Binomial Asset Pricing Model

The binomial asset pricing model is often used to price derivative securities for both stocks and bonds in the risk-neutral world (Beth). This model was introduced by Cox, Ross and Rubenstein in 1979 (Kim, S.). The binomial model is desirable because it is possible to check at any point in an option's life the possibility of early exercise (Michailidis). The model also does not depend on certain outcome probabilities, meaning the model is independent of investors that have subjective probabilities about an upward/downward movement in the underlying asset (Michailidis). Derivative securities are a financial agreement in which a buyer is given the right to purchase assets at a predetermined price, which can be exercised at any point before the expiration of the security (Beth). They are named as such due to the idea that derivatives themself have no inherent value: its value is derived from some other underlying asset (Odegbile). A security specifically is a "negotiable financial instrument that represents a type of financial value" (Niemiec). Traders can include both speculators and financial managers. Speculators are typically attracted to the options market because of the potential for high profits (Kim). Financial managers typically participate in the options market because it requires less capital than the stock market, and they can hedge risk in their portfolios (Kim). Options analysis in this sense "allows one to make better investment decisions because it can incorporate the value of flexibility of an investment into the initial evaluation of that investment" (Michailidis). Risk-neutral refers to the idea that "an investor is indifferent between a certain return and an uncertain return with the same expected value" (Odegbile). In this state, investors require no compensation for bearing risk, and as a result, the expected return on all securities is the risk-free interest rate (Odegbile). Interest rate is a "quantified property of the money market that yields 1

+ *r* dollars at a time for one dollar invested in the money market at time 0" (Niemiec). The binomial model follows a discrete binomial distribution where a random variable *X* has a binomial distribution with parameters *n* and *p* if *X* has a discrete distribution for which the probability function is

$$f(x|n,p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & \text{for } x = 0, 1, 2, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

where *n* is some positive integer, and *p* is between 0 and 1 (Beth). *p* is referred to as the risk-neutral probability (Odegbile). In regards to the Binomial Asset Pricing Model, there are two mutually exclusive outcomes: upstate or upward movement, meaning the given variable rises in value, or a downstate or downward movement, meaning the given variable drops in value. To denote the factor by which an asset rises, we use *u*, and to denote the factor by which an asset drops in value, we use *d* (Beth). Additionally, the Random Walk Theory has been generally applied to the Binomial Asset Pricing Model, meaning that differences in stock price over time have an identical price distribution and are self-reliant on each other (Wu). Each trial is referred to as a period, and we assume these periods are independent of one another (Beth). The Binomial Asset Pricing Model is "simple" due to the fact that the number of nodes of options increases linearly with time (Ho).

The development of this model has been greatly shaped by the development of financial and mathematical methodologies. Key influences include Bernoulli's contributions as well as Arrow's, which showed that "by using the temporal structure of the economy, equilibrium can be attained with a more limited number of markets" (Dimson). This is more commonly known as Arrow's theory of general equilibrium with incomplete asset markets. His claims provide the constitutional framework for asset pricing. Markowitz also revolutionized the ways investors view assets when he demonstrated that “the portfolio with maximum expected return is not necessarily the one with minimum variance. There is a rate at which the investor can gain expected return by taking on variance, or reduce variance by giving up expected return” (Dimson). He also points out that “in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among themselves” (Dimson). Tobin then takes Markowitz’s analysis one step further and shows how to identify which portfolio should be held by an investor, and considers how an investor should distribute their funds between safe liquid assets and risky assets (Dimson). Sharpe then determined that stocks are likely to co-move in the market (Dimson). Finally, an assortment of scholars continued to develop asset pricing models throughout the 20th century until 1973, when Black and Scholes modeled the distinction between American and European options with a closed-form solution (Dimson). This model and the Binomial Asset Pricing Model are the most commonly known and utilized option pricing models. To compare each of them, the Binomial Asset Pricing Model is a simple statistical method, and the Black Scholes Model “requires a solution of a stochastic differential equation” (Ahmad Dar). The assumptions of the Black Scholes Model are that the assets follow a lognormal distribution, there are no taxes and the risk-free rate of interest, r, is constant (Ahmad Dar). The model also assumes that the volatility (σ) of the underlying asset is constant and known and uses risk neutral possibilities (p) rather than subjective probabilities (Michailidis). Volatility can otherwise be defined as the “instantaneous standard deviation” of the asset, hence why it is denoted by σ (Michailidis). The Black Scholes Model is used for pricing European options and follows the format:

$$c = S_t N(d_1) - Ke^{-rT} N(d_2) - - - - [7]$$
$$p = Ke^{-rT} N(-d_2) - S_t N(-d_1) - - - - [8]$$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

