# Codification et Représentation de l'Information (CRI)

MI – USTHB – TD

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1- Etablir les tables de vérité des fonctions suivantes :

$$F1 = (X + Y)(\overline{X} + Y + Z)$$

 $V \perp V7 - (V \perp V)(V \perp 7)$ 

$$F2 = (\overline{X}Y + X\overline{Y}) \overline{Z} + (\overline{X} \overline{Y} + XY)Z$$

2- Démontrer à l'aide de tables de vérité les équivalences suivantes :

$\lambda + 12 - (\lambda + 1)(\lambda + 2)$		
$(\bar{X} + Y)(X + Z)(Y)$	$(Z' + Z) = (\bar{X})$	$+Y)(X \dashv$

X	Υ	Z	$\bar{X}$	X+Y	$ar{X}$ +Y+Z	F1
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	0	1	1	1
1	1	1	0	1	1	1

1- Etablir les tables de vérité des fonctions suivantes :

$$F1 = (X + Y)(\overline{X} + Y + Z)$$

$$F2 = (\overline{X}Y + X\overline{Y}) \overline{Z} + (\overline{X} \overline{Y} + XY)Z$$

X	Υ	Z	$\overline{X}$	¥	Z	$\overline{X}Y$	ΧŸ	$\overline{X}Y+X\overline{Y}$	$(\overline{X}Y+X\overline{Y})\overline{Z}$	$\overline{X}\overline{Y}$	XY	$\overline{X}\overline{Y}$ +XY	$(\overline{X}\overline{Y}+XY)Z$	F2
0	0	0	1	1	1	0	0	0	0	1	0	1	0	0
0	0	1	1	1	0	0	0	0	0	1	0	1	1	1
0	1	0	1	0	1	1	0	1	1	0	0	0	0	1
0	1	1	1	0	0	1	0	1	0	0	0	0	0	0
1	0	0	0	1	1	0	1	1	1	0	0	0	0	1
1	0	1	0	1	0	0	1	1	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0	0	1	1	0	0
1	1	1	0	0	0	0	0	0	0	0	1	1	1	1

2- Démontrer à l'aide de tables de vérité les équivalences suivantes :

$$X + YZ = (X+Y)(X+Z)$$

$$(\overline{X} + Y)(X + Z)(Y + Z) = (\overline{X} + Y)(X + Z)$$

Х	Υ	Z	YZ	X + YZ	X + Y	X + Z	(X+Y)(X+Z)	
0	0	0	0	0	0	0	0	
0	0	1	0	0	0	1	0	
0	1	0	0	0	1	0	0	
0	1	1	1	1	1	1	1	
1	0	0	0	1	1	1	1	
1	0	1	0	1	1	1	1	
1	1	0	0	1	1	1	1	
1	1	1	1	1	1	1	1	

2- Démontrer à l'aide de tables de vérité les équivalences suivantes :

$$X + YZ = (X+Y)(X+Z)$$
  
 $(\bar{X} + Y)(X + Z)(Y + Z) = (\bar{X} + Y)(X + Z)$ 

$$PG = (\overline{X} + Y)(X + Z)(Y + Z) \qquad PD = (\overline{X} + Y)(X + Z)$$

X	Υ	Z	$\overline{X}$	$\overline{X} + Y$	X+Z	Y + Z	PG	$(\overline{X} + Y)$	PD
0	0	0	1	1	0	0	0	1	0
0	0	1	1	1	1	1	1	1	1
0	1	0	1	1	0	1	0	1	0
0	1	1	1	1	1	1	1	1	1
1	0	0	0	0	1	0	0	0	0
1	0	1	0	0	1	1	0	0	0
1	1	0	0	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1

Simplifier algébriquement les expressions suivantes :

$$(x+y+xy)(xy+xz+yz)$$
  
 $(x+y+z)(\bar{x}+y+z)+xy+yz$   
 $abcd+abchg+\bar{d}hg+abcdefh.$   
 $a\bar{c}de+\bar{d}+\bar{e}+c$ 

 $AB + \overline{B}C = (A + \overline{B})(B + C)$ 

Démontrer algébriquement les égalités suivantes :

A 
$$\overline{B}$$
 +  $\overline{A}$   $\overline{C}$   $\overline{D}$  +  $\overline{A}$   $\overline{B}D$  +  $\overline{A}$   $\overline{B}C$   $\overline{D}$  =  $\overline{A}$   $\overline{C}$   $\overline{D}$  +  $\overline{B}$  A.B+ $\overline{A}$ .C +B.C=A.B+ $\overline{A}$ .C AB + ACD +  $\overline{B}D$  = AB+  $\overline{B}D$ 

### Simplifier algébriquement les expressions suivantes :

1- 
$$(x+ \bar{y} + x \bar{y})(xy + \bar{x}z + yz)$$
 ///  $yz = yz(x+/x)$   
//  $xy + \bar{x}z + yz = xy+\bar{x}z + xyz + yz\bar{x} = xy+\bar{x}z$ 

= 
$$(x(1+\bar{y})+\bar{y})(xy+\bar{y})z$$
  
=  $xxy + x\bar{x}z + xy\bar{y} + \bar{x}\bar{y}z$   
=  $xy + x\bar{x}z + xy\bar{y} + \bar{x}\bar{y}z$   
=  $xy + x\bar{x}z + xy\bar{y} + \bar{x}\bar{y}z$   
=  $xy + \bar{x}\bar{y}z$ 

= y + z

### Simplifier algébriquement les expressions suivantes :

2- 
$$(x + y + z)(\bar{x} + y + z) + xy + yz$$
 //(a+b)(a+c) = a+(b.c)  
=  $((y+z)+\frac{(x\bar{x})}{(x\bar{x})}+xy+yz$   
=  $y+z+xy+yz$   
=  $y(1+x+z)+z$ 

```
Simplifier algébriquement les expressions suivantes : abcd + abchg + \bar{d}hg + abcdefh. = abcd(1+efh) + abchg + \bar{d}hg = abcd + abchg + \bar{d}hg /// abchg(d+/d) = abcd + abchgd + abchgd + abchgd + abchgd
```

- = abcd(1+hg) + hg $\bar{d}(1+abc)$
- = abcd + dhg

Simplifier algébriquement les expressions suivantes : // a +  $\bar{a}$  b = a + b

Démontrer algébriquement les égalités suivantes :

$$A \overline{B} + \overline{A} \overline{C} \overline{D} + \overline{A} \overline{B}D + \overline{A} \overline{B}C \overline{D} = \overline{A} \overline{C} \overline{D} + \overline{B}$$

$$A.B+\bar{A}.C +B.C=A.B+\bar{A}.C$$

$$AB + ACD + \overline{B}D = AB + \overline{B}D$$

$$AB + \overline{B}C = (A + \overline{B})(B + C)$$

Démontrer algébriquement les égalités suivantes : A B + A C D+ A BD + A BC D = A C D + B

