Indian Institute of Technology Hyderabad

FWC22025

ASSIGNMENT-MATRICES

Contents

1 Problem

If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that

ar(EFGH) $=\frac{1}{2}$ ar(ABCD)

2 Solution

1. Construct a parallelogram with vertices A,B,C and D.
2. Point mid-points E,F,G and H on sides AB,BC,CD and DA.

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{B}}{2}$$
$$\mathbf{F} = \frac{\mathbf{B}+\mathbf{C}}{2}$$
$$\mathbf{G} = \frac{\mathbf{C}+\mathbf{D}}{2}$$
$$\mathbf{H} = \frac{\mathbf{D}+\mathbf{A}}{2}$$

3. By joining the midpoints of adjacent sides of parallelogram ABCD, another parallelogram EFGH is formed.
4. Join EG.
5. Now draw a perpendicular from point H to line EG and mark as point P,similarly from point F to line EG and mark as point Q.

Then,

$$\mathbf{ar(AEGD)} = \|\mathbf{(E\text{-}G)(H\text{-}P)}\| - (1)$$
$$\mathbf{ar(EHG)} = \frac{1}{2}\ \|\mathbf{(E\text{-}G)(H\text{-}P)}\| - (2)$$

Similarly,

$$\mathbf{ar(BEGC)} = \|\mathbf{(E\text{-}G)(F\text{-}Q)}\| - (3)$$
$$\mathbf{ar(EFG)} = \frac{1}{2}\ \|\mathbf{(E\text{-}G)(F\text{-}Q)}\| - (4)$$

From (1) and (2),

$$\mathbf{ar(EHG)} = \frac{1}{2}\ \mathbf{ar(AEGD)}\ -(5)$$

From (3) and (4),

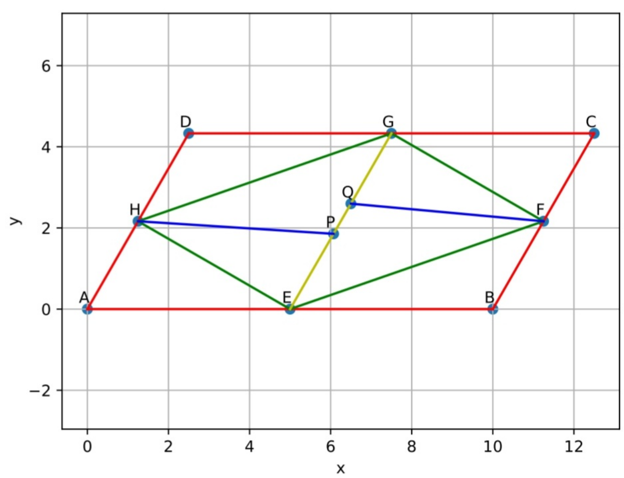
$$\mathbf{ar(EFG)} = \frac{1}{2}\ \mathbf{ar(BEGC)}\ -(6)$$

From (5) and (6),

$$\mathbf{ar(EHG)+ar(EFG)} = \frac{1}{2}\ \mathbf{ar(AEGD)}+\frac{1}{2}\ \mathbf{ar(BEGC)}$$

Hence,

$$\mathbf{ar(EFGH)} = \frac{1}{2}\ \mathbf{ar(ABCD)}$$



Figure

3 Construction

The parallelogram is constructed with m=10 and n=5,

Symbol	**Co-ordinates**	**Description**
m	10	AB
n	5	AD
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	point vector A
B	$\begin{pmatrix} m \\ 0 \end{pmatrix}$	point vector B
D	$\begin{pmatrix} 0 \\ n \end{pmatrix}$	point vector D
C	B+D	point vector C

The figure above is generated using python code provided in the below source code link.

https://github.com/madind5668/FWC/blob/main/assigment-4/codes/main.py