Course MM529, Matematiske metoder

Plan for week 47 and 48

Topics of the last lecture (week 47)

Applications of Integration (7.3, 9.3):
Lengths of curves, convergence criteria for infinite series
Complex numbers (A-1 – A-5):
Definition, arithmetic operations, representation in the complex plane.

Planned topics of the next lecture (week 48)

Complex numbers (A-5 – A-9):
Different representations of complex numbers, powers and roots
More on series (9.3, 9.4):
Ratio and root test, alternating series
Ordinary differential equations (7.9, 18.1):
Definition, order, classification, initial value problem, geometric interpretation of first order differential equations

Mandatory exercises

Here is the third mandatory exercise for the second quarter. You have to submit your solutions as a single pdf-file on blackboard until Monday, December 2, 9am. The electronic submission will not be possible after this deadline. There will be separate submission pages for the different subjects, please upload your solution on the page of your subject. Your solutions will be corrected by the teaching assistents. Each exercise is worth 20 points for a full and correct solution. For successfully passing the course you need to receive 50% of the possible points in each of the three quarters.

Mandatory exercises are meant as an individual work. You are welcome to work together with other students, but you must write up the solutions in your own words. It is not allowed just to copy the solutions of other students.

In your solution, you also have to document the way in which you obtained the solution, it is not sufficient just to write down the final result. Now the mandatory exercise follows:

(3) For the following problems, indicate which integration rules you apply.

a) Find the following antiderivatives:

$$\int x \sin x \, dx; \qquad \int 2x \sin(x^2) \, dx; \qquad \int x^2 e^x \, dx, \qquad \int \frac{2x-1}{x^2+x-6} \, dx.$$

b) Calculate the following definite or improper integrals. Make a sketch and explain which area is measured by the integral.

$$\int_{-2}^{2} (x+1)^3 \, dx; \qquad \int_0^1 \frac{e^x}{1+e^x} \, dx, \qquad \int_0^\infty x e^{-x^2} \, dx.$$

Topics and exercises for examinatorier

(1) Substitution method (substitute a function, one more exercise of this type):

Use the substitution $\ln t = x$ to solve the integral

$$\int \frac{e^{2x}}{e^x - 1}\, dx.$$

(2) Calculate the length of a graph

a) Exercise 7.3: 2, 3.

b) Describe the curve on the unit circle with $y \geq 0$ as the graph of a suitable function and calculate its length.

(3) Summation

a) Determine by the integral test for which values of $\alpha > 0$ the series

$$\sum_{k=1}^{\infty} \frac{1}{k^\alpha}$$

is convergent or divergent.

b) Determine an upper and a lower bound for

$$\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}.$$

How can you get better estimates, if needed.

c) Exercise 9.3: 35.

(4) Elementary operations on complex numbers

a) For the complex numbers $w = -2i$ and $z = 2 - 3i$ locate them in the complex plane and calculate $\mathrm{Re}(w)$, $\mathrm{Im}(w)$, $\bar{w}$, $\bar{z}$, w^2, z^2, $w + z$, $w \cdot z$, $\frac{w}{z}$ and $\frac{z}{w}$.

b) Find all solutions in the complex numbers of the quadratic equation $x^2 + x + 1 = 0$ by applying the standard algorithm for real numbers. Write the polynomial $x^2 + x + 1$ as a product of two polynomials of degree one with coefficients in the complex numbers.

c) Write the polynomial $x^2 + (1 + i)x + \frac{i}{2}$ as a product of two polynomials of degree one.

d) Exercise A1: 44.

Topics for studiegrupper (Studiecafé)

(1) Bounding sums

a) Discuss the proof of Theorem 8 in Section 9.3 of the book.

b) Check whether $\sum_{k=0}^{\infty} e^{-k}$ converges.

c) Is $\int_0^{\infty} \frac{1}{2^x}\, dx$ finite? If yes, find an upper bound.

(2) Length of the graph of a function

Exercises 7.3: 1, 3, 11.

(3) Complex numbers

a) For the complex numbers $w = 1 + i$ and $z = -2 - 3i$ locate them in the complex plane and calculate $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $\bar{w}$, $\bar{z}$, w^2, z^2, $w + z$, $w \cdot z$, $\frac{w}{z}$ and $\frac{z}{w}$.

Adress any problems, questions, that you could not solve or remained unclear in the next exercise courses.