= 
$$A B + A C D (B+B) + A BD + A BC D$$
  
=  $A B + A C D B + A C DB + A BD + A BC D$   
=  $B (A + A C D + AD + AC D) + A C D B$   
=  $B (A + A C D + AD + AC D) + A C D B$   
=  $B (A + C D + D + C D) + A C D B$   
=  $B (A + C + D + C) + A C D B$   
=  $B (A + D + 1) + A C D B$   
=  $B + A C D B$   
=  $B + A C D B$ 

# Démontrer algébriquement les égalités suivantes : $A.B+\bar{A}.C+B.C=A.B+\bar{A}.C$

= 
$$AB+\bar{A}C +BC (A+\bar{A})$$
  
=  $AB+\bar{A}C +ABC+\bar{A}CB$   
=  $AB (1+C) + \bar{A}C(1+B)$   
=  $AB+\bar{A}.C$ 

$$//$$
 a+ ab = a

## Démontrer algébriquement les égalités suivantes :

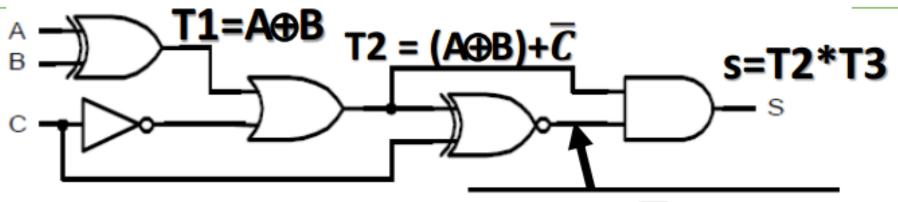
AB + ACD + 
$$\bar{B}$$
D = AB+  $\bar{B}$ D  
= AB + ACD(B+ $\bar{B}$ ) +  $\bar{B}$ D  
= AB + ACDB+ACD $\bar{B}$  +  $\bar{B}$ D // a+ ab = a

$$=AB+\bar{B}D$$

Démontrer algébriquement les égalités suivantes :  $AB + \overline{B}C = (A + \overline{B})(B + C)$ 

$$(A + \bar{B})(B + C) = AB + AC + B\bar{B} + \bar{B}C$$
  
=  $AB + AC(B + \bar{B}) + \bar{B}C$   
=  $AB + ACB + AC\bar{B} + \bar{B}C$   
=  $AB + \bar{B}C + \bar{B$ 





Α	В	С	T1	T2	Т3	S
0	0	0	0	1	0	0
0	0	1	0	0	0	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	1	0	0
1	0	1	1	1	1	1
1	1	0	0	1	0	0
1	1	1	0	0	0	0

T3= 
$$((A \oplus B) + \overline{C}) \oplus C$$

$$F(a,b,c) = /abc + a/bc$$

$$F(a,b,c) = /a/b/c + /a/bc +....$$

Exo 5 - 1

ab cd ,	00	01	11	10
00	1	1.	1	
01		1	1	
11		1	1	
10	1 .	11	1 /	_ 1 j

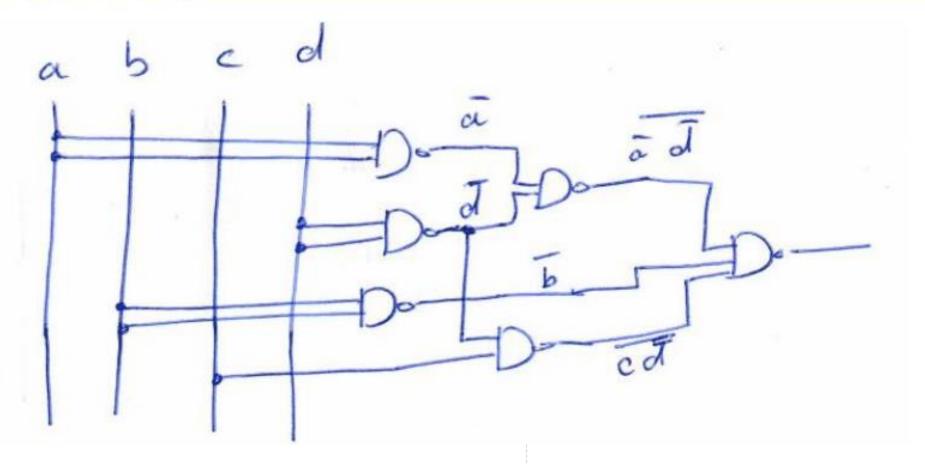
$$F(a,b,c,d) = \overline{a}\overline{d} + b + c\overline{d}$$
 La forme disjonctive

$$F(a,b,c,d) = \overline{a}\overline{d} + b + c\overline{d}$$

$$F(a,b,c,d) = \overline{\overline{a}} \overline{\overline{d}} + b + c\overline{\overline{d}}$$

$$F(a,b,c,d) = \overline{a}\overline{d} \cdot \overline{b} \cdot \overline{c}\overline{d}$$

# $F(a,b,c,d) = \overline{a}\overline{d} \cdot \overline{b} \cdot \overline{c}\overline{d}$



Exo 5 - 1

ab	00	01	11	10
cd ,				
00	1	1	1	0
01	0	1	1	, o _
11	_ 0, .	1	1	, lo
10	1	1	1	1

$$\overline{F(a,b,c,d)} = \overline{b}d + a\overline{b}\overline{c}$$

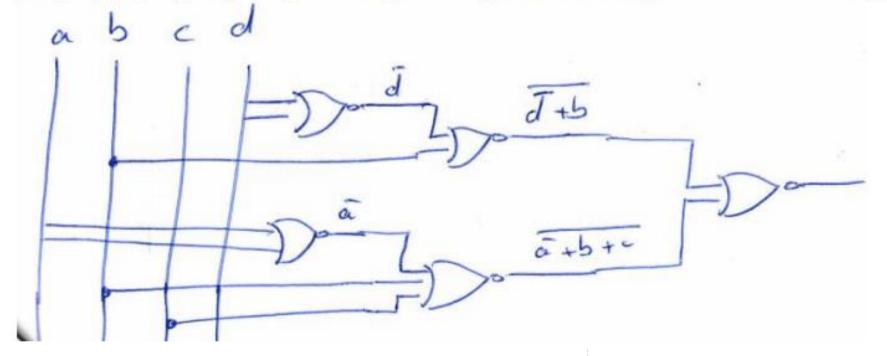
$$F(a,b,c,d) = (b+\overline{d})(\overline{a}+b+c)$$

