

## Sine

$$\begin{aligned}f(x) &= \sin(x) \\ -1 &\leq \sin(x) \leq 1 \\ \sin(0) &= 0\end{aligned}$$

For all integer multiples of  $\pi$ ,  $\sin$  attains 0

$$\sin(k\pi) = 0, \text{ Where } k \text{ is an integer}$$

The graph of sine is periodic with a period of  $2\pi$ , meaning it repeats itself every interval of  $2\pi$

## Derivative of $\sin(x)$

If  $y = \sin(x)$

$$y' = \cos(x)$$

Proof

$$\begin{aligned}
 y' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\
 &\therefore \frac{\sin(x+h) - \sin(x)}{h} \\
 &= \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} \\
 &= \frac{\sin(\cos(h) - 1) + \cos(x)\sin(h)}{h} \\
 y' &= \lim_{h \rightarrow 0} \frac{\sin(\cos(h) - 1) + \cos(x)\sin(h)}{h} \\
 y' &= \lim_{h \rightarrow 0} \frac{\sin(\cos(h) - 1)}{h} + \frac{\cos(x)\sin(h)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(\cos(h) - 1)}{h} + \lim_{h \rightarrow 0} \frac{\cos(x)\sin(h)}{h}
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} (f(x) + g(x))$$

$$= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h}$$

$$\therefore \lim_{x \rightarrow a} (kf(x)) = k \lim_{x \rightarrow a} f(x)$$

$$\begin{aligned}
 \lim_{h \rightarrow 0^-} \frac{\cos(h) - 1}{h} &= 0 \\
 \lim_{h \rightarrow 0^+} \frac{\cos(h) - 1}{h} &= 0 \\
 \therefore \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} &= 0 \\
 &= \sin(x) \times 0 + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
 &= \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
 \lim_{h \rightarrow 0^-} \frac{\sin(h)}{h} &= 1 \\
 \lim_{h \rightarrow 0^+} \frac{\sin(h)}{h} &= 1 \\
 \therefore \lim_{h \rightarrow 0} \frac{\sin(h)}{h} &= 1 \\
 \therefore \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} &= \cos(x) \times 1
 \end{aligned}$$

$$= \cos(x)$$

## Cosine

$$\begin{aligned}f(x) &= \cos(x) \\ -1 &\leq \cos(x) \leq 1 \\ \cos(0) &= 1\end{aligned}$$

## Tangent

$$\begin{aligned}f(x) &= \tan(x) \\ y = \tan(x) &= 0 \\ \sin(x) &= 0\end{aligned}$$

### Vertical Asymptote

A vertical line  $(x, 0)$  where the values of a function rise or fall infinitely.

The line  $x = a$  is a vertical asymptote of  $f(x)$  if

$$\lim_{x \rightarrow a} f(x) = \pm\infty$$

The zero points of  $\cos(x)$  create a vertical asymptote in relation to  $\tan(x)$

### Derivative of $\tan(x)$

If  $y = \tan(x)$

$$y' = \sec^2(x)$$

### Proof

$$y = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$y' = \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2}$$

$$\therefore y' = \frac{v \times u' - u \times v'}{v^2}$$

$$y' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$y' = \frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)}$$

$$y' = 1 + \left(\frac{\sin(x)}{\cos(x)}\right)^2$$

$$y' = 1 + \tan^2(x)$$

$$= \sec^2(x)$$

## Secant

$$f(x) = \sec(x)$$

$$\sec(0) = 1$$

## Derivative of $\sec(x)$

If  $y = \sec(x)$

$$y' = \tan(x) \sec(x)$$

## Relationships

$$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$$

## Identities

### Reciprocal Identities

$$\sin(\theta) = \frac{1}{\csc(\theta)} \text{ or } \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \text{ or } \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \text{ or } \cot(\theta) = \frac{1}{\tan(\theta)}$$

### Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\csc^2(\theta) = 1 + \cot^2(\theta)$$

### Ratio Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

### Sum and Difference of Angles

$$\sin(\alpha + \beta) = \sin(\alpha) \times \cos(\beta) + \cos(\alpha) \times \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \times \cos(\beta) - \cos(\alpha) \times \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \times \cos(\beta) - \sin(\alpha) \times \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \times \cos(\beta) + \sin(\alpha) \times \sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \times \tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \times \tan(\beta)}$$

