

OPTIMIZATION-BASIC

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1 PROBLEM

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q, costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A,B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having minimum cost per bag? What is the minimum cost of the mixture per bag?

2 SOLUTION

Let x and y be the number of bags of brand P and Q respectively. Obviously $x \geq 0, y \geq 0$. Mathematical formulation of the given problem is as follows:

$$\text{Minimize } Z = \min_{x,y} 250x + 200y (\text{Cost per bag}) \quad (1)$$

Subject to the constraints:
constraint on element A

$$3x + 1.5y \geq 18 \implies 2x + y \geq 12 \quad (2)$$

constraint on element B

$$2.5x + 11.25y \geq 45 \implies 2x + 9y \geq 36 \quad (3)$$

constraint on element C

$$2x + 3y \geq 24 \quad (4)$$

which can be expressed in vector form as

$$Z = \min_{\mathbf{x}} (250 \ 200) \mathbf{x} \quad (5)$$

$$\begin{pmatrix} 2 & 1 \\ 2 & 3 \\ 2 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 24 \\ 36 \end{pmatrix} \quad (6)$$

Solving using cvxpy, we get

$$Z_{\min} = 1950 \quad (7)$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad (8)$$

Hence, the minimum cost is $Z=1950$ which occurs at $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

Thus, the farmer should produce a mixture of cattle feed with 3 bags of brand P and 6 bags of brand Q to have minimum cost per bag.

<https://github.com/madind5668/FWC/blob/main/optimization/basic/codes/main.py>