

**informs<sup>®</sup>** ANNUAL MEETING

2021 ANAHEIM, CALIFORNIA



**Title:** Guaranteeing a physically realizable battery dispatch without charge-discharge complementarity constraints

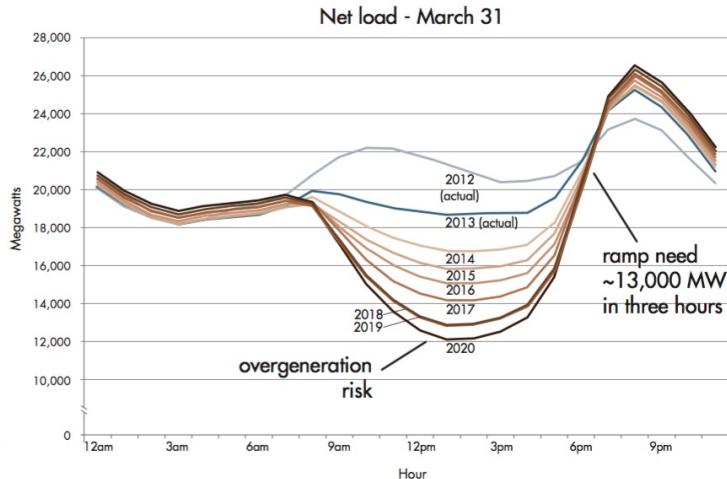
**Panel Session:** Operation research for emerging resources: Hybrid power plants, Virtual power plants, Batteries and beyond

**Presenters:** Nawaf Nazir (Pacific Northwest National Lab) and Mads Almassalkhi (University of Vermont)

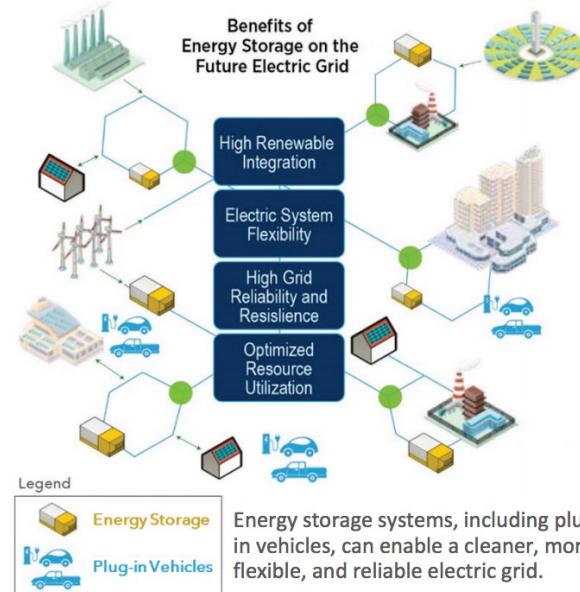
# Increased renewable generation and battery storage

- States like California and Vermont have ambitious targets → >50% of their energy met through renewable generation

Figure 2: The duck curve shows steep ramping needs and overgeneration risk



California duck curve showing a snapshot of a 24-hour period and steep ramping, source: CAISO



Energy storage provides an integrated solution to some critical energy needs,  
source: [1] Solving challenges in energy storage, Department of Energy report,  
Office of technology transitions

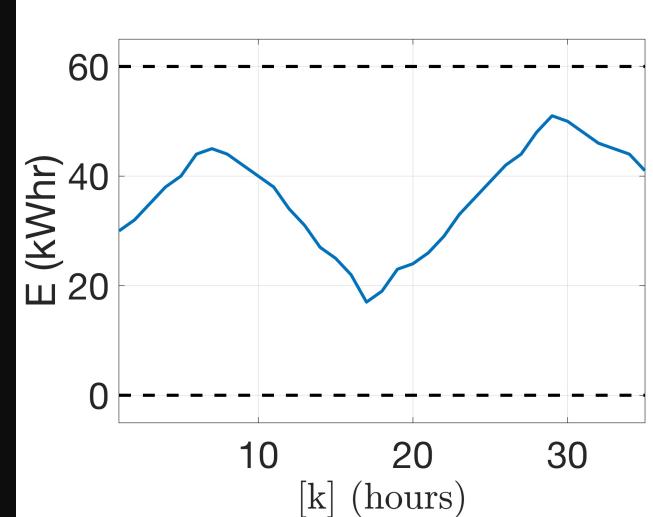
# Challenge in battery optimization

- Batteries cannot simultaneously charge and discharge
- Results in non-convex complementarity constraints
- Can be solved as a mixed-integer program
- Not scalable, computationally expensive

