SMP Eksamen

OPGAVE 1

1) **Marginal sandsynlighed for Y:**

$$f_Y(y=5) = f_{Y,X}(y=5, x=1) + f_{YX}(y=5, x=2) + f_{YX}(y=5, x=3) = 0 + \frac{1}{12} + 0 = \underline{\underline{\frac{1}{12}}}$$

$$f_Y(y=6) = f_{YX}(y=6, x=1) + f_{YX}(y=6, x=2) + f_{YX}(y=6, x=3) = \frac{2}{12} + 0 + \frac{2}{12} = \underline{\underline{\frac{4}{12}}}$$

$$f_Y(y=7) = f_{YX}(y=7, x=1) + f_{YX}(y=7, x=2) + f_{YX}(y=7, x=3) = \frac{2}{12} + \frac{1}{12} + \frac{2}{12} = \underline{\underline{\frac{5}{12}}}$$

$$f_Y(y=8) = f_{YX}(y=8, x=1) + f_{YX}(y=8, x=2) + f_{YX}(y=8, x=3) = 0 + \frac{2}{12} + 0 = \underline{\underline{\frac{2}{12}}}$$

Marginal sandsynlighed for X:

$$f_X(x=1) = f_{YX}(y=5, x=1) + f_{YX}(y=6, x=1) + f_{YX}(y=7, x=1) + f_{YX}(y=8, x=1) = 0 + \frac{2}{12} + \frac{2}{12} + 0 = \underline{\underline{\frac{4}{12}}}$$

$$f_X(x=2) = f_{YX}(y=5, x=2) + f_{YX}(y=6, x=2) + f_{YX}(y=7, x=2) + f_{YX}(y=8, x=2) = \frac{1}{12} + \frac{1}{12} + \frac{2}{12} = \underline{\underline{\frac{4}{12}}}$$

$$f_X(x=3) = f_{YX}(y=5, x=3) + f_{YX}(y=6, x=3) + f_{YX}(y=7, x=3) + f_{YX}(y=8, x=3) = 0 + \frac{2}{12} + \frac{2}{12} + 0 = \underline{\underline{\frac{4}{12}}}$$

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2) Forventningsværdi af X:

$$E[X] = \sum_{i=1}^{n} x_i f_X(x = x_i)$$

$$= \left(1 \cdot \frac{4}{12} + 2 \cdot \frac{4}{12} + 3 \cdot \frac{4}{12}\right) = \underline{\underline{2}}$$

Forventningsværdi af Y:

$$E[Y] = \sum_{i=1}^{n} y_i f_Y(y = y_i)$$

$$= 5 \cdot \frac{1}{12} + 6 \cdot \frac{4}{12} + 7 \cdot \frac{5}{12} + 8 \cdot \frac{2}{12} = \underline{\underline{\frac{20}{3}}}$$

Forventningsværdi af produkt af X og Y:

$$E[XY] = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i y_j f_{YX}(y_j, x_i)$$

$$= 1 \cdot 6 \cdot \frac{2}{12} + 1 \cdot 7 \cdot \frac{2}{12} + 2 \cdot 5 \cdot \frac{1}{12} + 2 \cdot 7 \cdot \frac{1}{12} + 2 \cdot 8 \cdot \frac{2}{12}$$

$$+ 3 \cdot 6 \cdot \frac{2}{12} + 3 \cdot 7 \cdot \frac{2}{12} = \underline{\underline{\frac{40}{3}}}$$

2. moment af X:

$$E[X^2] = \sum_{i=1}^{n} x_i^2 f_X(x_i)$$

$$= \left(1^2 \cdot \frac{4}{12} + 2^2 \cdot \frac{4}{12} + 3^2 + \frac{4}{12}\right) = \underline{\underline{\frac{14}{3}}}$$

2. moment af Y:

$$E[Y^2] = \sum_{i=1}^{n} y_i^2 f_Y(y_i)$$

$$= \left(5^2 \cdot \frac{1}{12} + 6^2 \cdot \frac{4}{12} + 7^2 \cdot \frac{5}{12} + 8^2 \cdot \frac{2}{12}\right) = \underline{\underline{\frac{271}{6}}}$$

