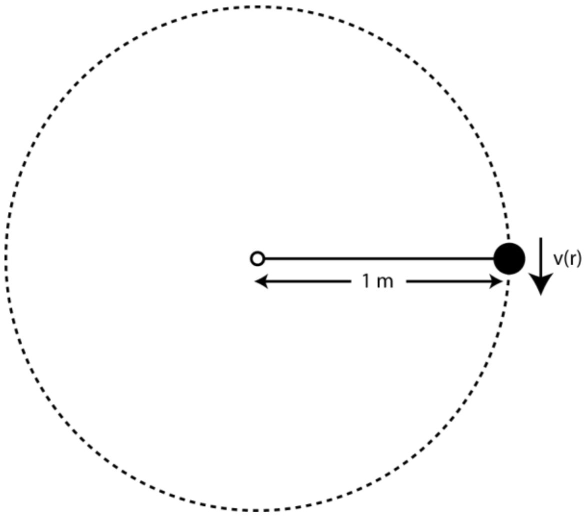
Project #1:



An aluminum rod (diameter = 5 cm, E = 70 GPa, density = 2700 kg/m^3) swings a 1 kg mass around a circle with a radius of 1 m at 2000 RPM. With a coordinate system fixed along the length of the rod, the angular acceleration is equivalent to the centripetal body force

$$b(x) = \frac{\rho A v^2}{x}$$

where the velocity $v = \omega x$ is given by the distance from the center of rotation and the angular velocity. Write a finite element program in MATLAB to approximate the displacement and stress in the rod and compare with analytical solutions and an approximate solution from ABAQUS (or any other commercial finite element code).

Report format

Reports must be typed and submitted **as a PDF file** to Gradescope prior to the due date.

Grading scale:

Points will be awarded for correct completion of each of the tasks listed below with partial credit awarded for incomplete or incorrect attempts. Plots must contain axis labels and units as appropriate for full credit. You may attempt more points than required (however, your maximum score will be capped at 100%).

- [5] Restate the problem that you are solving in your own words.
- [10] Plot displacement fields for analytical, ABAQUS and MATLAB solutions all on the same plot. Use a fine enough mesh so that you can demonstrate that all three approaches have the same solution.
- [10] Plot strain fields for analytical, ABAQUS and MATLAB solutions all on the same plot.

Analytical solution points

- [10] Analytical solution for the displacement field (show derivation). You may use a symbolic math package such as Mathematica.
- [10] Analytical solution for the strain field.

MATLAB solution points

- [10] Plot of the displacement field for linear element solution with a mesh containing two and four elements.
- [10] Plot of the strain field for linear element solution with a mesh containing two and four elements.
- [10] Plot of the displacement field for quadratic element solution with a mesh containing two and four elements.

- [10] Plot of the strain field for quadratic element solution with a mesh containing two and four elements.
- [15] Plot of the displacement field for cubic element solution with a mesh containing two and four elements.
- [15] Plot of the strain field for cubic element solution with a mesh containing two and four elements.
- [10] Comparison of times to (a) assemble and (b) solve the system of equations with 1000 elements for full storage and sparse storage of the stiffness matrix. (*Hint: Use tic() and toc() MATLAB functions for timing*)
- [15] Plot of L2 displacement error norm vs element size on a log-log plot and computed rate of convergence while using linear elements.
- [15] Plot of L2 displacement error norm vs element size on a log-log plot and computed rate of convergence while using quadratic elements.
- [15] Plot of energy error norm vs element size on a log-log plot and computed rate of convergence with linear elements.
- [15] Plot of energy error norm vs element size on a log-log plot and computed rate of convergence with quadratic elements.

ABAQUS solution points

- [20] Clearly explain how you modelled the problem in ABAQUS: explain what type of elements you used, what assumptions you made, and how loads and boundary conditions were implemented.
- [5] Plot displacement and strain fields for linear elements.
- [5] Plot displacement and strain fields for quadratic elements.

Side project (up to 100 points given)

Imagine that a second mass, m, is added at a coordinate x_m. One way to consider this is to add the body force

$$b_m(x) = m\omega^2(x_m)\delta(x - x_m)$$

where a delta function is used to represent a concentrated force. Note that in integration the delta function

$$\int f(x)\delta(x - x_0)\,dx = f(x_0)$$

Note that the result will be somewhat similar to a prescribed traction (but evaluated internally within the rod).

Given a 2 kg mass added at the midpoint of the rod (x_m = 0.5 m), determine whether or not your code converges to the exact solution with the optimal convergence rate in the following cases:

a) Linear elements, # of elements is always odd
b) Linear elements, # of elements is always even
c) Quadratic elements, # of elements is always odd
d) Quadratic elements, # of elements is always even

Your report should show how you determine the exact analytical solution and discuss the meaning of your results.