

Data sampling, transformation, multivariate random variables and CLT

GROUP ASSIGNMENT 5

Data sampling

You need testdata (virtuel measurements) for test or simulation of a flight simulator. The testdata should simulate the altitude of a plane and are supposed to be Rayleigh distributed:

$$f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

- a) Find the cdf for the Rayleigh distribution $F_X(x)$.
- b) Find the inverse of the Rayleigh cdf (ie. x as a function of $y = F_X(x)$).
- c) Make a Matlab programme that makes 10000 random Rayleigh distributed testdata with $\sigma=7$.
- d) Make a scatter-plot of the testdata.
- e) Make a histogram of the data. Does it look like a Rayleigh distribution?
- f) Repeat a) – e) for exponential distributed data: $f_X(x) = \lambda e^{-\lambda x}$; $x \geq 0$, with $\lambda = \frac{1}{8}$.

Transformation of random variable

A stochastic variable Y are given by: $Y = a \cdot X + b$, where a and b are constants and $X \sim \mathcal{U}[0,1]$.

- g) Determine the pdf $f_Y(y)$, and make a draw of it together with $f_X(x)$.
- h) Calculate $E[X]$, $E[Y]$, $\text{Var}[X]$ and $\text{Var}[Y]$.
- i) Calculate the correlation coefficient ρ_{XY} .
- j) Are X and Y independent?
- k) Make a Matlab programme that draws 10 random numbers between -2 and +2 by use of the *rand* function. Make a plot of the 10 random numbers.

Multivariate random variables

A resistor R in an analog filter is made of two resistors in series: $R_1 = 2.4k\Omega$ and $R_2 = 100\Omega$. Both resistors are uniform distributed $\pm 5\%$.

- l) Determine and draw the pdf $f_R(r)$ for the sum of the two resistors $R = R_1 + R_2$.
- m) Find the mean and standard deviation of $R = R_1 + R_2$.
- n) What would the mean and standard deviation be if we instead used one uniform distributed 5%-resistor $R_0 = 2.5k\Omega$?
- o) If you could chose any two uniform distributed 5%-resistors to build a $2.5k\Omega$ resistor, which resistor-values will you chose to get the most accurate total resistor? What will the standard deviation be? Determine and draw the pdf $f_R(r)$ for the sum of the two resistors.
- p) Make a simulation in Matlab of l) – o).

Central Limit Theorem (CLT)

Make a Matlab simulation to illustrate the Central Limit Theorem (CLT):

Make 9 random variables Y_1, \dots, Y_9 as the average of 1 – 9 i.i.d. random variables X_n (ie. $Y_1 = X_1$, $Y_2 = \frac{1}{2}(X_1 + X_2)$ etc.) and make a plot of the 9 random variables Y_i . Does the random variables approach a normal distributed when more terms are included? Make the simulation for:

- q) $X_n \sim \mathcal{U}[0,1]$
- r) $X_n \sim \text{Rayleigh distributed with } \sigma = 1$