SMP

Opgave 1 V17/18

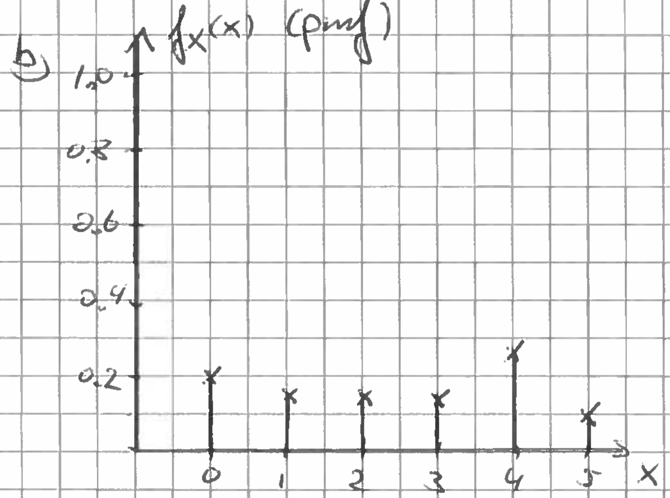
$X : \{0,1,2,3,4,5\}$

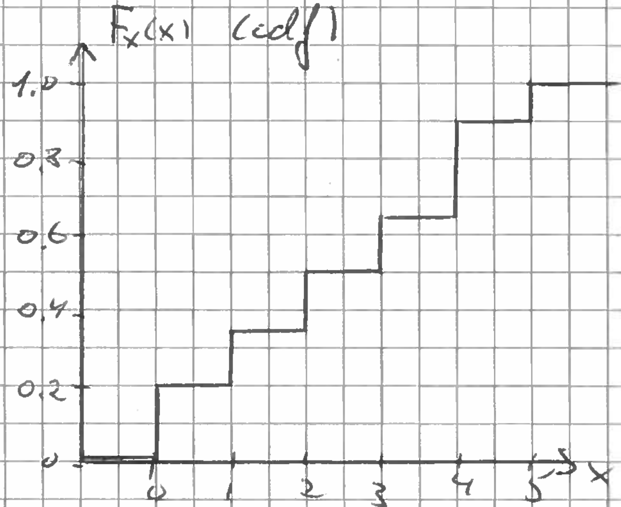
$Pr(1) = Pr(2) = Pr(3) = 0.15$, $Pr(4) = 0.25$, $Pr(5) = 0.10$

a) $\sum_{i=0}^{5} Pr(X=i) = 1 \Rightarrow \underline{\underline{Pr(X=0)}} = 1 - \sum_{i=1}^{5} Pr(X=i) = 1 - (3 \cdot 0.15 + 0.25 + 0.10)$

$= 1 - 0.80 = \underline{\underline{0.20}}$

b)





c) $\underline{\underline{E[X]}} = \sum_{i=0}^{5} i \cdot f_X(i) = 0 \cdot 0.2 + 1 \cdot 0.15 + 2 \cdot 0.15 + 3 \cdot 0.15 + 4 \cdot 0.25 + 5 \cdot 0.10$

$= 0.90 + 1.0 + 0.5 = \underline{\underline{2.40}}$

d) $E[X^2] = \sum_{i=0}^{5} i^2 \cdot f_X(i) = 0 \cdot 0.2 + 1 \cdot 0.15 + 4 \cdot 0.15 + 9 \cdot 0.15 + 16 \cdot 0.25 + 25 \cdot 0.10$

$= 2.10 + 4.00 + 2.50 = 8.60$

$\underline{\underline{Var[X]}} = E[X^2] - E[X]^2 = 8.60 - 2.40^2 = 8.60 - 5.76 = \underline{\underline{2.84}}$

SMP

Opgave 2 V17/18

Æg:

f_{XY}		X: Størrelse				
Y: Type		1	2	3	4	f_Y
1	Bær	2/25	2/25	1/25	1/25	6/25
2	Skrabe	2/25	3/25	3/25	2/25	10/25
3	Øko	1/25	2/25	3/25		
	f_X	5/25	7/25	7/25		1

a) $\sum_{x=1}^{4}\sum_{y=1}^{3} f_{XY}(x,y) = 1 \Rightarrow \underline{\underline{f_{XY}(x=4, y=3)}} = \text{Pr}(\text{Øko størrelse } 4)$

