# Revisiting PV Hosting Capacity: the Role of Dynamic Curtailment and Marginal $CO_2$ Emissions

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***Abstract*—Rapid deployment of residential solar PV panels, driven by lower costs and decarbonization policies, risks overloading substation transformers in electric distribution systems during low-load, sunny days. This paper explores how selectively curtailing solar photovoltaic (PV) energy allows for greater PV capacity deployment without overloading transformers. The study presents intelligent curtailment strategies that balance increased PV capacity with live grid conditions, ensuring residential transformers can handle the load. It also considers the financial impact of curtailment on PV owners, such as lost revenue and potential benefits from marginal CO2 emissions. The curtailment approach proposed prioritizes fairness among PV array owners, ensuring equitable distribution of curtailment across the PV assets. This method aims to boost clean energy generation while managing the technical and economic challenges of integrating more solar PV into the grid. The benefits are illustrated with simulation-based analysis that employs realistic minutely solar PV data and synthetic demand data over a full year for a hypothetical feeder with N=300 houses and varying PV assets.**

***Index Terms*—solar PV, hosting capacity, curtailment, CO2 emissions.**

## I. Introduction

As the cost of solar PV decreases, the penetration of residential solar PV arrays is increasing in distribution systems. At large penetrations, these solar PV systems can create complex technical challenges for distribution systems operations, impact reliability of power distribution and transmission, or require expensive capital expenditures for utilities [1]. For example, feeding too much solar power to the grid could cause voltage overloads in the distribution system or overload distribution substation transformers, leading to blackouts. Over-voltages may also compromise PV systems, negatively impacting PV power delivery reliability and PV owner revenue [2]. To avoid these challenges, distribution system operators (DSOs) have produced solar interconnection studies and developed methods to determine solar PV hosting capacities (HCs) for different feeders in their system [3].

Such HC ratings are generally static (i.e., fixed from year to year) and computed under conservative grid operating conditions (e.g., low demand, no reactive power compensation, maximum PV output conditions). Once the HC limit has been reached, households hoping to install solar PV must shoulder the cost of grid upgrades, or be rejected from installing panels altogether. In addition, the overriding assumption is that solar PV must not be curtailed [4].

There exists a variety of approaches for overcoming (conservative) hosting capacities. Conventionally, HC has been increased by upgrading the grid infrastructure with larger transformers or new conductors to transmit more power [5]. However, new distribution substation transformers are expensive and challenging for utilities to install and represent another static solution to a fundamentally dynamic problem (i.e., solar PV installations are escalating and sun irradiance varies based on time and location). Many studies recommend preventing voltage overloads caused by solar PV by controlling reactive power injections via grid-connected inverters or on-load tap-changing (OLTC) transformers [6], [7]. These methods effectively treat active power generated from solar PV as an uncontrollable disturbance and compute reactive-power set-points for inverters (e.g., volt-var control [8]). Some of these methods are centralized (and employ a utility-centric view with DER and feeder data) while others are more distributed and leverage local sensing, control and communication for coordination.

Besides controlling reactive power to overcome grid challenges associated with exceeding the conservative, static HC ratings, an active power curtailment strategy can also be employed. In active power schemes, the PV array's active power injection into the grid is reduced by the inverter [9]. Fully decentralized active power control can be achieved by deploying local control policies on PV inverters that reduce the output of the PV array when the grid frequency increases or if locally measured line voltage overloads occur (e.g., volt-watt control) [7].

Much of the literature has focused on actively controlling grid-connected resources to regulate grid conditions [10]. The motivation for this work is based on recent studies showing that low levels of active power curtailment (e.g., 5% of annual energy) can lead to a large increases in PV generation capacity (PVGC) (e.g., $>$200% increase) [11] Thus, this paper presents a novel coordination scheme for dynamically curtailing active power from residential solar PV systems, while limiting net monetary losses among households (i.e., reduce the necessary incentives for any such scheme). One foundational purpose of curtailment is to avoid the situation of a single household having to shoulder the substantial cost of grid upgrades. Therefore, ensuring that households are curtailed equitably is crucial in any dynamic curtailment scheme and minimizes

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lost revenue among participants (from curtailing energy).

