Subtraction of $e^+e^- \to u\bar{u}d\bar{d}$ with distributed soft counterterms

Simone Lionetti

April 5, 2019

1 Counterterms

For the process $e^+e^- \to u_1\bar{u}_2d_3\bar{d}_4$ as a real correction to $e^+e^- \to jj + X$, the elementary limits that need to be regulated are

$$C_{12}, \quad C_{34}, \quad C_{134}, \quad C_{234}, \quad C_{123}, \quad C_{124}, \quad S_{12}, \quad S_{34}. \tag{1}$$

At most three of these limits have a common overlap and need to be considered simultaneously. Due to the lack of a tree-level diagram for $e^+e^- \to gg$, the limit of two collinear pairs $C_{12}C_{34}$ is regular. The maximal overlaps that we need to consider are therefore

$$C_{123}S_{12}C_{12}, \quad C_{124}S_{12}C_{12}, \quad C_{134}S_{34}C_{34}, \quad C_{234}S_{34}C_{34}. \tag{2}$$

These are all of the same type, so it will be sufficient to consider the representative $C_{123}S_{12}C_{12}$.

1.1 C(1,2)

We subtract the limit of a $q\bar{q}$ pair going collinear using the current

$$C_{12}^{\mu\nu} = \frac{T_R}{s_{12}}\left[-g^{\mu\nu} + 4z_{1,2}z_{2,1}\frac{k_{1,2}^{\mu}k_{1,2}^{\nu}}{k_{1,2}^2}\right], \tag{3}$$

in combination with the generalised rescaling mapping and ? variables.

1.2 C(1,2,3)

The triple-collinear counterterm for $q'\bar{q}'q$ is determined using the current [1]

$$C_{123} = \frac{C_F T_R}{2s_{123}^2}\left[-\frac{t_{12,3}^2}{s_{12}^2} + \frac{s_{123}}{s_{12}}\left(\frac{4z_{3,12} + (z_{1,23} - z_{2,13})^2}{z_{12,3}} + (1-2\epsilon)z_{12,3}\right) - (1-2\epsilon)\right], \tag{4}$$

where

$$t_{12,3} \equiv 2\frac{z_{1,23}s_{23} - z_{2,13}s_{13}}{z_{12,3}} + \frac{z_{1,23} - z_{2,13}}{z_{12,3}}s_{12}. \tag{5}$$

Once more we use ? variables and the generalised rescaling mapping.

1.3 S(1,2)

In order to construct the $q\bar{q}$ soft counterterm, we start from the form of the current used in [2] which reads

$$\frac{T_R}{s_{12}^2}\sum_{i,j}\frac{s_{1i}s_{2j} + s_{1j}s_{2i} - s_{12}s_{ij}}{s_{(12)i}s_{(12)j}}\mathbf{T}_i \cdot \mathbf{T}_j, \tag{6}$$

where the sum runs over all coloured partons of the reduced process and includes the case $i = j$. Before discussing partial fractioning, we carry out two modifications.

The first one, which may be inessential, is the replacement of the invariants $s_{(12)i} = 2p_{12} \cdot p_i$ and $s_{(12)j} = 2p_{12} \cdot p_j$ in the denominator with s_{12i} and s_{12j}. This operation modifies the counterterm by terms which are of higher order in the double-soft limit S_{12}, and therefore does not spoil the cancellation in the counterterm's defining limit. Using the triple invariants s_{12i} and s_{12j} seems convenient because they make denominators naturally match the ones of collinear counterterms (which cannot be changed because the modification $s_{123} \to s_{(12)3}$ is *not* higher-order in the triple-collinear limit).

The second modification is more important, and is related to the fact that the global factor s_{12}^{-2} may cause the counterterm to diverge in limits which do not require either s_{12i} nor s_{12j} to go to zero. More concretely, the contribution from the terms with $i = j$ reads

$$\frac{T_R}{s_{12}^2} \sum_i \frac{2s_{1i}s_{2i}}{s_{(12)i}^2} \mathbf{T}_i^2, \tag{7}$$

and *all* terms are divergent in the triple-collinear limit $12j$ for *any* j. Thus, although from eq. (7) one might be tempted to assign the i-th term to $12i$-collinear kinematics, every term needs to be distributed among all $12j$-collinear kinematics. To this end, we use colour conservation to move all terms off the diagonal

$$\sum_i \frac{2s_{1i}s_{2i}}{s_{(12)i}^2} \mathbf{T}_i^2 = -\sum_{i \neq j} \frac{2s_{1i}s_{2i}}{s_{(12)i}^2} \mathbf{T}_i \cdot \mathbf{T}_j = -\sum_{i \neq j} \left[\frac{s_{1i}s_{2i}}{s_{(12)i}^2} + \frac{s_{1j}s_{2j}}{s_{(12)j}^2} \right] \mathbf{T}_i \cdot \mathbf{T}_j. \tag{8}$$

In this sense, the kinematics that we assign do not follow the divergent structure of the invariant poles but rather the colour, in a similar way as proposed for geometric subtraction [4].