$$\begin{aligned} E[k+1] &= E[k] + \Delta t \eta_c P_c[k] - \frac{\Delta t}{\eta_d} P_d[k], \quad \forall k \in \mathcal{T} \\ E[0] &= E_0 \\ 0 \leq P_c[k] \leq P_{\max}, \quad &\forall k \in \mathcal{T} \\ 0 \leq P_d[k] \leq P_{\max}, \quad &\forall k \in \mathcal{T} \\ 0 \leq E[k+1] \leq E_{\max}, \quad &\forall k \in \mathcal{T} \\ P_c[k]P_d[k] &= 0 \quad \forall k \in \mathcal{T}. \end{aligned}$$

The resulting SoC trajectory can be expressed as

$$\mathbf{E}(\mathbf{P}_c, \mathbf{P}_d) = \mathbf{1}_T E_0 + \eta_c \mathbf{A} \mathbf{P}_c - \frac{1}{\eta_d} \mathbf{A} \mathbf{P}_d,$$



# Traditional approaches to scalable battery optimization

## Relaxed model

$$E^r[k+1] = E^r[k] + \Delta t \eta_c P_c^r[k] - \frac{\Delta t}{\eta_d} P_d^r[k],$$

$$E^r[0] = E_0$$

$$0 \leq P_c^r[k] \leq P_{\max},$$

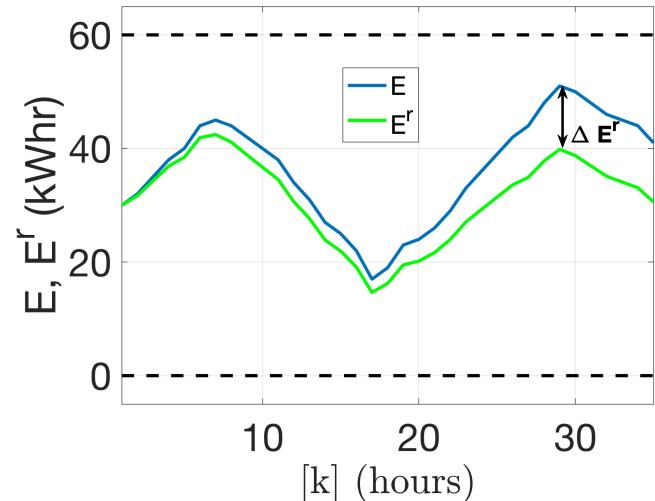
$$0 \leq P_d^r[k] \leq P_{\max},$$

$$0 \leq E^r[k+1] \leq E_{\max},$$

$$E^r(P_c^r, P_d^r) = \mathbf{1}_T E_0 + \eta_c \mathbf{A} P_c^r - \frac{1}{\eta_d} \mathbf{A} P_d^r$$