La forme conjonctive

$$F(a,b,c,d) = (b+\overline{d})(\overline{a}+b+c)$$

$$F(a,b,c,d) = \overline{(b+\overline{d})(\overline{a}+b+c)}$$

$$F(a,b,c,d) = (b+\overline{d}) + (\overline{a}+b+c)$$



Exo 5 - 2

ab cd ,	00	01	11	10
00	.1.7			(1,
01		(i	1	
11		11.	_1;	.~.
10	1			<b>1</b>

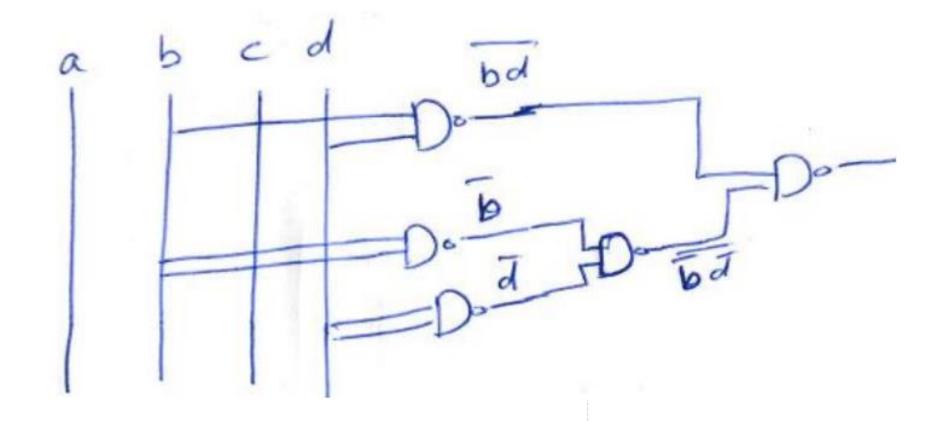
$$F(a,b,c,d) = bd + \overline{b}\overline{d}$$

La forme disjonctive

$$F(a,b,c,d) = bd + \overline{bd}$$

$$F(a,b,c,d) = bd + \overline{b}\overline{d}$$

$$F(a,b,c,d) = \overline{bd} \cdot \overline{\overline{bd}}$$



Exo 5 - 2

ab cd ,	00	01	11	10
00	1	0.1	٥ <sup>^</sup> .	1
01	0 \	1	1	0
11 .	.0	1	. 1	0
10	1	0	0 .	1

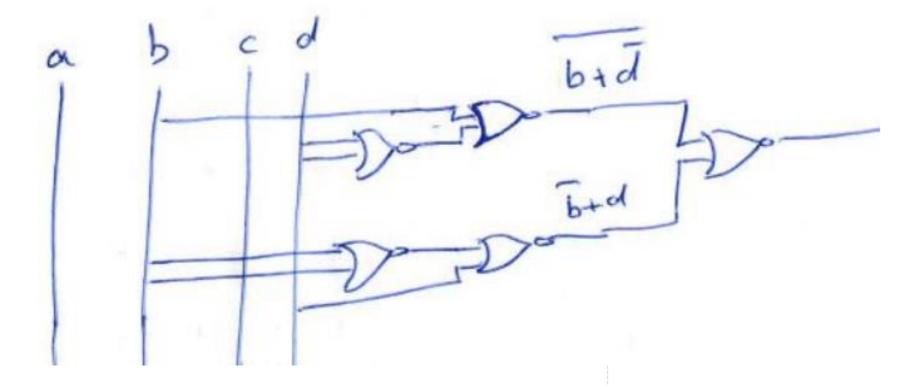
$$\overline{F(a,b,c,d)} = \overline{b}d + b\overline{d}$$

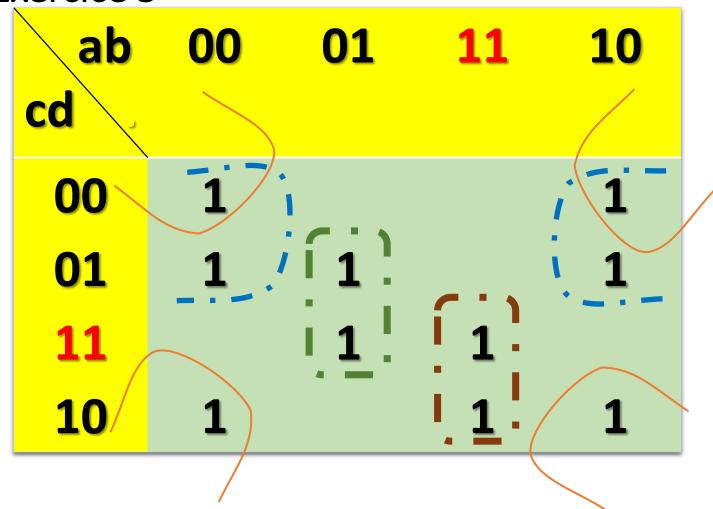
$$F(a,b,c,d) = (\mathbf{b} + \overline{d})(\overline{b} + d)$$

La forme conjonctive

$$F(a,b,c,d) = (\mathbf{b} + \overline{d})(\overline{b} + d)$$

$$F(a,b,c,d) = \overline{(b+\overline{d})(\overline{b}+d)} = \overline{(b+\overline{d})} + \overline{(\overline{b}+d)}$$





$$F(a,b,c,d) = /b/c + /b/d + /abd + abc$$

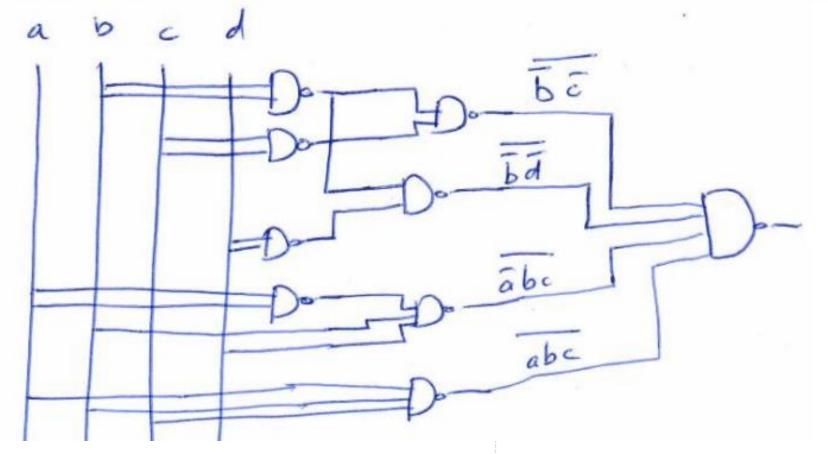
$$F(a,b,c,d) = \overline{b}\overline{c} + \overline{b}\overline{d} + \overline{a}bd + abc$$

