Mandatory excercises for week 48

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S17

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a)

$\boldsymbol{\int xsin(x)dx}$

Using integration by parts with $f(x) = x$ and $g'(x) = sin(x)$ and the formula $\int f(x) \cdot g'(x) = f(x) \cdot g(x) - \int f'(x) \cdot g(x)dx$

$\int xsin(x)dx = -xcos(x) + \int cos(x)dx = \underline{-xcos(x) + sin(x) + C}$

$\boldsymbol{\int 2xsin(x^2)dx}$

Using u-substitution with $u = x^2$ we get $du = 2x$ and $\int 2xsin(x^2)dx \Rightarrow \int sin(u)du = -cos(u)$ resubstituting u we get $\int 2xsin(x^2)dx = -\underline{cos(x^2) + C}$

$\boldsymbol{\int x^2 e^x dx}$

Using integration by parts with $f(x) = x^2$ and $g'(x) = e^x \Rightarrow \int x^2 e^x dx = x^2 e^x - \int 2xe^x dx$ applying integration by parts again with $f(x) = 2x$ and $g'(x) = e^x \Rightarrow \int x^2 e^x dx = x^2 e^x - 2xe^x + \int 2e^x dx = \underline{x^2 e^x - 2xe^x + 2e^x + C}$

$\boldsymbol{\int \frac{2x-1}{x^2+x-6} dx}$

Using u-substitution with $u = x^2 + x - 6$, $du = 2x + 1 \Rightarrow \int \frac{du}{u} = \int \frac{1}{u} du = ln|u| + C$ resubstituting u $\Rightarrow \int \frac{2x-1}{x^2+x-6} dx = \underline{ln|x^2 + x - 6| + C}$

b)

$\boldsymbol{\int_{-2}^{2} (x+1)^3 dx}$

First the indefinite integral is found by using u-substitution with $u = x+1$ thus $du = 1dx \Rightarrow \int u^3 du = \frac{1}{4}u^4$ resubstituting $u \Rightarrow \int (x+1)^3 dx = \frac{1}{4}(x+1)^4$ then inserting the upper-limit and subtracting the lower-limit inserted the definite integral is $= \underline{\frac{1}{4}(2+1)^4 - \frac{1}{4}(-2+1)^4 = 20,25 - 0,25 = 20}$ see figure on last page.

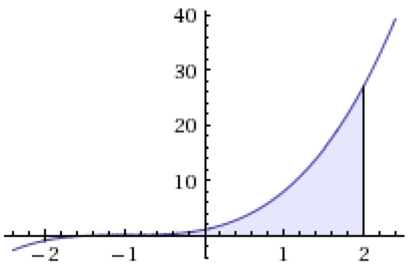
$\boldsymbol{\int_0^1 \frac{e^x}{1+e^x} dx}$

First the indefinite integral is found by using u-substitution with $u = 1 + e^x$ thus $du = e^x dx \Rightarrow \int \frac{du}{u} = \frac{1}{u} du = ln|u|$ resubstituting u $\Rightarrow \int \frac{e^x}{1+e^x} dx = ln|1 + e^x|$ then inserting the upper-limit and subtracting the lower-limit inserted the definite integral is $= \underline{ln|1 + e^1| - ln|1 + e^0| = ln(1+e) - ln(2) = 0,620114507}$ see figure on last page.

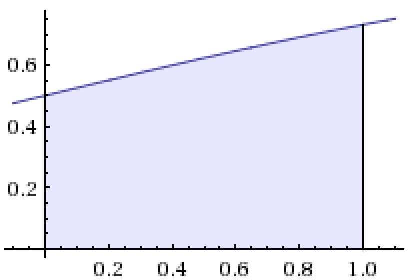
$\boldsymbol{\int_0^\infty xe^{-x^2} dx}$

$\int_0^\infty xe^{-x^2} dx = \lim_{n\to\infty} \int_0^n xe^{-x^2} dx$ to solve this, the definite integral is first found by substituting $u = -x^2$ thus $du = -2x \Rightarrow -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u$ resubstituting u $\Rightarrow \int xe^{-x^2} dx = -\frac{1}{2} e^{-x^2}$ now inserting the lower limit and evaulating the limit of the upper limit expression $\Rightarrow -\frac{1}{2} e^{0^2} - \lim_{n\to\infty} -\frac{1}{2} e^{-n^2}$ since $\lim_{n\to\infty} -\frac{1}{2} e^{-n^2}$ goes towards 0 as n goes toward infinity we get: $\underline{0 + \frac{1}{2} e^{0^2} = 0,5}$ see figure on last page.

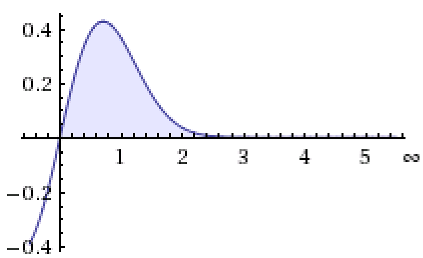
The area in blue is the calculated area



$$\int_{-2}^{2}(x+1)^3 dx$$



$$\int_0^1 \frac{e^x}{1+e^x} dx$$



$$\int_0^{\infty} ex^{-x^2} dx$$