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OPGAVE 1

3) Korrelationskoefficient for $\bar{X}$ og $\bar{Y}$:

$$\rho_{x,y} = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{0}{\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{13}{18}}} = \underline{\underline{0}}$$

da

$$Cov(x,y) = E[XY] - E[X]E[Y] = 0$$

$$\sigma_x = \sqrt{E[X^2] - E[X]^2} = \sqrt{\frac{14}{3} - 4} = \sqrt{\frac{2}{3}} = 0.816$$

$$\sigma_y = \sqrt{E[Y^2] - E[Y]^2} = \sqrt{\frac{271}{6} - \frac{400}{9}} = \sqrt{\frac{813-800}{18}} = \sqrt{\frac{13}{18}} = 0{,}850$$

4) Da korrelationskoefficienten er 0, er X og Y $\underline{\underline{\text{ukorrelerede}}}$.

5) Vi opskriver $f_x(x) \cdot f_y(y)$:

y\x	1	2	3
5	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
6	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
7	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{5}{36}$
8	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$

Da $f_x(x) \cdot f_y(y)$ ikke er identisk med $f_{xy}(x,y)$, er $\bar{X}$ og $\bar{Y}$ $\underline{\underline{\text{ikke uafhængige}}}$.

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6) Den betingede sandsynlighed $f_{X|Y}(x \mid y=6)$:

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

derved

$$f_{X|Y}(x=1 \mid y=6) = \frac{f_{X,Y}(x=1, y=6)}{f_Y(y=6)} = \frac{\frac{2}{12}}{\frac{4}{12}} = \underline{\underline{\frac{1}{2}}}$$

$$f_{X|Y}(x=2 \mid y=6) = \frac{f_{X,Y}(x=2, y=6)}{f_Y(y=6)} = \frac{0}{\frac{4}{12}} = \underline{\underline{0}}$$

$$f_{X|Y}(x=3 \mid y=6) = \frac{f_{X,Y}(x=3, y=6)}{f_Y(y=6)} = \frac{\frac{2}{12}}{\frac{4}{12}} = \underline{\underline{\frac{1}{2}}}$$

$f_{X|Y}(x \mid y=6)$ kan derved opskrives:

x	1	2	3
$f_{X\|Y}(x\|y=6)$	$\frac{1}{2}$	0	$\frac{1}{2}$

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OPGAVE 2

1) se vedlagt bilag A.

2) Ensemble middelværdi for $X(n)$

$$E[X(n)] = E[w(n)]$$
$$= \frac{1}{2}(10+0) = \underline{\underline{5}}$$

da for uniforme fordelinger gælder:

$$w \sim U(a,b) \qquad E[w] = \frac{1}{2}(a+b)$$

varians er:

$$\text{var}(x) = E[X(n)^2] - E[X(n)]^2 = \frac{1}{2}(10-0)^2 = \underline{\underline{\frac{25}{3}}}$$

da for uniforme fordelinger gælder:

$$w \sim U(a,b) \qquad \text{Var}(W) = \frac{1}{12}(b-a)^2$$

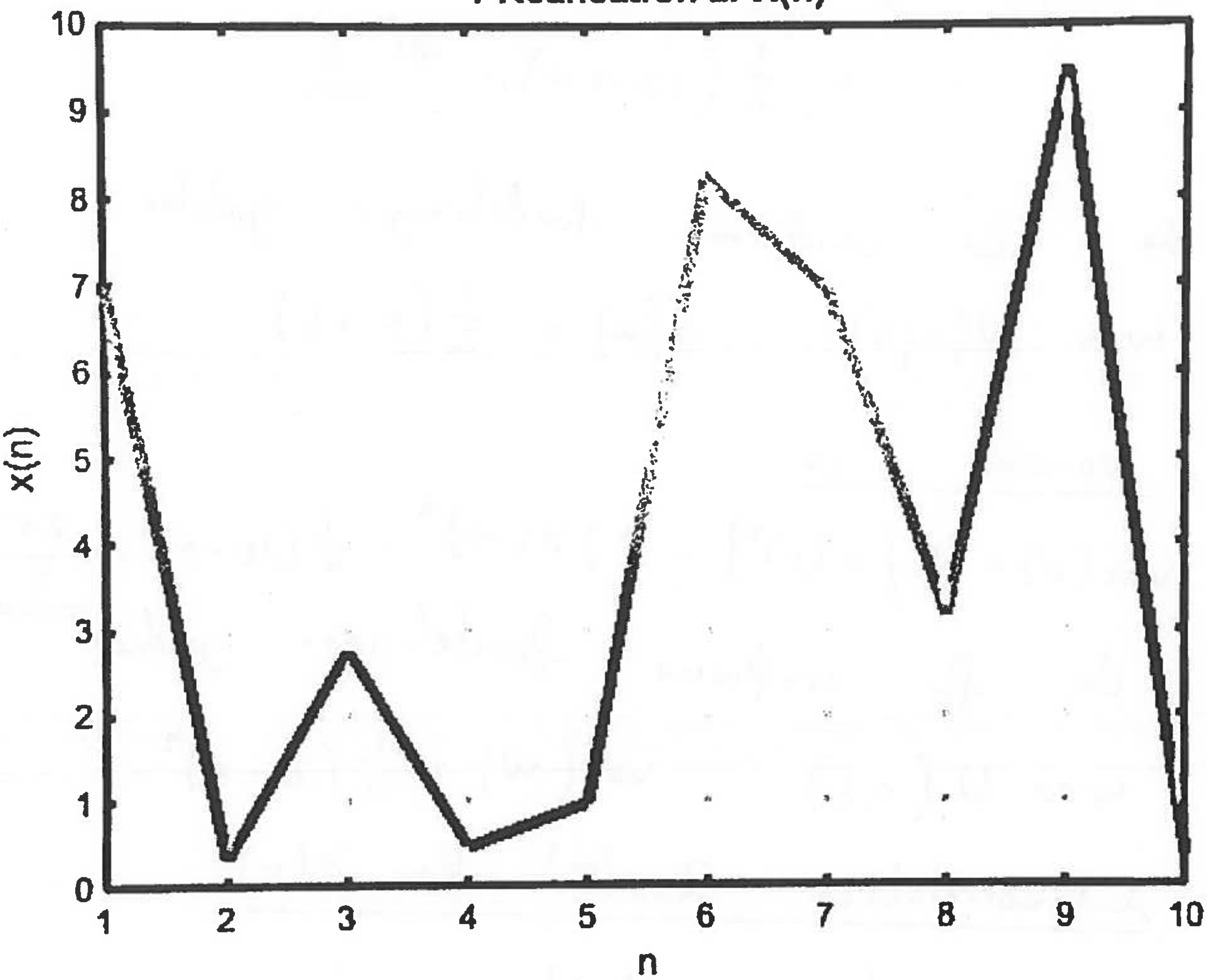
3) Autokorrelation $R_{xx}(\tau)$ for $X(n)$:

$$R_{xx}(0) = E[w(n)\cdot w(n)]$$
$$= \text{var}(w(n)) + E[w(n)]^2$$
$$= \frac{25}{3} + 5^2 = \underline{\underline{\frac{100}{3}}}$$

$$R_{xx}(1) = E[w(n)\cdot w(n+1)]$$
$$= E[w(n)]\cdot E[w(n+1)] \overset{[1]\text{-note}}{=} 5^2 = \underline{\underline{25}}$$

Bilag A

1 Realisation af X(n)



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OPGAVE 2

3)

(fortsat)

[1]-note.
Vi har at $w(n)$ og $w(n+1)$ er angivet til at være uafhængige (i.i.d.)

$$R_{xx}(2) = E[w(n)\cdot w(n+2)]$$
$$= E[w(n)]\cdot E[w(n+2)] = 5^2 = \underline{\underline{25}}$$

$$R_{xx}(3) = E[w(n)\cdot w(n+3)]$$
$$= E[w(n)]\,E[w(n+3)] = 5^2 = \underline{\underline{25}}$$

4)
Da middelværdi $E[X(n)]$ er uafhængig af n, og $E[X(n)^2]$ er uafhængig af n, kan $X(n)$ siges at være stationær i den brede forstand altså: WSS.