To justify these curtailment costs, we consider marginal emissions and the (socialized) value of $CO_2$ reductions from additional solar PV generation capacity, which has not been examined in solar PV curtailment to the authors knowledge. We explicitly consider residential PV systems that are each allotted a fixed budget for total curtailment over some period (e.g., monthly allotment). This concept of a *curtailment budget* ensures that the utility can easily estimate the available aggregate curtailment resources while bounding the PV system owner's total curtailment risk (e.g., maximum lost revenue). Thus, we integrate this budget into a novel active PV curtailment scheme with a goal of increasing the PVGC.

Next, we present our system model for the curtailment scheme in Section II.

## II. System Model

Consider a distribution substation transformer with an apparent power limit $L$ supplying $N$ households, each of which has a solar PV array of power capacity $\overline{P}_n \geq 0$. The total PV array capacity deployed under the transformer is $\overline{P} = \sum_{n=1}^{N} \overline{P}_n$. The HC is an immutable value determined by the transformer power limit, $L$, therefore, to ensure that the transformer is never overloaded, the following conservative condition must hold:

$$\overline{P} \leq L. \tag{1}$$

Clearly, (1) is conservative since it neglects any demand and assumes no PV generation can be curtailed. When considering more dynamic conditions, it is possible that $\overline{P} > L$ without overloading the transformer. The difference between HC and PVGC, is analyzed in in this paper to characterize how increasing $\overline{P}$ affects necessary curtailment and corresponding energy cost and CO2 impacts.

At each time-step $k$ of step size $\Delta t$ seconds, each household $n$ has net injection of $P_n[k] - d_n[k]$, where $P_n[k] \in [0, \overline{P}_n]$ is the active solar PV power injected and $d_n[k] \geq 0$ is the active demand consumed. Thus, within the conservative HC limits, it is straightforward to compute the net injection (expected) from solar PV and demand, since no curtailment is necessary:

$$P^L_{\text{net}}[k] := \sum_{n=1}^{N} P_n[k] - d_n[k], \tag{2}$$

where the conventional limit on total deployed solar PV generation capacity is $\overline{P}^L := L + \min_k \sum_n d_n[k]$.

To facilitate active power curtailment of solar PV, PV owners opt in to the dynamic curtailment scheme and are guaranteed that their total monthly energy losses (from curtailment) will be limited to some acceptable budget $A_n$, the monthly curtailment budget (or allotment) (e.g., \$/month).

A curtailment signal is sent by the DSO to all PV arrays (e.g., inverters) to beget curtailment at time $k$, $C_n[k] \in [0, P_n[k]]$. Clearly, if $\overline{P}_n = 0$, then house $n$ has no PV array and $P_n[k] = 0 = C_n[k]$ for all $k$. Thus, the curtailment signal depends on the (expected) overload on the transformer, which is a function of the total (net) injection seen by the transformer:

$$P_{\text{net}}[k] := \sum_{n=1}^{N} P_n[k] - C_n[k] - d_n[k], \tag{3}$$

which holds for any solar PV generation capacity $\overline{P}$. Thus, if $\overline{P} > L$ and $C_n[k] = 0\ \forall\ n,k$, then $P_{\text{net}}[k] > P^L_{\text{net}}[k]$ (even if the transformer may be overloaded in the case of $\overline{P}$).

The transformer is then overloaded at time step $k$, if $O[k] := P_{\text{net}}[k] - L > 0$. To mitigate this overload, we can dynamically curtail solar PV generation to reduce any (expected) overload, which means that the aggregate system curtailment necessary from the fleet of PV arrays is:

$$C[k] := \sum_{n=1}^{N} C_n[k]. \tag{4}$$

Clearly, if $O[k] > 0 \Rightarrow C[k] > 0$, then the curtailment scheme mitigates any overload on the transformer.

Since curtailment command $C_n[k] > 0$ represents lost energy at time-step $k$ to PV array owner $n$, there is a cost associated with curtailment. To quantify the cost of curtailment, we characterize the cost of curtailment at time-step $k$ as

$$X_n[k] := p C_n[k] \Delta t / 3600, \tag{5}$$

where $p$ is the marginal price of energy (in \$/kWh). Clearly, over a time interval of $K$ time-steps the total cost of curtailment for PV array $n$ is given as $\sum_{k=1}^{K} X_n[k] \leq A_n$, i.e., the total cost of curtailment over the time interval (e.g., one month) must be within the (monthly) budget, $A_n$.