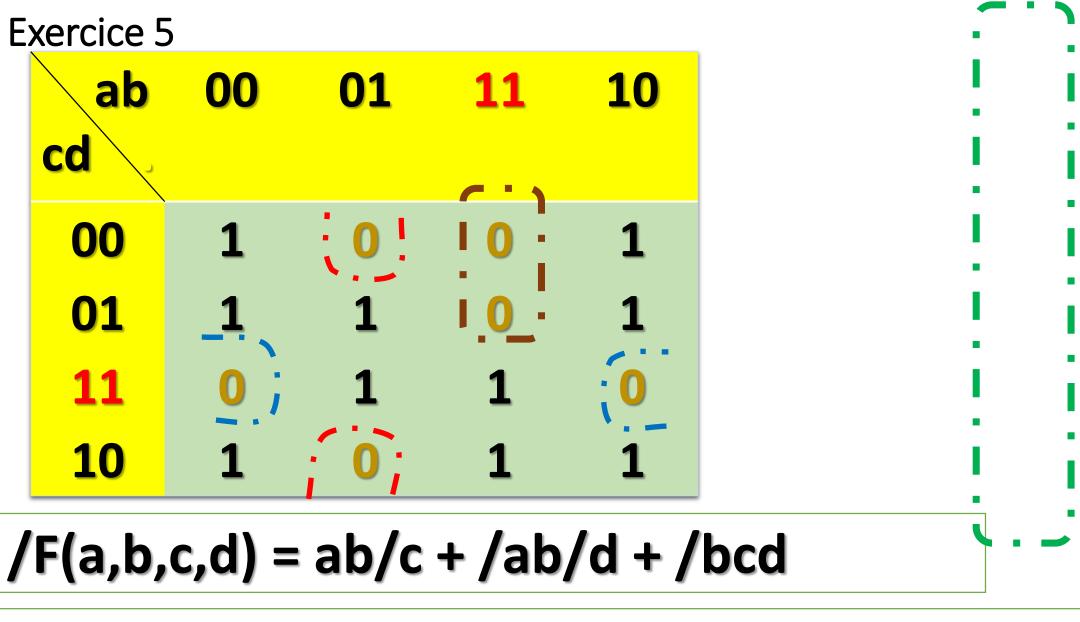
$$F(a,b,c,d) = \overline{b}\overline{c} + \overline{b}\overline{d} + \overline{a}bd + abc$$

$$F(a,b,c,d) = \overline{b}\overline{c} \cdot \overline{b}\overline{d} \cdot \overline{a}bd \cdot \overline{a}bc$$

Exercice 5







$$F(a,b,c,d) = (/a+/b+c)(a+/b+d)(b+/c+/d)$$

$$\overline{F(a,b,c,d)} = \overline{bcd} + \overline{abd} + ab\overline{c}$$

### La forme conjonctive

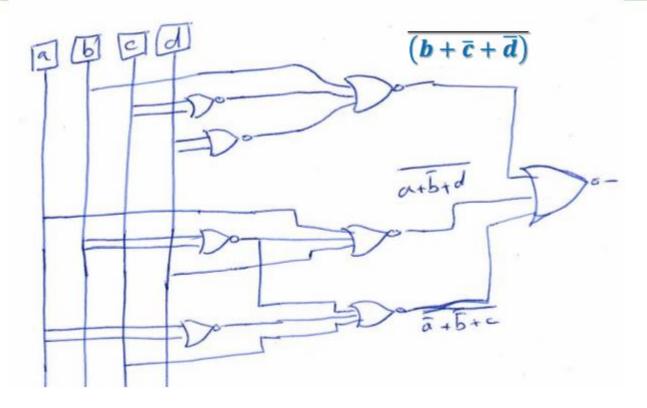
$$F(a,b,c,d) = (b + \overline{c} + \overline{d})(a + \overline{b} + d)(\overline{a} + \overline{b} + c)$$

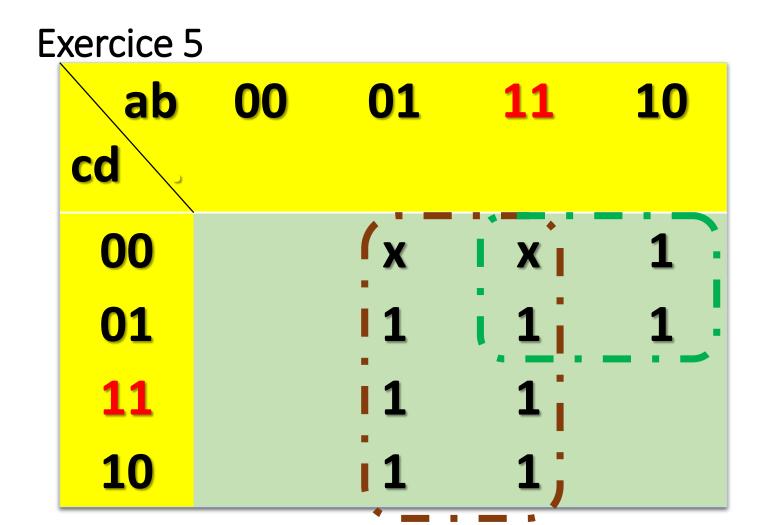
$$F(a,b,c,d) = \overline{(b+\overline{c}+\overline{d})(a+\overline{b}+d)(\overline{a}+\overline{b}+c)}$$

$$F(a,b,c,d) = \overline{\left(b+\overline{c}+\overline{d}\right)} + \overline{\left(a+\overline{b}+d\right)} + \overline{\left(\overline{a}+\overline{b}+c\right)}$$

Exo 5 - 3

$$F(a,b,c,d) = \overline{\left(b+\overline{c}+\overline{d}\right)} + \overline{\left(a+\overline{b}+d\right)} + \overline{\left(\overline{a}+\overline{b}+c\right)}$$





$$F(a,b,c,d) = b + a/c$$

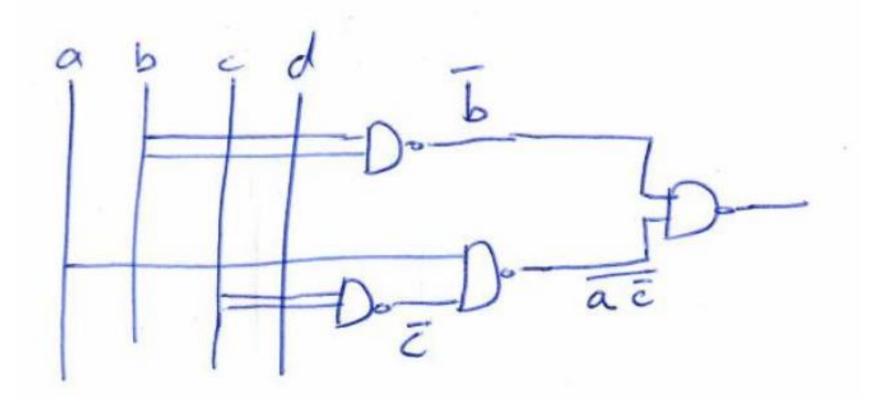
$$F(a,b,c,d) = b + a\overline{c}$$

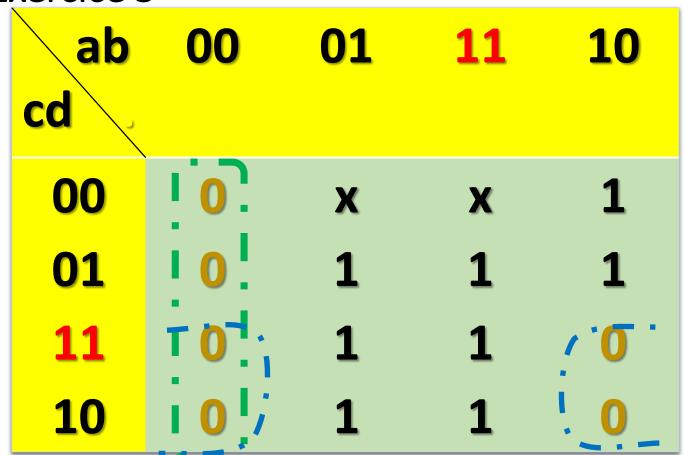
$$F(a,b,c,d) = \overline{b + a\overline{c}}$$

$$F(a,b,c,d) = \overline{\overline{b}} \cdot \overline{a}\overline{\overline{c}}$$

