SMP

Opg. 1 S18

Hændelser: A = Defekt RFID $\quad$ B = Registreret defekt

$\bar{A}$ = Ikke-defekt RFID $\quad$ $\bar{B}$ = Registreret ikke-defekt

Vi ved: $Pr(A) = 0.01$

$Pr(\bar{B} \mid \bar{A}) = 0.4$

$Pr(B \mid A) = 0.999$

a) $\underline{\underline{Pr(\bar{A})}} = 1 - Pr(A) = 1 - 0.01 = \underline{\underline{0.99}}$

b) $Pr(B \mid \bar{A}) = 1 - Pr(\bar{B} \mid \bar{A}) = 1 - 0.4 = 0.6$

$\underline{\underline{Pr(B)}} = Pr(B \mid A) \cdot Pr(A) + Pr(B \mid \bar{A}) \cdot Pr(\bar{A})$

$= 0.999 \cdot 0.01 + 0.6 \cdot 0.99$

$\underline{\underline{= 0.604 = 60.4\%}}$

c) $\underline{\underline{Pr(A \mid B)}} = \dfrac{Pr(B \mid A) \cdot Pr(A)}{Pr(B)} = \dfrac{0.999 \cdot 0.01}{0.604} \underline{\underline{= 0.0165 = 1.7\%}}$

d) $\underline{\underline{Pr(\bar{A} \mid B)}} = 1 - Pr(A \mid B) = 1 - 0.0165 = \underline{\underline{0.9835 = 98.4\%}}$

eller

$\underline{\underline{Pr(\bar{A} \mid B)}} = \dfrac{Pr(B \mid \bar{A}) \cdot Pr(\bar{A})}{Pr(B)} = \dfrac{0.6 \cdot 0.99}{0.604} = \underline{\underline{0.9834}}$

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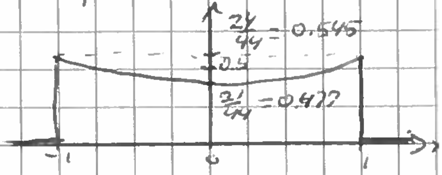
Opg. 2 S18

$$f_X(x) = \begin{cases} k\cdot(x^2+7) & -1<x<1 \\ 0 & \text{ellers} \end{cases}$$

a) $$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-1}^{1} k\cdot(x^2+7)\,dx = k\cdot\left[\tfrac{1}{3}x^3+7x\right]_{-1}^{1} = k\cdot 2\cdot\left(\tfrac{1}{3}+7\right) = k\cdot\tfrac{44}{3}$$

$$=1 \quad\Rightarrow\quad \underline{\underline{k=\tfrac{3}{44}}}$$



b) $$\underline{\underline{E(X)}} = \int_{-\infty}^{\infty} x\cdot f_X(x)\,dx = \frac{3}{44}\int_{-1}^{1}(x^3+7x)\,dx = \frac{3}{44}\left[\tfrac{1}{4}x^4+\tfrac{7}{2}x^2\right]_{-1}^{1}$$

$$= \frac{3}{44}\left(\left(\tfrac{1}{4}+\tfrac{7}{2}\right)-\left(\tfrac{1}{4}+\tfrac{7}{2}\right)\right) = \underline{\underline{0}}$$

c) $$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = k\cdot\int_{-1}^{1}(x^4+7x^2)\,dx = \frac{3}{44}\left[\tfrac{1}{5}x^5+\tfrac{7}{3}x^3\right]_{-1}^{1}$$

$$= \frac{3}{44}\,2\left(\tfrac{1}{5}+\tfrac{7}{3}\right) = \frac{3}{22}\cdot\frac{3+35}{15} = \frac{19}{55} = 0.345$$

$$\sigma^2 = E(x^2)-E(x)^2 = \frac{19}{55}-0 = \frac{19}{55} \Rightarrow \underline{\underline{\sigma}} = \sqrt{\frac{19}{55}} = \underline{\underline{0.588}}$$

d) $$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx = \int_{-1}^{x}\frac{3}{44}(x^2+7)\,dx = \frac{3}{44}\left[\tfrac{1}{3}x^3+7x\right]_{-1}^{x}$$