Since all $N$ customers receive a monthly allotment, the total realized cost of the dynamic curtailment scheme can be defined as

$$X = \sum_{n=1}^{N} \sum_{k=1}^{K} X_n[k] \leq \sum_{n=1}^{N} A_n. \tag{6}$$

This upper bound on realized costs of curtailment allows the utility (or public utility commission) to manage costs or risks. That is, if insufficient budgets are allocated to each PV array owner, then the total curtailment resource would become saturated in its ability to mitigate the transformer overload, i.e., $C[k] < O[k]$ (despite the curtailment scheme). However, even in this (extreme) case, the total budget allocations $\sum_n A_n$ would still inform us on how to flatten (i.e., minimize) the overload over a day to reduce risks (of transformer damage). In addition, if $X << \sum_n A_n$ and the budget for curtailment is too large, then the utility can correct billing at the end of fiscal cycles to reflect realized curtailment costs, $X$. This reduces the financial risks of the curtailment scheme to the utility.

Now, under the assumption that the budget allocation is large enough to guarantee that overload $O[k] = 0$ for all $k$ with $C[k] \geq 0$, we need to consider how curtailment $C[k] > 0$ can be cooperatively achieved by the $N$ PV array owners. Towards that objective, we consider the proportional coefficients

$$\alpha_n := \overline{P}_n / \overline{P} > 0, \tag{7}$$

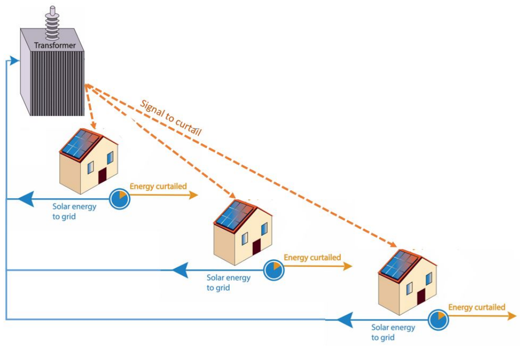


Fig. 1. Direct Control curtailment system diagram

where $\overline{P} := \sum_{n=1}^{N} \overline{P}_n$ and $\sum_{n=1}^{N} \alpha_n = 1$. Next, we use the coefficients $\alpha_n$ to motivate a direct control curtailment (DCC) scheme.

## III. Method: Direct Control Curtailment (DCC)

Given a measured overload at time-step $k$, $O[k] > 0$, a corrective action must be taken that involves curtailment $C[k] > 0$ to drive $O[k] \to 0$. This, $C[k] = O[k]$ is broadcast to all PV arrays. A system diagram of the direct control curtailment feedback system is depicted in Fig. 1. To determine how much to curtail from a single PV array, we use coefficients $\alpha_n$ to disaggregate $C[k]$ into $C_n[k]$ for all $n$:

$$C_n[k] = \alpha_n C[k]. \tag{8}$$

Curtailment takes place on a minute by minute basis. If the calculated amount of energy to curtail exceeds the amount of energy the household is producing, $C_n[k] > P_n[k]$, all the household's energy produced for the given time step is curtailed ($C_n[k] = P_n[k]$) as long as curtailing this amount does not exceed the households allotment, i.e., $\sum_{i=1}^{k} X_n[i] \geq A_n$. However, if $\sum_{i=1}^{k} X_n[i] \geq A_n$ is achieved at time $k$ the household does not participate in the dynamic curtailment scheme. Equation (7) is then updated and re-scaled such that the coefficients $\alpha_n$ reflect the updated capacities of all available PV arrays. PV array $n$ enters back into the curtailment scheme once its allotment is replenished.

## IV. Numerical Case Study

We use real, minutely solar PV data from a single 330 W solar panel over a full year (i.e., 1440 time-steps per 24-hour day for 365 days with $K = 1440 \times 365 = 525,600$). An example of data from the single solar panel for a few specific days is depicted in Fig. 2. The solar PV panel data is scaled up to $N_{\text{pnl}} = 15 - 35$ panels per PV array (for $N = 300$ PV arrays) to form time-varying $P_n[k]$ and capacity $\overline{P}_n = 0.33 N_{\text{pnl}}$ to achieve a PVGC in the range of 1.5-3.5 MW. The transformer limit is fixed to $L = 1$ MW across all scenarios. We consider two demand scenarios, where high demand represents 1.33 times nominal demand. The average minutely demand of each household is interpolated from EPRI's hourly representative residential demand data [12] with a first-order hold. Finally, to add variability in the demand data, a $\pm 10\%$ band is added