$= 1 - \frac{2+2+1+1+2+3+3+2+1+2+3}{25}$

$= 1 - \frac{22}{25} = \underline{\underline{\frac{3}{25}}}$

b) $\underline{\underline{\text{Pr}(X=3) = f_X(x=3)}} = \sum_{y=1}^{3} f_{XY}(x=3, y) = \frac{1+3+3}{25} = \underline{\underline{\frac{7}{25}}}$

c) $\underline{\underline{\text{Pr}(Y=1)}} = f_Y(y=1) = \sum_{x=1}^{4} f_{XY}(x, y=1) = \frac{2+2+1+1}{25} = \underline{\underline{\frac{6}{25}}}$

d) $\underline{\underline{\text{Pr}(Y=2 \mid X=2)}} = \frac{f_{XY}(x=2, y=2)}{f_X(x=2)} = \frac{3/25}{7/25} = \underline{\underline{\frac{3}{7}}}$

$= \frac{\text{Pr}(Y=2 \wedge X=2)}{\text{Pr}(X=2)}$

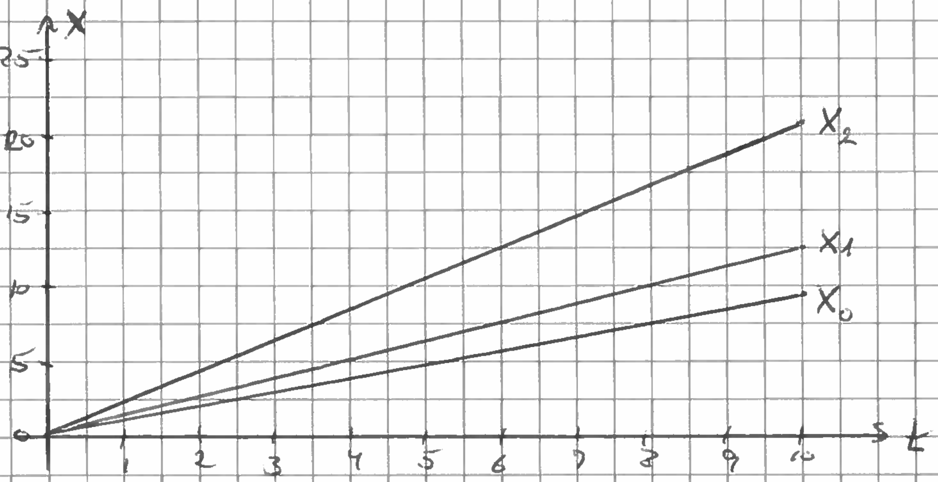
SMP

Opgave 3 V17/18

$X(t) = \alpha \cdot t \quad , \ t \geq 0 \quad , \ \alpha \sim \mathcal{N}(1, \tfrac{1}{4}) \Rightarrow \mu_\alpha = 1, \sigma_\alpha^2 = \tfrac{1}{4}$

a) 3 realiseringer : $\alpha_0 = 0.940$, $\alpha_1 = 1.278$, $\alpha_2 = 2.096$
(se bilag)

$X_1(t) = 0.940 \cdot t$; $X_2(t) = 1.278 \cdot t$; $X_3(t) = 2.096 \cdot t$



b) $\underline{\underline{E[X]}} = E[\alpha \cdot t] = E[\alpha] \cdot t = 1 \cdot t = \underline{\underline{t}}$

$\underline{\underline{Var[X]}} = Var[\alpha \cdot t] = t^2 \cdot Var[\alpha] = \underline{\underline{\tfrac{1}{4} t^2}}$

c) Da $E[X]$ og $Var[X]$ ikke er konstante uafhængig af t, er $\underline{\underline{X \text{ ikke WSS}}}$.

Da X ikke er WSS, er X heller $\underline{\underline{\text{ikke ergodisk}}}$.

