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problem 2.32

$Z = X + Y - c$

Variance of Z .

X and Y are independent

$\operatorname{cov}(x, y) = 0$

$\operatorname{Var}(c) = 0$

thus

$\operatorname{Var}(z) = \operatorname{var}(X) + \operatorname{Var}(Y) = \sigma_x^2 + \sigma_y^2$

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problem 2.35

$$f_X(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

a) Find the pdf for Y, when:

$$Y = X^2$$

1) Inverse $X = \sqrt{Y}$ for $X > 0$
$X = -\sqrt{Y}$ for $X \leq 0$

2) differentiate $\frac{d}{dy}\sqrt{y} = \frac{1}{2} y^{-\frac{1}{2}}$

$$\frac{d}{dy} -\sqrt{y} = -\frac{1}{2} y^{-\frac{1}{2}}$$

3) limits for pdf:

from $-\infty < x < \infty$ to $0 < Y < \infty$

4) New pdf for Y:

$$f_Y(y) = \left|\frac{1}{2} y^{-\frac{1}{2}}\right| \cdot \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{\sqrt{y}^2}{2\sigma_y^2}}$$

$$+ \left|-\frac{1}{2} y^{-\frac{1}{2}}\right| \cdot \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{(-\sqrt{y})^2}{2\sigma_y^2}}$$

$$= \left|y^{-\frac{1}{2}}\right| \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{y}{2\sigma_y^2}}$$

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5) Find the pdf for Y when

$Y = |X|$

1) inverse $X = Y$ for $X > 0$

$X = -Y$ for $X \leq 0$

2) differentiate $\frac{d}{dy} y = 1$

$\frac{d}{dy} -y = -1$

2) limits:

$-\infty < x < \infty$ to $0 < y < \infty$

3) New pdf for Y:

$$f_Y(y) = |1| \cdot \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_x^2}} + |-1| \frac{1}{\sigma_x\sqrt{2\pi}} e^{\frac{-(-y)^2}{2\sigma_x^2}}$$

$$= \frac{2}{\sigma_x\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_x^2}}$$