Our complete soft currents thus reads

$$S_{12} = \frac{T_R}{s_{12}^2} \sum_{i \neq j} \left[\frac{s_{1i}s_{2j} + s_{1j}s_{2i} - s_{12}s_{ij}}{s_{12i}s_{12j}} - \frac{s_{1i}s_{2i}}{s_{12i}^2} - \frac{s_{1j}s_{2j}}{s_{12j}^2} \right] \mathbf{T}_i \cdot \mathbf{T}_j. \tag{9}$$

It is easy to partial-fraction this expression into collinear kinematics; in the present implementation we use

$$1 = \frac{s_{12i}}{s_{12i} + s_{12j}} + \frac{s_{12j}}{s_{12i} + s_{12j}}, \tag{10}$$

for each term in the dipole sum, which leads to

$$S_{12}^{(i)} = \frac{T_R}{s_{12}^2} \sum_{i \neq j} \frac{s_{12i}}{s_{12i} + s_{12j}} 2is_{12i} + s_{12j} \left[\frac{s_{1i}s_{2j} + s_{1j}s_{2i} - s_{12}s_{ij}}{s_{12i}s_{12j}} - \frac{s_{1i}s_{2i}}{s_{12i}^2} - \frac{s_{1j}s_{2j}}{s_{12j}^2} \right] \mathbf{T}_i \cdot \mathbf{T}_j. \tag{11}$$

1.4 C(S(1,2),3)

$$C_{123}S_{12} = -\frac{2T_R}{s_{12}^2} \mathbf{T}_3^2 \left[\frac{s_{13}z_{2,13} + s_{23}z_{1,23} - s_{12}z_{3,12}}{s_{123}z_{12,3}} - \frac{s_{13}s_{23}}{s_{123}^2} - \frac{z_{1,23}z_{2,13}}{z_{12,3}^2} \right] \tag{12}$$

$$C_{123} - C_{123}S_{12} = \frac{C_F T_R}{s_{123}^2} \left[\frac{s_{123}}{s_{12}} \frac{z_1^2 + z_2^2}{z_1 + z_2} - 1 + \epsilon \left(1 + \frac{s_{123}}{s_{12}} (z_1 + z_2) \right) \right] \tag{13}$$

1.5 C(C(1,2),3)

$$\Pi^{\alpha\beta}(p, n) = -g^{\alpha\beta} + \frac{p^\alpha n^\beta + p^\beta n^\alpha}{p \cdot n}, \tag{14}$$

$$C_{(12)3}^{\alpha\beta,ss'} = \frac{C_F}{s_{123}} \delta_{ss'} \left[\frac{z_{12,3}}{2} \Pi^{\alpha\beta} - 2\frac{z_{3,12}}{z_{12,3}} \frac{k_{3,12}^\alpha k_{3,12}^\beta}{k_{3,12}^2} \right] \tag{15}$$

$$-g_{\alpha\beta}\Pi^{\alpha\beta} = (d - 2) - 2 = -2\epsilon \tag{16}$$

$$\begin{aligned} C_{(12)3}^{\alpha\beta,ss'} C_{12,\alpha\beta} = \frac{C_F T_R}{s_{12}s_{123}} \Bigg[& - z_{12,3}(\epsilon + 2z_{1,2}z_{2,1} + \text{gauge terms?}) \\ & + 2\frac{z_{3,12}}{z_{1,23}} \left(1 - z_{1,2}z_{2,1} \frac{(2k_{1,2} \cdot k_{3,12})^2}{k_{1,2}^2 k_{3,12}^2} \right) \Bigg] \end{aligned} \tag{17}$$

1.6 S(C(1,2))

$$S_{12}^{\mu\nu} = \sum_{(j,k)} 2\frac{p_j^\mu p_k^\nu + p_j^\nu p_k^\mu}{s_{12j}s_{12k}}\mathbf{T}_j\cdot\mathbf{T}_k = \sum_{(j,k)\neq 1,2}\left[\frac{p_j^\mu p_k^\nu + p_j^\nu p_k^\mu}{s_{12j}s_{12k}} - \frac{p_j^\mu p_j^\nu}{s_{12j}^2} - \frac{p_k^\mu p_k^\nu}{s_{12k}^2}\right]2\mathbf{T}_j\cdot\mathbf{T}_k, \tag{17}$$

$$S_{12}C_{12} = \frac{T_R}{s_{12}}\sum_{(j,k)\neq 1,2}\left[\frac{s_{jk}}{s_{12j}s_{12k}} - z(1-z)\left(\frac{2s_{j\perp}s_{k\perp}}{s_{12j}s_{12k}n_\perp^2} - \frac{s_{j\perp}^2}{s_{12j}^2 n_\perp^2} - \frac{s_{k\perp}^2}{s_{12k}^2 n_\perp^2}\right)\right]\mathbf{T}_j\cdot\mathbf{T}_k, \tag{18}$$

1.7 C(S(C(1,2)),3)

References

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[4] F. Herzog, Geometric subtraction for real radiation at NNLO, `https://www.ggi.infn.it/talkfiles/slides/slides4304.pdf`