Physica A 392 (2013) 6125–6134

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Detecting overlapping communities by seed community in weighted complex networks

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HIGHLIGHTS

- Introduce seed communities and absorbing degree.
- Propose a new algorithm.
- Find overlapping vertices.
- Give excellent experimental results.

ARTICLE INFO

Article history:
Received 4 March 2013
Received in revised form 24 June 2013
Available online 7 August 2013

Keywords:
Seed community
Weighted networks
Overlapping community
Absorbing degree

ABSTRACT

Detection of community structures in the weighted complex networks is significant to understand the network structures and analysis of the network properties. We present a unique algorithm to detect overlapping communities in the weighted complex networks with considerable accuracy. For a given weighted network, all the seed communities are first extracted. Then to each seed community, more community members are absorbed using the absorbing degree function. In addition, our algorithm successfully finds common nodes between communities. The experiments using some real-world networks show that the performance of our algorithm is satisfactory.

1. Introduction

Systems that comprise many interacting parts with the ability of generating a new quality of macroscopic collectively can be characterized as complex systems [1]. Complex systems are commonly modeled as complex networks or graphs [2–6]. Here the entities of the system are represented by the vertices of the graph and the interactions between the vertices are represented by the links.

Recently, the characterization of community (or module) structures in complex networks has received considerable attention, and it is widely believed that large networks in the real world are composed of many communities, whose nodes often cluster into tightly knit groups with high density of within-group edges and low density of between-group edges [7–10]. The vertices can be into group or cliques according to their structural position in the networks. In some community, vertices sharing a large number of links with other vertices may have important role enforce stability within the community; vertices settled at the boundaries between communities play an important role of mediation and lead to the relationships and exchange between different communities [7].

Up to now, a large number of computer algorithms have been proposed to detect community structures in complex networks. Loosely we can classify community detection algorithms into greedy algorithm, spectral graph partitioning and clustering methods, for example, the Kernighan–Lin algorithm which divides the networks according to the optimization

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0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.07.066

of the number of within and between-community edges using a greedy algorithm [11] and a spectral bisection algorithm based on the eigenvectors of the Laplacian matrix of a graph [12]. Another category is Hierarchical clustering which detects a community based on the similarity or intensity between vertices, where it is divided into agglomerative method and divisive method by adding or removing edges [13].

In 2002, the most popular algorithm for community detection called GN algorithm was proposed by Girvan and Newman [8]. This algorithm recursively removes edges with the highest edge betweenness of shortest path between pairs of vertices that run along the edge, and then constructs a hierarchical tree. To estimate the accuracy of this method, they introduced a quantitative measure for the quality of network division, called modularity (represented by the Q function) [14]. In order to deal with large real-world networks, Newman proposed a fast algorithm based on the optimization of the modularity in Ref. [15] with the time complexity $O(n^2)$.

Various other related approaches have been proposed based on the analysis of successive neighborhoods, extracting the initial seed and many different ideas in Refs. [16–24]. The computational complexity of more accurate algorithms is often high. Based on this deduction, here we propose a fast and efficient algorithm for detecting overlapping community structures in the weighted networks. The key strategy of our algorithm is to mine all the seed communities. Using the absorbing degree function, our algorithm computes the absorbing degree between the neighboring vertices of the seed community and the seed community. Then according to the value of the absorbing degree, the initial seed communities constantly enlarge. The other advantage of our algorithm is that common vertices can be found accurately during the community structure detection.

We organized this paper as follows. In Section 2 we explain the crucial concepts of our new overlapping community detection algorithm. In Section 3 we describe how to extract the seed community and propose a new algorithm in detail. Experimental results of synthetic and some real-world networks are shown in Section 4. Finally, our conclusions appear in Section 5.

2. Basic concepts

2.1. Community structure and common vertex

Along with the physical significance of the network and statistical properties, the community structure is a common property of many complex networks [8]. The most distinguishing feature of complex networks is connectivity degree density. Generally community structures in real networks, the nodes within a group have higher edge density than the nodes among groups. Two communities may overlap when a node joins different communities. For example, in Fig. 1, there are four communities, denoted by circle, square, triangle, and octagon, respectively. Node 6 belongs to the circle community as well as the square community. It is a common phenomenon that a vertex should belong to several communities in a real-world complex network at the same time. We call these kinds of vertices and their corresponding communities as common vertices and overlapping community structures respectively. In unweighted networks, a common vertex has the same number of edges to different communities. For example Fig. 2 shows the sketch of a weighted network with such a community structure, where the dotted circles represent a vertex, and each edge characterizes the relations between a pair of vertices with edge weight.

