SMP

Opgave 1

$$X:\ F_X(x) = \begin{cases} k\cdot(2-2\cdot e^{-\frac{1}{2}x}) & x\geq 0 \\ 0 & x<0 \end{cases}$$

a) $\lim_{x\to-\infty} F_X(x) = 0$ o.k.

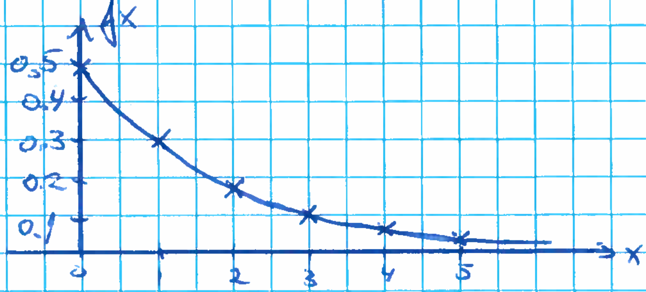
$\lim_{x\to+\infty} F_X(x) = k\cdot(2-0) = 1 \Rightarrow \underline{\underline{k=\frac{1}{2}}}$

$$F_X(x) = \begin{cases} 1-e^{-\frac{1}{2}x} & ; x\geq 0 \\ 0 & ; x<0 \end{cases}$$

$F_X(x)$ kontinuert i $x=0$:

$$\lim_{x\to 0^+} F_X(x) = k\cdot(2-2\cdot e^{-\frac{1}{2}\cdot 0}) = k\cdot(2-2) = 0 = \lim_{x\to 0^-} F_X(x) \quad \text{o.k.}$$

b) $$f_X(x) = \frac{dF_X}{dx} = \begin{cases} -(-\frac{1}{2})e^{-\frac{1}{2}x} = \underline{\underline{\frac{1}{2}e^{-\frac{1}{2}x}}} & x\geq 0 \\ \underline{\underline{0}} & x<0 \end{cases}$$



c) $$\underline{\underline{Pr(1\leq x\leq 2)}} = F_X(2) - F_X(1) = 1-e^{-\frac{1}{2}\cdot 2} - (1-e^{-\frac{1}{2}\cdot 1}) = e^{-\frac{1}{2}} - e^{-1}$$
$$\underline{\underline{= 0{,}239}}$$

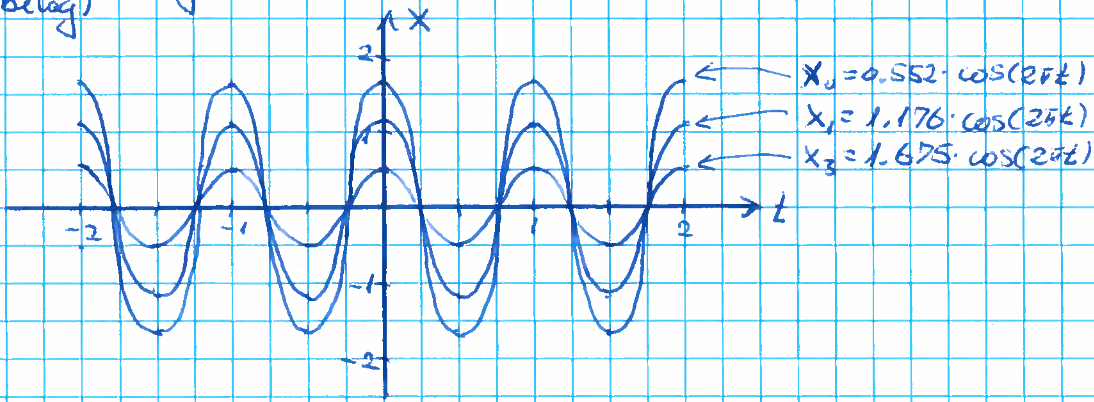
d) $$\underline{\underline{E[X]}} = \int_{-\infty}^{\infty} x\cdot f_X(x)\,dx = \int_0^{\infty} \frac{1}{2}x\, e^{-\frac{1}{2}x}\,dx = \left[-x e^{-\frac{1}{2}x}\right]_0^{\infty} + \int_0^{\infty} e^{-\frac{1}{2}x}\,dx$$
$$= \left[-(x+2)e^{-\frac{1}{2}x}\right]_0^{\infty} = 0+2 = \underline{\underline{2}}$$

