Optimization

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March 2023

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Chapter 1

Maximum and Minimum

Note:-

Local - Subsection of range
Global - Whole range

1.1 Maximize

$$R(x) = 45 - \frac{x^2}{3},\ 0 \leqslant x \leqslant 1$$

Find all local maximum values
Find the global maximum value

1.2 Minimize

- Local Minimum - Minimum in specified range
- Global Minimum - Overall minimum

1.3 Local/Relative Maximum/Minimum (Optimum)

- Find the critical values of the function:

  Stationary points, i.e. $f'(?) = 0$

  Undefined points, i.e. $f'(?) = \emptyset$

- Assess them for potential local maximum/minimum:

  Find the first derivative, input values from the left and right of the critical points and check the change in signs:

  $+$ to $-$: Maximum

  $-$ to $+$: Minimum

  Find second derivative, input the critical values and check the sign:

  $-$: Minimum

  $+$: Maximum

1.4 Global/Absolute Maximum/Minimum (Optima)

To find the Absolute Optima of a function whose domain is unrestricted:

$$\lim_{x\to\infty} f(x) \quad \lim_{x\to-\infty} f(x)$$

1.4.1 Conditions for finding the Absolute Optima easily

1. Closed Domain, i.e. $[x_1, x_2]$
2. Function is continuous for the duration of the closed domai

Theorem 1.4.1 Extreme Value Theorem

If a real valued function f is continuous on the closed interval $[a, b]$, the f must attain a maximum and minimum at least once.

$$f(c) \geqslant f(x) \geqslant F(d)$$
$$\forall x \in [a, b]$$

Where $f(c)$ is the function's minimum value and $F(d)$ is the function's maximum value.

Example 1.4.1

$$f(x) = x^3 \text{ on } [-1, 10]$$

- $f(x)$ is continuous due to it being a polynomial
- The function's domain is closed due to the end values being included in the domain

By EVT(**??**) $f(x)$ must attain absolute maximum and minimum at least once on the interval. Possibly at:

1. End points of the domain
2. Critical values of $f(x)$

$$\begin{aligned} f(x) &= x^3 \\ f'(x) &= 3x^2 \\ 0 &= 3x^2 \\ \frac{0}{3} &= x^2 \\ 0 &= x \end{aligned}$$

$$\begin{aligned} f(-1) &= -1 \\ f(10) &= 1000 \end{aligned}$$

$$\begin{aligned} \therefore \text{Absolute Maximum is } 1000 \\ \text{Absolute Minimum is } -1 \end{aligned}$$

Chapter 2

Concavity

Let f be a function that is differentiable over an open interval I

- If f' is increasing over I, we say f is concave up over I, i.e. $f'' > 0$
- If f' is decreasing over I, we say f is concave down over I, i.e $f'' < 0$

2.1 Inflection

A point where a function switches concavity, i.e:

$$\begin{gathered} f''(x^-) = +\text{ve} \ \text{ to } \ f''(x^+) = -\text{ve} \\ \text{or} \\ f''(x^-) = -\text{ve} \ \text{ to } \ f''(x^+) = +\text{ve} \end{gathered}$$

2.2 Curvature

2.2.1 Concave Up

The <u>cave</u> is facing up

2.2.2 Concave Down

The <u>cave</u> is facing down