When conducting a paired t-test between the Binomial Model and the Black Scholes Model, it was found that there is no significant difference between the asset price calculated by each respective model (Ahmad Dar). In fact, when using both models to compute the price of a put option, the results were perfectly correlated (Ahmad Dar). A Tukey-pairwise comparison reached the same conclusion (Ahmad Dar). It is generally said that the binomial model requires a less intensive mathematical background and skill set in comparison to the Black-Scholes model (Michailidis).

A key element of the binomial model is assuming there is no arbitrage: there is no possibility to begin with zero wealth, have a zero-possibility of losing money, and positive probability of earning money through investments (Beth). That being said, in order to rule out arbitrage, we assume $0 < d < 1 + r < u$ where r represents the fixed interest rate of the money market (Beth). The inequality ensures that even in the downstate, the value of a stock price remains positive, rules out arbitrage, and gives incentive for asset investment (Beth). To break this down further, the first part of the inequality, $0 < d$ specifically ensures that the value of a stock price remains positive, regardless of the factor of the downstate. Additionally, $d < 1 + r$ rules out arbitrage. If on the other hand we assume $d > 1 + r$, there is a possibility to begin with no initial wealth and to borrow an initial amount x to invest in, and in the event of a failure, the downstate price, xd, would be larger than the amount owed to the bank, $x(1 + r)$. This would lead to zero possibility of losing money because we would keep a profit after repaying the loan including interest (Beth). Lastly, $1 + r < u$ incentivizes asset investment. If this inequality were not the case, the money market would yield greater returns with less risk (Beth). Determining the potential price of an asset is necessary due to the fact that short selling is a common practice: “an investor borrows the stock, sells it, and uses the proceed to make some other investments and then repurchase the stock and return it to the owner with any dividends due” (Odegbile).

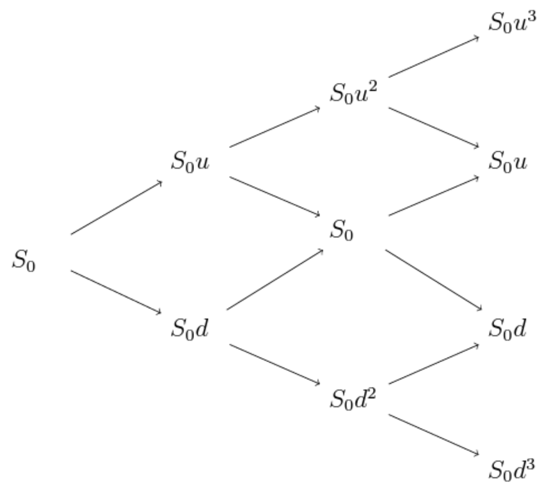
As discussed earlier, a derivative is some financial agreement that has value dependent on some agreed upon asset (Beth). The payoff and price of a derivative depends on the type of security and differs for the call-options. A call option specifically is an investor as a buyer, and a put option is an investor from the perspective of a seller (Niemiec). In this model explanation, the frame will be from the perspective of an investor looking to buy (call-option). An option gives its owner the choice to trade a given risky asset (stock) at a certain price by a certain date (Odegbile). Generally, the price of a derivative, V_0, is recursively based on the expected payoff, V_t, of the security where t is the future time period (Beth). Pricing derivatives requires the use of “risk-neutral measures” (Beth). These are similar to traditional probability measures but are adjusted for the risk taken on when purchasing assets (Beth). This enables us to price derivatives with consideration to a buyer’s risk aversion and aids in pricing derivatives for stock options and bond securities (Beth).

Many call options can be modeled with the Binomial Asset Pricing Model. Call options are specifically when the holder’s right is to buy (Odegbile). However, the holder is not obliged to exercise the right at the option’s maturity; it is only done so if considered economically advantageous (Odegbile). American options can be exercised at any point up to maturity, while European options can only be acted on at maturity (Odegbile). Note that this distinction is not geographical. The intrinsic value of an option is the value of an option if it were to be exercised immediately; however, the time value of an option is the value an option can take if it is left to mature (Odegbile). Bearing that in mind, the intrinsic value is the minimum that an option will be worth at any given time, and the total price of an option with time left to maturity consists of its intrinsic value in addition to its time value.

