SMP V 18/19

Opg. 1

R = Røg , $\overline{R}$ = Ikke røg $\quad$ $R \cap A$ = Røg og alarm

A = Alarm , $\overline{A}$ = Ikke alarm $\quad$ $\overline{R} \cap A$ = Ikke-røg og alarm

$Pr(R) = 0.33 \Rightarrow Pr(\overline{R}) = 1 - Pr(R) = 0.67$

$Pr(R \cap A) = 0.32$, $Pr(\overline{R} \cap A) = 0.07$

a) Alarm: $\underline{\underline{Pr(A)}} = Pr(R \cap A) + Pr(\overline{R} \cap A) = 0.32 + 0.07 = \underline{\underline{0.39 = 39\%}}$

b) Alarm uden røg (falske alarmer):

$$\underline{\underline{Pr(A \mid \overline{R})}} = \frac{Pr(A \cap \overline{R})}{Pr(\overline{R})} = \frac{0.07}{0.67} = 0.105 = \underline{\underline{10.5\%}}$$

c) Røg uden alarm (alarm svigt)

$$\underline{\underline{Pr(\overline{A} \mid R)}} = 1 - Pr(A \mid R) = 1 - \frac{Pr(A \cap R)}{Pr(R)}$$

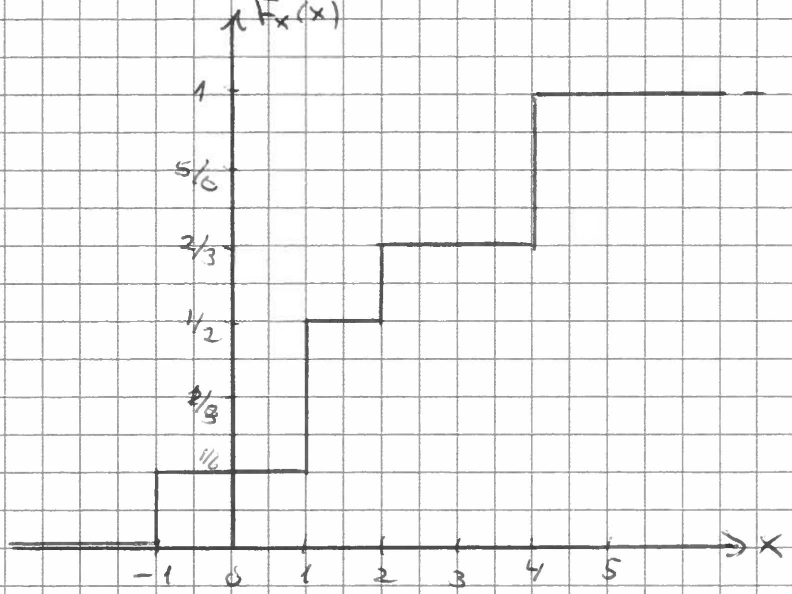
$$= 1 - \frac{0.32}{0.33} = 1 - 0.96 = \underline{\underline{0.04 = 4\%}}$$

SMP V18/19

Opg. 2

a) $\sum_{x_i} f_X(x_i) = 1 \Rightarrow a + 2a + a + 2a = 6a = 1 \Rightarrow \underline{a = \frac{1}{6}}$

b) $\underline{\underline{F_X(x) = \sum_{x_i \le x} f_X(x_i)}} = \begin{cases} 0 & x < -1 \\ a & -1 \le x < 1 \\ 3a & 1 \le x < 2 \\ 4a & 2 \le x < 4 \\ 6a & x \ge 4 \end{cases} = \underline{\underline{\begin{cases} 0 & x < -1 \\ 1/6 & -1 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 2/3 & 2 \le x < 4 \\ 1 & x \ge 4 \end{cases}}}$



c) $\underline{\underline{E[X] = \sum_{x_i} x_i f_X(x_i)}} = -1 \cdot \frac{1}{6} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{6} + 4 \cdot \frac{1}{3}$

$= -\frac{1}{6} + \frac{2}{6} + \frac{2}{6} + \frac{8}{6}$

$\underline{\underline{= \frac{11}{6} = 1.833}}$

d) $E[X^2] = \sum_{x_i} x_i^2 \cdot f_X(x_i) = (-1)^2 \cdot \frac{1}{6} + 1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{6} + 4^2 \cdot \frac{1}{3}$

$= \frac{1}{6} + \frac{2}{6} + \frac{4}{6} + \frac{32}{6} = \frac{39}{6} = \frac{13}{2}$

$\underline{\underline{Var[X] = \sigma_X^2}} = \underline{\underline{E[X^2] - E[X]^2}} = \frac{13}{2} - \left(\frac{11}{6}\right)^2 = \frac{234 - 121}{36}$

$\underline{\underline{= \frac{113}{36} = 3.139}} \underline{\underline{= 3.139}}$

SMP V18/19

Opg. 3

$X[n] = 2 \cdot Y[n] + W \quad , \quad Y[n] \sim \mathcal{N}(5,2) \quad , \quad W \sim \mathcal{U}[-2,2]$

a) X = 2 * (√2 * randn(11,1) + 5) + 4 * rand - 2

Realisering: Se bilag

b) $\underline{\underline{E[X]}} = 2 \cdot E[Y] + E[W] = 2 \cdot 5 + 0 = \underline{\underline{10}}$

$\underline{\underline{Var[X]}} = 4 \cdot Var[Y] + Var[W] = 4 \cdot 2 + \frac{(2-(-2))^2}{12}$

