Solutions

1 Let X denote the number of customers that rate the brand first quality. Then $X \sim binomial(n = 100, p = 0.35)$.

a. $\Pr(X > 40) = 1 - \Pr(X \leq 40) = 1 - binocdf(40{,}100{,}0.35) = 0.1250$
b. $\Pr(X \leq 30) = \Pr(X \leq 30) = binocdf(30{,}100{,}0.35) = 0.1730$
c. $\Pr(X = 45) = binopdf(45{,}100{,}0.35) = 0.0096$

2 If $X \sim binomial(n = 10, p)$, then the mean is $E[X] = np = 10p$ and the variance is $Var(X) = np(1-p) = 10p(1-p)$. The standard deviation (SD) is $\sqrt{Var(X)}$.

a. Results

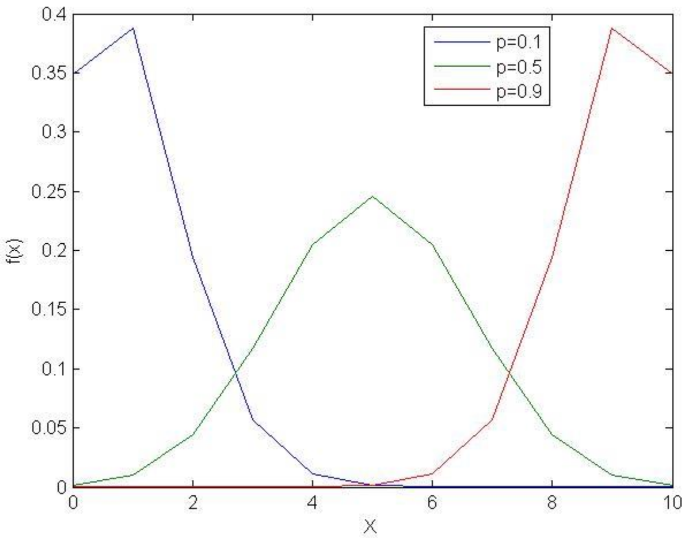
i. p=0.1: $E[X] = 10 \cdot p = 10 \cdot 0.1 = 1, SD = \sqrt{Var(X)} = \sqrt{10p(1-p)} = \sqrt{10 \cdot 0.1 \cdot (1 - 0.1)} = 0.9487$.

ii. p=0.5: $E[X] = 10 \cdot p = 10 \cdot 0.5 = 5, SD = \sqrt{Var(X)} = \sqrt{10p(1-p)} = \sqrt{10 \cdot 0.5 \cdot (1 - 0.5)} = 1.5811$.

iii. p=0.9: $E[X] = 10 \cdot p = 10 \cdot 0.9 = 9, SD = \sqrt{Var(X)} = \sqrt{10p(1-p)} = \sqrt{10 \cdot 0.9 \cdot (1 - 0.9)} = 0.9487$.

b. Matlab code

```
n = 10;
x = 0:n;
plot(x,binopdf(x,n,0.1),...
     x,binopdf(x,n,0.5),...
     x,binopdf(x,n,0.9))
legend('p=0.1','p=0.5','p=0.9')
xlabel('X')
ylabel('f(x)')
```



c. Matlab code generating 20 samples by histogram matching:

```
p = 0.1;
Fx = binocdf(x,n,p);
y = rand(1,20);
for i = 1:20
    dist = abs(Fx-y(i));
    [minval,minix] = min(dist);
    sample(i) = x(minix);
end
```

3 Let X denote the number of consumers that rate the company's brand first quality.

a. Statistical model: We have $X \sim binomial(n,p)$, with observation x = 30 and n = 100 trials.

b. The company assumes that $p = 0.35$. Hence, we have $H_0: p = 0.35$ and $H_1: p \neq 0.35$.

c. According to the hypothesis, the ideal value of X is 35. Hence, the observation x = 30 deviates from the ideal value by 5. For a two-tailed test, we need to consider the events $\{X \geq 35+5\}$ and $\{X \leq 35-5\}$. The exact p-value is

$$2 \cdot min\{Pr(X \geq x), Pr(X \leq x)\}$$

$$= 2 \cdot min\{Pr(X \geq 40), Pr(X \leq 30)\}$$

$$= 2 \cdot min\{1 - Pr(X \leq 40), Pr(X \leq 30)\}$$

$$= 2 \cdot min\{1 - \text{binocdf(40,100,0.35)}, \text{binocdf(30,100,0.35)}\}$$

$$= 2 \cdot min\{1 - 0.8750, 0.1730\} = 0.3460$$

And we fail to reject the null hypothesis.

d. Standardizing the observation, x=30, we get

$$z = \frac{x - np}{\sqrt{np(1-p)}} = \frac{30 - 100 \cdot 0.35}{\sqrt{100 \cdot 0.35 \cdot (1 - 0.35)}} = -1.0483$$

The approximate p-value is

$$2 \cdot |1 - \Phi(|z|)| = 2 \cdot |1 - \Phi(1.0483)| = 0.2945$$

And we fail to reject the null hypothesis.

4 Statistical model: We have $X \sim binomial(n,p)$, with observation x = 15 and n = 50 trials.

a. The maximum-likelihood estimate of p is $\hat{p} = \frac{x}{n} = 15/50 = 0.3$.

b. The endpoints of the 95% confidence interval are

$$p_{-} = \frac{1}{n + 1.96^2}\left[x + \frac{1.96^2}{2} - 1.96\sqrt{\frac{x(n-x)}{n} + \frac{1.96^2}{4}}\right]$$
$$= \frac{1}{50 + 1.96^2}\left[15 + \frac{1.96^2}{2} - 1.96\sqrt{\frac{15(50-15)}{50} + \frac{1.96^2}{4}}\right] = 0.1910$$

$$p_{+} = \frac{1}{n + 1.96^2}\left[x + \frac{1.96^2}{2} - 1.96\sqrt{\frac{x(n-x)}{n} + \frac{1.96^2}{4}}\right]$$
$$= \frac{1}{50 + 1.96^2}\left[15 + \frac{1.96^2}{2} + 1.96\sqrt{\frac{15(50-15)}{50} + \frac{1.96^2}{4}}\right] = 0.4375$$

If our null hypothesis states that $H_0\colon p = 0.35$, we fail to reject the null hypothesis, because the hypothesized value p lies within the confidence interval.