$$= \frac{3}{44}\left(\tfrac{1}{3}x^3+7x+\tfrac{1}{3}+7\right) = \frac{1}{44}(x^3+21x)+\frac{1}{2}, \quad -1<x<1$$

$$F_X(1) = \frac{1}{44}(1+21)+\frac{1}{2} = \frac{22}{44}+\frac{1}{2} = \frac{1}{2}+\frac{1}{2} = 1 \quad \text{o.k.}$$

$$\underline{\underline{F_X(x) = \begin{cases} 0 & x<-1 \\ \frac{1}{44}(x^3+21x)+\frac{1}{2} & -1<x<1 \\ 1 & x>1 \end{cases}}}$$

SMP

Opg. 3 S18

Kontinuert stokastisk proces: $y(t) = x(t) + w$

$x(t) \sim \mathcal{N}(0, t^2)$

$w \sim \mathcal{N}(3, 1)$

a) Én realisation i Matlab:

```
w = randn + 3;
t = 0:1:10;
x = randn(1,11).*t;
y = x + w
```

Plot: Se bilag.

b) Ensemble: $\underline{\mu_Y} = E(Y) = E(x) + E(w) = 0 + 3 = \underline{\underline{3}}$

c) Ensemble: $\underline{\underline{\sigma_Y^2}} = \sigma^2(x(t) + w) = \sigma^2(x) + \sigma^2(w) = \underline{\underline{t^2 + 1}}$

d) Temporal:

$$\underline{\hat{\mu}_{Y_1} = \langle Y_1 \rangle_T} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} y_1(nT) = \underline{\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} x_1(n) + w_1} = 0 + w_1 = \underline{\underline{w_1}}$$

$$\underline{\underline{\hat{\sigma}_{Y_1}^2}} = \underline{\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} \left(y_1^2(nT) - \hat{\mu}_{Y_1}^2\right)} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} (x_1(n) + w_1)^2 - w_1^2$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} \left(x_1(n)^2 + w_1^2 + 2 w_1 x_1(n)\right) - w_1^2$$

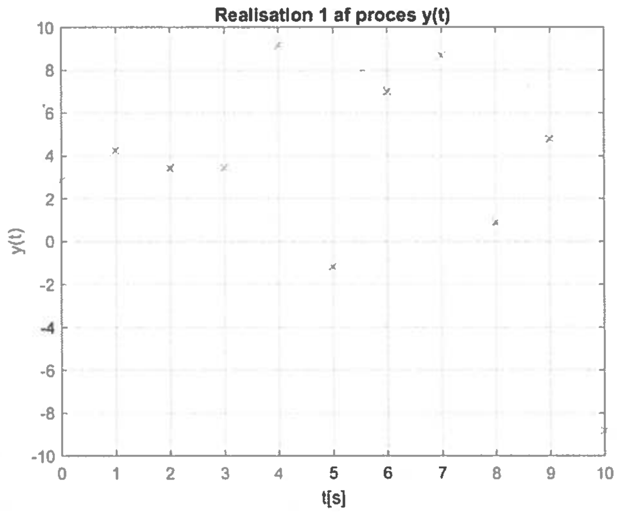
$$= \lim_{N\to\infty} \left(\frac{1}{N} \sum_{n=0}^{N} x_1(n)^2 + 2 w_1 \underbrace{\frac{1}{N} \sum_{n=0}^{N} x_1(n)}_{= \hat{\mu}_X = 0}\right)$$

