# Course MM529, Matematiske metoder

Plan for week 50 and 51

## Topics of the last lecture (week 50)

**Ordinary differential equations (ODEs)** (3.7, 7.9, 18.4-5):
Linear ODEs, structure of solutions, finding solutions of first order inhomogeneous linear differential equations, second order linear differential equations

## Planned topics of the next lecture (week 51)

**Ordinary differential equations (ODEs)** (3.7, 18.6, 9.4-5):
Second order inhomogeneous linear differential equations, initial value problems, sketch for higher order linear ODEs, more on infinite series, power series

## Mandatory exercises

Here is the final mandatory exercise for the second quarter. You have to submit your solutions as a single pdf-file on blackboard until Monday, December 23, 9am. The electronic submission will not be possible after this deadline. There will be separate submission pages for the different subjects, please upload your solution on the page of your subject. Your solutions will be corrected by the teaching assistents. Each exercise is worth 20 points for a full and correct solution. For successfully passing the course you need to receive 50% of the possible points in each of the three quarters.

Mandatory exercises are meant as an individual work. You are welcome to work together with other students, but you must write up the solutions in your own words. It is not allowed just to copy the solutions of other students.

In your solution, you also have to document the way in which you obtained the solution, it is not sufficient just to write down the final result. Now the mandatory exercise follows:

(4) Complex numbers

a) For the complex numbers $z = 3 - 2i$ and $w = 1 + 3i$, determine $z + w$, $z \cdot w$ and $\frac{z}{w}$ and $\overline{z}$ in algebraic form.

b) Rewrite $z = 1 + \sqrt{3}i$ and $w = 2 - 2i$ in polar and in exponential representation.

c) For $z = -2\sqrt{3} + 2i$ and $w = \sqrt{2}e^{i3\pi/4}$ calculate $|z|$, $\overline{w}$, $z \cdot w$, $z^{10}$, and $z + w$.

d) Calculate all third roots of the number $w = \sqrt{2}e^{i3\pi/4}$ and sketch their locations in the complex plane.

e) Calculate all zeroes of the polynomial $p(x) = x^3 - 4x^2 + 7x$.

(5) Ordinary differential equations

a) Solve the following differential equations (find the general solution):

(i) $y' - \frac{x^2}{y} = 0$, (ii) $y' = yxe^{x^2}$, (iii) $y' = (y + 1)\sin x$.

b) Explain which of the differential equations in a) are linear, homogeneous, inhomogeneous.

c) Solve the differential equation $y' = y^2x^2$ with initial value $y(1) = 1$.

## Topics and exercises for examinatorier

(1) Solve inhomogeneous first order equations

Exercise 7.9: 12, 14, 16, 18.

(2) For a second order linear inhomogeneous differential equation let $y_1, y_2, y_3$, be functions with the property that $y_1, y_2$ are solutions of the corresponding homogeneous differential equation and $y_3$ is a solution of the inhomogeneous differential equation, and $y_1 \neq cy_2$ for all $c \in \mathbb{R}$. Which of the following three statements are always true? (explain positive and negative answers)

a) $y_1 + y_2$ solves the homogeneous equation.

b) $y_3 + y_3$ solves the inhomogeneous differential equation.

c) $y_3 - y_3$ solves the homogeneous equation.

d) Find the (somewhat tricky) exception to the statement: $y = y_3 + c_1y_1 + c_2y_2$, $c_1, c_2 \in \mathbb{R}$ is the general solution of the inhomogeneous differential equation.

e) Find the general solution of the corresponding inhomogeneous differential equation.

(3) Solve homogeneous second order ODEs

Exercises 3.7: 2, 4, 6, 10, 12.

(4) Find homogeneous linear differential equations that have the following general solution:

a) $ce^{-x^2}$, $c \in \mathbb{R}$

b) $c_1e^{-x} + c_2$, $c_1, c_2 \in \mathbb{R}$

c) $c_1e^{2x}\cos(-x) + c_2e^{2x}\sin(-x)$, $c_1, c_2 \in \mathbb{R}$

d) $c_1e^{2x}\cos x + c_2e^{2x}\sin x$, $c_1, c_2 \in \mathbb{R}$

## Topics for studiegrupper (Studiecafé)

(1) Solve inhomogeneous first order ODEs

Exercise 7.9: 11, 13, 15, 17.

(2) Solve homogeneous ODEs of second order

Exercises 3.7: 1, 3, 5, 7, 9, 11.

Have a nice chrismas!

**Adress any problems, questions, that you could not solve or remained unclear in the next exercise courses.**