Exo 5 - 4

# $F(a,b,c,d) = \overline{b} \cdot \overline{a}\overline{c}$





$$/F(a,b,c,d) = /a/b + /bc$$

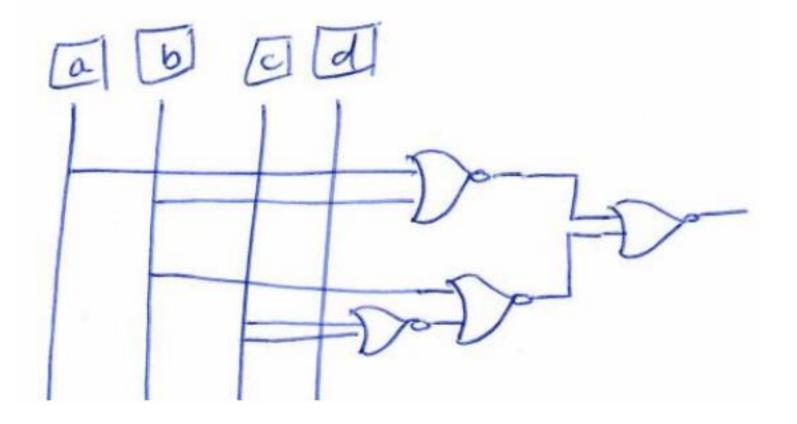
$$/F(a,b,c,d) = (a+b) (b+/c)$$

$$F(a,b,c,d) = (a+b)(b+\overline{c})$$

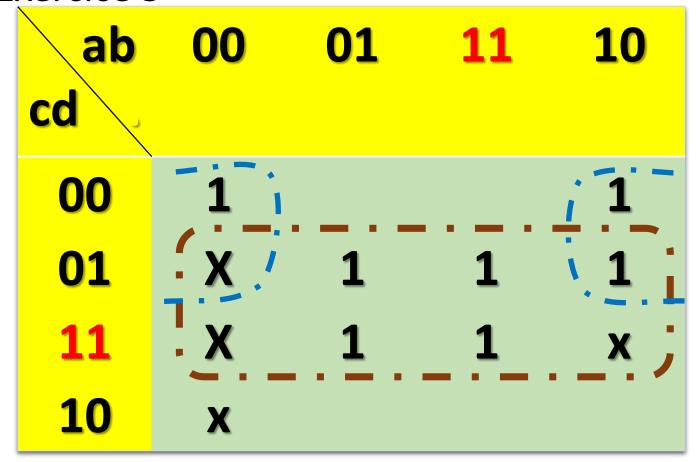
$$F(a,b,c,d) = \overline{(a+b)(b+\overline{c})}$$

$$F(a,b,c,d) = \overline{(a+b)} + \overline{(b+\overline{c})}$$

## $F(a,b,c,d) = \overline{(a+b)} + \overline{(b+\overline{c})}$



Exercice 5



$$F(a,b,c,d) = d + /b/c$$

Exo 5 - 5

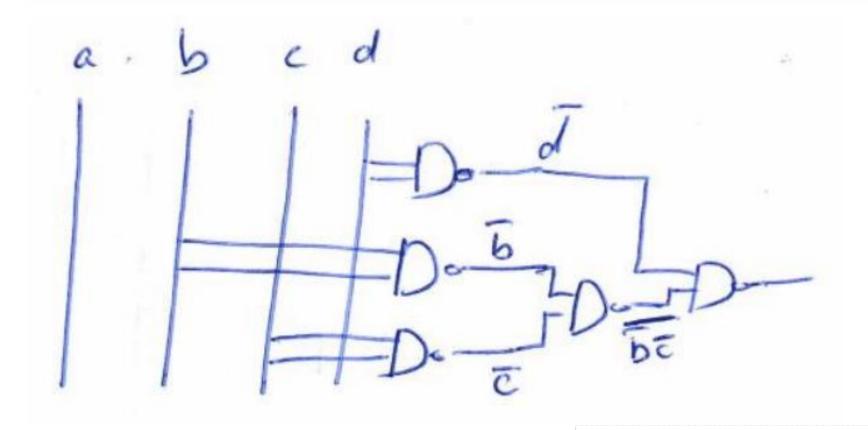
$$F(a,b,c,d) = d + \overline{b}\overline{c}$$

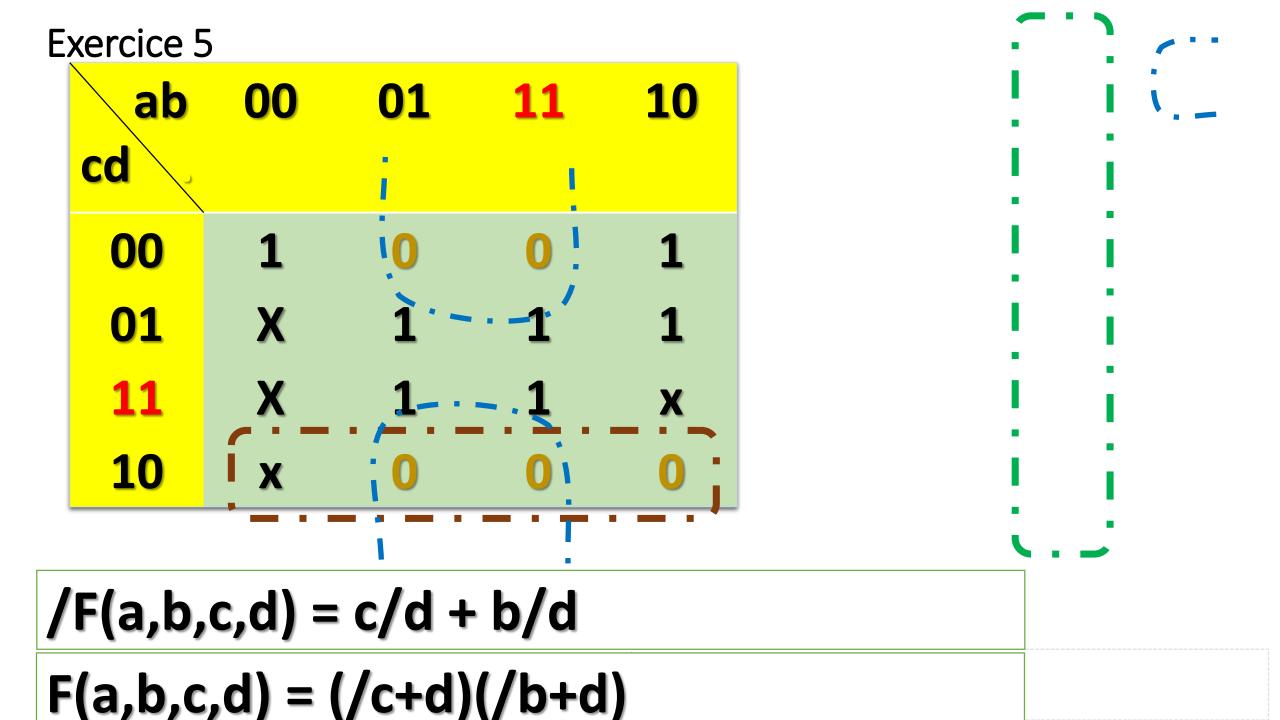
$$F(a,b,c,d) = \overline{d + \overline{b}\overline{c}}$$