The first and most intuitive model is a Single Period Binomial Model. This begins with an initial stock price, S_0, and takes on the value of the upstate or downstate depending on the outcome of a trial (Beth). This trial is represented with a coin-toss. The outcome of the coin-toss is random, and each outcome is mutually exclusive (Beth). If the toss lands on heads, the price of the upstate, $S_1(H)$, is the product of the original price and the up-factor. In other words, $S_1(H) = uS_0$ (Beth). The downstate can be modeled similarly. The call option can be bought for a price, V_0, and allows the buyer to purchase stock for an agreed upon stock price, K. At any time before the expiration of this call option, this price may be exercised for a payoff, V_t (Beth). That being said, for a single period model, the payoff of the option can be represented by the maximum of $S_1 - K$ (Beth). If $S_1 > K$, the option is "in the money" and can be exercised for a profit at time t. If $S_1 < K$, the option is "out of money" and cannot be immediately exercised for profit (Beth). Under this model, an agent's initial wealth, X_o, is held either in stocks, the money market, or a combination of both. The buyer invests in Δ_0 shares of stock at time 0 for a stock price of S_0. Bearing that in mind, there is a total of $\Delta_0 S_0$ invested in stocks at time 0 which will generate a profit of $\Delta_0 S_1$ at time 1 (Beth). Δ_0 is referred to as the hedge ratio or delta (Odegbile). Hedging as a financial method is the overall attempt to minimize risk by investing in a way that controls market fluctuations (Niemiec). This method has caused considerable increases in derivative investments (Wu). The remaining money, $X_0 - \Delta_0 S_0$ is invested into the money market and makes a return of $1 + r$. Therefore, the wealth of the agent at time 1 can be represented by $X_1 = \Delta_0 S_1 + (1+r)(X_0 - \Delta_0 S_0) I$ (Beth). The goal when pricing an option is to choose an initial stock investment and wealth such that $X_1(H)=V_1(H)$ and $X_1(T)=V_1(T)$ in order to price our call option (Beth). Bearing this in mind, after completing the derivation and utilizing past assumptions of the model, given our initial investment totals X_0 and our investment into stock totals Δ_0, the portfolio is worth either $V_1(H)$ or $V_1(T)$. This results in the pricing of the derivative security at

$$V_0 = \frac{1}{1+r}[\tilde{p}V_1(H) + \tilde{q}V_1(T)].$$, where $\tilde{p} = \frac{1+r-d}{u-d}$ and $\tilde{q} = \frac{u-1-r}{u-d}$.

Moving onto a multi-period model, the single period model must be extended to include multiple finite periods. The Binomial Asset Pricing Model assumes that prices can only move up and down at each step by predetermined amounts, and this may seem unrealistic at first glance. However, it is especially important when moving to a multi-period model to remember that this assumption remains true due to the fact that steps remain very small and are combined together (Odegbile). This is what makes the overall presumption reasonable, along with the idea that the model converges and can therefore be estimated in this fashion. The assumption of convergence is dependent on the corollary that states: "the payoff and value of a twice-exercisable option converge to their continuous limits as *n* increases" (Ho). The multi-period model can be visualized with a tree model that shows the possible outcomes.



This is often referred to as a tree or a lattice structure (Kim). To make the model more flexible, a "tilt parameter" can be introduced to the tree which recalibrates nodes relative to the strike price or barrier of an option (Tian). However, this is not commonly used and is a recent adaptation. The tree represents all possible paths an asset value could take during the life of the option, and at the expiration of the option, "all the terminal values for each of the final possible asset values are known, as they simply equal their intrinsic values" (Michailidis). As a result, the value of the option at each step is calculated working from the expiration to the beginning timestamp. Because the model follows a binomial distribution, the stock price can adhere to 2^n possible paths throughout the model, where n is the number of periods (Beth). The wealth equation for a general time *n* is given by $X_n = \Delta_{n-1}S_n + (1 + r)(X_{n-1} - \Delta_{n-1}S_{n-1})$, and the payoff of a call-option at time *n* is given by the maximum of $S_n - K$ (Beth). Lastly, the price of a call option a time *n* based on the expected payoff of the option in the time $n + 1$ is

