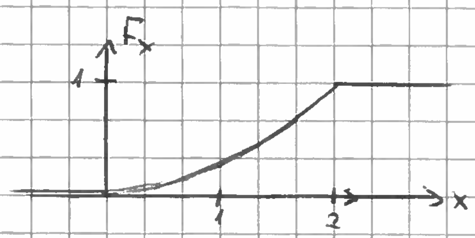
SMP

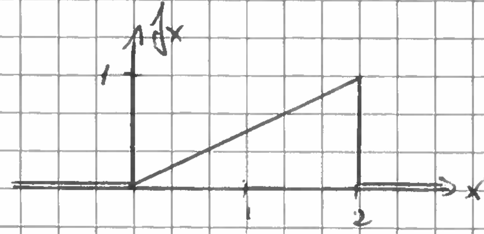
Opg. 1 S17re

$$F_X(x) = \begin{cases} 0 & x<0 \\ ax^2 & 0 \le x \le 2 \\ 1 & x>2 \end{cases}$$



a) F_X kontinuert $\Rightarrow \begin{cases} F_X(0)=0 \\ F_X(2)=1 \Rightarrow 4a=1 \Rightarrow \underline{\underline{a=\frac{1}{4}}} \end{cases}$

b) $\underline{\underline{f_X(x)}} = \frac{dF_X}{dx} = \underline{\underline{\begin{cases} 0 & x<0 \\ 2a\cdot x = \frac{1}{2}x & 0\le x\le 2 \\ 0 & x>2 \end{cases}}}$



c) $\underline{\underline{Pr(1\le X\le 2)}} = F(2) - F(1) = 1 - \frac{1}{4} = \underline{\underline{\frac{3}{4} = 0.75}}$

d) $\underline{\underline{E(X)}} = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_0^2 \frac{1}{2}x^2\,dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{8}{6} = \underline{\underline{\frac{4}{3} = 1.33}}$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^2 \frac{1}{2}x^3\,dx = \left[\frac{1}{8}x^4\right]_0^2 = \frac{16}{8} = 2$

$\Downarrow$

$\underline{\underline{Var(X)}} = E(X^2) - E(X)^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{18-16}{9} = \frac{2}{9} = \underline{\underline{0.22}}$

SMP

Opg 2 S17 re

$A_1 : 1k\Omega \quad , \quad A_{10} : 10k\Omega \quad , \quad A_{100} : 100k\Omega$

$B_1 : 1\% \quad , \quad B_5 : 5\%$

$N_1 = 1000 \quad , \quad N_{10} = 800 \quad , \quad N_{100} = 400 \quad \Rightarrow N = N_1 + N_{10} + N_{100} = 2200$

$Pr(B_1 | A_1) = \frac{1}{4} \quad ; \quad Pr(B_1 | A_{10}) = \frac{1}{2} \quad ; \quad Pr(B_1 | A_{100}) = \frac{3}{4}$

a) $\underline{\underline{Pr(A_1)}} = \frac{N_1}{N} = \frac{1000}{2200} = \underline{\underline{\frac{5}{11} = 0.455 = 45.5\%}}$

$\underline{\underline{Pr(A_{10})}} = \frac{N_{10}}{N} = \frac{800}{2200} = \underline{\underline{\frac{4}{11} = 0.364 = 36.4\%}}$

$\underline{\underline{Pr(A_{100})}} = \frac{N_{100}}{N} = \frac{400}{2200} = \underline{\underline{\frac{2}{11} = 0.182 = 18.2\%}}$

b) $\underline{\underline{Pr(A_{10} \cap B_1)}} = Pr(B_1 | A_{10}) \cdot Pr(A_{10}) = \frac{1}{2} \cdot \frac{4}{11} = \underline{\underline{\frac{2}{11} = 0.182 = 18.2\%}}$

c) $\underline{\underline{Pr(B_1)}} = Pr(B_1 | A_1) \cdot Pr(A_1) + Pr(B_1 | A_{10}) \cdot Pr(A_{10}) + Pr(B_1 | A_{100}) \cdot Pr(A_{100})$

$= \frac{1}{4} \cdot \frac{5}{11} + \frac{1}{2} \cdot \frac{4}{11} + \frac{3}{4} \cdot \frac{2}{11} = \underline{\underline{\frac{19}{44} = 0.432 = 43.2\%}}$

d) $Pr(B_5) = 1 - Pr(B_1) = \frac{25}{44} \quad , \quad Pr(B_5 | A_{100}) = 1 - Pr(B_1 | A_{100}) = \frac{1}{4}$

$\underline{\underline{Pr(A_{100} | B_5)}} = \frac{Pr(B_5 | A_{100}) \cdot Pr(A_{100})}{Pr(B_5)}$

