Course MM529, Matematiske metoder

Plan for week 40 and 41

Topics of the last lecture (week 40)

Applications of differentiation (4.3, 2.8, 4.9, 4.2, 4.10, 9.6):
Repeated application of L'Hospital's rule, Mean value theorem, Linear approximation, Newton method for finding zeroes, Taylor approximation (Taylor polynomial), Taylor series

Planned topics of the next lecture (week 41)

Partial differentiation (12.1 - 12.4):
Functions of several variables, partial derivatives, higher order partial derivatives

Mandatory exercises

Here is the second series of mandatory exercises of the first quarter. You have to submit your solutions as a single pdf-file on blackboard until next Friday, October 11, 9am. The electronic submission will not be possible after this deadline. There will be separate submission pages for the different subjects, please upload your solution on the page of your subject. Your solutions will be corrected by the teaching assistents. Each exercise is worth 20 points for a full and correct solution. For successfully passing the course you need to receive 50% of the possible points in each of the three quarters.

Mandatory exercises are meant as an individual work. You are welcome to work together with other students, but you must write up the solutions in your own words. It is not allowed just to copy the solutions of other students.

In your solution, you also have to document the way in which you obtained the solution, it is not sufficient just to write down the final result. Now the three mandatory exercises follow:

(1) For the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - 2x$ determine

a) the difference quotient at x_0 and for $x_0 = 1$ the derivative at x_0 as the limit of difference quotients,

b) the derivatives f', f'', and f''', and

c) the extreme points, the inflection points and the intervals where the function is concave or convex.

d) Sketch the graph of the function, marking the extreme points and inflection points.

(2) Determine the derivatives of the following functions. Indicate the rules that you apply.

a) $f(x) = \sin x \cos x$; b) $g(x) = e^{x \ln x}$; c) $h(x) = (x^2 + \sqrt{x+1})^2$;
d) the inverse function k^{-1} of $k(x) = 1 + 2x^3$.

(3) Calculate the following limits:

a) $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$; b) $\displaystyle\lim_{x\to\infty} \frac{e^x}{x}$; c) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x}$; d) $\displaystyle\lim_{x\to 0} \frac{x^2}{2\sin(x^2)}$.

Topics and exercises for examinatorier

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 1$ and $f(3) = 4$. Find a function value of the derivative f' that must definitely occur and localize the range where it must occur. Determine a differentiable function f with $f(0) = 1$ and $f(3) = 4$ where f' has no other function value than this..

(2) For the function

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0. \\ 1 & \text{if } x = 0, \end{cases}$$

use linear approximation at $a = 1$, $a = 0.1$ and $a = 0.01$ to estimate the function value at $x = 0$. Compare the results with $f(0)$. Interpret the results.

(3) a) Approximate with the Newton method the largest zero of $f(x) = x^4 - x^3 - x - 1$.

b) Explain the relation between the Newton method and linear approximation.

(4) Some unknown function f has at $x = 0$ the function value $f(0) = 1$, the derivative $f'(0) = 2$, the second derivative $f''(0) = 0$ and the third derivative $f'''(0) = -1$. Do your best to approximate $f(1)$, $f'(1)$ and $f''(1)$.

(5) a) Determine the Taylor polynomials $P_n(x)$ of order n of $f(x) = x^4 - x^3 - x - 1$ at $a = 0$ for all n.

b) For $n = 2$, use the Lagrange remainder to determine limits for the error that you make if you approximate $f(x)$ by $P_2(x)$ for $x = -\frac{1}{2}$ and $x = 2$.

(6) Calculate the following sums:

$$\sum_{k=0}^{\infty} \frac{1}{3^k} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots$$

$$\sum_{k=0}^{\infty} \frac{1}{k!} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \dots$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \dots$$

(7) a) Determine the Taylor series of $\cos x$ at $a = 0$..

b) Use the Taylor series of $\sin x$ at $a = 0$ to argue, why $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

Topics for studiegrupper (Studiecafé)

(1) Linear approximation

Exercises 4.9: 5, 15, 21.

(2) Newton method

Exercise 4.2: 7, 9, 21.

(3) Taylor polynomials

a) For the function $f(x) = \sqrt{x}$ calculate the first, second and third order Taylor polinomial at $a = 49$.

b) Approximate $\sqrt{50}$ by the function value for $x = 50$ of the Taylor polynomials and estimate the error you make by the Lagrange remainder.

Adress any problems, questions, that you could not solve or remained unclear in the next exercise courses.