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problem 2.6

We have a joint discrete probabilities

	B_1	B_2	B_3
A_1	$3/36$	*	$5/36$
A_2	$5/36$	$4/36$	$5/36$
A_3	*	$6/36$	*
$P(B_i)$	$12/36$	$14/36$	*

a)

$$P(A_1, B_2) = P(B_2) - P(A_2, B_2) - P(A_3, B_2)$$
$$= 14/36 - 4/36 - 6/36 = 4/36$$

$$P(A_3, B_1) = P(B_1) - P(A_1, B_1) - P(A_2, B_1)$$
$$= 12/36 - 5/36 - 3/36 = 4/36$$

$$P(B_3) = 1 - P(B_1) - P(B_2)$$
$$= 1 - 12/36 - 14/36 = 10/36$$

$$P(A_3, B_3) = P(B_3) - P(A_1, B_3) - P(A_2, B_3)$$
$$= 10/36 - 5/36 - 5/36 = 0$$

	B_1	B_2	B_3	$P(A_i)$
A_1	$3/36$	$4/36$	$5/36$	$12/36$
A_2	$5/36$	$4/36$	$5/36$	$14/36$
A_3	$4/36$	$6/36$	0	$10/36$

Shanmugan

problem 2.6 (cont'd)

b) Find $P(B_3 \mid A_1)$ and $P(A_1 \mid B_3)$

$P(B_3) = 10/36 \quad P(A_1) = P(A_1, B_1) + P(A_1, B_2)$
$+ P(A_1, B_3)$
$= 12/36$

$P(A_1, B_3) = 5/36$

$$P(B_3 \mid A_1) = \frac{P(A_1, B_3)}{P(A_1)} = \frac{5/36}{12/36} = \frac{5}{12}$$

$$P(A_1 \mid B_3) = \frac{P(A_1, B_3)}{P(B_3)} = \frac{5/36}{10/36} = \frac{5}{10}$$

c) Are A_1 and B_1 independent?

if independent $P(A_1, B_1) = P(A_1)P(B_1)$

$P(A_1, B_1) = 3/36 = 1/12$

$P(A_1)P(B_1) = 12/36 \cdot 12/36 = \frac{1}{9}$

thus not independent.

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problem 2.10

Tossing die four times

a) all possible outcome

1	H H H H
2	H H H T
3	H H T H
4	H T H H
5	T H H H
6	H H T T
7	H T H T
8	T H H T
9	H T T H
10	T H T H
11	T T H H
12	H T T T
13	T H T T
14	T T H T
15	T T T H
16	T T T T

ordered with replacement

$4^2 = 16$

b) stochastic variable X

	X
H H H H	4
H H H T	3
H H T H	3
H T H H	3
T H H H	3
H H T T	2
H T H T	2
T H H T	2
H T T H	2
T H T H	2
T T H H	2
H T T T	1
T H T T	1
T T H T	1
T T T H	1
T T T T	0

pmf:

$p(x=4) = 1/16$

$p(x=3) = 4/16$

$p(x=2) = 6/16$

$p(x=1) = 4/16$

$p(x=0) = 1/16$

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problem 2.11

tossing of two dice.

X - number of sum of eyes.

6 - side dice

$$P(X) = \begin{cases} \dfrac{6-|(x-(6+1))|}{6^2} & \text{for } x \in \{2, \dots, 12\} \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \dfrac{6-|x-7|}{36} & \text{for } x \in \{2, \dots 12\} \\ 0 & \text{otherwise} \end{cases}$$

$P(x=2) = \frac{1}{36}$ $P(x=3) = \frac{2}{36}$ $P(x=4) = 3/36$

$P(x=5) = 4/36$ $P(x=6) = 5/36$ $P(x=7) = 6/36$

$P(x=8) = \cancel{3}5/36$ $P(x=9) = 4/36$ $P(x=10) = 3/36$

$P(x=11) = 2/36$ $P(x=12) = 1/36$

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problem 2.17

joint mass function

Y \ X	−1	0	1	P(Y)
−1	1/4	1/8	0	3/8
0	0	1/4	0	1/4
1	0	1/8	1/4	3/8
P(X)	1/4	1/2	1/4	

(a) find $P(Y=1 \mid X=1)$

$P(X=1) = 1/4$

$P(Y=1) = 3/8$

$P(Y=1, X=1) = 1/4$

$$P(Y=1 \mid X=1) = \frac{P(Y=1, X=1)}{P(X=1)} = \frac{1/4}{1/4} = 1$$

b) Find $P(X=1 \mid Y=1)$

$$P(X=1 \mid Y=1) = \frac{P(Y=1, X=1)}{P(Y=1)} = \frac{1/4}{3/8} = \frac{2}{3}$$

c) find ρ_{XY}.

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problem 2.17

c) continued

$$E[x] = \sum_i x_i \, p(x_i) = (-1)\cdot \tfrac{1}{4} + 0\cdot \tfrac{1}{2} + 1\cdot \tfrac{1}{4} = 0$$

$$E[y] = \sum_i y_i \, p(\cancel{x_i}) = (-1)\cdot \tfrac{3}{8} + 0\cdot \tfrac{1}{4} + 1\cdot \tfrac{3}{8} = 0$$

$$E[xy] = \sum_i \sum_m x_i y_m \, p(x_i, y_m)$$

$$= (-1)\cdot(-1)\cdot \tfrac{1}{4} + 0\cdot 0\cdot \tfrac{1}{4} + 1\cdot 1\cdot \tfrac{1}{4}$$

$$+ 0\cdot(-1)\cdot \tfrac{1}{8} + 0\cdot 1\cdot \tfrac{1}{8} = 0$$

$$E[x^2] = \sum_i x_i^2 \, p(x_i) = 1^2\cdot \tfrac{1}{4} + 0^2\cdot \tfrac{1}{2} + (-1)^2\cdot \tfrac{1}{4}$$

$$= \tfrac{1}{2}$$

$$E[y^2] = \sum_i y_i^2 \, p(y_i) = 1^2\cdot \tfrac{3}{8} + 0^2\cdot \tfrac{1}{4} + (-1)^2\cdot \tfrac{3}{8}$$

$$= \tfrac{3}{4}$$

$$\rho_{xy} = \frac{E[xy] - E[x]\,E[y]}{\sqrt{E[x^2] - E[x]^2}\,\sqrt{E[y^2] - E[y]^2}}$$

$$= \underline{\underline{0}}$$