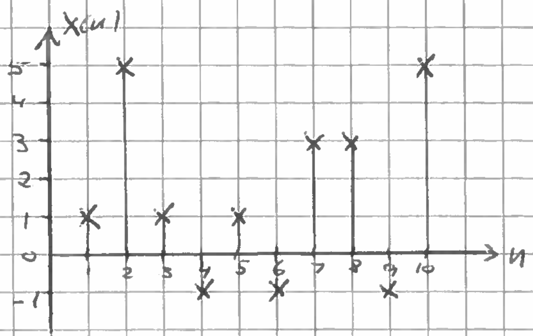
$= \frac{1/4 \cdot 2/11}{25/44} = \underline{\underline{\frac{2}{25} = 0.08 = 8.0\%}}$

SMP

Opg. 3 S17 re

$X(n) = 2\cdot W(n) - 1$, $W(n) \sim \{0, 1, 2, 3\}$, $f_W(i) = \frac{1}{4}$, $i = 0, .., 3$

a) $X(n) \sim \{-1, 1, 3, 5\}$



En realisering - 10 samples $(n=1,..,10)$:

Matlab: x(n) = 2*(unidrnd(4,1,10)-1)-1

$x(1)=1$, $x(2)=5$, $x(3)=1$, $x(4)=-1$, $x(5)=1$

$x(6)=-1$, $x(7)=3$, $x(8)=3$, $x(9)=-1$, $x(10)=5$

b) $\underline{\underline{\hat{\mu}_i}} = E(2W_i(n)-1) = 2E(W_i(n)) - 1 = 2\cdot\frac{0+3}{2} - 1 = 3 - 1 = \underline{\underline{2}}$

$\underline{\underline{\hat{\sigma}_i^2}} = Var(2W_i(n)-1) = 4\cdot Var(W_i(n)) = 4\cdot\left(E(W_i^2(n)) - E(W_i(n))^2\right)$

$= 4\cdot\left(\frac{0^2+1^2+2^2+3^2}{4} - \left(\frac{3}{2}\right)^2\right) = (1+4+9-9) = \underline{\underline{5}}$

c) $\underline{\underline{\mu_x}} = E(2W(n)-1) = 2E(W(n)) - 1 = 2\cdot\frac{0+3}{2} - 1 = 3 - 1 = \underline{\underline{2}}$

$\underline{\underline{\sigma_x^2}} = Var(2W(n)-1) = 4\cdot Var(W(n)) = 4\cdot\left(\frac{(3-0+1)^2-1}{12}\right) = 4\cdot\frac{16-1}{12}$

$= \frac{15}{3} = \underline{\underline{5}}$ (Diskret uniform fordeling)

d) μ_x og σ_x^2 uafh. af tiden (n) $\rightarrow$ $\underline{\underline{\text{WSS}}}$

$\mu_x = \hat{\mu}_i$ og $\sigma_x^2 = \hat{\sigma}_x^2$ $\rightarrow$ $\underline{\underline{\text{Ergodisk}}}$

SMP

Opg. 4 S17 re

Specifikation: $T \sim \mathcal{N}(3000, 10000)$

Test:

i	1	2	3	4	5	6	7	8	9	10	11	12
T_i	3148	2956	2803	2933	2869	3111	2789	2995	2909	2929	3148	2867

a) Hypotese: $H_0: \hat{\mu}_T = T = 3000$

$H_1: \hat{\mu}_T \neq T = 3000$

b) $$\underline{\underline{\hat{\mu}_T}} = \frac{1}{n}\sum_{i=1}^{n} T_i = \frac{1}{12}\sum_{i=1}^{12} T_i = \frac{35457}{12} = \underline{\underline{2954{,}75}} \text{ timer}$$

c) $$z = \frac{\overline{T} - \mu_T}{\sigma/\sqrt{n}} = \frac{2954{,}75 - 3000}{100/\sqrt{12}} = -1.5675$$

$$p = 2\cdot|1-\phi(|z|)| = 2\cdot(1-\phi(1.5675)) = 2\cdot(1-0.942) = 2\cdot 0.058$$

$$= 0.116 > 0.05$$

$\Rightarrow$ <u>H_0 kan ikke afvises.</u>

d) $$\Delta T = 1.96\cdot\frac{\sigma}{\sqrt{n}} = 1.96\,\frac{100}{\sqrt{12}} = 56{,}58$$

$$T_- = \overline{T} - \Delta T = 2954{,}75 - 56{,}58 = 2898{,}2$$

$$T_+ = \overline{T} + \Delta T = 2954{,}75 + 56{,}58 = 3011{,}3$$

$\Downarrow$

95% konfidens interval: $\underline{\underline{[2898\,;\,3011] \text{ timer}}}$

Da $T = 3000 \in [2898\,;\,3011]$, kan H_0 ikke afvises.