2.2. Edge weight

A network can be modeled as $G(V, E)$, where V represents the set of n vertices or nodes, and E is a set of m links. In this work we shall use unweighted or weighted, but undirected complex networks. For given vertices $i \in V$ and $j \in V$, the edge weight E_{ij} is defined as

$$E_{ij} = \begin{cases} w_{ij}, & \text{if } i \text{ and } j \text{ are connected in weighted networks} \\ 1, & \text{if } i \text{ and } j \text{ are connected in unweighted networks} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

2.3. Vertex weight

The vertex weight V_i of vertex i in networks is defined as

$$V_{i(i\in V)} = \begin{cases} \sum_{j\in V} E_{ij}, & \text{if } j \text{ is the neighboring vertices of } i \text{ in weighted networks and unweighted networks} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

For example, in Fig. 2, the vertex weight of vertex 3 equals $0.2 + 0.5 + 0.6 + 0.7 = 2.0$, the vertex weight of vertex 6 is 1.4, etc. If the network is unweighted, the vertex weight of vertex 3 is 4. The vertex weight of a vertex in an unweighted network is equal to the total number of edges connected with this vertex, which is identical to the vertex degree.

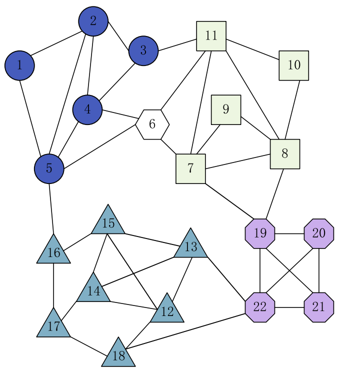

Fig. 1. A small network with overlapping community structure.

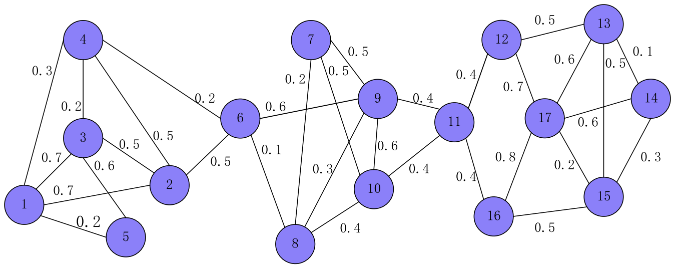

Fig. 2. A weighted network with overlapping community structure.

2.4. Absorbing degree

If we extract some vertices from the real networks through some methods as the seed communities $[C_s^x]$, we can find neighboring vertices of each seed community and we develop them to achieve the community structures. Each seed community absorbs its neighbors by different degrees. We can define the absorbing degree $A\left(C_s^x, i\right)$ with the simple expression

$$A(C_s^x, i) = \frac{\sum\limits_{i \in V, j \in C_s^x} E_{ij}}{V_i} \tag{3}$$

where $\sum_{i \in V, j \in C_s} E_{ij}$ is the total number of edges between the seed communities $[C_s^x]$ and its neighboring vertex i, V_i is the degree vertex i, and C_s^x is one of the seed communities.

From Eq. (3), $A(C_s^x, i)$ reflects how degree the neighboring vertex i belongs to C_s^x. In order to explain the process of computing, consider Figs. 1 and 2, which are an unweighted complex network with 22 vertices and a weighted complex network with 17 vertices. The data tables have three columns describing the network as shown in Tables 1 and 2. Each row of this table contains the seed communities C_s^x (in Section 3.1 we explain how to extract the seed communities), the neighbors of the seed communities *Nei* and the absorbing degree $A(C_s^x, i)$.

