Probabilistic Surface Reconstruction with Unknown Correspondence

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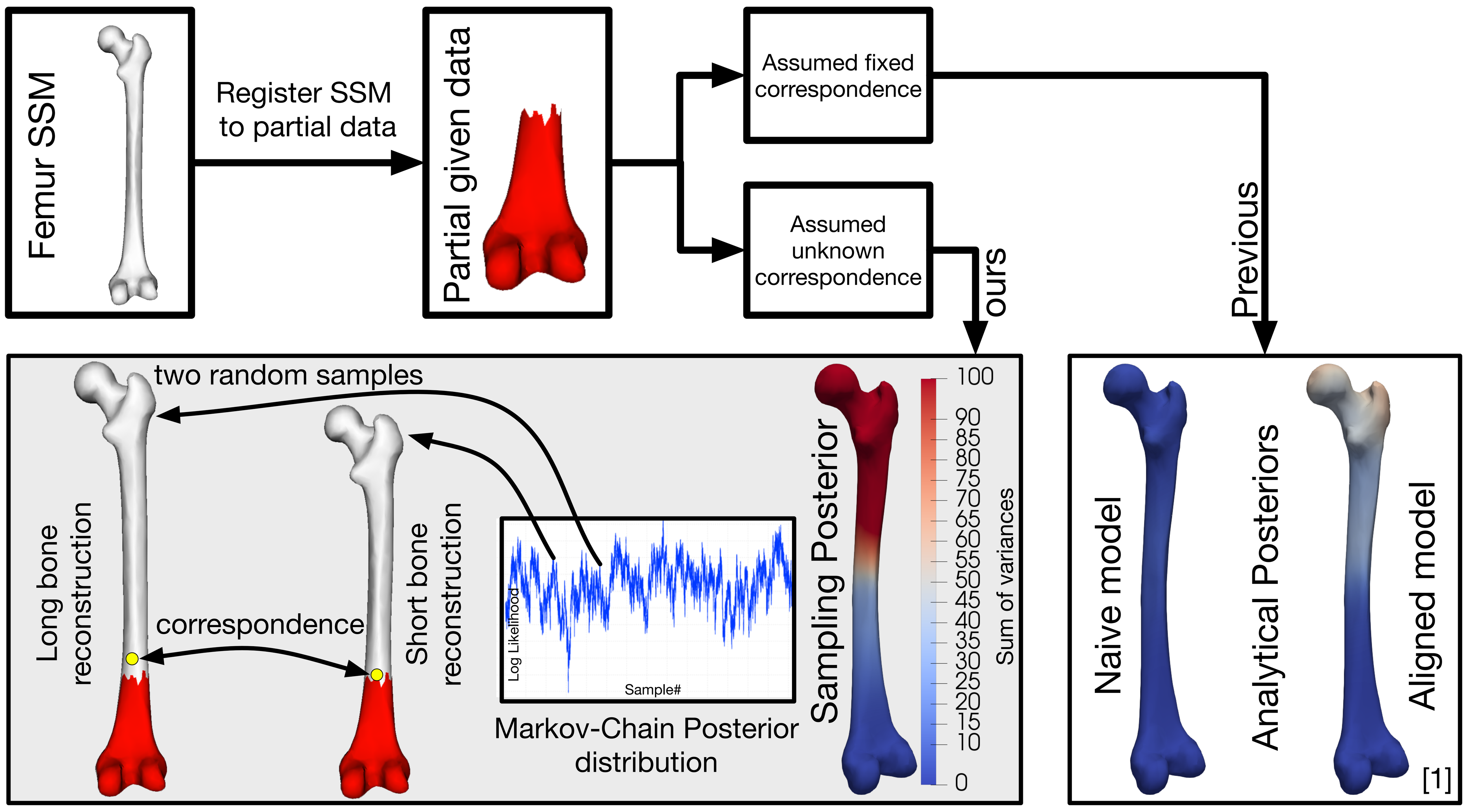
Problem

Modelling the posterior distribution of triangulated surface reconstructions from partial data.

