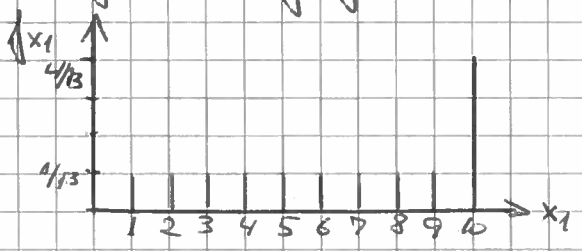
SMP

GA3: Black Jack II

X_n: Værdien af det n'te kort

1) Marginal pmf f_{X_1}: $\underbrace{X_1(1) = X_1(2) = \cdots = X_1(9) = \frac{1}{13}}_{=0.077}$, $\underbrace{X_1(10) = \frac{4}{13}}_{=0.308}$



$\sum_{i=1}^{10} x_1(i) = 9 \cdot \frac{1}{13} + \frac{4}{13} = \frac{13}{13} = 1$ O.K.

2) $\underline{E[X_1] = \sum_{i=1}^{10} x_1(i) \cdot f_{X_1}(x_1(i)) = (1+2+\cdots+9)\cdot\frac{1}{13} + 10\cdot\frac{4}{13} = \frac{45+40}{13} = \frac{85}{13} = 6.54}$

3) $f_{X_2|X_1}(x_2 \mid x_1 = 10) = \begin{cases} \frac{4}{51} = 0.078 & x_2 = 1,2,\cdots,9 \\ \frac{15}{51} = \frac{5}{17} = 0.294 & x_2 = 10 \end{cases}$

Der er 51 kort tilbage
$4 \times \{1,2,\cdots,9\}$
$+15 \times \{10\}$

4) $f_{X_1|X_2}(x_1 = 10 \mid x_2) = \dfrac{f_{X_2|X_1}(x_2 \mid x_1 = 10) \cdot f_{X_1}(x_1 = 10)}{f_{X_2}(x_2)}$

$$f_{X_2}(x_2) = f(x_2 \mid x_1 = 1)\cdot f(x_1 = 1) + f(x_2 \mid x_1 = 2)\cdot f(x_1 = 2) + \cdots + f(x_2 \mid x_1 = 9)\cdot f(x_1 = 9) + f(x_2 \mid x_1 = 10)\cdot f(x_1 = 10)$$

$$= \begin{cases} \frac{3}{51}\cdot\frac{4}{52} + 8\cdot\frac{4}{51}\cdot\frac{4}{52} + \frac{4}{51}\cdot\frac{4}{13} = \frac{3+32+16}{51\cdot 13} = \frac{51}{51\cdot 13} = \frac{1}{13} & x_2 = 1,2,\cdots,9 \\ 9\cdot\frac{16}{51}\cdot\frac{4}{52} + \frac{15}{51}\cdot\frac{4}{13} = \frac{144+60}{51\cdot 13} = \frac{204}{51\cdot 13} = \frac{4}{13} & x_2 = 10 \end{cases}$$

$$\Downarrow$$

$$f_{X_1|X_2}(x_1 = 10 \mid x_2) = \begin{cases} \frac{4/51 \cdot 4/13}{1/13} = \frac{16}{51} = 0.314 > f_{X_1}(x_1 = 10) & x_2 = 1,2,\cdots,9 \\ \frac{5/17 \cdot 4/13}{4/13} = \frac{5}{17} = 0.294 < f_{X_1}(x_1 = 10) & x_2 = 10 \end{cases}$$

7) Første 8 kort: $x_1 = 1, x_2 = 2, x_3 = 10, x_4 = 10, x_5 = 1, x_6 = 3, x_7 = 7, x_8 = 7$

Tilbage i stokken: 44 kort ($2\times1, 3\times2, 3\times3, 4\times4, 4\times5, 4\times6, 2\times7, 4\times8, 4\times9, 14\times10$)

9. kort: x_9: 1 2 3 4 5 6 7 8 9 10

$f_{X_9|X_1\cdots X_8}$: $\frac{2}{44}$ $\frac{3}{44}$ $\frac{3}{44}$ $\frac{4}{44}$ $\frac{4}{44}$ $\frac{4}{44}$ $\frac{2}{44}$ $\frac{4}{44}$ $\frac{4}{44}$ $\frac{14}{44}$ $\quad \sum \frac{44}{44} = 1$ O.K.

$$\underline{E[X_9] = \sum_{i=1}^{10} x_9(i)\cdot f_{X_9|X_1\cdots X_8} = \frac{1\cdot2 + 2\cdot3 + 3\cdot3 + 4\cdot4 + 4\cdot5 + 4\cdot6 + 2\cdot7 + 4\cdot8 + 4\cdot9 + 14\cdot10}{44} = \frac{299}{44} = 6.8}$$

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Simultan pmf $f_{X_1X_2}(x_1,x_2)$: Alle tal: $\times \frac{4}{52\cdot 51} = \frac{1}{3\cdot 13\cdot 17} = \frac{1}{663}$

x_1 \ x_2	1	2	3	4	5	6	7	8	9	10	f_{X_1}	$(\times \frac{1}{663})$
1	3	4	4	4	4	4	4	4	4	16	51	$\frac{1}{13}$
2	4	3	4	4	4	4	4	4	4	16	51	$\frac{1}{13}$
3	4	4	3	4	4	4	4	4	4	16	51	$\frac{1}{13}$
4	4	4	4	3	4	4	4	4	4	16	51	$\frac{1}{13}$
5	4	4	4	4	3	4	4	4	4	16	51	$\frac{1}{13}$
6	4	4	4	4	4	3	4	4	4	16	51	$\frac{1}{13}$
7	4	4	4	4	4	4	3	4	4	16	51	$\frac{1}{13}$
8	4	4	4	4	4	4	4	3	4	16	51	$\frac{1}{13}$
9	4	4	4	4	4	4	4	4	3	16	51	$\frac{1}{13}$
10	16	16	16	16	16	16	16	16	16	60	204	$\frac{4}{13}$
f_{X_2}	51	51	51	51	51	51	51	51	51	204	663	
$(\times \frac{1}{663})$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{4}{13}$		1

3) $f_{X_2|X_1}(x_2|x_1=10) = \frac{f_{X_1X_2}(x_1=10,x_2)}{f_{X_1}(x_1=10)} = \begin{cases} \frac{16/663}{4/13} = \frac{4}{51} = 0.078 & x_2 = 1,2,\dots,8 \\ \frac{60/663}{4/13} = \frac{15}{51} = \frac{5}{17} = 0.294 & x_2 = 10 \end{cases}$

4) $f_{X_1|X_2}(x_1=10|x_2) = \frac{f_{X_1X_2}(x_1=10,x_2)}{f_{X_2}(x_2)} = \begin{cases} \frac{16/663}{1/13} = \frac{16}{51} = 0.314 & x_2 = 1,2,\dots,9 \\ \frac{60/663}{4/13} = \frac{15}{51} = \frac{5}{17} = 0.294 & x_2 = 10 \end{cases}$