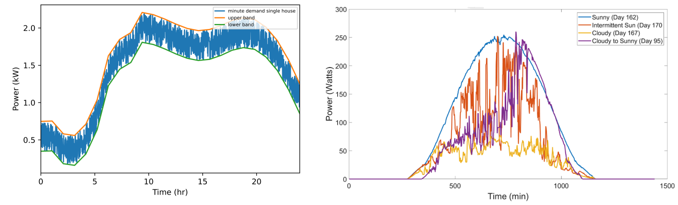


Fig. 2. Minutely data used in case study for representative days. Left: Household demand data. Right: Solar PV data for a panel.

to randomly perturb time-series demand data. An example of minutely time-series demand data over a day is depicted for a single household in Fig. 2 along with upper and lower bands.

To simplify computations, we simulate each day separately, allocate (an oversized) monthly allotment to each day evenly, and re-sample realizations of randomized demand time-series data for each household. Solar PV power is curtailed on a minute by minute basis per proportional policy in (8). Lastly, to measure fairness of the dynamic curtailment scheme, Jain's Fairness Index (JFI) is applied to the set of curtailment costs $\{\sum_{k=1}^{1440} X_n[k]\}_{n=1}^{N}$ each day. Not surprisingly (given the proportional allocation in (8)), the average JFI of the year was 0.999 and consistently high across all scenarios, which indicates a fair utilization of curtailable PV assets.

### A. Dynamic Curtailment Scheme over a Full Year

Traditionally the DSO determines the HC by considering a clear-sky day at noon, when solar PV output $P_n[k] \to \overline{P}_n$ and it is assumed to coincide with the minimum aggregate household energy demand, $\underline{d}_n := \min_k d_n[k]$. However, if excess power produced can be curtailed, it is possible for the installed capacity of solar PV to be greater than the HC limit, $L$. Therefore, as aforementioned, $\overline{P}$ becomes a dynamic value, and the condition in (1) no longer applies. We now compare this dynamic PVGC to the traditional conservative HC.

Fig. 3 depicts the percent increase in PVGC in comparison with the percent energy curtailed for both nominal demand and high demand scenarios. Annual energy gain from curtailment increases with percent curtailed because increasing curtailment facilitates an increased capacity to produce energy. With greater demand there is a greater percent increase in PVGC for the same amount curtailed because overload power decreases.

From Fig. 3, it is shown that with only 5% annual energy curtailed, the percent increase in solar PV generation capacity is 87% under nominal demand and 115% for high demand. Thus, the proposed dynamic curtailment scheme provides a huge increase in PVGC for just a nominal annual energy curtailment.

Fig. 4 depicts the annual net energy production vs. PVGC. As the PVGC increases, annual net energy produced also increases. Since curtailment facilitates increased PVGC, the cost of paying out net metering bills increases. Therefore the cost of curtailment is fronted by the DSO in this scenario. It is clear that the uncurtailed energy produced after demand (orange line) is linearly increases as PVGC increases. When

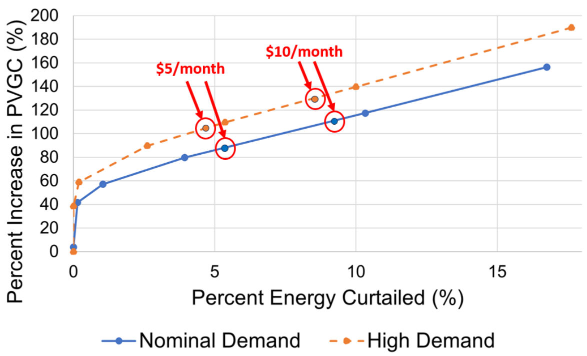


Fig. 3. Annual energy curtailed vs. PVGC. The markings indicate respective \$5/month and \$10/month allotments for different demand cases.

