Solutions

1 The true parameter is estimated $\lambda = 10$ (i.e., 10 events per hour on average). The parameter, given the observation time t = 4, is $\lambda_t = \lambda t = 10 \cdot 4 = 40$.

a. $\Pr(X \geq 45) = 1 - \Pr(X \leq 45) = 1 - poisscdf(45{,}40) = 0.1903$

b. $\Pr(X \leq 38) = poisscdf(38{,}40) = 0.4160$

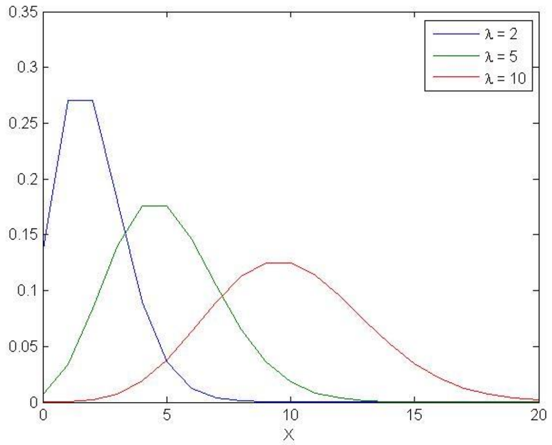
c. $\Pr(X = 45) = poisspdf(45{,}40) = 0.0440$

2 We have $X \sim poisson(\lambda)$, so $E[X] = \lambda$ and $Var(X) = \lambda$.

a. For $\lambda = 2, E[X] = 2$ and $Var(X) = 2$. For $\lambda = 5, E[X] = 5$ and $Var(X) = 5$. For $\lambda = 10, E[X] = 10$ and $Var(X) = 10$.

b. Matlab code

```
x = 0:20;
plot(x,poisspdf(x,2),...
     x,poisspdf(x,5),...
     x,poisspdf(x,10))
xlabel('X')
legend('\lambda = 2','\lambda = 5','\lambda = 10')
```



c. Matlab code generating 20 samples by histogram matching:

```
lambda = 2;
Fx = poisscdf(x,lambda);
y = rand(1,20);
for i = 1:20
    dist = abs(Fx-y(i));
    [minval,minix] = min(dist);
    sample(i) = x(minix);
end
```

3 Let X denote the number of cars passing the traffic light per hour.

a. Statistical model: We have $X \sim poisson(\lambda)$, with observation x = 45 and t = 4 hours. The parameter estimate is $\hat{\lambda} = x/t = 45/4 = 11.25$.

b. The management team's claim is that $\lambda = 10$. Hence, we have $H_0\colon \lambda = 10$ and $H_1\colon \lambda \neq \lambda$.

c. Standardizing the observation, x=45, we get

$$z = \frac{x - t \cdot \lambda}{\sqrt{t \cdot \lambda}} = \frac{45 - 4 \cdot 10}{\sqrt{4 \cdot 10}} = 0.7906$$

The approximate p-value is

$$2 \cdot \big(1 - \Phi(|z|)\big) = 2 \cdot \big(1 - \Phi(0.7906)\big) = 0.4292$$

And we fail to reject the null hypothesis.

4 Statistical model: We have $X \sim poisson(\lambda)$, with observation x = 25 and t = 2 hours.

a. The parameter estimate is $\hat{\lambda} = x/t = 25/2 = 12.50$.

b. The endpoints of the 95% confidence interval are

$$\lambda_{-} = \frac{1}{t}\left[x + \frac{1.96^2}{2} - 1.96\sqrt{x + \frac{1.96^2}{4}}\right] = \frac{1}{2}\left[25 + \frac{1.96^2}{2} - 1.96\sqrt{25 + \frac{1.96^2}{4}}\right] = 8.4672$$

$$\lambda_{+} = \frac{1}{t}\left[x + \frac{1.96^2}{2} + 1.96\sqrt{x + \frac{1.96^2}{4}}\right] = \frac{1}{2}\left[25 + \frac{1.96^2}{2} + 1.96\sqrt{25 + \frac{1.96^2}{4}}\right] = 18.4536$$

If our null hypothesis states that $H_0\!: \lambda = 10$, we fail to reject the null hypothesis, because the hypothesized value λ lies within the confidence interval.