Programming assignment #6

To quantify the error of a finite element approximations, error norms are often computed. In this assignment, write a function to compute error norms for a finite element solution, where the exact

displacement is $u^{ex}(x) = \sin(2\pi x^2)\exp(-x^2)$, and the finite element solutions are given as $u^h(x) = d_I N_I(x)$ and 3-node quadratic elements are used.

The two error norms to compare are the L2 and energy error norms of the function, written as:

$$\bar{e}_{L2} = \left(\frac{\int_0^L \left(u^{ex}(x) - u^h(x)\right)^2 dx}{\int_0^L \left(u^{ex}(x)\right)^2 dx} \right)^{1/2} \quad \text{and} \quad \bar{e}_{en} = \left(\frac{\int_0^L E\left(\frac{du^{ex}}{dx} - \frac{du^h}{dx}\right)^2 dx}{\int_0^L E\left(\frac{du^{ex}}{dx}\right)^2 dx} \right)^{1/2},$$

Use the four-point Gaussian quadrature rule to evaluate these integrals numerically within each element. Make sure that you either specify the quadrature points and locations with enough precision (16 significant figures) or symbolically (see https://en.wikipedia.org/wiki/Gaussian_quadrature#Gauss%E2%80%93Legendre_quadrature)

Instructions for programming and assignment submission:

```
function [e1, e2] = asurite_hw6(mesh, d)
    % Defines default inputs for testing.
    if nargin == 0
        ne = 4;
        nn = 2*ne + 1;
        mesh.x = linspace(-2, 2, nn);
        mesh.conn = [1:2:nn-2; 2:2:nn-1; 3:2:nn];
        d = (sin(2*pi*mesh.x.^2) .* exp(-mesh.x.^2))';
    end
    % Compute e1 and e2
end
```

- The order of the input variables and output variables must not be changed.
- **The input variables are:**
  - **mesh:** a structure containing fields mesh.x and mesh.conn that defines a mesh of 3-node quadratic element.
  - **d:** A N×1 vector of nodal displacements, where N is the number of nodes.
- **The output variable is:**
  - **e1:** the L2 error norm of the interpolated solution.
  - **e2:** the energy error norm of the interpolated solution.