SMP

Opgave 2

$$X(t) = A \cdot \cos(2\pi \cdot t) \quad ; \quad A \sim \mathcal{U}[0;2]$$

a) Tre realiseringer: $A_0 = 0.552$, $A_1 = 1.176$, $A_2 = 1.675$
(Se bilag)



b) $$\underline{\underline{\mu_T(x_1)}} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_1(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1.176 \cdot \cos(2\pi t)\,dt = \lim_{T\to\infty} \frac{1}{T}\cdot 0 = \underline{\underline{0}}$$

$$\underline{\underline{\sigma_T^2(x_1)}} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_1^2(t)\,dt - \mu_T(x_1)^2 = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1.176^2 \cdot \cos^2(2\pi t)\,dt$$

$$= 1.176^2 \cdot \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos^2(2\pi t)\,dt = 1.176^2 \cdot \frac{1}{2} = \underline{\underline{0.691}}$$

c) $$\underline{\underline{E[X]}} = E[A \cdot \cos(2\pi t)] = E[A] \cdot \cos(2\pi t) = \frac{2-0}{2} \cdot \cos(2\pi t) = \underline{\underline{\cos(2\pi t}}$$

$$\underline{\underline{Var[X]}} = Var[A \cdot \cos(2\pi t)] = \cos^2(2\pi t) \cdot Var[A] = \cos^2(2\pi t) \cdot \frac{2^2 - 1^2}{12}$$

$$= \underline{\underline{\frac{1}{4}\cos^2(2\pi t)}}$$

d) Da $E[X]$ og $Var[X]$ ikke er konstante, men afhænger af t, er $\underline{\underline{X \text{ ikke WWS}}}$.

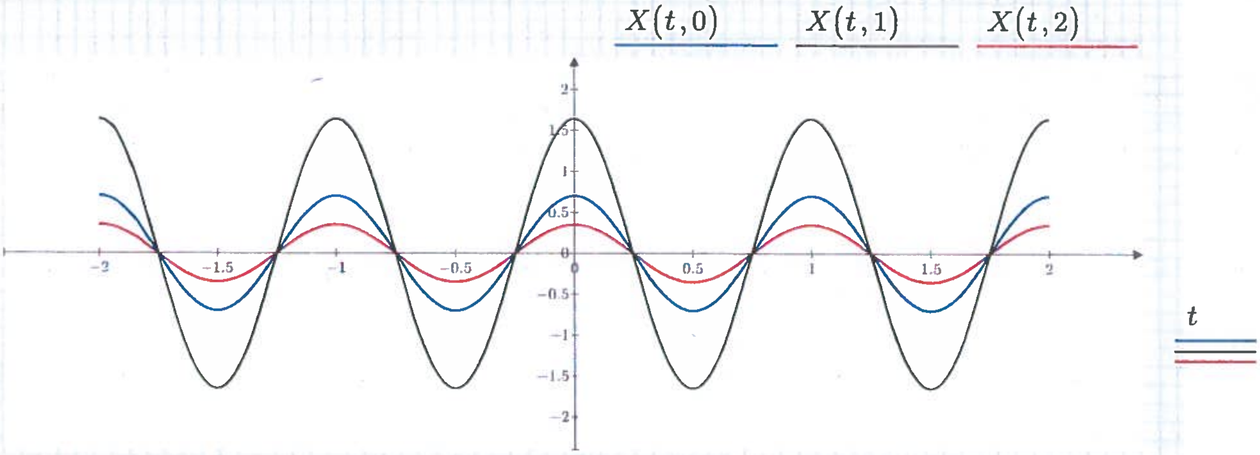
Da X ikke er WWS, er den heller <u>ikke ergodisk</u>

Opgave 2

a)

$$A := \frac{1}{6} \cdot \text{Uniform}(3) + 1 \qquad A_0 = 0.701 \qquad A_1 = 1.646 \qquad A_2 = 0.348$$

$$X(t,i) := A_i \cdot \cos(2\,\pi \cdot t)$$



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Opgave 3

Handelser: S : Overholder specifikation

$\overline{S}$: Overholder ikke specifikation

T : Positiv test (godkendes)

