Muon Lifetime

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The procedure and experimental results are discussed for the detection or the inference of muon particles in a scintillation detector, timing the particle decays, muon arrival statistics and calculating the collective particle's mean lifetime. Results included an acceptable muon decay constant and findings about the nature of muon arrival statistics. Particle detection events made by the scintillation detector were found to accurately represent characteristics of muons.

I. INTRODUCTION

There is historical relevancy to this experiment, in that the experiment was originally performed by Rossi and Hall in 1941. The flux of muons were measured on top of Mt. Washington in New Hampshire around altitudes of 2000m and its associated mountain base. The ratio of flux was found to be around 1.4, but it should have been around 22. To explain this result, time dilation needed to be considered. Where the expected flux value was considered muons traveling even at the speed of light, time dilation of muons actually held the subatomic particles around .994c. Cosmic rays or protons traveling with very high energies are usually the source of muons.

The purpose of this experiment is a determination of muon lifetime. This principle result is due to the distribution of decay times among the detected muons. The relationship shows that decays are independent, and because of the proportionality of decay rate to number of muons present, the mean muon lifetime is obtainable. The following exponential fit is the weighted average of the muon lifetimes. The moving average logarithm of this graph yields a smooth linear fit that is inversely proportional to the Muon decay constant.

If, instead, the mean lifetime of muons were determined using the average of all time delays measured, then the resulting experimental values for the mean lifetime would be highly susceptible to systematic errors. This is because a triggering pulse in the photomultiplier tube (PMT) would allow any form of energetic radiation within a period of 20μ seconds to signify a muon decay pulse. There would be much more false-positives.

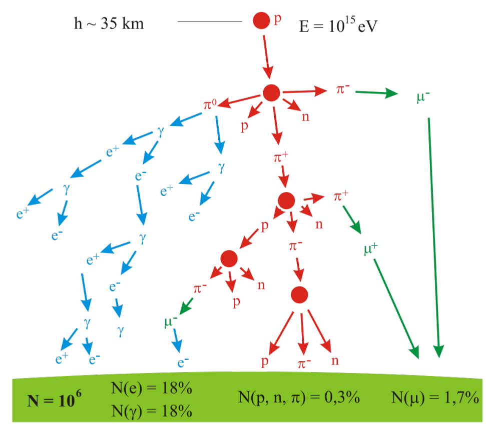

FIG. 1. A breakdown of different ionized particles and electromagnetic radiation as a result of a cosmic rays colliding with the upper atmosphere. Positive and negative muon particles are shown in green.

II. THEORY

A. Time Dilation Effect of Special Relativity

While the experimental apparatus used was not ideal for determining the effect of time dilation on the particles detected, the stopping rate as a function of altitude above sea level can be used to *somewhat* distinguish the differences between classical and quantum mechanical physics, but only if the experiment was repeated at two different altitudes. However, since this experiment was not performed in such a manner, relativistic time dilation could not be determined.

When cosmic ray protons impact with the upper atmosphere, pions are created that subsequently decay into muons and muon neutrinos. The lifetime of the muon would suggest attenuation after about 500m in the frame of the Earth without relativistic effects, but since the combined effects of time dilation and length contraction occur, muons may reach the surface of the Earth with relatively ease.[1] Time dilation is given by the following relationship

$$t = \frac{1}{\sqrt{[1 - \frac{v}{c}]}} t_o \qquad (2.1)$$

B. Cosmic Ray Background Radiation

The experimental limitations do present a fault in the investigative possibilities, but interestingly enough, by switching the multi-channel analyzer mode from pulse-height to multi-channel scalar, the data can be used to observe variations in cosmic radiation across space. This is because the particle's associated energies are no longer being sorted into bins based on their amplitudes, but rather by their arrival times.

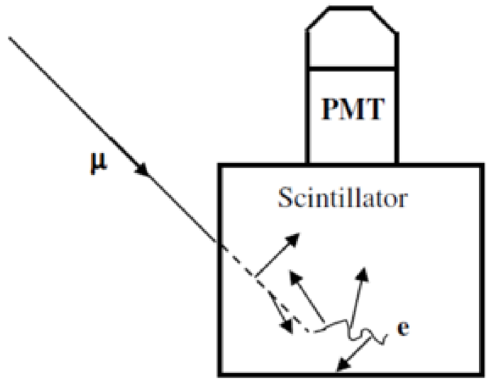

FIG. 2. Muons with less kinetic energy than the distance across the photomulitplier tube will stop and decay after some time t, ejecting electron-positron pair and neutrinos.

C. Muon Physics

Muons are subatomic particles that are very similar to electrons in nature, aside from their greater masses factoring in at about 200 times greater than the electron. Because of their sizes, muons are inherently unstable and will eventually decay into an electron/positron pair as well as two neutrinos.[2] This is shown by $\mu^- \rightarrow e^- \bar{\nu_e} \nu_u$ and $\mu^+ \rightarrow e^+ \bar{\nu_u} \nu_e$ Furthermore, consideration of the average lifetime and mass of the muon can be used to measure the strength of the weak interaction force.