$$P_c = \max\{0, P_c^r - P_d^r\}, \quad P_d = \max\{0, -(P_c^r - P_d^r)\},$$

$$P_c - P_d = P_c^r - P_d^r$$



# Traditional approaches to scalable battery optimization

## Single input model

$$E^s[k+1] := E^s[k] + \eta \Delta t P_b[k], \quad \forall k \in \mathcal{T}$$

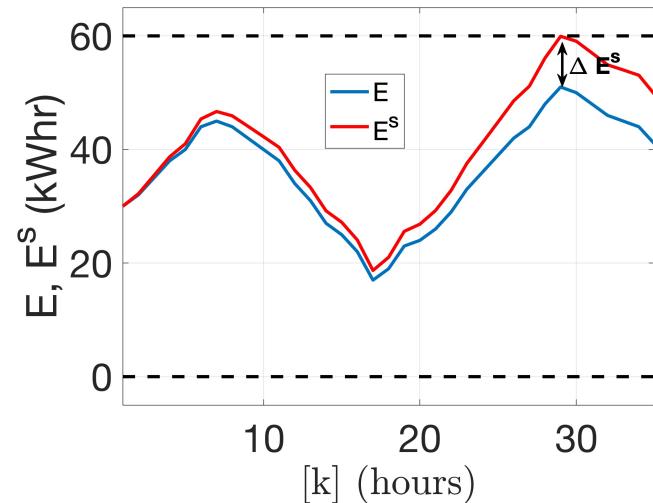
$$E^s[0] = E_0$$

$$-P_{\max} \leq P_b[k] \leq P_{\max} \quad \forall k \in \mathcal{T}$$

$$0 \leq E^s[k+1] \leq E_{\max}. \quad \forall k \in \mathcal{T}$$

The simplified model's SoC trajectory is then

$$\mathbf{E}^s(\mathbf{P}_b) = \mathbf{1}_T E_0 + \eta \mathbf{A} \mathbf{P}_b.$$

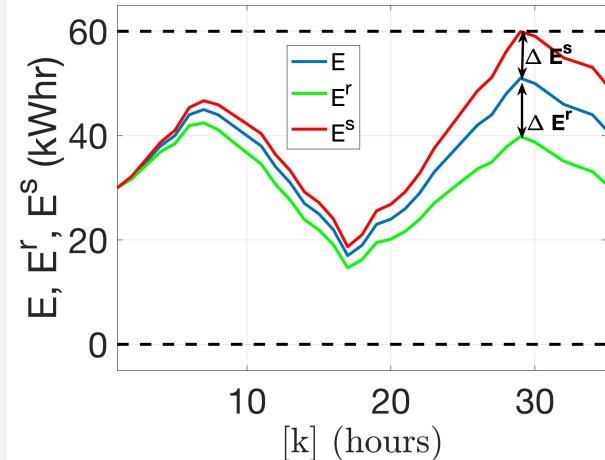


# Linear robust battery dispatch (RBD)

$$\begin{aligned}
 & \min_{\mathbf{P}_c - \mathbf{P}_d} \quad f(\mathbf{P}_c - \mathbf{P}_d) \\
 \text{s.t.} \quad & \mathbf{0} \leq \mathbf{1}_T E_0 + \eta_c \mathbf{A} \mathbf{P}_c - \frac{1}{\eta_d} \mathbf{A} \mathbf{P}_d \\
 & \mathbf{E}_{\max} \geq \mathbf{1}_T E_0 + \eta \mathbf{A} (\mathbf{P}_c - \mathbf{P}_d) \\
 & 0 \leq \mathbf{P}_c \leq \mathbf{1}_T P_{\max} \\
 & 0 \leq \mathbf{P}_d \leq \mathbf{1}_T P_{\max} \\
 & \mathbf{P}_c + \mathbf{P}_d \leq \mathbf{1}_T P_{\max}
 \end{aligned}$$

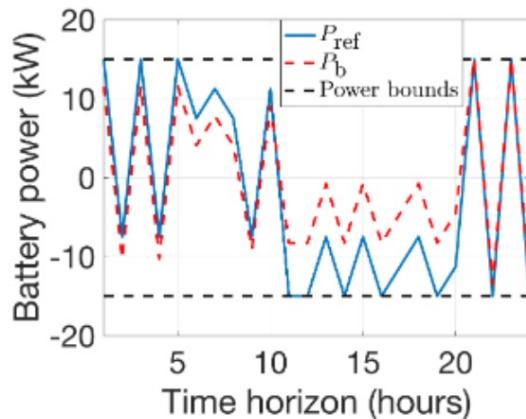
**Lemma III.1.** If inputs  $\mathbf{P}_b = \mathbf{P}_c - \mathbf{P}_d = \mathbf{P}_c^r - \mathbf{P}_d^r$  satisfy  $\mathbf{P}_c \cdot \mathbf{P}_d = \mathbf{0}$  and  $\mathbf{P}_c^r \cdot \mathbf{P}_d^r \geq 0$ , then  $\mathbf{E}^r(\mathbf{P}_c^r, \mathbf{P}_d^r) \leq \mathbf{E}(\mathbf{P}_c, \mathbf{P}_d) \leq \mathbf{E}^s(\mathbf{P}_b)$ .

