept: -2. $(f(0) = -2))$ $(0, -2)$

x-intercept: $0 = 3x^2 + 5x - 2$

$x = \frac{-5 \pm \sqrt{25 + 4\cdot 6}}{6} = \frac{-5 \pm 7}{6} = \begin{cases} -2 \\ \frac{1}{3} \end{cases}$.

$(-2, 0), \left(\frac{1}{3}, 0\right)$.

(32) a) $-1 \le 2x-5 < 7$

b) $x^2+7x+10<0$

c) $-6 < x-2 < 4$.

Sol-n:

a). $\begin{cases} 2x-5<7 \\ 2x-5 \ge -1 \end{cases} \Rightarrow \begin{cases} x<6. \\ 2x \ge 4 \Rightarrow x \ge 2. \end{cases} \Rightarrow x \in [2,6)$

b) $x^2+7x+10<0$

$(x+2)(x+5)<0.$

$\left[\begin{array}{l} \begin{cases} x<-2 \\ x>-5 \end{cases} \\ \begin{cases} x>-2 \\ x<-5 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} x \in (-5,-2) \\ x \in \phi \end{array}\right. \Rightarrow x \in (-5,-2).$

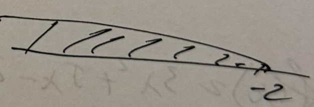
c). $\begin{cases} x-2<4 \\ x-2>-6 \end{cases} \Rightarrow \begin{cases} x<6 \\ x>-4 \end{cases} \quad x \in (-4,6)$

(33). $10-(2y+1) \le -4(3y+2)-3$

$9-2y \le -12y-11$

$10y \le -20.$

$y \le -2$



(34). $x(x+5)^2(x-4)<0$

$x(x-4)<0, \ x \ne -3$

$\left\{\begin{array}{l} \left[\begin{array}{l} \begin{cases} x<0 \\ x-4>0 \end{cases} \\ \begin{cases} x>0 \\ x-4<0 \end{cases} \end{array}\right. \\ x \ne -3 \end{array}\right. \quad \left\{\begin{array}{l} \left[\begin{array}{l} x \in \phi \\ x \in (0,4) \end{array}\right. \\ x \ne -3 \end{array}\right. \Rightarrow \quad x \in (0,4)$

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35. $2x^4 > 3x^3 + 9x^2$.

$2x^4 - 3x^3 - 9x^2 > 0$

$x^2(2x^2 - 3x - 9) > 0$

$\begin{cases} x \neq 0 \\ 2x^2 - 3x - 9 > 0 \end{cases}$ $\quad x = \frac{3 \pm \sqrt{9 + 4 \cdot 18}}{4} = \frac{3 \pm \sqrt{81}}{4} = \left[\begin{array}{l} 3 \\ -\frac{3}{2} \end{array}\right.$

$\begin{cases} x \neq 0 \\ (x-3)(x+\frac{3}{2}) > 0 \end{cases}$

$\begin{cases} x \neq 0 \\ \left[\begin{array}{l} \begin{cases} x > 3 \\ x > -\frac{3}{2} \end{cases} \\ \begin{cases} x < 3 \\ x < -\frac{3}{2} \end{cases} \end{array}\right. \end{cases} \Rightarrow \begin{cases} x \neq 0 \\ \left[\begin{array}{l} x > 3 \\ x < -\frac{3}{2} \end{array}\right. \end{cases} \Rightarrow \quad x \in (-\infty, -\frac{3}{2}) \cup (3, \infty)$

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36. $f(x) = -\frac{1}{2}|4x-5| + 3$.

x - ? for which $f(x) < 0$.

$-\frac{1}{2}|4x-5| + 3 < 0$

$\frac{1}{2}|4x-5| > 3$

$|4x-5| > 6$.

$\left[\begin{array}{l} 4x-5 > 6, \ 4x-5 \geq 0. \\ -4x+5 > 6, \ 4x-5 < 0 \end{array}\right.$

$\left[\begin{array}{l} x > \frac{11}{4}, \ x \geq \frac{5}{4} \\ x < -\frac{1}{4}, \ x < \frac{5}{4} \end{array}\right.$ $\quad x \in (-\infty, -\frac{1}{4}) \cup (\frac{11}{4}, \infty)$

(137). $13 - 2|4x-7| \leq 3$

$2|4x-7| \geq 10.$

$|4x-7| \geq 5$

$$\left[\begin{array}{l} 4x-7 \geq 5,\ 4x-7 \geq 0 \\ -4x+7 \geq 5,\ 4x-7 < 0 \end{array}\right.$$

$$\left\{\begin{array}{l} x \geq 3\ ,\ x \geq \frac{7}{4}. \\ x \leq \frac{1}{2} = \frac{2}{4},\ x < \frac{7}{4} \end{array}\right. \Rightarrow \left[\begin{array}{l} x \geq 3 \\ x \leq \frac{2}{4} = \frac{1}{2} \end{array}\right. \quad x \in (-\infty, \frac{1}{2}] \cup [3, \infty)$$

(138) – ?

wrong # in somewhere, solved all.