Mandatory excercises for week 41

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S17

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1

a)
$f(x) = x^3 - 2x$
Difference quotient at $x_0 = \frac{f(x_0+h)-f(x_0)}{h}$
The derivative at x_0 when $x_0 = 1$
$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^3-2(1+h)+1}{h} = \lim_{h\to 0} \frac{h^3+3h^2+h-1}{h} =$
$\lim_{h\to 0} h^2 + 3h - 1 = -1$
b)
$f' = 3x^2 - 2,\ f'' = 6x,\ f''' = 6$

Inflection points and extreme points for f(x).
There are no global extreme points for f(x) as it's range is $(-\infty, \infty)$ The inflection points for $f(x)$ is where $f''(x) = 0$ from above we know that $f'' = 6x$, thus we get that $6x = 0 <=> x = 0$ So in the interval $(-\infty, 0) f(x)$ is concave and in the interval $(0, \infty) f(x)$ is convex. Since in $(-\infty, 0) f''(x)$ is negative, and in $(0, \infty) f''(x)$ is positive.
d)
see last page.

2

a)
$f(x) = sin(x)cos(x) <=> f' = cos^2(x) - sin^2(x)$ by the product rule
b)
$f(x) = e^{xlnx}, f'(x) = e^{xlnx}(ln(x)+1) = x^x(ln(x)+1)$ by the chain rule and the product rule, e^x being the outer function and $xlnx$ being the inner.
c)
$f(x) = (x^2 + \sqrt{x+1})^2, f'(x) = 2(x^2 + \sqrt{x+1})(2x + \frac{1}{2\sqrt{x+1}})$ by the chain

rule and the addition rule, x^2 being the outer function and $x^2+\sqrt{x+1}$ the inner.

d)

The inverse function of $f(x)=1+x^3 => x=1+y^3 => y=\sqrt[3]{\frac{x-1}{2}}$

$f(x)=\sqrt[3]{\frac{x-1}{2}}$, $f'(x)=\frac{1}{3}(\frac{x-1}{2})^{\frac{-2}{3}}\frac{1}{2}$ by the chain rule.

3

a)

$\lim_{x\to 0}\frac{e^x-1}{x}$ since this results in $\frac{0}{0} => \lim_{x\to 0}\frac{d(e^x-1)}{dx}/\frac{dx}{dx}=\lim_{x\to 0}e^x=1$

b)

$\lim_{x\to\infty}\frac{e^x}{x}$ this results in $\frac{\infty}{\infty} => \lim_{x\to\infty}\frac{d(e^x)}{dx}/\frac{dx}{dx}=\lim_{x\to\infty}e^x=\infty$

c)

$\lim_{x\to 1}\frac{ln(x)}{x} => \lim_{x\to 1}\frac{ln(1)}{1}=0$

d)

$\lim_{x\to 0}\frac{x^2}{2sin(x^2)}$ since this results in $\frac{0}{0} => \lim_{x\to 0}\frac{d(x^2)}{dx}/\frac{d(2sin(x^2)}{dx} =>$
$\lim_{x\to 0}\frac{2x}{4xcos(x^2)}=\frac{1}{2cos(x^2)}=\frac{1}{2}$

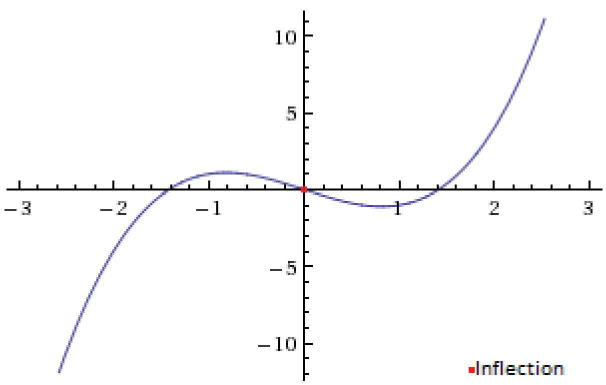

Figure 1: Assignment 1d) $x^3 - 2x$