Contributions

- Modelling the posterior distribution of surface reconstructions from a partial surface, without assuming a fixed point-to-point correspondence.
- New proposal using geometry information, for faster convergence.
- We show the limitations of the analytical posterior.

Overview



Statistical Shape Models

Statistical shape models (SSM) are linear models of shape variation learned from data. PCA leads to a parametric model of the form:

$$\vec{s} = \vec{\mu} + UD\vec{\alpha} = \vec{\mu} + \sum_{i=1}^{n} \alpha_i \sqrt{\lambda_i}\vec{u}_i,\ \ \alpha_i \sim \mathcal{N}(0,1)$$

The Analytical Posterior Distribution:
Albrecht et al. showed how to compute the analytical posterior distribution $P(\vec{\alpha}|\vec{s}_g)$ in [1] with $\vec{s}_g$ being the partial data information modelled with an SSM: $\vec{s}_g = \vec{\mu}_g + U_g D_g \vec{\alpha} + I_{3q}\epsilon$ and $\epsilon \sim \mathcal{N}(0,\sigma^2)$ modelling the noise of the observed partial data.

Shape and Pose Representation:

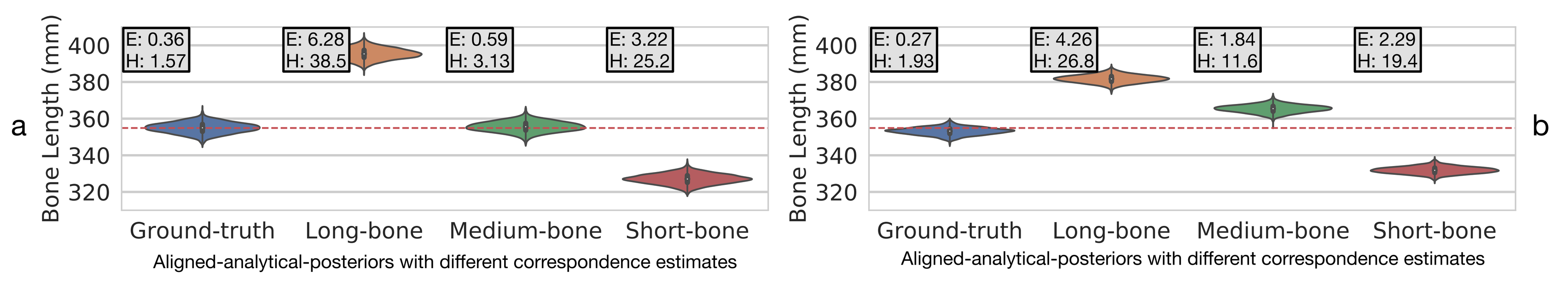
- Translation: $\vec{t} = (t_x, t_y, t_z)^T \in \mathbb{R}^3$.
- Rotation: $R(\phi,\psi,\rho) \in SO(3)$
- $\vec{\theta} = (\alpha_0, \dots, \alpha_{N-1}, \phi, \psi, \rho, t_x, t_y, t_z)^T$

Metropolis-Hastings

The *Sampling Posterior* is computed as a Markov-Chain using the Metropolis-Hastings algorithm:

$\vec{\theta}_0 \leftarrow$ arbitrary initialisation
for $i = 0$ to S **do**
  $\vec{\theta}' \leftarrow$ sample from $Q(\vec{\theta}'|\vec{\theta}_i)$
  $t \leftarrow \dfrac{q(\vec{\theta}_i|\vec{\theta}')p(\Gamma_T|\vec{\theta}')p(\vec{\theta}')}{q(\vec{\theta}'|\vec{\theta}_i)p(\Gamma_T|\vec{\theta}_i)p(\vec{\theta}_i)}$. {acceptance threshold}
  $r \leftarrow$ sample from $\mathcal{U}(0,1)$
  if $t > r$ **then**
    $\vec{\theta}_{i+1} \leftarrow \vec{\theta}'$
  else
    $\vec{\theta}_{i+1} \leftarrow \vec{\theta}_i$