$$F(a,b,c,d) = \overline{\overline{d} \cdot \overline{b}\overline{c}}$$

Exo 5 - 5

## $F(a,b,c,d) = \overline{\overline{d} \cdot \overline{b}\overline{c}}$





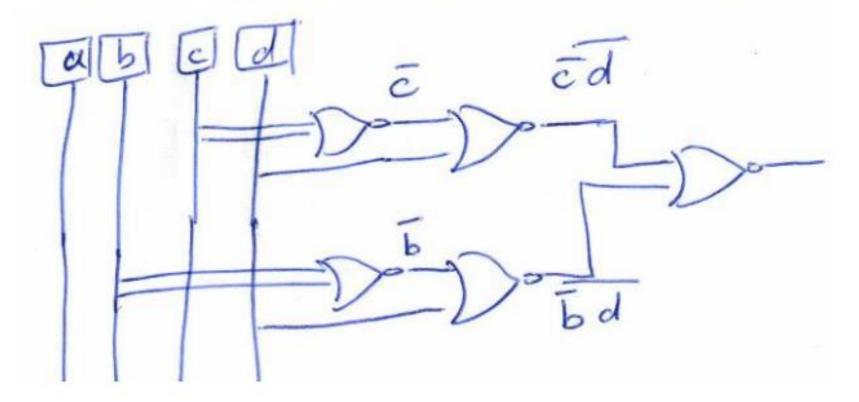
$$\overline{F(a,b,c,d)} = c\overline{d} + b\overline{d}$$

F(a,b,c,d) = 
$$(\overline{c} + d)(\overline{b} + d)$$
 La forme conjonction

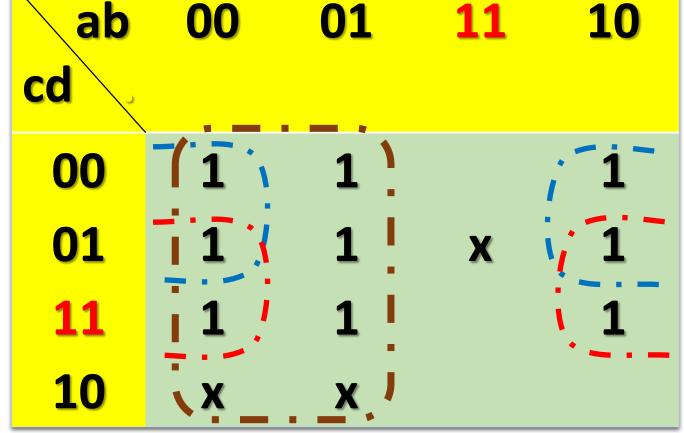
$$F(a,b,c,d) = \overline{(\overline{c}+d)(\overline{b}+d)}$$

$$F(a,b,c,d) = \overline{(\overline{c}+d)} + \overline{(\overline{b}+d)}$$

## $F(a,b,c,d) = \overline{(\overline{c}+d)} + \overline{(\overline{b}+d)}$



Exercice 5 cd



$$F(a,b,c,d) = /a + /b/c + /bd$$

Exo 5 - 6

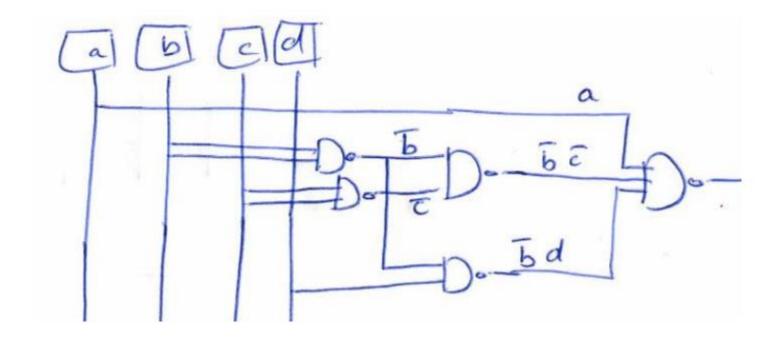
$$F(a,b,c,d) = \overline{a} + \overline{b}\overline{c} + \overline{b}d$$

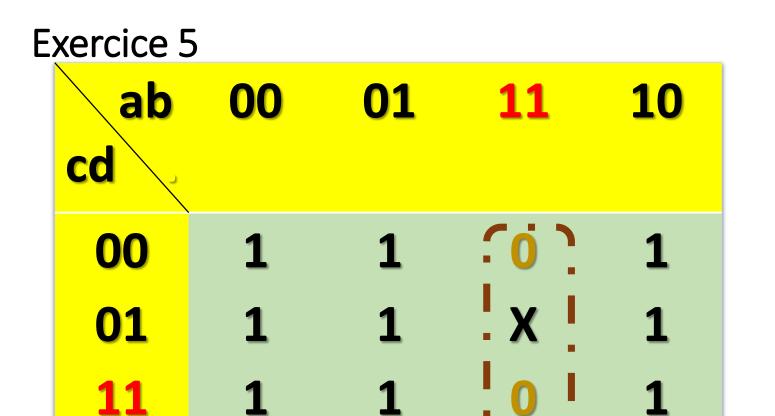
$$F(a,b,c,d) = \overline{\overline{a} + \overline{b}\overline{c} + \overline{b}d}$$

$$F(a,b,c,d) = \overline{\overline{a}} \cdot \overline{\overline{b}\overline{c}} \cdot \overline{\overline{b}d}$$

Exo 5 - 6

## $F(a,b,c,d) = \overline{\overline{a}} \cdot \overline{\overline{b}}\overline{c} \cdot \overline{\overline{b}}\overline{d}$