From Tables 1 and 2, we obtained all the seed communities first and then two seed communities absorbed new vertices by the $A(C_s^x, i)$ function. We can consider a threshold value μ for the value of the absorbing degree. In this paper, the vertex will be swallowed up by only one community or the vertex will be a common vertex. In Table 1, we got $A(C_s^1, 16) < A(C_s^3, 16)$ and $A(C_s^1, 6) = A(C_s^2, 6) = 0.5$, so vertex 16 belongs to C_s^3 and vertex 6 is a common vertex. Indeed, for $0.5 < \mu \leq 1, i \in C_s^x$; for $\mu = 0.5, i \in$ common vertex; for $\mu < 0.5, i \notin C_s^x$. Finally, we obtained four communities {1, 2, 3, 4, 5}, {7, 8, 9, 10, 11}, {12, 13, 14, 15, 16, 17, 18}, {19, 20, 21, 22} and one common vertex {6} in Fig. 1 with 22 vertices. In Fig. 2 with 17 vertices, we got three communities {1, 2, 3, 4, 5}, {7, 8, 9, 10}, {12, 13, 14, 15, 16, 17} and two common vertices {6, 11}.

2.5. Modularity

To find which algorithm performs well, Newman and Girvan [14] defined a measure of the quality of a particular division of a network called the modularity Q, computed using Eq. (4), e.g.

$$Q = \sum_i (e_{ii} - a_i^2) = Tre - \left\| e^2 \right\|, \tag{4}$$

Table 1
Absorbing degree for 22 vertices.

C_s^x	*Nei*	$A(C_s^x, i)$
$C_s^1 = \{2, 4, 5\}$	1	1
	3	0.667
	6	0.5
	16	0.333
$C_s^2 = \{7, 8, 11\}$	6	0.5
	9	1
	10	1
	19	0.25
$C_s^3 = \{12, 13, 14, 15\}$	7	0.2
	16	0.667
	17	1
	18	1
	22	0.25
$C_s^4 = \{19, 22\}$	8	0.2
	13	0.25
	20	1
	21	1

Table 2
Absorbing degree for 17 vertices.

C_s^x	*Nei*	$A(C_s^x, i)$
$C_s^1 = \{1, 2, 3, 4\}$	5	1
	6	0.5
$C_s^2 = \{9\}$	6	0.5
	7	1
	8	1
	10	1
	11	0.5
$C_s^3 = \{17\}$	12	0.857
	13	1
	14	1
	15	1
	16	0.765

where e_{ii} is the fraction of all edges in the network that link vertices in community i to vertices in community j, which the communities are newly generated. a_i is the fraction of edges which have one or both vertices inside i. The maximal modularity corresponds to the partition that comprises the most inner edges within modules and the least outer edges between modules. Modularity optimization fails to identify modules smaller than a scale which depends on the total number of links of the network and on the degree of interconnectedness of the modules, even in cases where modules are unambiguously defined [25], and cannot tackle the overlapping community structure. In the case of weighted networks, Newman [26] proposed an extended modularity defined as

$$Q = \frac{1}{2m} \sum_{i,j} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j), \tag{5}$$

where A_{ij} represents an adjacency matrix with the weight of the edge between i and j, $k_i = \sum_j A_{ij}$ is the sum of the weights of the edges attached to vertex i, c_i is the community to which vertex i is assigned, the δ-function $\delta(u, v)$ is 1 if $u = v$ and 0 otherwise and $m = \frac{1}{2} \sum_{ij} A_{ij}$.

We proposed another extended modularity based on the absorbing degree, to quantify the overlapping community structure in unweighted or weighted networks. Then we rewrite the modularity in Eq. (5) as

$$Q_a = \frac{1}{2m} \sum_{u} \sum_{i,j} \left[A_{ij} - \frac{V_i V_j}{2m} \right] A\left(C_s^x, i\right) A\left(C_s^x, j\right), \tag{6}$$

where A_{ij} represents an adjacency matrix with the weight of the edge between i and j, V_i is the vertex weight of vertex i in weighted networks, and C_s^x is one of the seed communities in the networks. $A\left(C_s^x, i\right)$ is 1 if vertex i completely belongs to C_s^x. If vertex i does not completely belong to C_s^x, obviously $A\left(C_s^x, i\right)$ satisfies the following conditions:

$$0 \le A\left(C_s^x, i\right) < 1.$$

Otherwise, $A\left(C_s^x, i\right)$ is 0; note that Q_a reduces to Q in Ref. [11] when each vertex belongs to only one community (readers can refer to Clauset et al. [18] for details).

