Discrete Systems

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Chapter 1

Introduction to Modelling

1.1 The Modelling Framework

Definition 1.1.1: System

Something in the real world we are interested in modelling

Definition 1.1.2: Abstraction

Deciding which elements of the real world to include and which parts to leave out

Definition 1.1.3: Model

The resulting abstracted system, which is a description of the system that includes only essential features. Models can be represented in diagrams, equations which can be used for mathematical analysis and computer programs which can run as *simulations*

Definition 1.1.4: Simulation

The process of designing a model of a real system, and conducting experiments with this model

The result of analysis and simulation could be a prediction about what the system will do, an explanation the system's behaviour, or a design intended to achieve a purpose.

Predictions can be validated and testing by taking measurements from the real world and comparing the data obtained with the results from analysis and simulation.

Some reasons to simulate a system include:

- Estimate some quantity or measure of effectiveness (MOE)
- Gain an understanding of the behaviour of the system
- Evaluate various alternative strategies

Definition 1.1.5: Iterative Modelling

Starting with a simple model, and adding on features gradually starting with the most essential ones.

Definition 1.1.6: Internal Validation

Comparing the results of successive models from iterative modelling.

Definition 1.1.7: External Validation

Comparing the results from the real world to generated models.

1.1.1 Types of Models

Physical models - Resemble the system being studied.

Scaled models - Resemble the system under study but at a different size, i.e. a scaled up model of an atom.

Analog models - A model where the properties of the real object is represented by a substituted property that often behaves in a similar manner, e.g. voltage through an electronic analogue computer network may represent flow of goods through a system.

Schematic model - A pictorial representation of a system, e.g. a blueprint or a graph.

Games / Man-Machine models - Management games, war games, planning competition.

Simulation models - Have no human interaction, an abstract model, such as a discrete-event system simulation model.

Mathematical models - A model where symbols represent entities.

Heuristic model - A collection of descriptors or decision rules, usually computer based which is not limited by physical, diagrammatic or mathematical bounds.

1.1.2 Characteristics of Models

Static Model - A model that does not change / move with time. Examples include Monte Carlo Sampling.

Dynamic Model - A model that changes with time. Examples include Time series regression.

Deterministic Model - A model that has a single solution for a given input state or configuration, like a pure function in functional programming.

Stochastic / Non-Deterministic Model - A model whose output mimics probabilistic/random phenomena.

Discrete Model - A model that represents the system in a discrete manner where variables change at distinct and countable points in time.

Continuous Model - A model that represents the system in a continuous manner where variables change continuously over time.

1.2 Testing the Falling Penny Myth

Definition 1.2.1: The Falling Penny Myth

A penny dropped from the top of the Empire State building would be going so fast when it hit the pavement that it would be embedded in the concrete, or if it hit a person it would break their skull

Note:-

The Standard Equations of Motion (SUVAT):

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$s = vt + \frac{1}{2}at^2$$

Where:
s - Distance
u - Initial Velocity
v - Velocity
a - Acceleration
t - Time

We can test this by considering two models. The first disregards the effect of air resistance, in which case the primary force acting on the penny is gravity, which causes the penny to accelerate downward.

If the initial velocity is 0 $u = 0$ and the acceleration a is constant the velocity after t seconds can be found using the one of the standard equations of motion:

$$v = u + at = at$$

and the distance the penny is dropped is:

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

To find the time the penny hits the sidewalk we can make t the subject of this equation:

$$s = \frac{1}{2}at^2$$
$$2s = at^2$$
$$\frac{2s}{a} = t^2$$
$$t = \sqrt{\frac{2s}{a}}$$

When $a = 9.8\text{m/s}^2$ due to gravity, and the height of the Empire State Building $s = 381\text{m}$, $t = 8.8\text{s}$. Computing the velocity we get 86.24m/s. However we must take into account that this model is based on simplifications, such as gravity is constant. Gravity is however different on different parts of the globe and it gets weaker as you move away from the surface of the earth. This however results in small negligible differences so ignoring them can be a good choice for this problem.

Disregarding air resistance however is not a good choice as it has substantial effects on the velocity of the penny, as when the penny reaches about 29m/s the upward force of air resistance equals the downward force of gravity and the penny stops accelerating, this point is known as the penny's terminal velocity in air.

This leads into the second model where the penny accelerates until it reaches terminal velocity of 29m/s and hits the sidewalk at this velocity, which is not fast enough to damage it.