Mandatory excercises for week 52

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S17

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4

a
$z + w = (3+1) + (-2+3)i = \underline{4+i}$
$z \cdot w = (3-2i)(1+3i) = 1(3-2i) + 3i(3-2i) = 3 - 2i + 9i - 6i^2 = 3 - 2i + 9i + 6 = \underline{9+7i}$
$\frac{z}{w} = \frac{3-2i}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{3-9i-2i+6i^2}{1+9} = \frac{-3-11i}{10} = \underline{-\frac{3}{10} - \frac{11}{10}i}$
$\overline{z} = \overline{3-2i} = \underline{3+2i}$
b
$z = 1+\sqrt{3}i$ this means that the magnitude of $z = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = 2$ and $\theta = tan^{-1}\frac{\sqrt{3}}{1} = 60^\circ = \frac{2}{3}\pi$ so z's polar form looks like: $\underline{2(cos\frac{2}{3}\pi + isin\frac{2}{3}\pi)}$ and it's exponential form looks like: $\underline{2e^{i\frac{2}{3}\pi}}$
$w = \underline{\sqrt{8}(cos - \frac{1}{2}\pi + isin - \frac{1}{2}\pi) = \sqrt{8}e^{i(-\frac{1}{2}\pi)}}$ by the same method used above.
c
$|z| = \sqrt{-2\sqrt{3}^2 + 2^2} = \sqrt{12+4} = \sqrt{16} = \underline{8}$
$\overline{w} = \overline{\sqrt{2}e^{i\frac{3}{4}\pi}}$ first, to convert the exponential form to regular, we have r $= \sqrt{2}$ and $\theta = \frac{3}{4}\pi$ this means that $a = rcos\theta = \sqrt{2} \cdot cos\frac{3}{4}\pi = -1$ and $b = rsin\theta = \sqrt{2} \cdot sin\frac{3}{4}\pi = 1$ so the regular form looks like: $-1 + i \Rightarrow \overline{w} = \underline{-1-i}$
$z \cdot w = (-2\sqrt{3} + 2i)(-1+i) = -2\sqrt{3}(-1+i) + 2i(-1+i) = 2\sqrt{3} - 2\sqrt{3}i - 2i + 2i^2 =$
$\underline{(2\sqrt{3} - 2) + (-2\sqrt{3} - 2)i \approx 1,464101615 - 5,464101615i}$
z^{10} is easy to solve if it is first converted to exponential form, thus r and θ is needed, r is found above to be 8, $\theta = tan^{-1}\frac{2}{2\sqrt{3}} = -30^\circ = -\frac{1}{6}\pi \Rightarrow z = 8e^{i-\frac{1}{6}\pi} \Rightarrow z^{10} = (8e^{i-\frac{1}{6}\pi})^{10} = 8e^{(-\frac{1}{6}\pi i)*10} = \underline{8e^{-\frac{10}{6}\pi i}}$
$z + w = (-2\sqrt{3} - 1) + (2+1)i = \underline{-2\sqrt{3} - 1 + 3i \approx -4,464101615 + 3i}$
d
First w is converted to polar form, and looks like: $\sqrt{2}(cos\frac{3}{4}\pi + isin\frac{3}{4}\pi)$ now all third roots can be found by De Moivre's Theorem, using the following formula: $r^{\frac{1}{3}}(cos(\frac{\theta+2\pi k}{n}) + isin(\frac{\theta+2\pi k}{n})), k = 0,1,2$ and n being the root i.e. 3 inserting into the formula the following roots are found:$k = 0 \Rightarrow \underline{2^{\frac{1}{6}}(cos\frac{1}{4}\pi + isin\frac{1}{4}\pi)}, k = 1 \Rightarrow \underline{2^{\frac{1}{6}}(cos\frac{11}{12}\pi + isin\frac{11}{12}\pi)}, k = 2 \Rightarrow \underline{2^{\frac{1}{6}}(cos\frac{19}{12}\pi + isin\frac{19}{12}\pi)}$ see last page for sketch.
e
First factor x out like so: $x(x^2 - 4x + 7)$ thus $\underline{x = 0}$ or $x^2 - 4x + 7 = 0 \Leftrightarrow x^2 - 4x = -7 \Leftrightarrow x^2 - 4x + 4 = -3 \Leftrightarrow (x-2)^2 = -3 \Leftrightarrow x - 2 = \sqrt{3}i$ or $x - 2 = \sqrt{3} - i \Leftrightarrow \underline{x = 2 + \sqrt{3}i}$ or $\underline{x = 2 - \sqrt{3}i}$

5

a
$\frac{dy}{dx} - \frac{x^2}{y} = 0 \Rightarrow \frac{dx}{dy} = \frac{x^2}{y} \Rightarrow ydy = x^2dx \Rightarrow \frac{y^2}{2} + c_1 = \frac{x^3}{3} + c_2 \Rightarrow \frac{y^2}{2} - \frac{x^3}{3} = c_2 - c_1 \Rightarrow \underline{3y^2 - 2x^3 - c = 0}$
with all constants consolidated to the final constant c.

$\frac{dy}{dx} = yxe^{2x} \Rightarrow y\frac{dy}{dx} = xe^{2x} \Rightarrow ydy = xe^{2x}dx \Rightarrow \frac{y^2}{2} + c = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c \Rightarrow \underline{y^2 = xe^{2x} - \frac{1}{2}e^{2x} + c}$
$\frac{dy}{dx} = (y+1)sinx \Rightarrow y + 1dy = sinxdx \Rightarrow \frac{y^2}{2} + y + c = -cosx + c \Rightarrow \underline{\frac{y^2}{2} + y + cosx + c = 0}$

b

(i) in a is non-linear and homogenous, (ii) is linear and homogenous and (iii) is linear and homogenous.

c

$\frac{dy}{dx} = y^2x^2 \Rightarrow y^2dy = x^2dx \Rightarrow \frac{y^3}{3} + c = \frac{x^3}{3} + c \Rightarrow y^3 - x^3 = c$ inserting $y(1) = 1 \Rightarrow 1 - 1 = c \Rightarrow 0 = c \Rightarrow$ $\underline{y^3 - x^3 = 0}$

Sketch of roots.