

1 Cake Meeting (from the exam E18) You're organizing the First Annual DTU Cake Meeting, to be held on three days in January. There will be lots of cake for everyone and a large number of bakers have applied to bake cakes for the event. You need to hire bakers according to the following constraints.

1. Each candidate baker has given you a list of types of cakes they can bake.
2. Each baker can bake at most four cakes during the entire event.
3. At most one of each type of cake (chocolate cake, strawberry cake, ...) can be produced by all the bakers in total.

1.1 Suppose there are b candidate bakers and t different types of cake. Give an algorithm that computes the maximum number of cakes that can be produced according to the constraints. Analyze the asymptotic running time of your algorithm. Remember to argue that your algorithm is correct.

1.2 It turns out that there were way too many cakes and not enough variation in style on each day. So you impose the following new constraints (in addition to constraint 1, 2, and 3 from above).

4. There must be produced exactly k cakes each day, and thus $3k$ cakes altogether.
5. Each baker can bake at most two cakes each day (and still at most 4 cakes in total).

Give an efficient algorithm that either assigns a baker and a cake to each of the $3k$ cake slots, or correctly reports that no such assignment is possible. Analyze the asymptotic running time of your algorithm. Remember to argue that your algorithm is correct.