Opgave ETSMP: Transformation of random Variables

$$Y = a \cdot X + b \qquad X \sim U[0,1]$$

(g) Find $f_Y(y)$

$$f_X(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{ellers} \end{cases}$$

$$f_Y(y) = \begin{cases} k & b \leq y \leq a+b \\ 0 & \text{ellers} \end{cases}$$

$$k = \frac{1}{a+b-b} = \frac{1}{a}$$

(h) Beregn $E[x]$:

$$E[x] = \int_{-\infty}^{\infty} x \cdot f_X(x) \, dx = \int_0^1 x \cdot 1 \, dx = \left[\tfrac{1}{2} x^2 \right]_0^1$$

$$= \tfrac{1}{2} \cdot 1^2 - \tfrac{1}{2} \cdot 0^2 = \tfrac{1}{2}$$

$$E[y] = \int_b^{a+b} y \cdot \frac{1}{a} \, dy = \frac{1}{2} \cdot \frac{1}{a} \left[y^2 \right]_b^{a+b}$$

$$= \frac{1}{2a} \left((a+b)^2 - b^2 \right) = \frac{1}{2a} \left(a^2 + b^2 + 2ab - b^2 \right)$$

$$= \frac{a + 2b}{2} = \frac{a}{2} + b$$

—— 1 ——

(h) fortsat

$$Var(x) = E[x^2] - E[x]^2$$

$$E[x^2] = \int_0^1 x^2 \cdot 1 \, dx = \left[\frac{1}{3} x^3\right]_0^1 = \frac{1}{3}$$

$$Var(x) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}}$$

$$Var(y) = \frac{1}{12}\left(a+b-b\right)^2 = \underline{\underline{\frac{a^2}{12}}}$$

(i)
$$\rho_{xy} = \frac{E[xy] - E[x]E[y]}{\sqrt{Var(x) \cdot Var(y)}}$$

$$E[xy] = \iint x \cdot y \cdot f_{xy}(x,y) \, dx \, dy$$

$$= \int_0^1 x \cdot (ax+b) \cdot 1 \, dx$$

$$= \left[\frac{a}{3} x^3 + \frac{b}{2} x^2\right]_0^1 = \frac{a}{3} + \frac{b}{2}$$

$$\rho_{xy} = \frac{\frac{a}{3} + \frac{b}{2} - \frac{1}{2}\cdot\left(\frac{a}{2} + b\right)}{\sqrt{\frac{1}{12} \cdot \frac{a^2}{12}}}$$

$$= \underline{\underline{sign(a)}}$$

———— 2 ————