

# Limit of a Function

Reporting about the behavior of a function within the range of its dangerous values.

$$f(x) = x^2 + \frac{1}{x}$$

Input variable -  $x$

Output -  $f(x)$

Name of function -  $f$

"Acceptable"/Permissible input values of  $x$  - All real numbers except zero

$$\begin{aligned} & (x, f(x)), (x+h, f(x+h)) \\ & \frac{f(x+h) - f(x)}{x+h-x} \\ & \frac{f(x+h) - f(x)}{h} \end{aligned}$$

## Proof

$$y = -16t^2 + 100t + 6$$

Points used:

$(0, 6), (1, 90), (3, 162)$

When  $t = 0$  and  $y = 6$

$$\begin{aligned} y &= at^2 + bt + c \\ 6 &= a(0)^2 + b(0) + c \\ c &= 6 \end{aligned}$$

When  $t = 1$  and  $y = 90$

$$\begin{aligned} 90 &= a(1)^2 + b + 6 \\ 90 &= a + b + 6 \\ 84 &= a + b \\ 84 - b &= a \end{aligned}$$

When  $t = 3$  and  $y = 162$

$$\begin{aligned} 162 &= a(3)^2 + 3b + 6 \\ 162 &= 9a + 3b + 6 \\ 162 &= 9(84 - b) + 3b + 6 \\ 162 &= 756 - 9b + 3b + 6 \\ -594 &= -6b + 6 \\ -600 &= -6b \\ b &= 100 \end{aligned}$$

$\therefore b = 100$

$$\begin{aligned} 84 - 100 &= a \\ a &= -16 \end{aligned}$$

Therefore  $a = -16$ ,  $b = 100$ , and  $c = 6$

**Given  $f(x) = x^2$ , find the Limit of  $f(x)$  at  $x = 3$**

$$\text{As } x \rightarrow 3^-, f(x) \rightarrow 9$$

$$\text{As } x \rightarrow 3^+, f(x) \rightarrow 9$$

Or

$$\lim_{x \rightarrow 3^-} f(x) = 9$$

$$\lim_{x \rightarrow 3^+} f(x) = 9$$

The first 9 is known as the left limit of  $f(x)$  and the other 9 is known as the right limit of  $f(x)$

The Limit of  $f(x)$  at  $x = 3$

$$\lim_{x \rightarrow 3} f(x) = 9$$

This is because the left limit and right limit converge.

**In the case where:**

$$\lim_{x \rightarrow 1^-} f(x) = 5$$

$$\lim_{x \rightarrow 1^+} f(x) = 4$$

The left and right limits do not converge so there is no limit of  $f(x)$  for  $x = 1$

$$\lim_{x \rightarrow 1} f(x) = \text{No such unique number}$$

$\therefore$  The limit of  $f(x)$  at  $x = 1$  does not exist

**In the case where the one limit does not exist (increasing without bounds):**

$$\lim_{x \rightarrow 1^-} f(x) \rightarrow \infty$$

$$\lim_{x \rightarrow 1^+} f(x) \rightarrow 4$$

The limit does not exist because the left limit does not exist.

$$\lim_{x \rightarrow 1} f(x) = \text{does not exist}$$

$\therefore$  the left limit does not exist

## *Graphical*

Given  $f(x) = x^2$ , evaluate the Limit of  $f(x)$  at  $x = 3$  using the graphical approach.