$\overline{T}$: Negativ test (godkendes ikke)

Data: $P_r(S|T) = 0.90$; $P_r(\overline{S}|\overline{T}) = 0.80$; $P_r(T) = 0.85$

a) $\underline{\underline{P_r(S|\overline{T})}} = 1 - P_r(\overline{S}|\overline{T}) = 1 - 0.80 = \underline{\underline{0.20 = 20\%}}$

b) $\underline{\underline{P_r(S \cap T)}} = P_r(S|T) \cdot P_r(T) = 0.90 \cdot 0.85 = \underline{\underline{0.765 = 76.5\%}}$

c) $P_r(S) = P_r(S \cap T) + P_r(S \cap \overline{T})$

$= P_r(S|T) \cdot P_r(T) + P_r(S|\overline{T}) \cdot P_r(\overline{T})$

$= P_r(S|T) \cdot P_r(T) + P_r(S|\overline{T}) \cdot (1 - P_r(T))$

$= 0.90 \cdot 0.85 + 0.20 \cdot (1 - 0.85)$

$= 0.765 + 0.030$

$= 0.795 = 79.5\%$

$\Downarrow$

$P_r(\overline{S}) = 1 - P_r(S) = 1 - 0.795 = 0.205 = \frac{N_{\overline{S}}}{N_{tot}}$

$\Downarrow$

$\underline{\underline{N_{\overline{S}}}} = P_r(\overline{S}) \cdot N_{tot} = 0.205 \cdot 1000 = \underline{\underline{205}}$

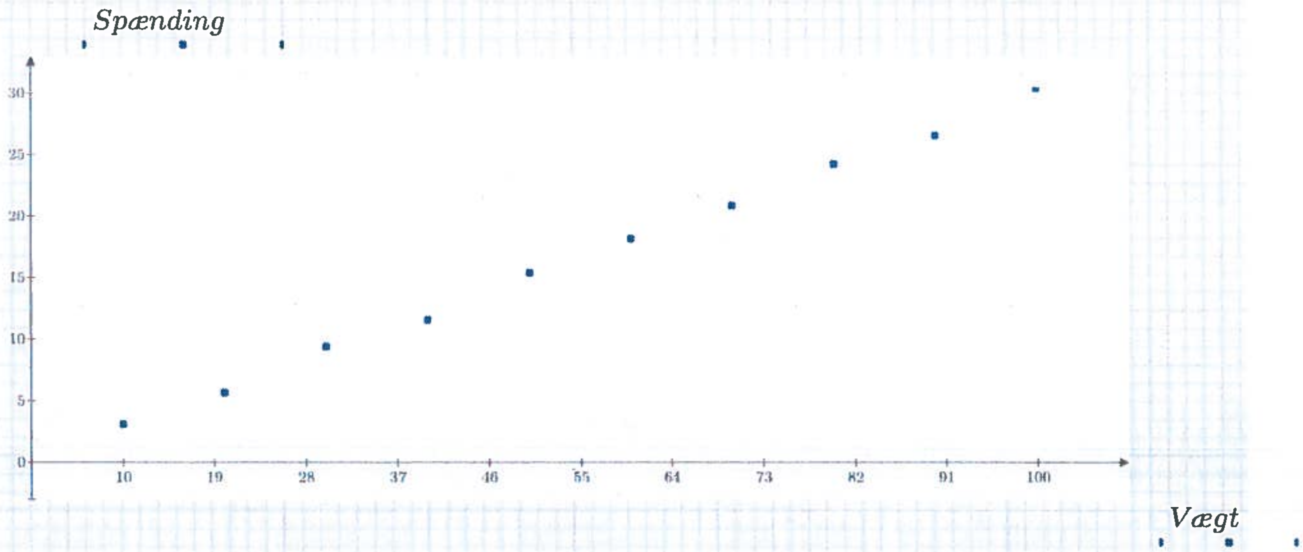
Opgave 4 : Se bilag

Opgave 4

$$Vægt := [10\ \ 20\ \ 30\ \ 40\ \ 50\ \ 60\ \ 70\ \ 80\ \ 90\ \ 100]$$

$$Spænding := [3.11\ \ 5.68\ \ 9.41\ \ 11.58\ \ 15.42\ \ 18.18\ \ 20.87\ \ 24.25\ \ 26.58\ \ 30.42]$$