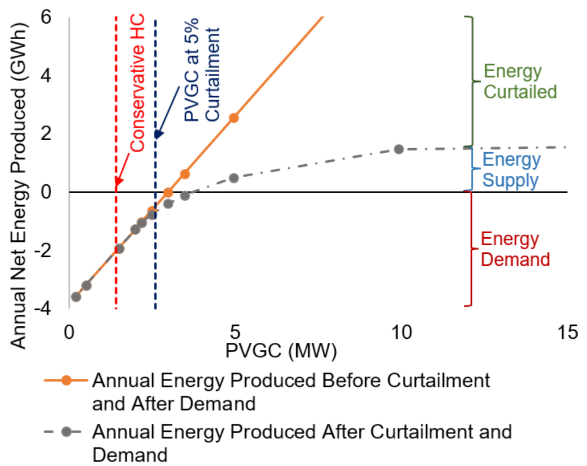


Fig. 4. Annual net energy production with and without curtailment for different PVGC values.

the PVGC is zero, the annual net energy "produced" is exactly equal to the total annual demand of all households in the curtailment scheme. As the hosing capacity increases, the annual net energy becomes positive, acting as an "energy supply." The energy that is curtailed for a given HC is equal to the difference between the annual energy produced before curtailment and the annual energy produced after curtailment (marked in green in Fig. 3).

As the PVGC increases, the annual net energy produced begins to plateau. As you add more solar PV generation capacity, the amount of energy to curtail will increase more above the power limit. However, the amount of energy to the grid can only increase incrementally because of the strict transformer power limit. After a certain point no more energy can be added to the grid while the limit remains the same.

### B. *Effects of Curtailment on Allotment Requirements*

Since the monthly allotment (or costs of curtailment) is a linear function of the monthly energy curtailed, it is straightforward to relate allotment to energy curtailed and corresponding increase in PVGC as shown in Fig. 3. For example, for less than \$5/month for each household, one can achieve greater than a 100% increase in PVGC in the case of high average demand and 87% increase in PVGC under nominal average demand conditions. Clearly, by doubling the allotment from \$5/mo to \$10/mo, the increase in PVGC does not double. This is because PVGC increases sub-linearly for linear increases in curtailed energy (after the first 2-3% of energy curtailment). As demand increases, more PVGC can be deployed for the *same* relative curtailment rate (e.g., 5%) but, as shown in Fig. 3, with higher monthly allotments. The higher monthly costs may seem non-intuitive, but are due the higher demand enabling a larger PVGC and creating a larger total clean energy output from which the same relative curtailment rate represents more energy and, thus, higher costs. Thus, it is no surprise that a fixed budget (e.g., \$5/mo) requires less relative curtailment with increased average demand.

### C. *Carbon Footprint Analysis*

While dynamic curtailment scheme have clear costs associated with curtailing solar PV energy, the benefit of the additional PVGC and the corresponding additional clean energy represents a viable value stream within the context of carbon credits or taxes. Today, there is a wide range of \$/tCO2 values. In a recent report by Brattle, it was found that the current federal estimate of the social cost of carbon is \$51/ton while a Nature study estimated the cost to be \$185/ton [13].

When considering a life cycle analysis of solar PV, natural gas, and coal, aspects such as mining for materials, manufacture of plants and products, as well as commissioning and decommissioning are considered. Solar PV is estimated to have a lifetime carbon footprint of $\beta_{\text{pv}} = 40$ gCO2e/kWh while natural gas and coal fossil fuel generators range from $\beta_{\text{fuel}} \in [500, 1000]$ gCO2e/kWh [14]. As more solar PV capacity is integrated into the grid and enabled by dynamic curtailment (i.e., $\overline{P} > L$), the additional renewable energy generated ($\Delta E$) will offset that from fossil fuel generators. Since solar PV has a smaller carbon footprint ($\beta_{\text{pv}} < \beta_{\text{fuel}}$), less CO2 will be emitted. This marginal CO2 reduction ($\beta_{\text{fuel}} - \beta_{\text{pv}}$) represents a socialized value that is determined by carbon credit markets or carbon taxes and today is estimated as $p_{\text{tCO2}} \in [51, 185]$ \$/tCO2e. Thus, to calculate the financial impact (or value) of any dynamic curtailment scheme, we consider the revenue generated from $CO_2$ offsets due to additional clean energy (enabled by the increased solar PV capacity) against the curtailed energy losses, as follows:

$$\Delta E = \frac{\Delta t}{3600} \sum_{k=1}^{K} P_{\text{net}}[k] - P_{\text{net}}^{L}[k] \tag{9}$$

$$\text{Revenue}_{\text{CO2}} = \Delta E (\beta_{\text{fuel}} - \beta_{\text{pv}}) p_{\text{tCO2}} \times 10^{-6} \tag{10}$$

with $\text{Profit} = \text{Revenue}_{\text{CO2}} - X$, where $X$ is from (6) and defines (utility) profits as a result of CO2 revenue vs. solar PV energy curtailment costs. Fig. 5 demonstrates total annual social profit and shows how profits are highly dependent on the fuel type that is replaced by solar PV. As the annual percent energy curtailed increases beyond 5%, the net annual profit begins to drop off. This is because the cost of the dynamic curtailment scheme increases linearly with curtailed energy, but revenues