3. Algorithm

3.1. Extracting the seed community $[C_s^x]$

Analyzing the structure of Fig. 1, the vertex degree of vertices 2, 4 and 5 is four, that is to say the total number of edges connected with these vertices is the same. Moreover, these three vertices constitute a fully connected graph. Beyond this, we also find that vertices 7, 8 and 11 have the same characteristics as vertices 2, 4 and 5. If some vertices have the two characteristics as follows:

(1) Some vertices have the same vertex degree.
(2) The vertex weight of their neighboring vertices is not bigger than their vertex weight.
(3) Some vertices constitute a fully connected graph

are likely belong to the core of some community. That is to say that they have an important role during detecting the community structures in real networks. With this idea in our assumption, we can call such vertices as the seed communities $[C_s^x]$. The number of seed communities $[C_s^x]$ may differ in different real networks with specific characteristics. The number of vertices in a seed community can be one or more than one, but is less than the total number of vertices in a complex network. The whole process of exploration overlapping community structure, primary task is to excavate of seed community $[C_s^x]$. The well-known Bron–Kerbosch [27] algorithm can find the maximal clique with highest link density. In this paper, we improved the Bron–Kerbosch algorithm to extract all the seed communities with two characteristics from real networks.

3.2. EM-BOAD algorithm

We describe our algorithm as the following.
Algorithm Name: Extended Modularity Based On Absorbing Degree (EM-BOAD) algorithm

Input: the vertices and edges of one complex network, and the edge weight E_{ij}
Output: community structures in the complex network
Begin
a. Compute the vertex weight $[V_i]$.
b. Extract the seed communities $[C_s^x]$ as shown in Section 3.1.
c. ***While*** seed community! = Null
1. Find all neighboring vertices of every seed communities.
2. Use Eq. (3) to compute the absorbing degree $A\left(C_s^x, i\right)$ between every seed community and its neighboring vertices, using $A\left(C_s^x, i\right)$ to judge whether expand the seed communities.
End While
If not absorbed vertices! = Null
1. Disconnect links between these vertices and absorbed vertices, then not absorbed vertices form a new relatively independent community.
2. ***Goto*** b.
End IF
Use Eq. (6) to calculate Q_a for all the new communities obtained after above steps.
End

3.3. Computational complexity

Comparing with the GN algorithm, in order to detect community, we need to find the seed communities from a complex network, and then find all neighboring vertices of every seed community. These two parallel steps have computational complexity of at most $O(2n) = O(n)$. In the EM-BOAD algorithm, the third routine is an essential step where the absorbing degree of the seed community and its neighboring nodes plays a vital role during extracting community structures in the weighted networks. Each vertex only decides its own fate to move preferably in a direction that leads to an increase of Q_a without a pre-existing global plan. It will take at most $O(n)$ computation time. Finally, the worst computing time is upper bound at most $O(n^2)$.

4. Applications and results

The detecting overlapping community algorithm proposed in this paper was implemented using Java programming language running on a PC with 2.67 GHz processor, 4 GB memory and Win7 operating system. Now we applied the EM-BOAD algorithm using computer-generated networks and some real-world networks, that is, the friendship network from Zachary's karate club study, dolphin's associations, college football and netscience-coauthor network etc.

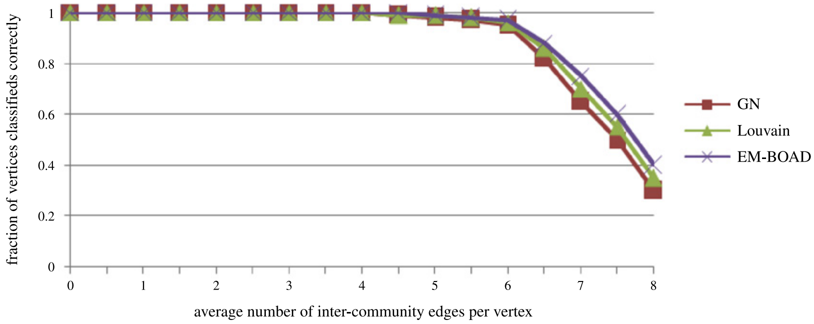

Fig. 3. The fraction of vertices correctly identified by three algorithms in the computer-generated networks.

