Mandatory excercises for week 45

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S17

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1

a)

$T_2(x)$ in $x_0 = 0$ for $f(x) = e^{-x} : T_2(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 = 1 - 1x + \frac{1}{2}x^2$

b)

$T_2(1) = 1 - 1 \cdot (1) + \frac{1}{2} \cdot 1^2 = \frac{1}{2}$

The error made using $T_k(x)$ instead of $f(x) = R_k(x) = \frac{f^{(k+1)}(c)}{k+1!}(x-a)^{k+1}$ where c is a point between x and 0, which means that the bounds for the error $f(1) - T_2(1) = R_2(0)$ and $R_2(1)$ $R_2(x) = \frac{f'''(c)}{(2+1)!}x^3 = \frac{-e^{-c}}{6}x^3, R_2(0) = \frac{-e^0}{6} = -\frac{1}{6}, R_2(1) = \frac{-e^{-1}}{6} = -\frac{1}{6e}$ thus the error is bounded by: $-\frac{1}{6} \leq R_2(1) \leq -\frac{1}{6e}$

2

a)

Difference quotient with respect to $x = \frac{f(x+h,y)-f(x,y)}{h}$ at point (1,2) $= \frac{f(1+h,2)-f(1,2)}{h}$

$f(1,2) = 1 \cdot 2^2 + e^{1+2} = 4 + e^3 \approx 24,0855,\ f(1+h,2) = (1+h)2^2 + e^{(1+h)+2} = 4 + 4h + e^{3+h}$

inserting we get: $\frac{4+4h+e^{3+h}-24,0855}{h}$

Difference quotient with respect to $y = \frac{f(x,y+k)-f(x,y)}{k}$ at point (1,2) $= \frac{f(1,2+k)-f(1,2)}{k}$

$f(1,2+k) = 1(2+k)^2 + e^{1+(2+k)} = k^2 + 4k + 4 + e^{3+k}$ inserting we get:

$\frac{k^2+4k+4+e^{3+k}-24,0855}{k}$

b)

There are 2 partial derivatives of our function $f(x,y)$ because it has 2 variables, $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$

$\frac{\partial f(x,y)}{\partial x}$ means we differentiate $f(x,y)$ with respect to x and treat y as a constant, this gives:

$\frac{\partial f(x,y)}{\partial x} = y^2 + e^{x+y}$

$\frac{\partial f(x,y)}{\partial y}$ is the same but with respect to y and treating x as a constant and gives: $\frac{\partial f(x,y)}{\partial y} = 2xy + e^{x+y}$

at $f(1,2)$ this gives us $\frac{\partial f(x,y)}{\partial y} = 2 \cdot 1 \cdot 2 + e^{1+2} = 4 + e^3 \approx 24,0855$, $\frac{\partial f(x,y)}{\partial x} = 2^2 + e^{1+2} = 4 + e^3 \approx 24,0855$