~~Da den tidslige middelværdi (over n)~~
~~er lig den tidslige ensemble middelværdien~~
~~Samt at stations den~~

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OPGAVE 2

4) (fortsat)

processen $X(n)$ er ergodisk, da den tidslige (over n) fordelingsfunktion er identisk med ~~den~~ fordelingsfunktionen for ensemblet.

SMP Eksamen

OPGAVE 3

Event A: Finne har laktoseintoleranse.

Event B: Test positiv.

Vi kender:

$$Pr(A) = 0,2$$

$$Pr(B \mid A) = 0,9$$

$$Pr(B \mid \bar{A}) = 0,3$$

1) Total sandsynlighed for positiv Test:

$$Pr(B) = Pr(B \mid A)Pr(A) + Pr(B \mid \bar{A})Pr(\bar{A}) = \underline{\underline{0,42}}$$

$$Pr(\bar{A}) = 1 - Pr(A) = 0,8$$

2) Find $Pr(A \mid B)$:

$$Pr(A \mid B) = \frac{Pr(B \mid A)\,Pr(A)}{Pr(B)} = \frac{0,9 \cdot 0,2}{0,42} = \underline{\underline{0,43}}$$

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OPGAVE 4

1) Find bedste hældning α:

Sample middel af Antal:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(5562 + \ldots + 268) = 2213$$

middel tid:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i = \frac{1}{10}(1901 + \ldots + 1991) = 1946$$

hældning α:

$$\alpha = \frac{\sum_{i=1}^{n} (t - \bar{t})(x - \bar{x})}{\sum_{i=1}^{n} (t - \bar{t})^2} = \underline{\underline{-59{,}5}}$$

skæring β:

$$\beta = \bar{x} - \alpha \cdot \bar{t} = \underline{\underline{118000}}$$

Lineær tilnærmelse $\hat{x} = \alpha \cdot t + \beta$

$\alpha = -59{,}5$

$\beta = 118000$

se bilag B for plot.

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OPGAVE 4

2) Residudtegning

$$res(t) = x(t) - (\alpha \cdot t + \beta)$$

se bilag C for plot

3) 95% konfidensinterval:

for α:

invers student t fordeling $n-2 = 8$ frihedsgrader, og 95% (0,975):

