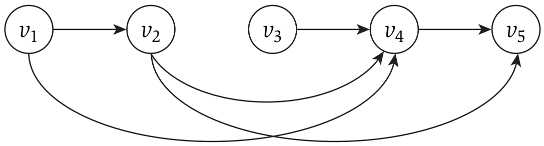

Figure 6.29 The correct answer for this ordered graph is 3: The longest path from v_1 to v_n uses the three edges (v_1, v_2),(v_2, v_4), and (v_4, v_5).

```
While there is an edge out of the node w
  Choose the edge (w, v_j)
     for which j is as small as possible
  Set w = v_j
  Increase L by 1
end while
Return L as the length of the longest path
```

In your example, say what the correct answer is and also what the algorithm above finds.

(b) Give an efficient algorithm that takes an ordered graph G and returns the *length* of the longest path that begins at v_1 and ends at v_n. (Again, the *length* of a path is the number of edges in the path.)

4. Suppose you're running a lightweight consulting business—just you, two associates, and some rented equipment. Your clients are distributed between the East Coast and the West Coast, and this leads to the following question.

Each month, you can either run your business from an office in New York (NY) or from an office in San Francisco (SF). In month i, you'll incur an *operating cost* of N_i if you run the business out of NY; you'll incur an operating cost of S_i if you run the business out of SF. (It depends on the distribution of client demands for that month.)

However, if you run the business out of one city in month i, and then out of the other city in month $i + 1$, then you incur a fixed *moving cost* of M to switch base offices.

Given a sequence of n months, a *plan* is a sequence of n locations—each one equal to either NY or SF—such that the i^{th} location indicates the city in which you will be based in the i^{th} month. The *cost* of a plan is the sum of the operating costs for each of the n months, plus a moving cost of M for each time you switch cities. The plan can begin in either city.

The problem. Given a value for the moving cost M, and sequences of operating costs $N_1, \ldots, N_n$ and $S_1, \ldots, S_n$, find a plan of minimum cost. (Such a plan will be called *optimal*.)

Example. Suppose $n = 4$, $M = 10$, and the operating costs are given by the following table.

	Month 1	Month 2	Month 3	Month 4
NY	1	3	20	30
SF	50	20	2	4

Then the plan of minimum cost would be the sequence of locations

$$[NY, NY, SF, SF],$$

with a total cost of $1 + 3 + 2 + 4 + 10 = 20$, where the final term of 10 arises because you change locations once.

(a) Show that the following algorithm does not correctly solve this problem, by giving an instance on which it does not return the correct answer.

```
For i = 1 to n
  If N_i < S_i then
    Output "NY in Month i"
  Else
    Output "SF in Month i"
End
```

In your example, say what the correct answer is and also what the algorithm above finds.

(b) Give an example of an instance in which every optimal plan must move (i.e., change locations) at least three times.

Provide a brief explanation, saying why your example has this property.

(c) Give an efficient algorithm that takes values for n, M, and sequences of operating costs $N_1, \ldots, N_n$ and $S_1, \ldots, S_n$, and returns the *cost* of an optimal plan.

5. As some of you know well, and others of you may be interested to learn, a number of languages (including Chinese and Japanese) are written without spaces between the words. Consequently, software that works with text written in these languages must address the *word segmentation problem*—inferring likely boundaries between consecutive words in the