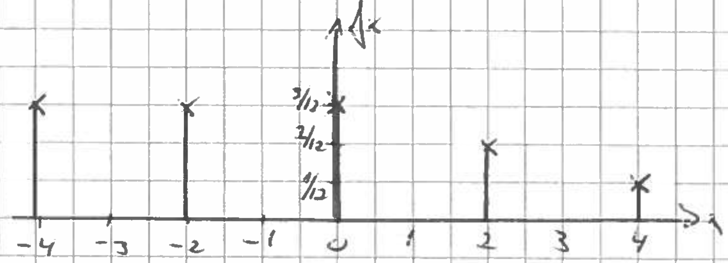
SMP, S17

Opg 1 S17

a) $\lim_{x \to \infty} F_X(x) = 12a = 1 \Rightarrow \underline{\underline{a = \frac{1}{12}}}$

b) pmf $f_X(x)$ er størrelsen af trinnene i cdf $F_X(x)$:

$$f_X(x) = \begin{cases} 3/12 & x = -4 \\ 3/12 & x = -2 \\ 3/12 & x = 0 \\ 2/12 & x = 2 \\ 1/12 & x = 4 \\ 0 & \text{ellers} \end{cases}$$



$$\sum_i f_X(x_i) = \frac{3+3+3+2+1}{12} = \frac{12}{12} = 1 \quad \text{o.k.}$$

c) $\underline{\underline{E(X)}} = \sum_i x_i f_X(x_i) = -4 \cdot \frac{3}{12} - 2 \cdot \frac{3}{12} + 0 \cdot \frac{3}{12} + 2 \cdot \frac{2}{12} + 4 \cdot \frac{1}{12} = \frac{-12-6+0+4+4}{12}$

$$= \frac{-10}{12} = \underline{\underline{-\frac{5}{6} = -0.833}}$$

d) $E(X^2) = \sum_i x_i^2 \cdot f_X(x_i) = (-4)^2 \cdot \frac{3}{12} + (-2)^2 \cdot \frac{3}{12} + 0^2 \cdot \frac{3}{12} + 2^2 \cdot \frac{2}{12} + 4^2 \cdot \frac{1}{12} = \frac{48+12+0+8+16}{12}$

$$= \frac{84}{12} = 7$$

$\underline{\underline{Var(X)}} = E(X^2) - E(X)^2 = 7 - \left(-\frac{5}{6}\right)^2 = 7 - \frac{25}{36} = \frac{252-25}{36} = \underline{\underline{\frac{227}{36} = 6.31}}$

SMP

Opg 2 S17

A : Regn , $\bar{A}$: Ikke regn

B : 1-15/6 , $\bar{B}$: 16-30/6 , $N_{Juni} = 30$

$Pr(A|B) = 0.20$, $Pr(A|\bar{B}) = 0.30$, $Pr(B) = Pr(\bar{B}) = \frac{1}{2}$

a) $Pr(A) = Pr(A|B) \cdot Pr(B) + Pr(A|\bar{B}) \cdot Pr(\bar{B}) = 0.2 \cdot \frac{1}{2} + 0.3 \cdot \frac{1}{2} = 0.25$

$= \frac{N_{Regn}}{N_{Juni}}$

$\Downarrow$

$\underline{\underline{N_{Regn}}} = 0.25 \cdot N_{Juni} = 0.25 \cdot 30 = \underline{\underline{7.5 \text{ dage}}}$

b) $\underline{\underline{Pr(\bar{B}|A)}} = \frac{Pr(A|\bar{B}) \cdot Pr(\bar{B})}{Pr(A)} = \frac{0.30 \cdot 0.5}{0.25} = \underline{\underline{0.60}}$

c) $\underline{\underline{Pr(\leq 1 \text{ dag med regn} | B)}} = Pr(0 \text{ dage med regn} | B) + Pr(1 \text{ dag med regn} | B)$

$p = Pr(A|B) = 0.2$

$1-p = Pr(\bar{A}|B) = 0.8$

$$= \binom{15}{0} \cdot p^0 \cdot (1-p)^{15} + \binom{15}{1} \cdot p^1 \cdot (1-p)^{14}$$

$$= 1 \cdot 0.2^0 \cdot 0.8^{15} + 15 \cdot 0.2 \cdot 0.8^{14}$$

$$= 0.8^{14} \cdot (0.8 + 3)$$

$$= 3.8 \cdot 0.8^{14} = \underline{\underline{0.167 = 16.7\%}}$$

SMP

Opg. 3 S17

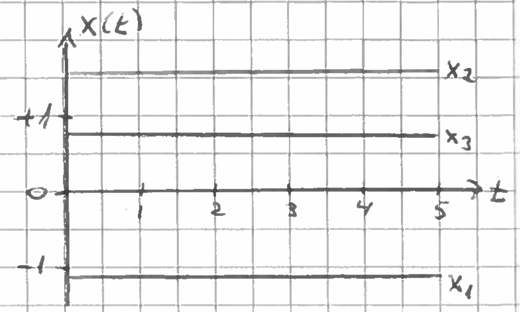
$$X(t) = (-1)^n + w = \begin{cases} 1 + w & (n=0) \\ -1 + w & (n=1) \end{cases} \qquad Pr(n=0) = Pr(n=1) = \frac{1}{2}$$