4.1. Synthetic data

In 2002, Girvan and Newman presented a method for benchmarking community detection algorithms using simulated data [8]. We have also tested the performance of our algorithm by applying it on a set of computer-generated networks. Each network was constructed with 128 vertices with four communities of 32 vertices. Edges were placed independently at random between vertex pairs with probability z_{in} for an edge to fall between vertices in the same community and z_{out} to fall between vertices in different communities. The values of z_{in} and z_{out} were chosen to make the expected degree of each vertex equal to 16. The accuracy of the method is evaluated by measuring the fraction of correctly identified nodes and the normalized mutual information. Girvan and Newman showed the modularity round 0.5 with Eq. (4) in 2004.

We have also validated our algorithm by applying it on synthetic data. From experimental results, the networks had four communities by the EM-BOAD algorithm, yielding the modularity Q_a value of 0.472. In Fig. 3 we show the fraction of vertices correctly detected into the four communities in the computer-generated network by GN algorithm [8], Louvain algorithm [28] and our EM-BOAD. As the figure shows, both the three algorithms are more than 90% of all vertices detected correctly from $z_{out} = 0$ to $z_{out} = 6$. When z_{out} is bigger than 6, the accuracy begins to deteriorate markedly. The EM-BOAD algorithm does however perform noticeably better than the GN algorithm and the Louvain algorithm, especially where more vertices are falling into different communities. Although some method had been tested on LFR benchmark graphs [29], the EM-BOAD algorithm has been successfully tested on the other benchmark proposed in Ref. [30]. The normalized mutual information is nearly 1 for the macro-communities with a mixing parameter k_3 up to 33. It reaches 0.47 when the mixing parameter is around 53.

4.2. Zachary

We now turn to applications of our EM-BOAD algorithm to some real-world networks. Our first of such example is the well-known "karate club" study of Zachary [31] which is widely used as a benchmark for new algorithms in detecting community structures. Zachary observed social interactions between the members of a karate club at an American university over the course of two years. He constructed the network of 34 members of the karate club as vertices and 78 edges representing friendships among members of the club. Accidentally, a dispute arose between the club's administrator and its principal karate teacher whether to raise club fees, and as a result the club eventually split into two groups, forming two smaller clubs, centered on the administrator and the teacher, respectively.

Many authors have tested their approaches to this unweighted network for community detection and accuracy evaluation. First we got two seed communities {1} and {34} from Zachary's karate club during testing our algorithm on this network. Then we provided the result of two overlapping communities mined and vertex 3 in common within less than 10 ms based on our EM-BOAD algorithm, yielding the Q_a value of 0.406 as shown in Fig. 4. This network is also discussed with two or four communities found by some other algorithms. The algorithm [32] detected four non-overlapping communities. In Ref. [33], there are two communities and one common vertex with 10, and the modularity Q_o is 0.4214. Q (Eq. (4)) is 0.35 in the GN algorithm of Ref. [14], and two communities and one common vertex 3 were detected. In this paper, we use the modularity Q_a in the GN algorithm, and find that Q_a in the GN algorithm is greater than in the EM-BOAD algorithm. So Q_a is more accurate when exploring the community structure. Not only are the two groups well separated according to the reality but also our algorithm can probe the common nodes. Our algorithm gives rise to more precise answers than other competing algorithms.

4.3. Dolphin's associations

This data set is taken from the social network of 62 dolphins living in Doubtful Sound, New Zealand, and was compiled by Lusseau [34]. It describes the associations between dolphin pairs being the statistically significant frequent association. Three overlapping communities are detected by the EM-BOAD algorithm with modularity Q_a of 0.521, which are represented by blue, green and red in Fig. 5. The experimental results show that two common vertices 9 (Double) and 40 (SN89) exist. In

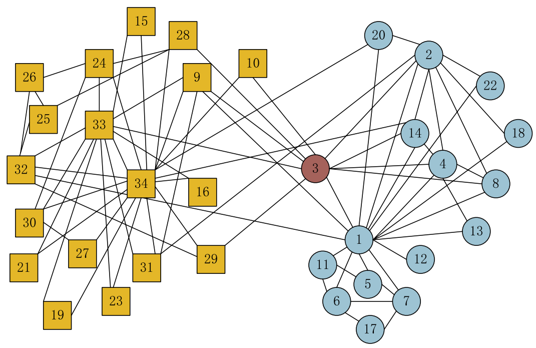

Fig. 4. Two overlapping communities in friendship network from Zachary's karate club study.