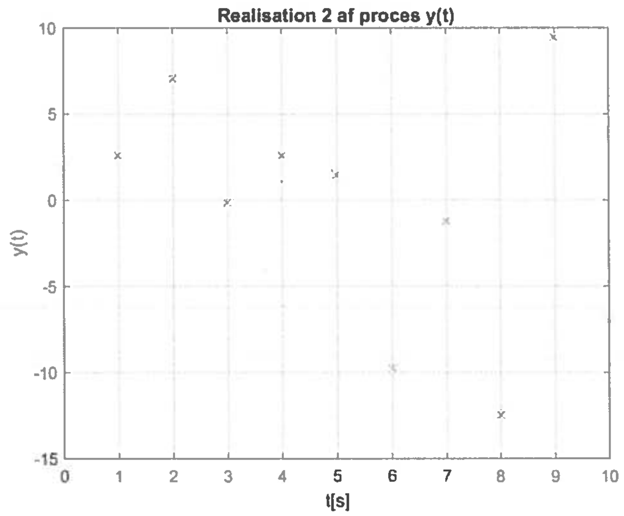
$$= \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} x_1(n)^2 = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} \left(x_1(n)^2 - \overset{=0}{\hat{\mu}_{X_1}^2}\right) = \hat{\sigma}_{X_1}^2 = \underline{\underline{\infty}} \quad (t^2)$$

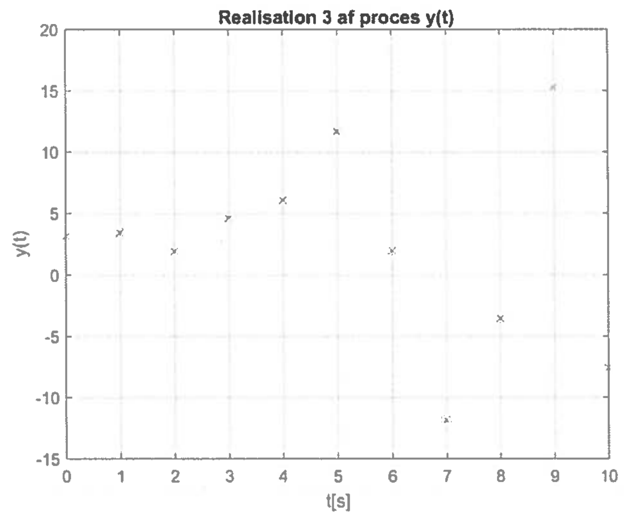
e) Da $\hat{\mu}_{Y_1} = w_1 \neq 3 = \mu_Y$ og $\hat{\sigma}_Y \neq \sigma_Y^2 = t^2 + 1$ (σ_Y^2 tidsafh.)

er Y __ikke WSS__, og derfor heller __ikke ergodisk__.

```
%Opgave 3a S18
%En realisation (11 samplinger) af den stokastiske proces
w=randn+3
t=0:1:10;
x=randn(1,11).*t
y=x+w
plot(t,y,'x')
grid
xlabel('t[s]')
ylabel('y(t)')
title('Realisation af proces y(t)')
```







Opgave 4

Produkt A: $x_A := \begin{bmatrix} 98 \\ 98 \\ 93 \\ 96 \\ 95 \\ 92 \\ 99 \\ 95 \\ 95 \\ 94 \\ 98 \\ 93 \\ 99 \\ 94 \\ 98 \end{bmatrix}$

$N_A := 15 \cdot 100 = 1500$

$N_{Akorrekt} := \sum_{i=0}^{14} x_{A_i} = 1437$

a) $p_{Aest} := \dfrac{N_{Akorrekt}}{N_A} = 0.958$

b) Test-statistik: **Binomialfordeling - Normal-approximation**

Betingelser: $N_A \cdot p_{Aest} = 1437$ > 5 --> o.k. $N_A \cdot (1 - p_{Aest}) = 63$ > 5 --> o.k.

Test-hypotese: H0: $p_0 := 0.95$ H1: $p_1 \neq 0.95$

c) $z := \dfrac{N_{Akorrekt} - N_A \cdot p_0}{\sqrt{N_A \cdot p_0 \cdot (1 - p_0)}} = 1.422$