$$\Delta \mathbf{E}^r \leq \left( \frac{1}{\eta_d} - \eta_c \right) \mathbf{A} \mathbf{1}_T \frac{P_{\max}}{2}.$$



- [2] Nawaf Nazir, Mads Almassalkhi, "Guaranteeing a physically realizable battery dispatch without charge-discharge complementarity constraints," IEEE PES Letters

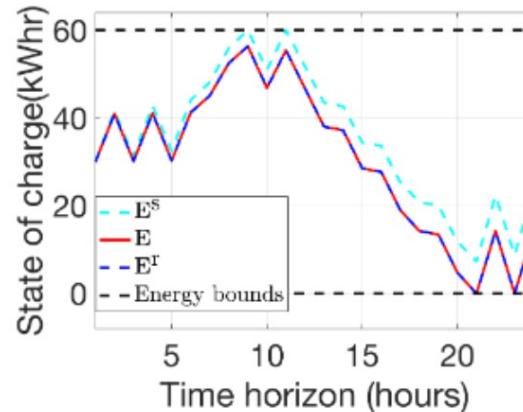
# Simulations results and comparisons



Tracking a battery reference power signal  $P_{ref}$  with the net battery output  $P_b \in [-P_{max}, P_{max}]$ .

TABLE I  
SOLVE TIME (SEC) AND POWER TRACKING RMSE (kW) COMPARISON WITH INCREASING BATTERIES FOR RBD VS MIP VS NLP

Batteries	RBD		MIP		NLP	
	Time	RMSE	Time	RMSE	Time	RMSE
10	1.7	47.8	16.3	43.7	5.1	54
100	3.1	478.7	271.8	437.8	50.5	478.7
200	6.3	957.4	1114	866	133.2	1190.2
500	11.5	2327.4	—	—	351.6	2415.2
1000	22.6	4787.1	—	—	1115	4787.1



Comparison between predicted SoC ( $E^S, E^r$ ) and actual SoC  $E$  resulting from optimized dispatch with the energy limits [0, 60].

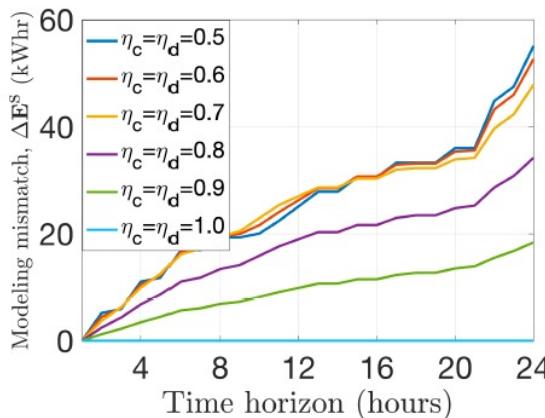
# Simulations results and conclusions

## Conclusions

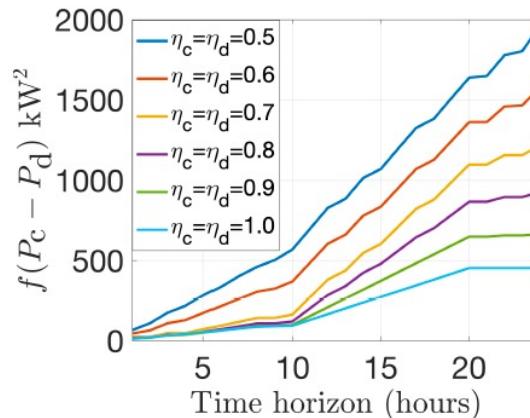
- A new linear formulation to optimally dispatch batteries
- Guaranteeing satisfaction of SoC constraints
- Avoiding non-convex/MIP formulations

## Future work:

- Incorporate battery constraints into optimal power flow and MPC formulations
- Study impact of conservativeness on practical case-studies



Modeling mismatch obtained for different  $\eta_c = \eta_d$  efficiencies.



Corresponding cumulative objective function values  $(P_{ref}[k] - P_b[k])^2$  showing reduced tracking performance with increased modeling mismatch (i.e., lower efficiencies).

# Thank you! Questions/comments?



Fall in Vermont, Groton, VT



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