### Double Angles

$$\begin{aligned}\sin(2\theta) &= 2 \sin(\theta) \cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2 \cos^2(\theta) - 1 \\ &= 1 - 2 \sin^2(\theta) \\ \tan(2\theta) &= \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}\end{aligned}$$

### Questions

$$y = (\sin x)^2$$

$$y' = 2 \cos(\sin x)$$

$$y = \cos(5x + 4)$$

$$y' = -5 \sin(5x + 4)$$

$$g(x) = 3 \sec(x) - 10 \tan(x)$$

$$h(x) = 3w^{-4} - w^2 \tan(w)$$

$$y = 5 \sin(x) \cos(x) + 9 \sec(x)$$

$$y = \frac{\sin(t)}{3 - 2 \cos(t)}$$

$$y = \sin(10x)$$

$$f(w) = \tan(w) \sec(w)$$

$$y = 2 \sin(3x + \tan(x))$$

$$h(z) = \sin(z^6) + \sin^6(2)$$

$$f(t) = \sin(2t) + \cos(4t)$$

$$f(x) = [\sqrt[3]{2x} + \sin^2(3x)]^{-\frac{1}{2}}$$

$$y = \frac{4 \sin(x^2)}{\cos(x^2)}$$

$$h(x) = x^2 \cos(x^3)$$

$$y = \sqrt{5z + \tan(4z)}$$

Q1.  $g(x) = 3 \sec(x) - 10 \tan(x)$

$$\begin{aligned}
 g'(x) &= 3(\tan(x) \sec(x)) - 10 \sec(x) \\
 &= 3 \tan(x) \sec(x) - 10 \sec^2(x) \\
 &= \sec(x)(3 \tan(x) - 10 \sec(x)) \\
 g'(x) &= \sec(x)(3 \tan(x) - 10 \sec(x))
 \end{aligned}$$

Q2.  $h(w) = 3w^{-4} - w^2 \tan(w)$

$$\begin{aligned}
 h'(w) &= -12w^{-5} - (2w)(\tan(w)) + (w^2)(\sec^2(w)) \\
 h'(w) &= -12w^{-5} - 2w \tan(w) - w^2 \sec^2(w)
 \end{aligned}$$

Q3.  $y = 5 \sin(x) \cos(x) + 4 \sec(x)$

$$\begin{aligned}
 y' &= 5[\cos(x) \times \cos(x) - \sin(x) \times \sin(x)] + 4 \tan(x) \sec(x) \\
 y' &= 5(\cos^2(x) - \sin^2(x)) + 4 \tan(x) \sec(x) \\
 y' &= 5 \cos(2x) + 4 \tan(x) \sec(x)
 \end{aligned}$$

Q4.  $y = \frac{\sin(t)}{3-2 \cos(t)}$

$$\begin{aligned}
 y' &= \frac{(\cos(t)(3-2 \cos(t))) - (2 \sin(t))(\sin(t))}{(3-2 \cos(t))^2} \\
 y' &= \frac{3 \cos(t) - 2 \cos^2(t) - 2 \sin^2(t)}{(3-2 \cos(t))^2} \\
 y' &= \frac{3 \cos(t) - 2(\cos^2(t) + \sin^2(t))}{(3-2 \cos(t))^2} \\
 y' &= \frac{3 \cos(t) - 2}{(3-2 \cos(t))^2}
 \end{aligned}$$

Q5.  $y = \frac{\sin(10z)}{z}$

$$\begin{aligned}
 y' &= \frac{(10 \cos(10z))(z) - (1)(\sin(10z))}{z^2} \\
 y' &= \frac{10z \cos(10z) - \sin(10z)}{z^2}
 \end{aligned}$$

Q6.  $f(w) = \tan(w) \sec(w)$



$$\begin{aligned}f'(w) &= (\sec^2(w))(\sec(w)) + (\sec(w)\tan(w))(\tan(w)) \\&= \sec^3(w) + \sec(w)\tan^2(w) \\f'(w) &= \sec(w)(\sec^2(w) + \tan^2(w))\end{aligned}$$

Q14.  $y = \sqrt{5z + \tan(4z)}$

$$\begin{aligned}y &= (5z + \tan(4z))^{\frac{1}{2}} \\y' &= \left(\frac{1}{2}\right)(5 + 4\sec^2(4z))(5z + \tan(4z))^{-\frac{1}{2}} \\y' &= \frac{\frac{5}{2} + 2\sec^2(4z)}{\sqrt{5z + \tan(4z)}}\end{aligned}$$