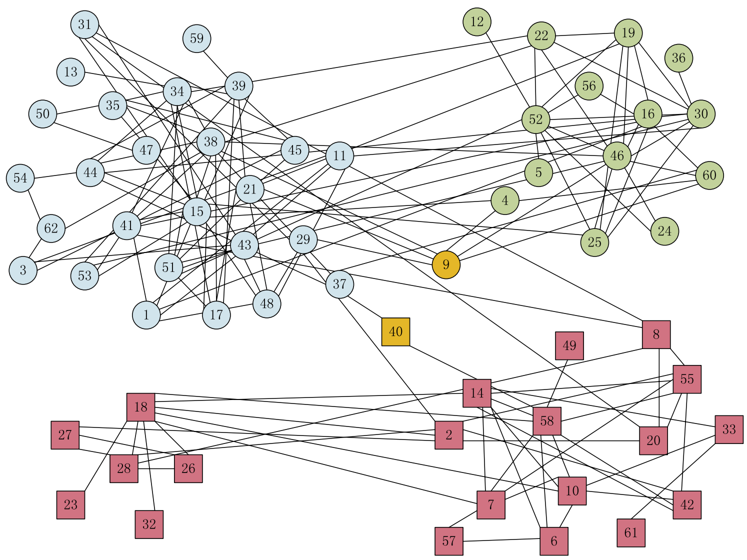

Fig. 5. Three overlapping communities of dolphin's living network. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the GN algorithm of Ref. [14] and the Chen algorithm of Ref. [33], this network splits naturally into two large communities and four communities with the modularity Q (Eq. (4)) of 0.38 and modularity Q_o of 0.5478 respectively.

4.4. College football

As a further demonstration of the EM-BOAD algorithm, we turn to a football network which we look at as a representation of the schedule of Division I games for the 2000 season. This network contains 115 nodes and 615 edges; the vertices in the graph represent teams (identified by their college names) and edges represent regular season games between the two teams they connect. The network can be divided into 12 communities according to athletic conferences. Games are more frequent between members of the same communities than between members of different communities.

Using this network as a benchmark to test the performance of the EM-BOAD algorithm, almost all teams are correctly grouped with the other teams in their conference. There are a few independent teams that do not belong to any conference; these tend to be grouped with the conference with which they are most closely associated. For example, the Sunbelt conference is broken into three pieces and grouped with members of the Western Athletic conference and Southeastern conference. This happens because the Sunbelt teams played nearly as many games against some Western Athletic teams as they did against teams in their own conference. 12 groups and two independent teams 81 and 43 with orange heptagon are detected by our EM-BOAD algorithm, as shown in Fig. 6, with the modularity Q_a of 0.562. In Ref. [15], the fast algorithm detected 6 communities with the modularity of partition of 0.546. The algorithm introduced in this paper has higher efficiency than the fast algorithm [18]. The Chen algorithm [33] splits naturally into 13 non-overlapping communities with the modularity Q_o of 0.5868. Naturally, our algorithm fails in cases like these where the network structure genuinely does not correspond to the conference structure. In all other respects however it performs remarkably satisfactory.