$= 8 + \frac{16}{12} = 8 + \frac{4}{3} = \frac{28}{3} = \underline{\underline{9.33}}$

c) $E[X]$ og $Var[X]$ er uafh. af $n \Longrightarrow$ $\underline{\underline{X \text{ er WSS}}}$

Da W er forskellig for de enkelte realisationer, er
<u>X ikke ergodisk</u>

(1 realisation $\hat{\mu}_X = 2 \cdot 5 + W = 10 + W$)

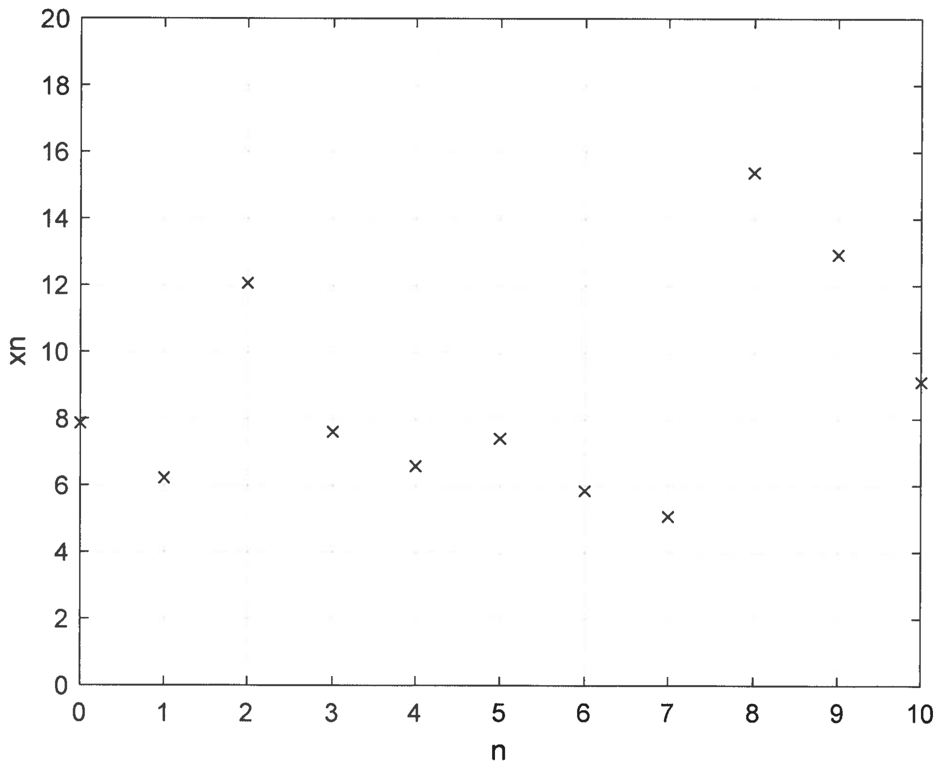
SMP V18/19 Opgave 3a

```
%%Opgave_3_V18/19 Realization of stochastic processes
clear all

%%Samples of a discrete stocastic process

yn=sqrt(2)*randn(11,1)+5; %11 samples of random normal numbers
                           with mean 5 and variation 2
w=2*rand-2 %A uniformly distributed random number between -2 and +2
xn=2*yn+w %11 samples (n=0,..,10) of the stochastic process: x(n)=2*y(n)+w

figure(1)
n=linspace(0,10,11);%11 samples between 0 and 10
plot(n,xn,'kx')
grid
axis([0,10,0,10])
xlabel('n')
ylabel('xn')
```



w = -1.7609

xn = 7.8623, 6.2181, 12.0614, 7.6034, 6.5731, 7.4083, 5.8408, 5.0709, 15.3837, 12.9216, 9.1090

SMP V18/19

Opg 4

a) Forbedringer: δ_i (Gl-Ny): 64, 137, -13, 0, 133, 123, 125, 87, 23, 128, 120, 73

Estimeret forbedring: $\hat{\delta} = \frac{\sum \delta_i}{12} = 83.3\,s$

b) Parret t-test

c) $H_0: \delta_0 = 120\,s$, $H_1: \delta_0 \neq 120\,s$

d) Samplevarians $s_d^2 = \frac{1}{n-1} \cdot \sum_i (\delta_i - \hat{\delta})^2 = \frac{1}{11} \sum_{i=1}^{12} (\delta_i - 83.3)^2 = 2941\,s^2$

e) t-værdi: $t = \frac{\hat{\delta} - \delta_0}{\sqrt{s_d^2/n}} = \frac{83.3 - 120}{\sqrt{2941/12}} = -2.3420$

p-værdi: $p = 2 \cdot (1 - t_{cdf}(|t|, 11)) = 2 \cdot (1 - 0.9805)$
$= 0.039 < 0.05$

f) $t_0 = invcdf(0.975, 11) = 2.201$

$\Delta\delta = t_0 \cdot \sqrt{\frac{s_d^2}{n}} = 2.201 \cdot \sqrt{\frac{2941}{12}} = 34.5$

$\delta_- = \hat{\delta} - \Delta\delta = 83.3 - 34.5 = 48.8$

$\delta_+ = \hat{\delta} + \Delta\delta = 83.3 + 34.5 = 117.8$

95% konfidensinterval: $[48.8 ; 117.8]$

g) Da $p < 0.05$ og $\hat{\delta} \notin [48.8 ; 117.8]$ må hypotesen om en forbedring på 2 minutter forkastes.