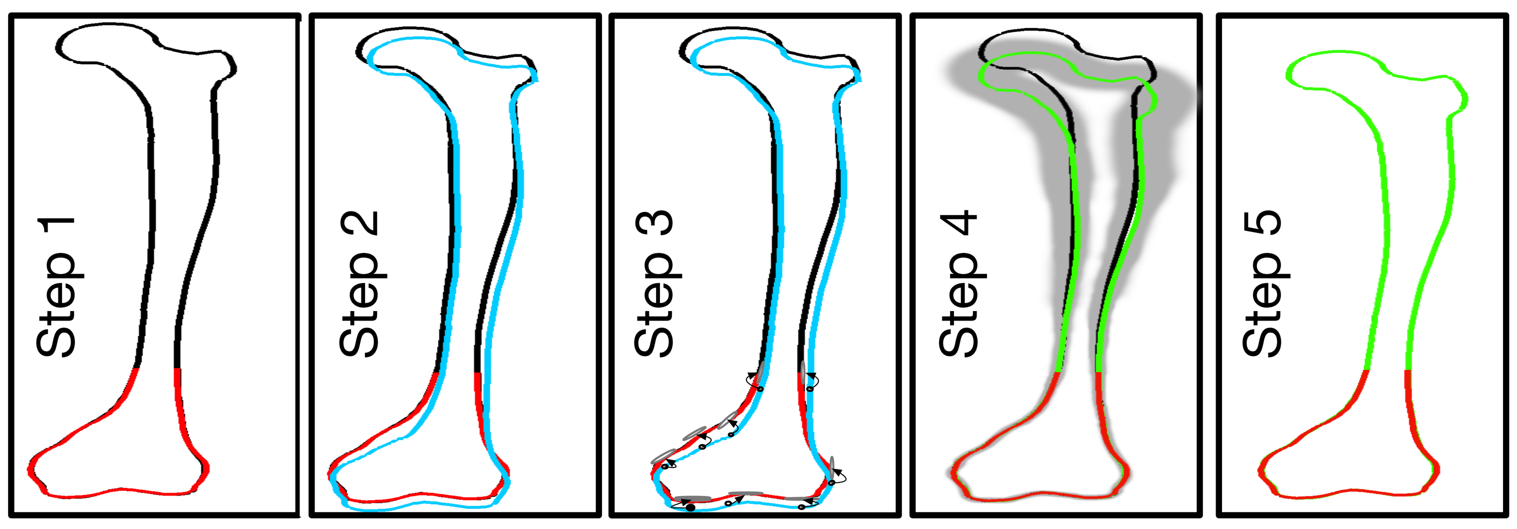
Results - Correspondence Sensitivity



Informed Projection Proposal

The standard random-walk proposal Q, requires a very small step size to keep the model fixed around the partial data $\vec{s}_g$. We introduce the informed projection proposal which keeps the model fixed at the known part of the model $\vec{s}_{g*}$.