a)



b) Middelværdier: $\mu_{Vægt} := \frac{1}{10} \cdot \sum_{i=0}^{9} Vægt_{0,i} = 55$

$$\mu_{Spænding} := \frac{1}{10} \cdot \sum_{i=0}^{9} Spænding_{0,i} = 16.55$$

Lineær regression: $\beta := \frac{\sum_{i=0}^{9} \left(\left(Vægt_{0,i} - \mu_{Vægt} \right) \cdot \left(Spænding_{0,i} - \mu_{Spænding} \right) \right)}{\sum_{i=0}^{9} \left(Vægt_{0,i} - \mu_{Vægt} \right)^2} = 0.301$

$$\alpha := \mu_{Spænding} - \beta \cdot \mu_{Vægt} = -0.014$$

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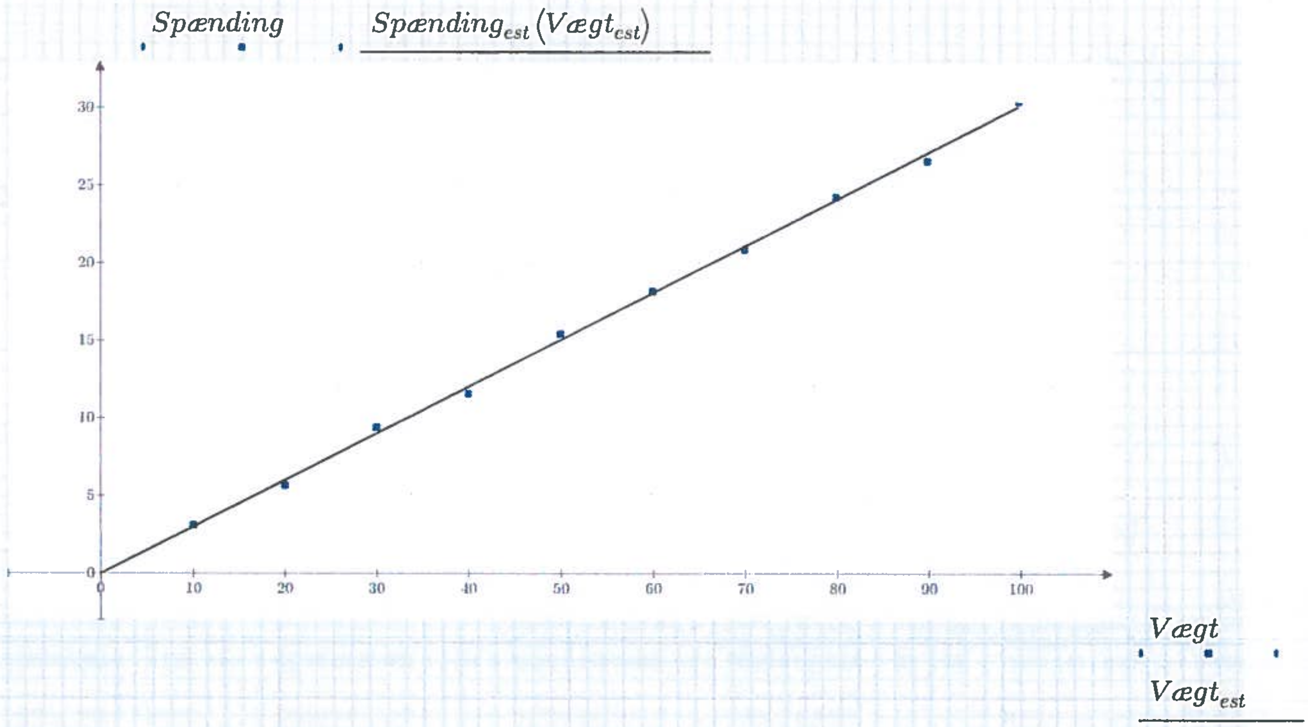
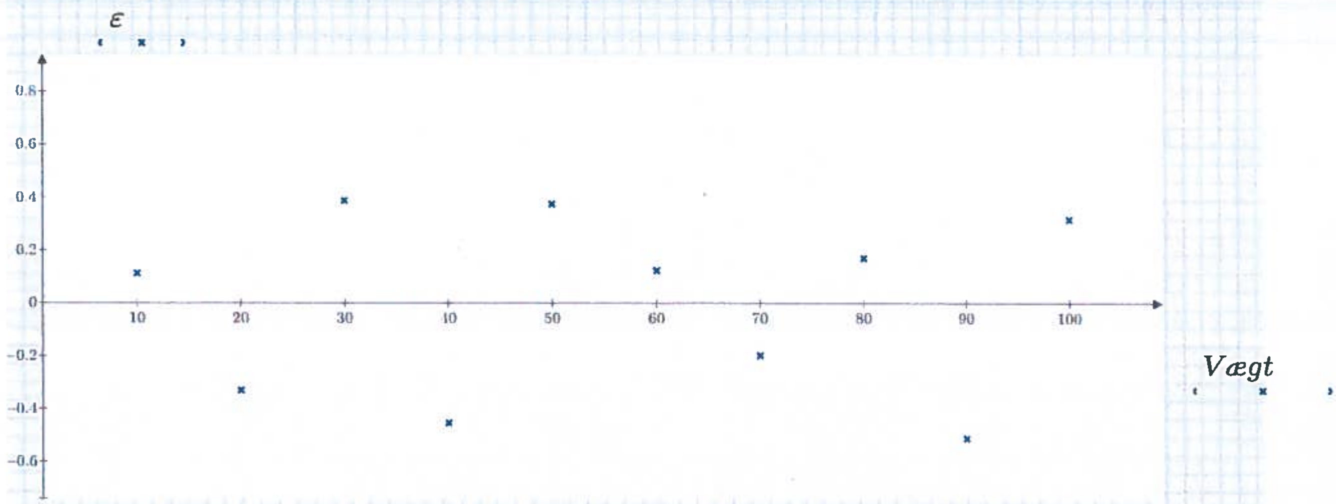
c) $Spænding_{est}(Vægt_{est}) := \alpha + \beta \cdot Vægt_{est} \xrightarrow{float, 5} 0.30116 \cdot Vægt_{est} - 0.014$