The muon decay probability adheres to the form of exponential decay and its distribution is given by

$$\frac{-dN}{N_o} = \frac{e^{\frac{-t}{\tau}}}{\tau} \tag{2.2}$$

The number of resulting muons after some time t is the integration of (2.2) and it is given by

$$N(t) = N_o e^{\frac{-t}{\tau_\mu}} \tag{2.3}$$

where τ_μ is the mean muon lifetime and N_o is the initial number or otherwise known as a normalization factor.

The flux of muons at altitudes near sea level is expected to be roughly $\frac{1}{min} cm^2$. Muons that enter a scintillation detector will stop if their kinetic energy is less than the energy required to travel the remaining distance of the scintillation detector. Once the muons stop, it will decay into its constituents after some time.

D. Experimental Design Flaws

The abstract refers to the *inference* of muon particles instead of their absolute existence for a couple reasons. Firstly, there is no lead shielding used in this experiment to provide a quality control of stopping power, so there is a very real and likely possibility that the scintillation detector is picking up other subatomic particles and other various background radiation. The lead shielding would ideally be constructed at two different altitudes to attenuate most if not all particles different aside from muons. This filtering would be based on their relativistic and quantum characteristics. However, it turns out that the finite size of the PMT restricts the ranges of energies allowed for muons to stop inside. Additionally, the muon stopping rate can be determined from the number of muon decays in some time interval. This stopping rate is proportional to the muon density flow, or flow per unit area, with the same finite energies required to stop in the PMT. In order to extrapolate an indication of time dilation, one would have to capture data at a different altitude.

The dependency on muon determination is on the experimental apparatus to understand that a triggering scintillation pulse observing an event will only count as a muon if the timing threshold of the subsequent decay falls within the $\tau = 20\mu$s.

Secondly, only some statistical analysis may be applied as the data collected is an average. In addition to this, while arrival times may suggest muon-groups, it does not necessarily mean that the muon sources are geospatially the same. As a result of these experimental disadvantages, several physical determinations of the muon are unable to be obtained. As for the scintillation crystal, almost any type of energetic radiation will cause it to become florescent. It's in these cases that a voltage discriminator becomes extremely important to filter out unwanted data. However, undesirable background radiation still exists.

E. Apparatus

This experiment was performed in three stages in order to better understand data collection for actual muon particles. In the first stage, a function generator was used to produce a pulse signal down the length of a long coaxial cable. The pulses were effective in mimicking the pulse creation mechanism of the photomultiplier tube. The coaxial cable served well for its purpose, as electromagnetic waves propagate inside the length of the wire and produce a delay effect to the initial wave. Once a double pulse was achieved, the timing between pulses was configured to mimic muon lifetimes.

Next, the pulses were routed through the PMT and preamplifier to a light emitting diode, or LED. This was mainly done as an equipment check to verify everything was in working order. When similar pulse results to the first stage were obtained, the third stage was ready to be initialized.

At this point, the pulse/function generator was disconnected and routed through to the light-free box containing the scintillator. A few adjustments were made to ensure the time-to-amplitude converter's (TAC) pulse detection was in working order by increasing the PMT voltages. The TAC was a key component of the experiment, as it was how double pulses were detected within the muon lifetime range of 20μ seconds.

F. Results

This experiment required a few corrections in order to better achieve acceptable values of the mean lifetime. Background radiation counts were subtracted from the total number of detected events. The exponential best fit line to this data was the mean lifetime curve of both positive and negative muons. Since the two share the same lifetime within the scopes of this experiment, it was acceptable to use a weighted fit. Background radiation was determined using the equation

$$B = \Gamma^2 T \Delta t \tag{2.4}$$

Where Γ is the counting rate and T is the total run time over the course of 5 days (roughly 91 hours or 5460 minutes).

Equation (2.4) was utilized in order to ensure correct bin width times.[4] Solving for Δt is useful in this respect, and if the positive and negative muons differed in lifetime decay, it could be substituted into the following equation to solve for the first order number of counts per bin using the correction methods described above. There would be no need for second order corrections since the highest degree is first order.

$$N_{bin}(t) = B - \Delta t \frac{d}{dt} N_o \left(\frac{R_\mu e^{-\lambda t}}{1 + R_\mu} + \frac{e^{-(\lambda + \Lambda)t}}{1 + R_\mu} \right) \tag{2.5}$$

Where B is the number of background counts per bin, N_o is the number of muons at $t = 0$, Λ is the capture rate of μ^-, the negative muons that interact via the weak force with protons in the scintillation crystal, Δt is the time associated to each bin, R_μ is the ratio $\frac{N_{\mu^+}}{N_{\mu^-}}$

However, since both muons share the same lifetime within the scope of this experiment, an exponential best fit is instead used over equation (2.5).