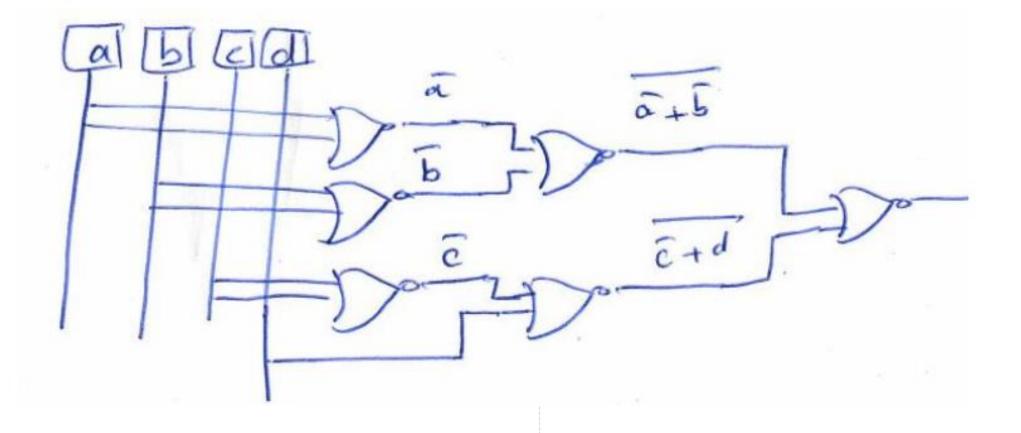
Exo 5 - 6

$$F(a,b,c,d) = (\overline{a} + \overline{b})(\overline{c} + d)$$

$$F(a,b,c,d) = (\overline{a} + \overline{b})(\overline{c} + d)$$

$$F(a,b,c,d) = \overline{(\overline{a} + \overline{b})} + \overline{(\overline{c} + d)}$$

$$F(a,b,c,d) = \overline{(\overline{a} + \overline{b})} + \overline{(\overline{c} + d)}$$



Simplifier à l'aide du Tableau de Karnaugh les fonctions suivantes

puis réaliser les circuits correspondants à l'aide de portes NOR ou NAND.

F(a, b, c)= 
$$\pi(0, 1, 2, 3, 4, 7)$$
  
G(a, b, c, d)= $\Sigma(2, 6, 7, 10, 11, 12, 14)$ 

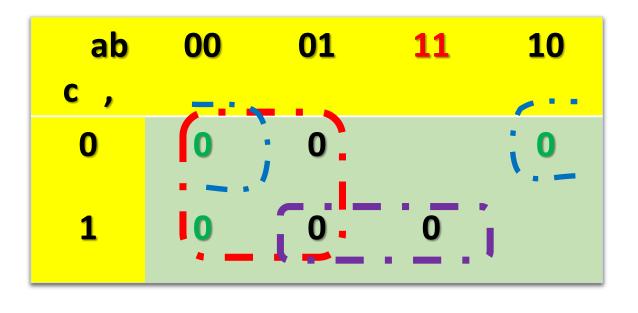
F(a, b, c)= 
$$\pi(0, 1, 2, 3, 4, 7)$$
  
//positions de 0  
G(a, b, c, d)= $\Sigma(2, 6, 7, 10, 11, 12, 14)$   
// positions de 1

	а	b	С	F
<u>0</u>	0	0	0	0
1	0	0	1	0
<u>2</u>	0	1	0	0
<u>3</u>	0	1	1	0
<u>4</u>	1	0	0	0
<u>5</u>	1	0	1	1
<u>6</u>	1	1	0	1
<u>7</u>	1	1	1	0

	a	b	С	d	G
<u>0</u>	0	0	0	0	0
<u>1</u>	0	0	0	1	1
<u>2</u>	0	0	1	0	0
<u>3</u>	0	0	1	1	0
<u>4</u>	0	1	0	0	0
<u>5</u>	0	1	0	1	0
<u>6</u>	0	1	1	0	1
<u>7</u>	0	1	1	1	1
<u>8</u>	1	0	0	0	0
<u>9</u>	1	0	0	1	0
<u>10</u>	1	0	1	0	1
<u>11</u>	1	0	1	1	1
<u>12</u>	1	1	0	0	1
<u>13</u>	1	1	0	1	0
<u>14</u>	1	1	1	0	1
<u>15</u>	1	1	1	1	0

 $F(a, b, c) = \pi(0, 1, 2, 3, 4, 7)$ 

	a	b	С	F
<u>0</u>	0	0	0	0
1	0	0	1	0
<u>2</u>	0	1	0	0
<u>3</u>	0	1	1	0
<u>4</u>	1	0	0	0
<u>5</u>	1	0	1	1
<u>6</u>	1	1	0	1
<u>7</u>	1	1	1	0



$$F(a,b,c) = ab/c + a/bc$$

$$/F(a,b,c) = /a + /b/c + bc$$

$$F(a,b,c) = a (b+c) (/b+/c)$$

$$F(a, b, c) = \pi(0, 1, 2, 3, 4, 7)$$

$$F(a,b,c) = \overline{ab\overline{c} + a\overline{b}c}$$

$$F(a,b,c) = \overline{ab\overline{c}} \cdot \overline{a\overline{b}c}$$

$$\overline{F(a,b,c)} = \overline{a} + bc + \overline{b}\overline{c}$$
  $F(a,b,c) = a(b+c)(\overline{b} + \overline{c})$ 

$$F(a, b, c) = \pi(0, 1, 2, 3, 4, 7)$$

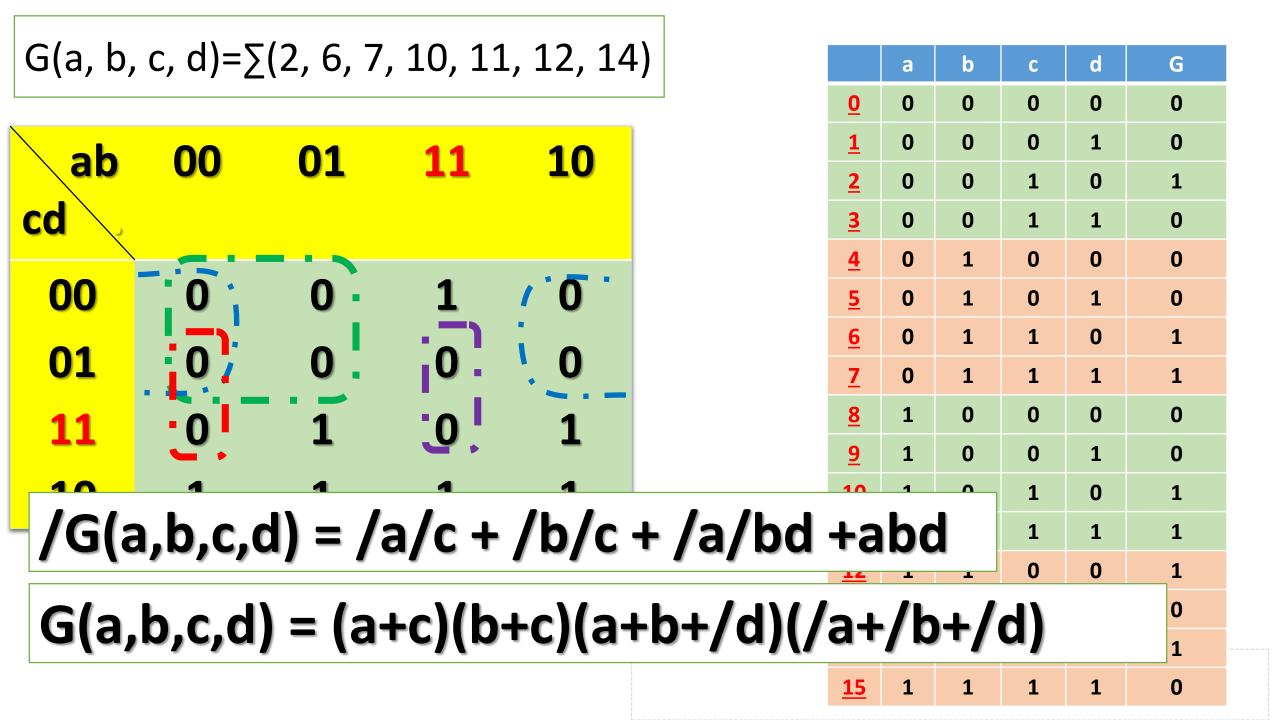
$$F(a,b,c) = \overline{a(b+c)(\overline{b}+\overline{c})}$$