$$V_n(\omega_1\omega_2 \ldots \omega_n) = \frac{1}{1+r}[\widetilde{p}V_{n+1}(\omega_1\omega_2 \ldots \omega_n H) + \widetilde{q}V_{n+1}(\omega_1\omega_2 \ldots \omega_n T)],$$

where the sequence $\omega_1, \omega_2 \ldots \omega_n$ denotes the result of the first n coin tosses (Beth). The theorem that governs this pricing model states: "Let V_n be a random variable derivative security paying off at time N, which depends on the first N coin tosses $\omega_1,\omega_2...\omega_n$. Define a sequence of random variables $V_{n-1},V_{n-2}...V_0$ recursively by the equation above so that each V_n depends on the first n coin tosses $\omega_1,\omega_2...\omega_n$ where n falls in the interval [0, N-1]". Next, define the delta-hedging formula to be

$$\Delta_n(\omega_1\omega_2\ldots\omega_n) = \frac{V_{n+1}(\omega_1\omega_2...\omega_n H)-V_{n+1}(\omega_1\omega_2...\omega_n T)}{S_{n+1}(\omega_1\omega_2...\omega_n H)-S_{n+1}(\omega_1\omega_2...\omega_n T)}.$$

If we set $X_0 = V_0$, then we will have that $X_n(\omega_1,\omega_2...\omega_n)=V_n(\omega_1,\omega_2...\omega_n)$ for all sequences $\omega_1,\omega_2...\omega_n$" (Beth). This theorem can be proven by induction, with case 1 being that ω_{n+1} = H and case 2 being that ω_{n+1} = T. As a result, we see that $X_{n+1}(\omega_1,\omega_2...\omega_{nn+1}) = V_{n+1}(\omega_1,\omega_2...\omega_n\omega_{n+1})$ (Beth). In order to solve a multi-step model numerically, oftentimes a Monte Carlo simulation is used which is based on the Law of Large numbers, meaning that as sample size *n* approaches infinity, the sample mean approaches its expectation (Wu).However, because Monte Carlo simulation is computationally intensive, it is most often reserved for research purposes (Kim). In practical applications, we must approximate the stochastic process by using small values for *n* (Ho). This allows for more rapid computation for option prices (Ho). In all cases, accuracy of the model increases as *n* increases.

Another application of the Binomial Asset Pricing Model entails utilizing real-world probabilities rather than risk neutral probabilities. This allows for three benefits: direct inference, practitioners using the model at corporations avoid anxiety that comes with using risk-neutral probabilities on inherently risky cash flows, and the model simplifies pricing when skewness and kurtosis appear (Arnold). This model also aims to safeguard against conceptual disparities that arise when computing risk-neutral probabilities: "it is difficult to understand why we need event probabilities from an event economy that does not compensate risk bearing [in risk-neutral models], even though we are pricing assets from a real-world economy that does compensate risk bearing" (Arnold). When developing a real-world probability Binomial Asset Pricing Model, a continuous time model requires a "stochastic risk-adjusted discount rate" (Arnold). A discretized version of the original Black-Scholes instantaneous Continuous Asset Pricing Model derivation allows for changing risk-adjusted discount rates. Discretization makes it possible to infer real-world parameters (Arnold). In this model, S_0 and S_T are the asset price at time $t = 0$ (Arnold). Assume the asset pays no dividends, and R_S, the ratio of S_T to S_0, is the total discretely-compounded return on the asset from time 0 to time T (Arnold). R_F is the total risk-free rate from time 0 to time T. Let r_F be the annualized continuously-compounded risk-free rate, and k_S be the annualized continuously-compounded expected return on the stock (Arnold). Using the definitions, the one-period option pricing formula can be defined by

$$\begin{aligned} V_0 &= \frac{1}{R_F}\left[E(V_T) - \left(\frac{V_u - V_d}{u - d}\right)(E(R_S) - R_F)\right] \\ V_0 &= e^{-r_F T}\left[E(V_T) - \left(\frac{V_u - V_d}{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)\left(e^{k_S T} - e^{r_F T}\right)\right] \end{aligned}$$