Figure 3 was the raw data obtained from the 5 day continual experimental run. Notice the exponential decay, which was to be expected. It represents the lifetime of muons, but only once they've entered the scintillator. However, the survival process looks the same either way.

Figure 4 substitutes the channel numbers for the appropriate calibrated values of decay times. The logarithm was obtained to show a relatively even distribution of randomized events over and below the line.

Figure 5 accounts for the experimental muon counts by subtracting the average number of background counts per bin. It's at this point where large statistical uncertainties begin to show in the logarithm of the original curve.

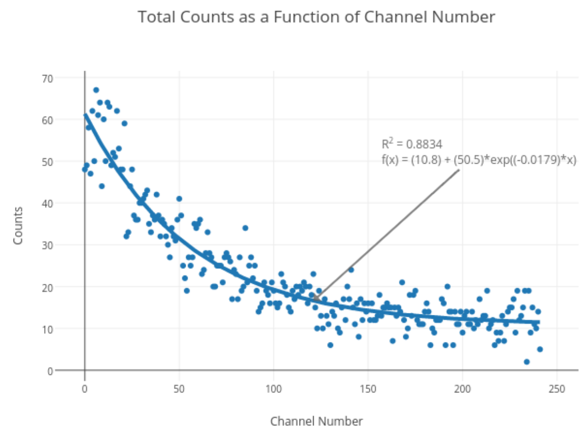

FIG. 3. Number of events as a function of channel number. Background counts are included.

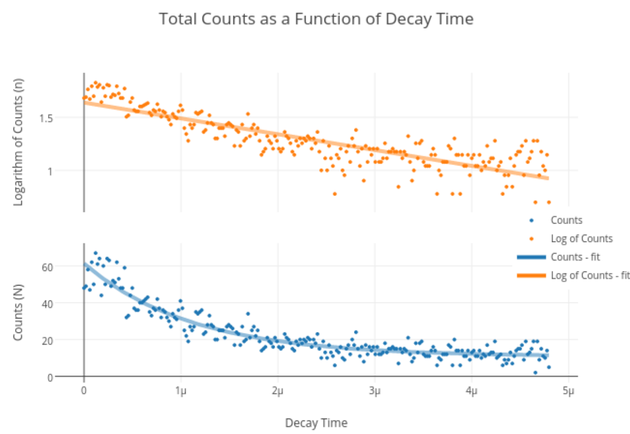

FIG. 4. Total events as a function of decay time. Background counts included.

Figures 6-8 deal with increased interval moving averages to smooth out the data. Results were gathered from the fourth moving average.

Results are tabulated as follows:

The mean muon lifetime was determined from the inverse of the slope generated by the logarithm of the 4th interval moving average of muon counts. The slope was $-4.5x10^5 \pm 1.5x10^4$ which yielded an experimental value of the mean muon lifetime once the inverse was applied.

Counts	Average Value	Uncertainty	Std Error
Total	22.4	13.7	.9
Muon	19.4	13.7	.9

TABLE I. Results of detected events.

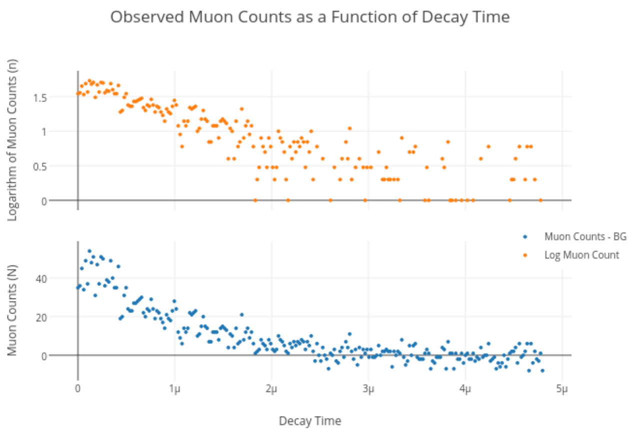

FIG. 5. Observed muon counts determined by removing the average background counts per bin. Decay Time is substituted in place of channel numbers, which was determined using a calibration equation.

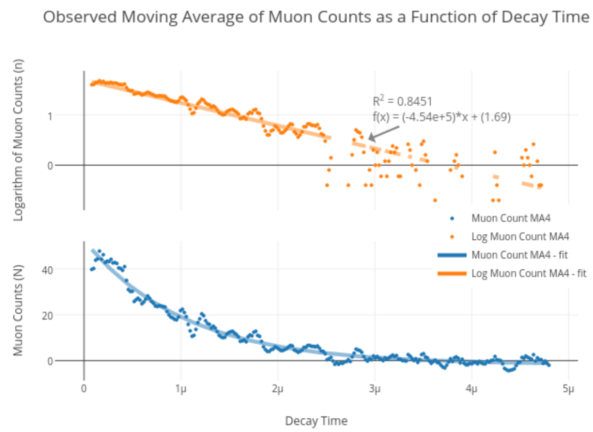

FIG. 7. Smoothing out the observed muon counts over intervals of 4 yields an R-squared value of .85, which is acceptable. Final results are based on the logarithm linear fit.