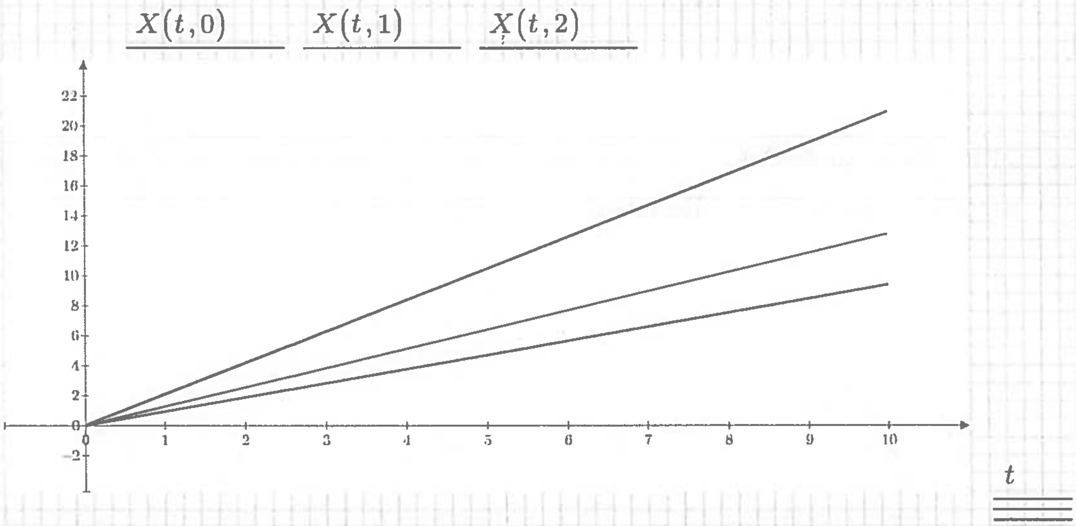
Opgave 3

Normalfordelte koefficienter: $\alpha := \frac{1}{2}\,\mathrm{Normal}(3) + 1$

$\alpha_0 = 0.94 \qquad \alpha_1 = 1.278 \qquad \alpha_2 = 2.096$

Stokastisk variabel: $X(t,i) := \alpha_i \cdot t$

Tre realiseringer:



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SMP

Opgave 4 V17/18

$X \sim \text{Poisson}(\lambda = 25/\text{time})$

a) $\underline{\underline{\Pr(X > 30/\text{time})}} = 1 - \Pr(X \leq 30/\text{time}) = 1 - \text{poisscdf}(30, 25)$

$= 1 - 0.865 = \underline{\underline{0.137 = 13.7\%}}$

b) $\underline{\underline{\Pr(X \leq 80/4\,\text{timer})}} = \text{poisscdf}(80, 4\lambda) = \text{poisscdf}(80, 100)$

$\underline{\underline{= 0.023 = 2.3\%}}$

c) $t = 24$ timer: $X = 653 \Rightarrow \underline{\underline{\hat{\lambda}}} = \frac{X}{t} = \frac{653}{24} = \underline{\underline{27.2/\text{time}}}$

d) $t \cdot \lambda = 24 \cdot 25 = 600 \gg 5 \rightarrow \sim \mathcal{N}$-approximation

$z = \frac{X - \lambda \cdot t}{\sqrt{\lambda \cdot t}} = \frac{653 - 600}{\sqrt{600}} = 2.164 \sim \mathcal{N}(0,1)$

NULL-hypotese: $H_0: \lambda = 25/\text{time}$, $H_1: \lambda \neq 25/\text{time}$

p-værdi: $p = 2 \cdot (1 - \phi(2.164)) = 2 \cdot (1 - 0.985) = 0.03$

Da $p = 0.03 < 0.05 = \alpha$ forkastes H_0. Dvs. $\underline{\underline{\lambda \neq 25/\text{time}}}$

e) $\lambda_- = \frac{1}{t}\left(X + \frac{1.96^2}{2} - 1.96 \cdot \sqrt{X + \frac{1.96^2}{4}}\right)$

$= \frac{1}{24}\left(653 + \frac{1.96^2}{2} - 1.96\sqrt{653 + \frac{1.96^2}{4}}\right) = \frac{1}{24}(654.9 - 50.1) = 25.2$

$\lambda_+ = \frac{1}{t}\left(X + \frac{1.96^2}{2} + 1.96 \cdot \sqrt{X + \frac{1.96^2}{4}}\right) = \frac{1}{24}(654.9 + 50.1) = 29.4$

$\Downarrow$

95% konfidensinterval: $\underline{\underline{[\lambda_-; \lambda_+] = [25.2; 29.4]}}$

Da $\lambda = 25 \notin [25.2; 29.4]$ forkastes H_0.

Opgave 4

a) $Pr\_30 := 1 - \text{ppois}(30, 25) = 0.137$

b) $Pr\_80 := \text{ppois}(80, 100) = 0.023$

c) $x := 653$ $t := 24$ $\lambda := 25$

$\lambda_{est} := \frac{x}{t} = 27.208$

d) $z := \frac{x - t \cdot \lambda}{\sqrt{t \cdot \lambda}} = 2.164$

$\text{pnorm}(z, 0, 1) = 0.985$

$p := 2 \cdot (1 - \text{pnorm}(z, 0, 1)) = 0.0305$

e) $a := x + \frac{1.96^2}{2} = 654.921$

$b := 1.96 \cdot \sqrt{x + \frac{1.96^2}{4}} = 50.122$

$\lambda_{min} := \frac{1}{t} \cdot (a - b) = 25.2$ $\lambda_{max} := \frac{1}{t} \cdot (a + b) = 29.377$

95% konfidensinterval: $[\lambda_{min};\ \lambda_{max}]$=[25.2; 29.4]

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