$Vægt_{est} := 0, 1 .. 100$



d) Residualer:

$\varepsilon := Spænding - Spænding_{est}(Vægt) = [0.112 \;\; -0.329 \;\; 0.389 \;\; -0.452 \;\; 0.376 \;\; 0.124 \;\; -0.197 \;\; 0.171 \;\; -0.51 \;\; 0.318]$



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e) $s2_{Vægt} := \frac{1}{9} \cdot \sum_{i=0}^{9} \left(Vægt_{0,i} - \mu_{Vægt}\right)^2 = 916.667$ $\qquad$ $s_{Vægt} := \sqrt{s2_{Vægt}} = 30.277$

$$s2_r := \frac{1}{8} \cdot \sum_{i=0}^{9} \left(Spænding_{0,i} - Spænding_{est}\left(Vægt_{0,i}\right)\right)^2 = 0.133 \qquad s_r := \sqrt{s2_r} = 0.365$$

$$s2 := \frac{1}{8} \cdot \sum_{i=0}^{9} {\varepsilon_{0,i}}^2 = 0.133$$

$$t_0 := \mathrm{qt}(0.975, 8) = 2.306$$

$$\Delta\beta := t_0 \cdot \frac{s_r}{\sqrt{9} \cdot s_{Vægt}} = 0.009$$

$$\beta_{lav} := \beta - \Delta\beta = 0.292$$

$$\beta_{høj} := \beta + \Delta\beta = 0.31$$

95% konfidensinterval: [β-; β+]=[βlav; βhøj]=[0.292; 0.310]

f) Residualplottet viser at residualerne er tilfældigt fordeltet uafhængig af vægten.

95% konfidensintervallet for hældningen b er meget lille ($\frac{\Delta\beta}{\beta} = 0.031$).

Begge dele betyder, at antagelsen om linearitet mellem vægtbelastningen og transducerspændingen er en god antagelse.

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