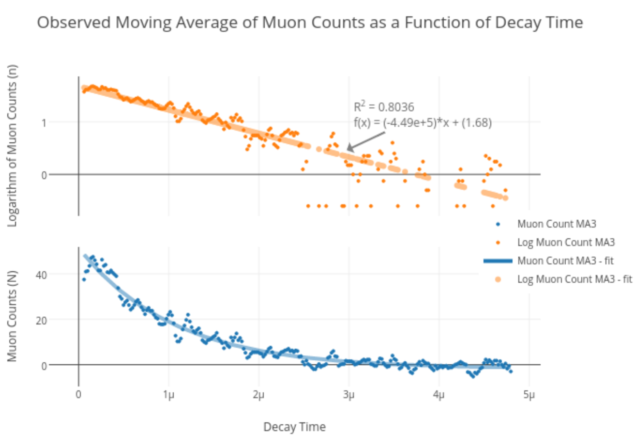

FIG. 6. Observed muon counts smoothed by 3 interval moving averages. Linear fit to the logarithm of muon counts yields the inverse of the muon's mean lifetime constant. Notice the logarithm of the curve begins to show large statistical uncertainties towards lower counts.

$(2.2 \pm .2)\mu s$ was the value obtained.

The counts as a function of arrival time were determined to be Poisson distributed, which coincides with how counting experiments are understood in terms of uncertainties. The measured number of counts in each bin should be N_i with uncertainties $\sigma_i = \sqrt{N_i}$. Therefore the maximum liklihood of some count N_i depends on $\tau \pm \delta\tau$, $B \pm \delta B$, and $N_o \pm \delta N_o$

Notice the shape of the data coincides more with Poisson distributed data than the Gaussian fit. This infers to the true randomness of the data obtained. It suggests that the number of muon counts were independent of each other in any given arrival time.

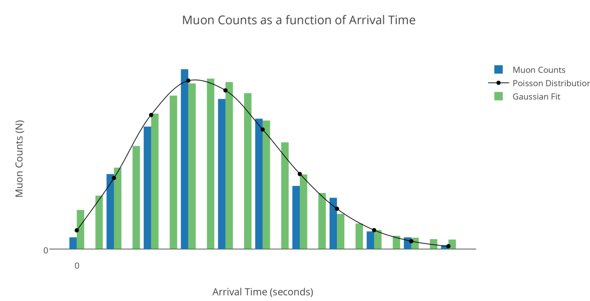

FIG. 8. Number of muon Counts per 1 second intervals. Blue lables the detected muon events while green is the gaussian fit and the curve is the poisson distribution.

If the counts were not independent of one another, the distribution would suggest a heavy relationship of muons originating from the same source. As it stands, the solid angle generated from the occurrence of muons travels a wide distributed area by the time it reaches sea level. Cosmic rays incident on larger atoms will shower subatomic particles in the same general direction of their initial path, but there are slight angle changes regardless. Therefore, it's highly likely that the detected muons come from different sources. This agrees with the data, as the number of muon counts per 1 second interval is random, and so it's unlikely they came from the same source.

G. Conclusion

Results from the experiment were good. $(2.2 \pm .2)\mu s$ was the value obtained for the mean muon lifetime which is within a standard deviation of the true value. The experimental setup proved to be difficult as there were very many working pieces. A large part of the experiment was understanding what each element of the apparatus did, and how it influenced the overall picture. The data obtained in this experiment came from another group of students as the original data had not come out as expected. This could be due to several reasons, the most likely being computer issues.

The level of understanding gained from this experiment was pretty significant. It was first believed that there wasn't a strong presence of proof that the detected particles would be muonic in nature, but several filtering mechanics allows the results to prove differently. By using the discriminator, statistics of background counts, the TAC for muon detection using timing standards, the mean lifetime of the muon was ascertained.

The arrival statistics portion of this experiment clearly showed a Poisson distribution, which works well with what we expect about randomness and decay statistics. Additionally, it was determined that the muon counts per interval also suggested that their origins were not likely singular in geospatial terms; rather, most likely detected muons from several different subtended angles. Of course, each of these particles would have been generated from cosmic ray collision in the upper atmosphere, anywhere from a perpendicular point of origin to some angle extended towards the horizontal and falls within the allowed travel distances as determined through length contraction and time dilation. [3]

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