Shanmugan

problem 2.7

Two bags of blue and red marbles

First bag: 7 red, 3 blue

Second bag: 4 red, 5 blue

Event A: transfered marple is blue

Draw a blue marple from first bag: and put it in second bag.

$Pr(A) = \frac{3}{10}$ $Pr(\bar{A}) = 1 - \frac{3}{10} = \frac{7}{10}$

Event B: draw a red marple from second bag.

$Pr(B|A)$ probability of drawing a red marple given that the transfered marple was blue

$Pr(B|A) = \frac{4}{10}$ (now 10 marples, still only 4 red)

$Pr(B|\bar{A}) = \frac{5}{10}$ probability of drawing a red marple given that the transfered marple was red.

problem 2.7 (continued)

total probability $Pr(B)$:

$$Pr(B) = Pr(B|A)^{Pr(A)} + Pr(B|\bar{A})\,Pr(\bar{A})$$

$$= \frac{4}{10} \cdot \frac{3}{10} + \frac{5}{10} \cdot \frac{7}{10} = \frac{47}{100}$$

then using Bayes Rule

$$Pr(A|B) = \frac{Pr(B|A)\,Pr(A)}{Pr(B)} = \frac{\frac{4}{10} \cdot \frac{3}{10}}{\frac{47}{100}} = \frac{12}{47}$$

Shanmugan

probl. 2.9

Event A : student pass

Event B: student studied

$Pr(A|B) = 0,9$

$Pr(A|\bar{B}) = 0,2$

$Pr(B) = 0,75$ $\quad Pr(\bar{B}) = 1 - 0,75 = 0,25$

Find $Pr(B|A)$?

total probability $Pr(A) = Pr(A|B)\,Pr(B) + Pr(A|\bar{B})\,Pr(\bar{B})$

$= 0,9 \cdot 0,75 + 0,2 \cdot 0,25$

$= 0,725$

Bayes Rule:

$$Pr(B|A) = \frac{Pr(A|B)\,Pr(B)}{Pr(A)} = \frac{0,9 \cdot 0,75}{0,725} = \underline{0,931}$$

Shanmugan

problem 2.12

Event A $\quad pr(A) = \frac{1}{2}$

Event B $\quad pr(B) = \frac{1}{4}$

Event C $\quad pr(C) = \frac{1}{4}$

3 trials (independent), number of all possible outcomes:

Ordered with replacement:

n^r $\quad n$ - objects (A, B or C)

r - trials or samples

total number $3^3 = 27$

Number of outcomes with all three events taking place:

Ordered without replacement:

$$\frac{n!}{(r-n)!} = \frac{3!}{0!} = 6$$

probability of each of these outcomes:

$P(ABC) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = 0,0313$

thus probability of any of these 6:

$6 \cdot 0,0313 = \underline{\underline{0,1875}}$

Shanmugam

problem 2.16

Modem with BER of 10^{-4}.

$Pr($ bit error of one bit$) = 0,0001$

Use Binomial distribution

$$Pr_{10000}(1) = \frac{10000!}{1!(10000-1)!} \cdot 0,0001^{1} (1-0,0001)^{10000-1}$$

$$= 10000 \cdot 0,0001 \cdot 0,3679$$

$$= 0,3679$$

$$Pr_{10000}(0) = \frac{10000!}{0! \cdot 10000!} \cdot 0,0001^{0} (1-0,0001)^{10000}$$

$$= 1 \cdot 1 \cdot 0,3679$$

$$= 0,3679$$

$$Pr_{10000}(k \leq 2) = Pr_{10000}(1) + Pr_{10000}(0) = \underline{0,736}$$