Assignment 2

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Question 1

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

Solution:

$$
\begin{aligned}
(p \rightarrow q) \wedge (p \rightarrow r) &\equiv (\neg p \vee q) \wedge (\neg p \vee r) && \text{Conditional Disjunction Equivalence} \\
&\equiv \neg p \vee (q \wedge r) && \text{By the First Distributive Law} \\
&\equiv p \rightarrow (q \wedge r) && \text{Conditional Conjunction Equivalence}
\end{aligned}
$$

Question 2

$$p \wedge p \equiv p$$

Solution:

$$
\begin{aligned}
p &\equiv p \wedge T && \text{By the First Identity Law} \\
&\equiv p \wedge (p \vee \neg p) && \text{By the First Negation Law} \\
&\equiv (p \wedge p) \vee (p \wedge \neg p) && \text{By the Second Distributive Law} \\
&\equiv (p \wedge p) \vee F && \text{By the Second Negation Law} \\
&\equiv p \wedge p && \text{By the Second Identity Law}
\end{aligned}
$$

Question 3

1. $\exists x N(x)$
2. $\forall x N(x)$
3. $\neg \exists x N(x)$
4. $\exists x \neg N(x)$
5. $\neg \forall x N(x)$
6. $\forall x \neg N(x)$

Solution:

1. There are some students in my school that have visited North Dakota

2. All students in my school have visited North Dakota

3. $\neg \exists x N(x) \equiv \forall x \neg N(x) \therefore$ All the students in my class have not visited North Dakota

4. There are some students in my school that have not visited North Dakota

5. $\neg \forall x N(x) \equiv \exists x \neg N(x) \therefore$ There are some students in my school that have not visited North Dakota

6. All the students in my class have not visited North Dakota