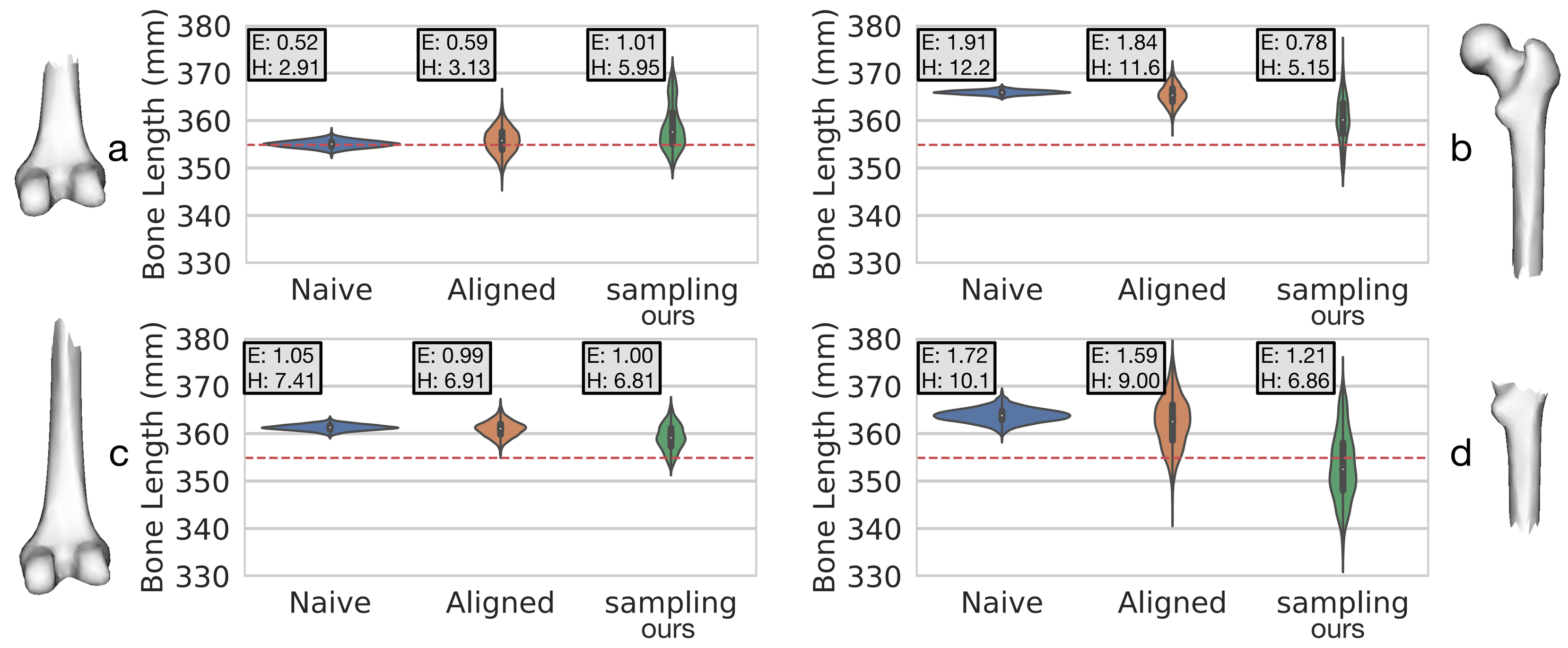
1. Compute corresponding points, $\vec{s}_{g*}$ (red).
2. Propose a random pose update $\vec{\theta}_o$ (blue), with fixed shape parameters $\vec{\alpha}$.
3. Compute analytical-posterior $p(\vec{\alpha}|\vec{\theta}_o, \vec{s}_{g*})$ based on $\vec{s}_{g*}$.
4. Draw a sample shape Γ_p (green) from the posterior distribution (gray).
5. Compute $\vec{\theta}'$ from Γ_p on the full SSM $p(\vec{\alpha})$ (green).



Transition probability:

- $q(\vec{\theta}_i|\vec{\theta}') \rightarrow$ sampling $\vec{\alpha}$ from $p(\vec{\alpha}|\vec{\theta}_o, \vec{s}_{g*})$ (from step 3)
- $q(\vec{\theta}'|\vec{\theta}_i) \rightarrow$ sampling $\vec{\alpha}'$ from $p(\vec{\alpha}'|\vec{\theta}', \vec{s}_{g*})$

Results - Analytical vs Sampling posterior



References

[1] T. Albrecht, M. Lüthi, T. Gerig, and T. Vetter, "Posterior shape models," *Medical image analysis*, 2013.