$t_0 = 2{,}3060$

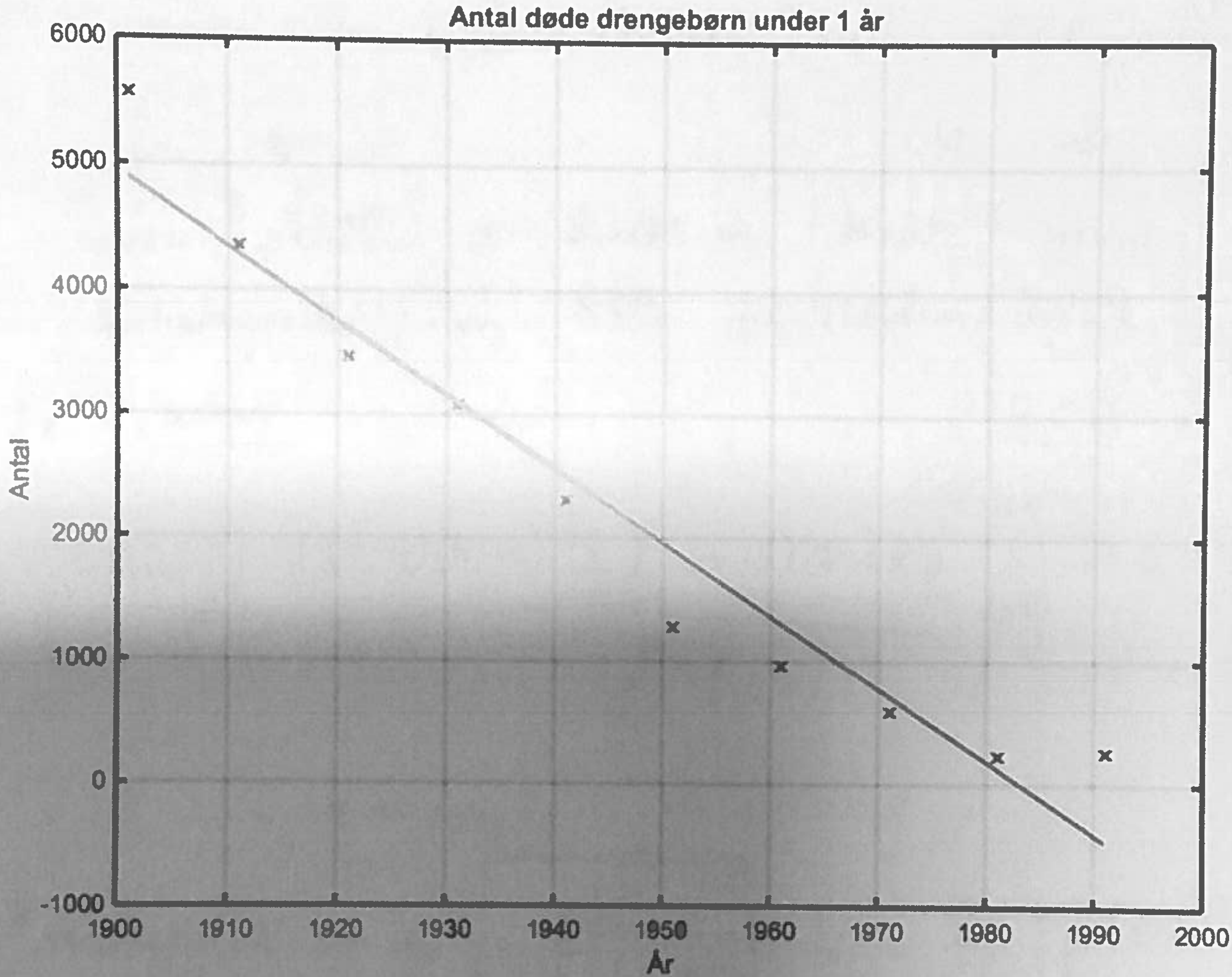
$$S_r^2 = b = \frac{1}{n-2}\left(\sum_{i=1}^{n}(x-\bar{x})^2 - \frac{\left(\sum_{i=1}^{n}(x-\bar{x})(t-\bar{t})\right)^2}{\sum_{i=1}^{n}(t-\bar{t})^2}\right) = \frac{1}{n-2}\sum_{i=1}^{n}(x-(\alpha \cdot t+\beta))^2$$

$$= 204150$$

$$\alpha_{nedre} = \alpha - t_0\sqrt{\frac{b}{\sum_{i=1}^{n}(t-\bar{t})^2}} = -70{,}97$$

