

Shammugan

Problem 2.1 a

Event A_1 : Ace first card
Event A_2 : Ace second card
Event A_3 : Ace third card
Event A_4 : Ace fourth card

$$\Pr(A_1) = \frac{4}{52} \quad (\text{four cards out of } 52)$$

$$\Pr(A_2) = \frac{4}{52} \quad (\text{again four out of } 52)$$

$$\Pr(A_3) = \frac{4}{52}$$

$$\Pr(A_4) = \frac{4}{52}$$

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4) =$$

Probability of drawing Ace in first
and probability of drawing Ace in second
and ...

because of replacement, events are
independent, we have:

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4) = \Pr(A_1) \Pr(A_2) \Pr(A_3) \Pr(A_4)$$

thus

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4) = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = 3.5 \cdot 10^{-5}$$

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~~1/130000~~

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Problem 2.16

Because of no-replacement

$$Pr(A_1 \cap A_2 \cap A_3 \cap A_4) = Pr(A_1) Pr(A_2|A_1) \cdot Pr(A_3|A_2, A_1) Pr(A_4|A_3, A_2, A_1)$$

$Pr(A_1)$ - ace in first card

$Pr(A_2|A_1)$ - ace in second when an ace was drawn in first card

$Pr(A_3|A_2, A_1)$ - ace in third when an ace was drawn in both first and second card

$Pr(A_4|A_1, A_2, A_3)$ - ace in fourth when an ace was drawn in both first, second and third.

$$Pr(A_1) = \frac{4}{52} \quad (\text{four out of 52 are aces})$$

$$Pr(A_2|A_1) = \frac{3}{51} \quad (\text{only 3 aces left and 51 cards left})$$

$$Pr(A_3|A_1, A_2) = \frac{2}{50}$$

$$Pr(A_4|A_3, A_2, A_1) = \frac{1}{49}$$

problem 2.1b (continued)

thus

$$\begin{aligned} \Pr(A_1 \cap A_2 \cap A_3 \cap A_4) &= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \\ &= \underline{\underline{3,69 \cdot 10^{-6}}} \end{aligned}$$

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Problem 2.2 a

Die tossing:

Event A_1 : Dots = 3

Event A_2 : Dots = even

Event A_3 : Dots = odd

$Pr(A_1) = \frac{1}{6}$ (one of ~~the~~ 6 sides on the die has 3 dots)

$Pr(A_1 \cap A_3) = \frac{1}{6}$ (one of 6 sides has 3 dots and are odd)

Problem 2.2 b

$$Pr(A_2 \cup A_3) = 1$$

sides that are either odd or even. $\frac{6}{6} = 1$

$$Pr(A_2 \cap A_3) = 0$$

empty set, sides that are both odd and even.

$$Pr(A_1 | A_3) = \frac{1}{3}$$

given the side are odd, what is the chance of a 3.
3 odd sides, 1 three.

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Problem 2.2 c

Are A_2 and A_3 disjoint?

Yes, no overlap for even and odd

Problem 2.2. d

Are A_2 and A_3 independent?

No, since $\Pr(A_2 \cap A_3) = 0$
and $\Pr(A_2) \Pr(A_3) = \frac{1}{4}$

$$\Rightarrow \Pr(A_2) = \frac{1}{2}$$

$$\Pr(A_3) = \frac{1}{2}$$

for independence, we have

$$\Pr(A_2 \cap A_3) = \Pr(A_2) \Pr(A_3)$$