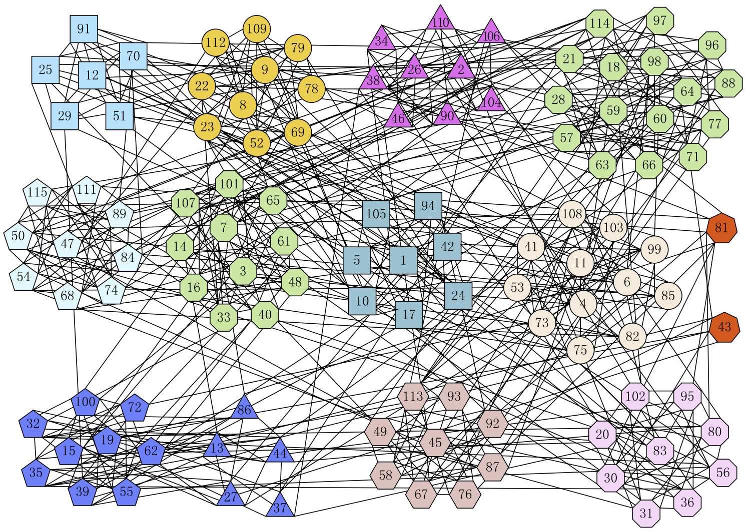

Fig. 6. Twelve overlapping communities of college football network.

Table 3
Execution time and modularity on some real-world networks.

Data	Unweighted networks			Weighted networks	
	Karate[a]	Dolphin[a]	Football[a]	Netscience[a]	Cond-mat-2003[a]
Nodes/links	34/78	62/159	115/615	1589/2742	31163/120029
GN algorithm	0.406/0 s	0.482/15 s	0.604/95 s	0.812/138 s	0.416/1256 s
Louvain algorithm	0.42/0 s	0.467/0 s	0.594/0 s	0.821/0 s	0.473/175 s
Chen algorithm	0.4214/0 s	0.5478/0 s	0.5868/0 s	0.8531/1 s	0.4521/192 s
Swam algorithm	0.425/0 s	0.536/0 s	0.571/0 s	0.863/0 s	0.474/8 s
EM-BOAD algorithm	0.418/0 s	0.521/0 s	0.562/0 s	0.842/9 s	0.4589/187 s

[a] http://www-personal.umich.edu/~mejn/netdata/.

4.5. Netscience-coauthor network

This benchmark is taken from a weighted network of scientists working on network theory and experiment, as compiled by M.E.J. Newman in May 2006, which was about coauthorships between scientists who are themselves publishing on the topic of networks. There are a total of 1589 scientists and 2742 coauthorships in this collaboration network. We just take these scientists previously published as the test benchmark. 282 overlapping communities are detected by running our EM-BOAD algorithm with the modularity Q_a of the partition as high as 0.842, and many of these common vertices between communities are cross-disciplinary researchers. In Fig. 7, we just showed the test results of 397 scientists. The dashed edges denote some scientists being not in a same community had cooperation with each other and vertex 311 is common vertex.

In order to further verify the performance of our algorithm, we have applied it on a number of test-case networks that are commonly used for efficiency comparison and we have compared it with four community detection algorithms (see Table 3). This table gives the performances of the GN algorithm [14], Louvain algorithm [28], Chen algorithm [33], Swam algorithm [32], and our EM-BOAD algorithm for detecting community structures in networks of various sizes. For each algorithm/network, the table displays the modularity that is achieved and the computation time. The Chen algorithm used the modularity Q_o [33], and the other algorithm used Eq. (6) for computing the modularity during the experiments.

5. Conclusions

A new EM-BOAD algorithm for detecting overlapping community structures from weighted networks has been proposed, which has considerable improvements in comparison with the previous algorithms. The key aspect of the proposed algorithm is extracting the seed community with two characteristics. During computation of our algorithm, the critical step was to compute absorbing degree between the seed community and its neighbors. Then the seed community absorbs new members and a larger seed community is expanded. The EM-BOAD algorithm successfully mined common vertex. Some classical real-world networks were used to test the performance of the EM-BOAD algorithm. For Karate club's friendship network, two overlapping communities with one common vertex were found correctly by the EM-BOAD algorithm. For a dolphin associate network, three overlapping communities were founded, whose modularity is 0.521. We detected 12 communities from a college football network, whose modularity is 0.562. We have also tested our algorithm by applying it