$$F(a,b,c) = \overline{a} + \overline{(b+c)} + \overline{(b+\overline{c})}$$

 $G(a, b, c, d) = \sum (2, 6, 7, 10, 11, 12, 14)$ 

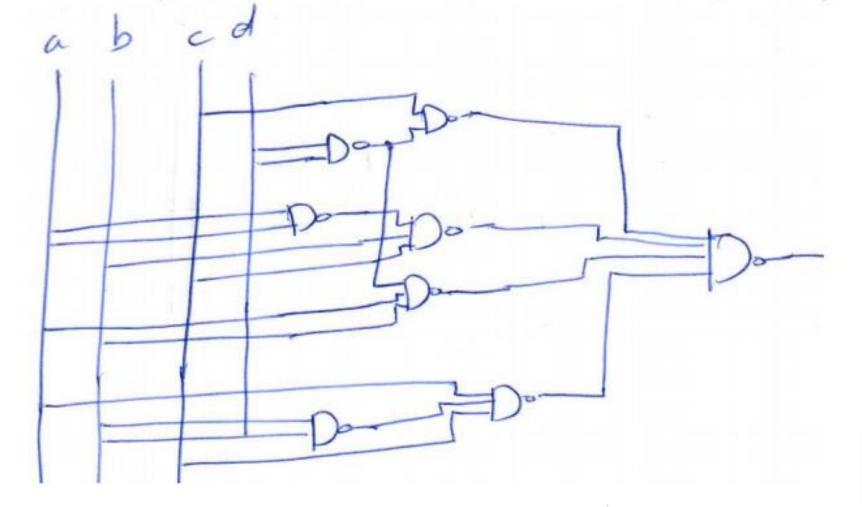
ab	00	01	11	10
cd				
00	0	0	1,	0
01	0	0	0	0
11	o_	(1)	0_	1.
10	. 1	1	11:	1

// 5/ 5/ 2/2	, , , , , , , , , , , , , , , , , , , ,	,,						
					а	b	С	d
00	01 11	10		<u>0</u>	0	0	0	0
	01 11	10		<u>1</u>	0	0	0	1
				<u>2</u>	0	0	1	0
				<u>3</u>	0	0	1	1
0	0 1	0		<u>4</u>	0	1	0	0
				<u>5</u>	0	1	0	1
0	0 0	0		<u>6</u>	0	1	1	0
0	1 0 1	1		<u>7</u>	0	1	1	1
Ÿ _ !	<u> </u>			<u>8</u>	1	0	0	0
1	1 1	1 1		<u>9</u>	1	0	0	1
	• • • • • • • • • • • • • • • • • • • •	-		<u>10</u>	1	0	1	0
				11	1	0	1	1
b.c.d)	= c/d + ab	/d +	labc + a	a/Ł	<b>)</b> C	1	0	0
						1	0	1
				14	1	1	1	0



$$G(a,b,c,d) = c\overline{d} + \overline{a}bc + ab\overline{d} + a\overline{b}c$$

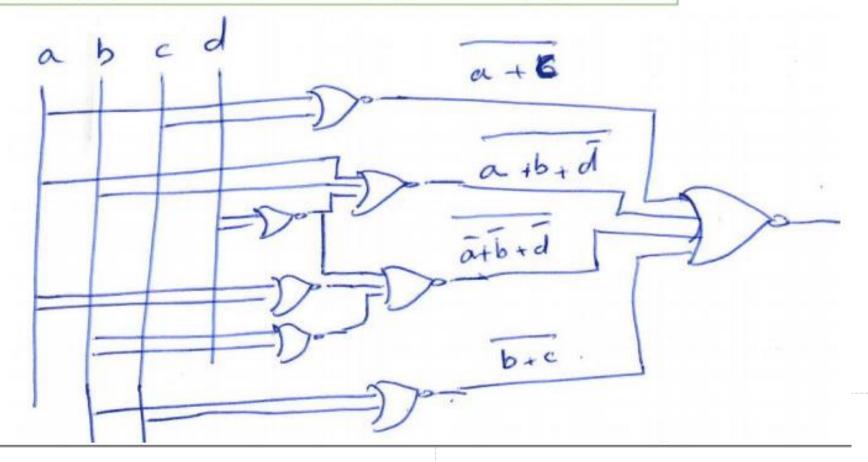
 $G(a,b,c,d) = \overline{c}\overline{d} \cdot \overline{a}bc \cdot ab\overline{d} \cdot \overline{a}\overline{b}c$ 



Exo 6 
$$\overline{G(a,b,c,d)} = \overline{a}\overline{c} + \overline{a}\overline{b}d + abd + \overline{b}\overline{c}$$

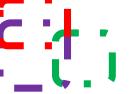
G(a,b,c,d)= 
$$(a+c)(a+b+\overline{d})(\overline{a}+\overline{b}+\overline{d})(b+c)$$

G(a,b,c,d)=
$$\overline{(a+c)}+\overline{(a+b+d)}+\overline{(a+b+d)}+\overline{(b+c)}$$



Exo 7  $F = \overline{(x + y + z)} + (\overline{x + y + \overline{z}}) + \overline{\overline{x} + y + z}$   $F = \overline{(x + y + z)} \overline{(\overline{x + y + \overline{z}})} \overline{\overline{x} + y + z}$   $F = (x + y + z)(x + y + \overline{z})(\overline{x} + y + z)$ 

X	У	Z	x + y + z	$x + y + \bar{z}$	$\bar{x} + y + z$	F
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				



ab	00	01	11	10
С,				
0				
1				

F =
-----

F =	
-----	--

X	У	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Soit la fonction F(A,B,C) définie comme suit:

 $F(A,B,C) = 1 \text{ si } (ABC)_2 \text{ comporte un nombre impair de 1};$ 

F(A,B,C) = 0 sinon.

Etablir la table de vérité de F

A	В	С	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	