$$w \sim \mathcal{N}(0, \tfrac{1}{4}) \quad (\mu = 0,\ \sigma^2 = \tfrac{1}{4}\ (\sigma = \tfrac{1}{2}))$$

a) 3 realisationer

Matlab:
```
n = randi([0,1],1,3);
w = 0.5 · randn(1,3)
```

$x_1 = (-1)\wedge n(1) + w(1) = -1.103$
$x_2 = (-1)\wedge n(2) + w(2) = 1.508$
$x_3 = (-1)\wedge n(3) + w(3) = 0.756$



b) $\underline{\underline{\hat{\mu}_{x_1}}} = x_1 = -1 - 0.103 = \underline{\underline{-1.103}}$

$\underline{\underline{\hat{\sigma}^2_{x_1} = 0}}$ (X_1 konstant)

c) $\underline{\underline{E(X)}} = E((-1)^n + w) = E((-1)^n) + E(w) = (-1)^0 \cdot Pr(n=0) + (-1)^1 \cdot Pr(n=1) + E(w)$

$= 1 \cdot \frac{1}{2} + (-1)\frac{1}{2} + 0 = 1 - 1 + 0 = \underline{\underline{0}}$

$\underline{\underline{Var(X)}} = Var((-1)^n + w) = Var((-1)^n) + Var(w)$

$= E(((-1)^n)^2) - E((-1)^n)^2 + Var(w)$

$= E(1) - 0^2 + \frac{1}{4} = 1 + \frac{1}{4} = \underline{\underline{\frac{5}{4} = 1.25}}$

d) $E(X)$ og $Var(X)$ uafh. af t $\rightarrow$ $\underline{\underline{\text{WSS}}}$

$E(X) \neq \hat{\mu}_{x_1}$ og $Var(X) \neq \hat{\sigma}^2_{x_1}$ $\rightarrow$ $\underline{\underline{\text{Ikke ergodisk}}}$

SMP

Opg 4 S17

Type 1: $n_1 = 10$, $\hat{\mu}_1 = 5.21\,m$, $s_1^2 = 1.33\,m^2$

Type 2: $n_2 = 12$, $\hat{\mu}_2 = 4.18\,m$, $s_2^2 = 0.89\,m^2$

a) $H_0: \delta = \hat{\mu}_1 - \mu_2 = 0$

$H_1: \delta = \mu_1 - \mu_2 \neq 0$

b) $\underline{\underline{\hat{\delta}}} = \hat{\mu}_1 - \hat{\mu}_2 = 5.21 - 4.18\,m = \underline{\underline{1.03\,m}}$

c) $\underline{\underline{\hat{s}^2}} = \frac{1}{n_1+n_2-2}\left((n_1-1)\hat{s}_1^2 + (n_2-1)\hat{s}_2^2\right) = \frac{1}{20}\cdot(9\cdot 1.33 + 11\cdot 0.89)\,m^2 = \underline{\underline{1.088\,m^2}}$

d) $t = \frac{\hat{\delta} - 0}{\sqrt{\hat{s}^2\cdot\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{1.03}{\sqrt{1.088\cdot\left(\frac{1}{10}+\frac{1}{12}\right)}} = 2.306 \sim t(20)$

$t_{cdf}(2.306, 20) = 0.9840$

p-værdi $= 2\cdot(1-0.9840) = 2\cdot 0.0160 = 0.032 < 0.05$

$\Downarrow$

$\underline{\underline{H_0 \text{ afvises}}}$. Dvs. de to typer gps'er er forskellige.

e) $t_0 = t_{inv}(0.975, 20) = 2.086$

$\Delta\delta = t_0\cdot\sqrt{\hat{s}^2\cdot\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = 2.086\cdot\sqrt{1.088\cdot\left(\frac{1}{10}+\frac{1}{12}\right)} = 0.932$

$\delta_- = \hat{\delta} - \Delta\delta = 1.03 - 0.93 = 0.10$

$\delta_+ = \hat{\delta} + \Delta\delta = 1.03 + 0.93 = 1.96$

95% konfidens interval for $\delta = \underline{\underline{[0.10, 1.96]}}$

$\delta = 0 \notin [0.10, 1.96] \Rightarrow H_0$ afvises.