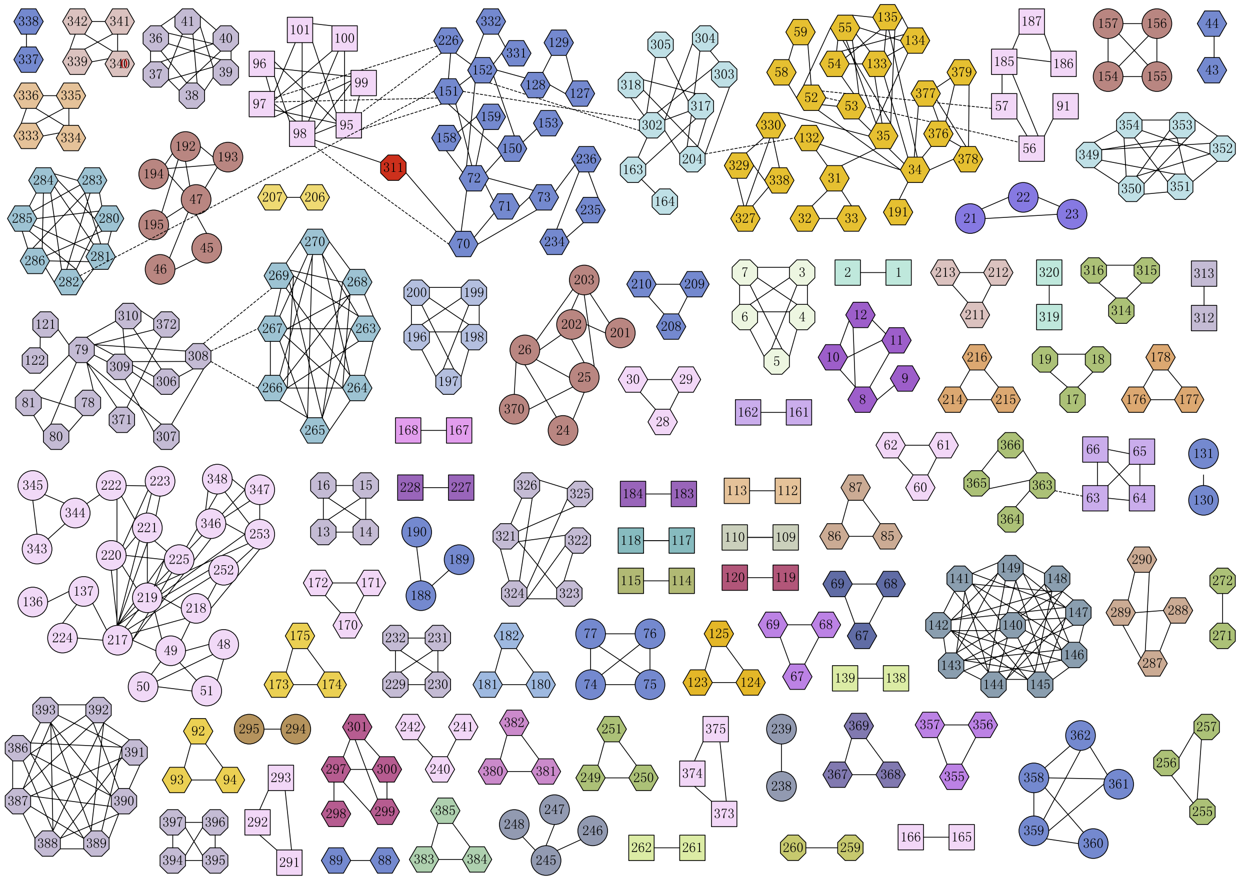

Fig. 7. 83 overlapping communities from netscience-coauthor network.

on ad-hoc networks that are composed of 128 vertices, which are split into four communities of 32 vertices each. The results show that the EM-BOAD algorithm performed noticeably better than the GN algorithm and the Louvain algorithm, especially more vertices are falling into different communities. Some large networks were also tested by the EM-BOAD algorithm and acceptable results were obtained, such as Cond-mat-2003. Experimental results demonstrate that the proposed EM-BOAD algorithm is rather efficient for detecting overlapping communities in the weighted networks.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (Nos: 61173183, 60973152, and 60573172), the Doctoral Program Foundation of Institution of Higher Education of China (No: 20070141014), Program for Liaoning Excellent Talents in University (No: LR2012003), the National Natural Science Foundation of Liaoning province (No: 20082165) and the Fundamental Research Funds for the Central Universities (No: DUT12JB06).

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