maximum in $S'$, which takes time $O(n)$. Thus the running time $T(n)$ satisfies

$$T(n) \le 2T\left(\frac{n}{2}\right) + O(n),$$

as desired.

We note that this is not the best running time achievable for this problem. In fact, one can find the optimal pair of days in $O(n)$ time using dynamic programming, the topic of the next chapter; at the end of that chapter, we will pose this question as Exercise 7.

# Exercises

**1.** You are interested in analyzing some hard-to-obtain data from two separate databases. Each database contains $n$ numerical values—so there are $2n$ values total—and you may assume that no two values are the same. You'd like to determine the median of this set of $2n$ values, which we will define here to be the $n^{\text{th}}$ smallest value.

However, the only way you can access these values is through *queries* to the databases. In a single query, you can specify a value $k$ to one of the two databases, and the chosen database will return the $k^{\text{th}}$ smallest value that it contains. Since queries are expensive, you would like to compute the median using as few queries as possible.

Give an algorithm that finds the median value using at most $O(\log n)$ queries.

**2.** Recall the problem of finding the number of inversions. As in the text, we are given a sequence of $n$ numbers $a_1, \ldots, a_n$, which we assume are all distinct, and we define an inversion to be a pair $i < j$ such that $a_i > a_j$.

We motivated the problem of counting inversions as a good measure of how different two orderings are. However, one might feel that this measure is too sensitive. Let's call a pair a *significant inversion* if $i < j$ and $a_i > 2a_j$. Give an $O(n \log n)$ algorithm to count the number of significant inversions between two orderings.

**3.** Suppose you're consulting for a bank that's concerned about fraud detection, and they come to you with the following problem. They have a collection of $n$ bank cards that they've confiscated, suspecting them of being used in fraud. Each bank card is a small plastic object, containing a magnetic stripe with some encrypted data, and it corresponds to a unique account in the bank. Each account can have many bank cards