# Course MM529, Matematiske metoder

Plan for week 43 and 44

## Topics of the last lecture (week 43)

**Vectors in** $\mathbb{R}^n$ (10.2):
Vectors, length and direction, multiplication by a scalar and addition of vectors, dot product
**Applications of partial differentiation** (12.6, 12.7):
the gradient, tangent planes and linear approximation, directional derivatives

## Planned topics of the next lecture (week 44)

**Applications of partial differentiation** (12.7, 13.1, 13.2):
Geometric interpretation of the gradient, maxima and minima in the interior
**Integration** (5.2, 5.3):
Areas and Riemann sums, the definite integral

## Mandatory exercises

Here is the first series of mandatory exercises of the second quarter. You have to submit your solutions as a single pdf-file on blackboard until Tuesday, November 5, 9am. The electronic submission will not be possible after this deadline. There will be separate submission pages for the different subjects, please upload your solution on the page of your subject. Your solutions will be corrected by the teaching assistents. Each exercise is worth 20 points for a full and correct solution. For successfully passing the course you need to receive 50% of the possible points in each of the three quarters.

Mandatory exercises are meant as an individual work. You are welcome to work together with other students, but you must write up the solutions in your own words. It is not allowed just to copy the solutions of other students.

In your solution, you also have to document the way in which you obtained the solution, it is not sufficient just to write down the final result. Now the three mandatory exercises follow:

(1) Taylor approximation

a) Determine the Taylor polynomial $T_2(x)$ of degree 2 in $x_0 = 0$ for the function $f(x) = e^{-x}$.

b) Use the Taylor polynomial to find the estimate $T_2(1) \approx f(1)$ and give an upper bound for the error $|f(1) - T_2(1)|$, using the Lagrange remainder. Compare this estimate with the actual error.

(2) Let $f(x, y) = xy^2 + e^{x+y}$.

a) Calculate the difference quotient with respect to $x$ and with respect to $y$ at the point $(a, b) = (1, 2)$.

b) Determine all partial derivatives of first and second order. and calculate their values in $(a, b) = (1, 2)$.

## Topics and exercises for examinatorier

(1) Let $\mathbf{a} = (2, 1)$ and $\mathbf{b} = (-1, 4)$ be vectors in $\mathbb{R}^2$, and $\mathbf{c} = (-1, 2, 3, -4)$ and $\mathbf{d} = (1, 1, 1, 1)$ in $\mathbb{R}^4$.

a) Calculate the sums and the differences $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $\mathbf{b} - \mathbf{a}$, $\mathbf{c} + \mathbf{d}$, $\mathbf{c} - \mathbf{d}$, and the dot products $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{c} \cdot \mathbf{d}$.

b) Discuss the geometric interpretation of $2 \cdot \mathbf{a}$, $-\mathbf{b}$, $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ in $\mathbb{R}^2$.

c) Calculate the lengths of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\sqrt{2} \cdot \mathbf{d}$.

d) Why wouldn't it make sense to calculate the length of $\mathbf{a} \cdot \mathbf{b}$?

e) Determine the cosine of the angles between $\mathbf{a}$ and $\mathbf{b}$, and between $\mathbf{c}$ and $\mathbf{d}$.

(2) Gradient and tangent spaces

Exercises 12.7: 2, 4 (only (a) and (b)).

(3) Linear approximation

Exercises 12.6: 4, 6.

(4) Directional derivatives

a) For the function $f(x, y) = 2x + xy^2$ find the directional derivatives in the direction of $\mathbf{v} = (3, 4)$ at the point $\mathbf{a} = (1, 1)$. Find the unit vectors $\mathbf{v_0}$ of all directions in which the directional derivative is $-1$ in $\mathbf{a}$.

b) For the function $f(x, y, z) = x^2 + y^2 + z^2$ find the directional derivative in the direction of $\mathbf{v} = (1, 1, 1)$ at $\mathbf{a} = (0, 1, 2)$.

## Topics for studiegrupper (Studiecafé)

(1) Vectors in $\mathbb{R}^n$. Let $\mathbf{a} = (1, 1, 1)$ and $\mathbf{b} = (-1, 2, -3)$, and $\mathbf{c} = (6, 0, -2)$ in $\mathbb{R}^3$.

a) Calculate the three pairwise sums and the six pairwise differences of the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, and the three pairwise dot products. Determine for the three pairwise angles between the vectors whether they are smaller, equal to, or bigger than $90^o$.

b) Calculate the lengths of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, respectively.

(2) Gradient and tangent spaces

Exercises 12.7: 3, 5 (only (a) and (b)).

(3) Gradients and directional derivatives

Find the directional derivative of $f(x, y) = x^2 y$ at the point $(-1, -1)$ in the direction of the vector $(1, 2)$ (Exercises 12.7: 11).

**Adress any problems, questions, that you could not solve or remained unclear in the next exercise courses.**