

Aggregate Form Suite

$$1) b \leq \#count\{ Y : f(Y) \}$$

Since $b \leq \#count\{ Y : f(Y) \}$ is inherently disjoint to $\#count\{ Y : f(Y) \} < b$, we can instead express the count aggregate in a negated form:

$$2) \text{ not } \#count\{ Y : f(Y) \} < b$$

Note that for the rewriting to be valid, there must be no occurrence of the counting variable (Y) in the rule outside the counting functions/comparisons. Therefore the occurrence of the variable Y is equivalent to the '_' anonymous variable since any valid value for that location satisfies the criterion of Y.

This gives us the following aggregate forms:

$$3) b \leq \#count\{ f(_) \}$$

$$4) \text{ not } \#count\{ f(_) \} < b$$

ASP competitors repeatedly use count aggregates of the form

$$b \leq \#count\{ Y : f(Y) \} \leq b$$

$$b = \#count\{ Y : f(Y) \} \quad (\text{syntactically equal})$$

We have empirical evidence from the Spring indicating that this form solves significantly faster than the others. Thus...

The expression

$$b \leq \#count\{ Y : f(Y) \}$$

is equivalent to the expansion

$$\begin{aligned} & (b = \#count\{ Y : f(Y) \}) \quad \vee \\ & (b + 1 = \#count\{ Y : f(Y) \}) \quad \vee \\ & (\dots = \#count\{ Y : f(Y) \}) \quad \vee \\ & (\text{infinity} = \#count\{ Y : f(Y) \}) \end{aligned}$$

This is of course an absurdly (infinitely) large implementation, so we instead expand the negated form of the count aggregate:

So we instead use the expression

$$\text{not } \#count\{ Y : f(Y) \} < b$$

which is equivalent to the expansion

$$\begin{aligned} & \text{not } (\\ & \quad (b - 1 = \#count\{ Y : f(Y) \}) \vee \\ & \quad (\dots = \#count\{ Y : f(Y) \}) \vee \\ & \quad (0 = \#count\{ Y : f(Y) \}) \\ &) \end{aligned}$$

by deMorgan's Law we have an equivalence to

$$\begin{aligned} & (\text{not } b - 1 = \#count\{ Y : f(Y) \}) \wedge \\ & (\text{not } \dots = \#count\{ Y : f(Y) \}) \wedge \\ & (\text{not } 0 = \#count\{ Y : f(Y) \}) \end{aligned}$$

The most common usage of this rewriting is $b = 2$.
Giving us

$H :- f(Y), f(Y'), Y < Y', B.$

equivalent to

$H :- \text{not } 1 = \#count\{ Y: f(Y) \},$
 $\text{not } 0 = \#count\{ Y: f(Y) \}, B.$

where H represents the head of the rule and B represents the rest of the body. H and B must have no occurrence of Y or Y' .

This would give us the general forms:

5) $\text{not } b - 1 = \#count\{ Y : f(Y) \},$

\dots
 $\text{not } 0 = \#count\{ Y : f(Y) \}$

6) $\text{not } b - 1 = \#count\{ f(_) \},$

\dots
 $\text{not } 0 = \#count\{ f(_) \}$