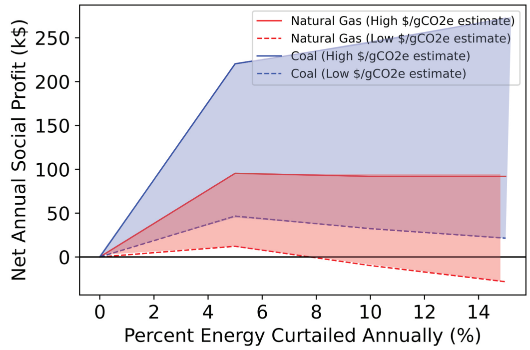


Fig. 5. Net annual social profit vs. percent curtailed

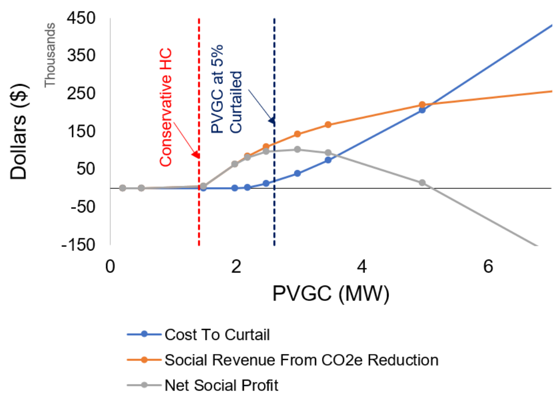


Fig. 6. Annual net social revenue and profit vs. PVGC

plateau since the increasing PVGC results in diminishing returns due to the transformer limit (as seen in Fig. 4).

This trade-off is further illustrated in Fig. 6, which depicts the total dynamic curtailment costs, carbon-based revenue, and profit as a function of PVGC for the nominal demand case with $\beta_{\text{fuel}} = 500$ gCO2e/kWh and $p_{\text{tCO2}} = 185$. As such, after the conservative HC line marked in red, the net societal revenue and profit from CO2e reduction as well as the annual cost to curtail begin to increase. The societal profit from CO2e reduction plateaus at PVGC $\overline{P} = 3$ MW and profit becomes negative at $\overline{P} = 3$ MW because energy curtailment grows super-linearly due to the transformer limit. However, even for a reasonable 5% of annual energy curtailed, the net societal profit can become a significant \$100,000, which is \$27/mo-household and more than enough to incentivize utilities to cover the curtailment costs for PV array owners and any nominal infrastructure necessary to implement the dynamic curtailment scheme (and calm PUCs). Even with a more modest $p_{\text{tCO2}}$ =\$51/tCO2e, the monthly profit would still be \$7.50/mo-household, which is enough to cover curtailment budgets.

Note that instead of curtailing solar PV to mitigate transformer overloads, (expensive) batteries could be used to store otherwise curtailed energy until a later time with nominal 10% round-trip losses. However, it is unclear if they provide a more economical alternative due to high costs of installation.

## V. Conclusion and Future Work

In this paper, we show how dynamic solar PV curtailment can enable greater deployment of PV generation behind a distribution substation transformer, which reduces $CO_2$ emissions by lessening reliance on fossil fuel generators. We also explore how such a curtailment program could be largely subsidized by incorporating the social cost of carbon. Simulations with nominal demand and high demand cases on a feeder with a 1MW transformer showed that even a minimal 5% annual energy curtailment can significantly increase PV generation capacity by more than 80% with household (energy) losses not exceeding \$10 per month allotments, which can be made up from socialized $CO_2$ savings. These benefits are amplified with increased demand, which aligns well with ongoing utility electrification programs. Thus, the simulation study recommends a more dynamic approach to account for BTM solar PV and the grid's hosting capacity. Future research will focus on equitable monthly allotments relative to installed solar array capacities, accounting for uncertain scenarios, and the impact of the distribution feeder itself in limiting solar PV generation capacity (beyond just the transformer copper-plate model studied herein).

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