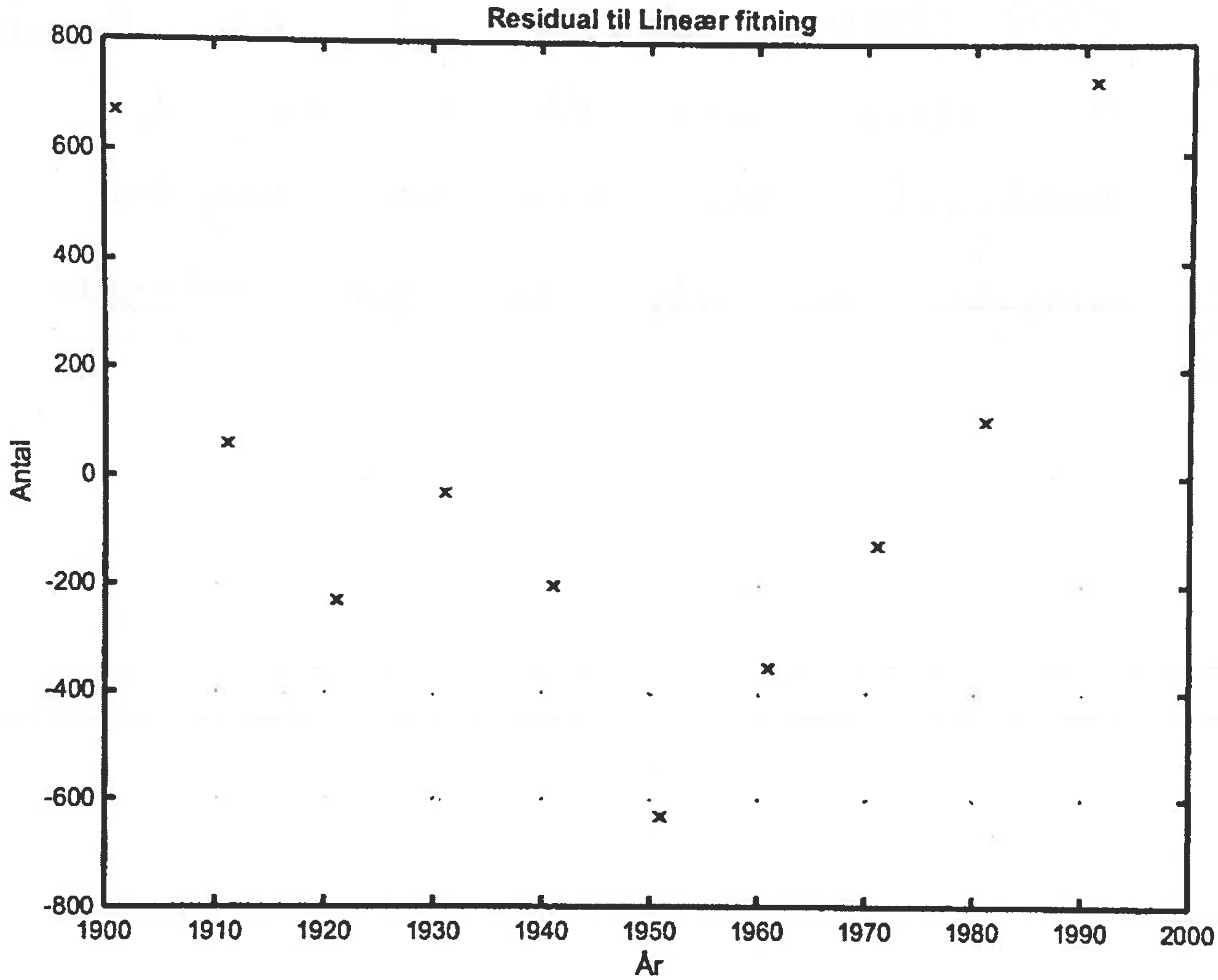
$$\alpha_{øvre} = \alpha + t_0\sqrt{\frac{b}{\sum_{i=1}^{n}(t-\bar{t})^2}} = -48{,}03$$

Bilag B



Antal døde drengebørn under 1 år
6000
5000
4000
3000
2000
1000
0
-1000
Antal
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
År

Bilag C


Residual til Lineær fitning
800
600
400
200
0
-200
-400
-600
-800
Antal
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
År

JMP EKSAMEN

OPGAVE 4

4)

Der er umiddelbart for få målepunkter. ~~Dog er~~ Der en lineær tendens, men residualplottet viser at residualerne systematisk ligger under 0 mellem år 1920 - 1970, desuden vil data forvente at afvige mere efter år 1991, da dødeligheden ikke kan være negativ. Linearitet er ikke en god antagelse.