where E is the expected value. Although these equations entail discounting at the risk-free rate, they are not risk-neutral pricing. There is no change of probability measure, and the expected cash flow (E(V_T)) is in the real-world, not a risk-neutral world (Arnold). This risk-adjusted cash flow is the real-world expected cash flow minus a risk premium (Arnold). To apply the generalized one-period option pricing model in an iterative manner to create a multi-stage binomial tree, the real-world underlying security discount rate is used in place of the risk-free rate (Arnold). This allows the underlying security price to increase and decrease at a given stage in the binomial tree. Then, option prices can be calculated recursively. We let i be the number of upward price movements and j be the number of downward price movements. For stage (i,j) where $i+j$ is less than the terminal stage, the option price $V(i,j)$ follows the equation

$$\begin{aligned} V(i,j) &= e^{-r_F T}\Bigg\{ [pV(i+1,j) + (1-p)V(i,j+1)] \\ &- \left(\frac{V(i+1,j) - V(i,j+1)}{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)\left(e^{k_S T} - e^{r_F T}\right) \Bigg\} \end{aligned}$$

If the model were to be generated using the risk free rate, r_F, rather than the underlying security's discount rate, k_S, the model becomes risk-neutral (Arnold). This model is set to match European style options. To adapt this model to American-style options, much like the risk-neutral model, the maximum between the solution *V(i,j)* and the option's immediate exercise value must be taken (Arnold). This model can then give traders "real-time, real-world probabilities that individual American-style options will finish in the money" (Arnold). This model, for example, builds on "Bloomberg's BETA function to get the expected return" (Arnold). That being said, this model "allows parameter inference from the real-world probability density function of the underlying security," and "allows real-world statistical information (e.g. historical or forecast volatility) to be incorporated into option pricing" (Arnold). Lastly, the model allows stochastic parameters throughout the option's life (Arnold).

Another subset of financial methodology are fuzzy-stochastic models. Fuzzy refers to the level of uncertainty present in input data (Zmeškal). This would affect the probability *p* parameter in our model, and a fuzzy measure would be introduced instead. Fuzzy-stochastic methodologies pay particular attention to option valuation models such as the Binomial Asset Pricing Model due to the fact that there are such a wide breadth of possible applications (Zmeškal). Fuzzy stochastic models are generally only used to evaluate American options. All input parameters are given fuzzy volatility such as the up index, down index, risk-free rate, growth rate, initial underlying asset price, and exercise price as these parameters are often difficult to concretely define (Zmeškal). To summarize the creation of the fuzzy model, a fuzzy random variable is introduced that utilizes random intervals and fuzzy sets to define a range of possibilities for the inputs detailed earlier. The rest of the model functions the same; however, the outputs take into account that there may be slight perturbations in the input parameters that are fed to the model. “Fuzzy” parameters are marked with a tilde. These parameters are advantageous as they ultimately make the model more risk-comprehensive. Following model evaluation, when it is desirable to reduce the derivative price to a crisp value for decision purposes, “defuzzyfication” methods such as center of gravity area, first of maxima, last of maxima, mean of maxima, center of maxima (median), centroid method, or the bisector method can be performed (Zmeškal).

A Binomial Asset Pricing Model can also be extended into a Trinomial Asset Pricing Model using subordination (Chang). The classic trinomial pricing model is defined in the risk-neutral world and takes the following form:

$$S(\mathrm{E}^{(TR,t+\Delta t)},\ t+\Delta t) = S(\mathrm{E}^{(TR,t)},\ t)\begin{cases} 1+\frac{3}{2}\sigma^2\Delta t+\sigma\sqrt{3\Delta t}, & if\ \varepsilon^{(TR,t+\Delta t,\Delta t)}=1, \\ 1, & if\ \varepsilon^{(TR,t+\Delta t,\Delta t)}=0, \\ 1+\frac{3}{2}\sigma^2\Delta t-\sigma\sqrt{3\Delta t}, & if\ \varepsilon^{(TR,t+\Delta t,\Delta t)}=-1. \end{cases}$$