$pvalue := 2 \cdot (1 - \text{cnorm}(|z|)) = 0.155$ --> pvalue > 0.05 --> **H0 afvises ikke.**

d) 95% konfidensinterval [p_{min} , p_{max}]:

$$p_{min} := \frac{1}{N_A + 1.96^2} \cdot \left(N_{Akorrekt} + \frac{1.96^2}{2} - 1.96 \cdot \sqrt{\frac{N_{Akorrekt} \cdot (N_A - N_{Akorrekt})}{N_A} + \frac{1.96^2}{4}} \right) = 0.947$$

$$p_{max} := \frac{1}{N_A + 1.96^2} \cdot \left(N_{Akorrekt} + \frac{1.96^2}{2} + 1.96 \cdot \sqrt{\frac{N_{Akorrekt} \cdot (N_A - N_{Akorrekt})}{N_A} + \frac{1.96^2}{4}} \right) = 0.967$$

p0=0.95 er indeholdt i konfidensintervallet --> **H0 kan ikke afvises.**

Produkt B: $x_B := \begin{bmatrix} 99 \\ 94 \\ 89 \\ 92 \\ 90 \\ 91 \\ 92 \\ 96 \\ 92 \\ 90 \\ 90 \\ 95 \end{bmatrix}$

$N_B := 12 \cdot 100 = 1200$

$N_{Bkorrekt} := \sum_{i=0}^{11} x_{B_i} = 1110$

$p_{Best} := \dfrac{N_{Bkorrekt}}{N_B} = 0.925$

e) **Uparret sammenligningstest af to populationer med ukendt varians (uparret t-test):**
Er det gennemsnitlige antal korrekt monitorerede enheder pr. måling (100 enheder) ens for de to produkter?
Uparret fordi, der ikke er en relation mellem en enhed fra det ene produkt til en enhed fra det andet produkt. Desuden er der et forskelligt antal målinger i de to tests (15 og12 hhv.).
Desuden antages de to polulationer at have ens, men ukendt, varians.

f) Antal korrekte pr. måling (100 enheder): $n_A := \frac{N_{Akorrekt}}{15} = 95.8$ $n_B := \frac{N_{Bkorrekt}}{12} = 92.5$

Forskel: $d := n_A - n_B = 3.3$

Sample-varians: $S2_A := \frac{1}{14} \cdot \sum_{i=0}^{14} \left(x_{A_i} - n_A\right)^2 = 5.6$ $S2_B := \frac{1}{11} \cdot \sum_{i=0}^{11} \left(x_{B_i} - n_B\right)^2 = 8.818$

Pooled varians: $S2_{AB} := \frac{1}{25} \cdot \left(14 \cdot S2_A + 11 \cdot S2_B\right) = 7.016$

$$t := \frac{d}{\sqrt{S2_{AB}} \cdot \sqrt{\frac{1}{15} + \frac{1}{12}}} = 3.217$$

$pvalue := 2 \cdot (1 - \mathrm{pt}(|t|, 15 + 12 - 2)) = 0.004$ --> pvalue < 0.05 --> **H0 afvises**
--> Overvågningssystemet **virker forskelligt** på de to typer produkter

<u>Alternativ</u>: Test af succesrate pr. enhed

Testresultat: $\delta := p_{Acst} - p_{Bcst} = 0.033$

Test-hypotese: H0: $\delta_0 := 0$ H1: $\delta_0 \neq 0$

Sample-varians (pr. måling): $S2_A := \frac{1}{14} \cdot \sum_{i=0}^{14} \left(x_{A_i} - \frac{N_{Akorrekt}}{15}\right)^2 = 5.6$

$S2_B := \frac{1}{11} \cdot \sum_{i=0}^{11} \left(x_{B_i} - \frac{N_{Bkorrekt}}{12}\right)^2 = 8.818$

Pooled varians (pr. enhed): $S2_{AB} := \frac{1}{100} \cdot \frac{1}{15 + 12 - 2} \cdot \left(14 \cdot S2_A + 11 \cdot S2_B\right) = 0.07016$

$$t := \frac{\delta - \delta_0}{\sqrt{S2_{AB}} \cdot \sqrt{\frac{1}{N_A} + \frac{1}{N_B}}} = 3.217$$

$pvalue := 2 \cdot (1 - \mathrm{pt}(|t|, 15 + 12 - 2)) = 0.004$ --> pvalue<0.05 --> **H0 afvises**
--> Overvågningssystemet **virker forskelligt** på de to typer produkter