Trinomial tree matching can then be conducted to produce:

$$S(\mathrm{E}^{(U,t+\Delta t)},\ t+\Delta t) = S(\mathrm{E}^{(U,t)},\ t)\begin{cases} 1+\left(\mu+\frac{\sigma^2}{4}\right)\Delta t+\sqrt{\frac{3}{2}}\sigma\sqrt{\Delta t}, & if\ \varepsilon^{(U,t+\Delta t,\Delta t)}=1 \\ 1+\left(\mu-\frac{\sigma^2}{2}\right)\Delta t, & if\ \varepsilon^{(U,t+\Delta t,\Delta t)}=0 \\ 1+\left(\mu+\frac{\sigma^2}{4}\right)\Delta t-\sqrt{\frac{3}{2}}\sigma\sqrt{\Delta t}, & if\ \varepsilon^{(U,t+\Delta t,\Delta t)}=-1. \end{cases}$$

And finally, the corresponding risk-neutral tree has the form:

$$S^{\mathbf{Q}}(\mathrm{E}^{(U,t+\Delta t)},\ t+\Delta t) = S^{\mathbf{Q}}(\mathrm{E}^{(U,t)},\ t)\begin{cases} 1+\left(r+\frac{\sigma^2}{4}\right)\Delta t+\sqrt{\frac{3}{2}}\sigma\sqrt{\Delta t},\ \ w.\,p.\ \ q^{(u)}=\frac{1}{3} \\ 1+\left(r-\frac{\sigma^2}{2}\right)\Delta t,\ 1,\ \ w.\,p.\ \ q^{(n)}=\frac{1}{3} \\ 1+\left(r+\frac{\sigma^2}{4}\right)\Delta t-\sqrt{\frac{3}{2}}\sigma\sqrt{\Delta t}\ \ w.\,p.\ \ q^{(d)}=\frac{1}{3}\ . \end{cases}$$

Finally, the Binomial Asset Pricing Model can be extended to be multivariate. In this case, “the payoff on the option depends on the outcome of two or more random variables” (Ho). Multivariate analysis is also frequently used for American options considering that the intermediate value of the option depends on the underlying price of the asset and other variables such as interest rate, r (Ho). Bearing this in mind, we must model the covariance of the asset price and the interest rate, as well as the time series properties of each (Ho). Multi-Period and multivariate models are also possible. With that in mind, in a bivariate case (X, Y) for example, the up and down movements for each random variable must be computed (X_1, X_2, Y_1, Y_2), as well as the conditional probability of an increase in the second variable given an increase in the first ($Y_1|X_1$), the conditional probability of an increase in the first variable at the second time step ($X_2|X_1$), and finally, the conditional probability of the second variable at the second time stamp given its value at the first time and the value of the first variable at the second time stamp ($Y_2|Y_1,X_2$) (Ho). However, it is important to consider the limitations on accuracy when natural limits are placed on the conditional probabilities (Ho).

To summarize, option pricing using the Binomial Asset Pricing Model can be distinguished by the following steps:

- modeling an evolution of underlying asset in accordance with the observed volatility
- computation of intrinsic value (payoff function)
- at maturity day T, option price is equal to intrinsic value

- the American option can be exercised at any point during the pre-allocated time period, and the asset's price can be defined by the Bellman dynamic programming optimal equation (alternate form of the equation detailed above)

  $f_t = \max(g_t; F_t)$, where $F_t = (1+R)^{-dt} \cdot [f^u_{t+dt} \cdot (p) + f^d_{t+dt} \cdot (1-p)]$ and $p = \frac{\frac{1+R}{1+C} - D}{U - D}$.

There are also some guiding principles for investors looking to utilize the model. A seller is thought to be satisfied if they have a hedging portfolio that has enough value to pay off the options contract when it is exercised and enough value to match the value of the options contract at any given time (Niemiec). To pay off a call when exercised, the portfolio must be valued at the maximum of (S_n−K) (Niemiec). For a put, the portfolio should be valued at a maximum of (K-S_n). A binomial model is generally thought of as the benchmark for more complex models, particularly the American call option, while the standard for the European call option is the Black Scholes Model (Chance). The binomial model has many variations, and no particular form seems to dominate the rest. However, the necessary principles include prohibiting arbitrage for a finite number of time steps and recovering the correct volatility (Chance). The choice of the actual risk-neutral probability is meaningless when it comes to the limit of the function; however, a risk-neutral probability of ½ offers the fastest convergence (Chance). Bearing this in mind, the Binomial Asset Pricing Model is more correctly thought of as a family of models, rather than a single model, being that there are no less than 11 different binomial models that can accurately predict the correct asset price at an equal level of accuracy (Chance).

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