Implicit Application

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Height of the wall is $y(t)$

Distance from the wall is $x(t)$

From Pythagoras $y(t)^2 + x(t)^2 = 10^2$

Known

$$\frac{dy}{dx} = 10, \;\; x(t) = 6$$

Unknown

$$\frac{dx}{dt}$$

$$y^2 + x^2 = 10^2$$

$$2y\frac{dy}{dt} + 2x\frac{dy}{dt} = 0$$

$$2y(10) + 2x\frac{dy}{dt} = 0$$

$$20y + 2x\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{20y}{2x}$$

$$y^2 = \sqrt{10^2 - x^2}$$

$$\frac{dx}{dt} = -\frac{20(\sqrt{10^2 - (6)^2})}{2(6)}$$

$$\frac{dx}{dt} = -\frac{40}{3} \; m/sec$$

2

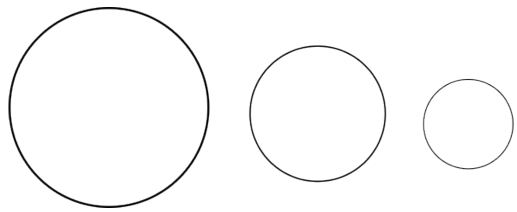


Figure 1: Balloon deflating

Known

$$A = 4\pi r^2, \quad \frac{dV}{dt} = 2, \quad r = 12, \quad V = \frac{4}{3}\pi r^3,$$

Unknown

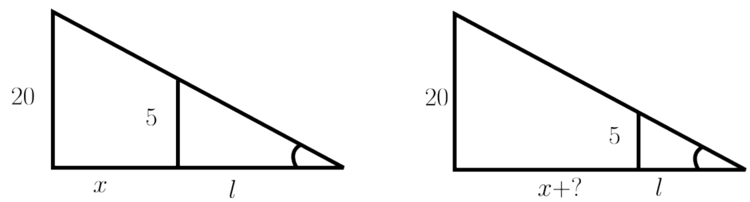
$$\frac{dr}{dt}$$

$$\begin{aligned} 4\pi r^2 \frac{dr}{dt} &= \frac{dV}{dt} \\ 4\pi r^2 \frac{dr}{dt} &= -2 \\ \frac{dr}{dt} &= -\frac{2}{4\pi r^2} \end{aligned}$$

$$\begin{aligned} 4\pi r^2 &= A \\ 8\pi r \frac{dr}{dt} &= \frac{dA}{dt} \\ 8\pi r(-\frac{2}{4\pi r^2}) &= \frac{dA}{dt} \end{aligned}$$

$$\frac{dA}{dt} = -\frac{1}{3} ft^2/min$$

3



Known

$$\frac{dx}{dt} = 4$$

Unknown

$$\frac{dl}{dt}$$

$$\frac{x+l}{20} = \frac{l}{5}$$
$$x + l = 4l$$
$$x = 3l$$
$$\frac{x}{3} = l$$
$$\frac{dl}{dx} = \frac{1}{3}$$

$$\frac{dl}{dt} = \frac{dl}{dx} \times \frac{dx}{dt}$$
$$\frac{dl}{dt} = \frac{4}{3}$$

3.1

Let L be the sum of the distance from the lamppost and the length of the boy's shadow.

$$\frac{dL}{dt} = \frac{dl}{dt} + \frac{dx}{dt} \therefore \frac{dL}{dt} = \frac{16}{3}\ ft/sec$$

3.2

$$\frac{dl}{dt} = \frac{4}{3}\ ft/sec$$

4

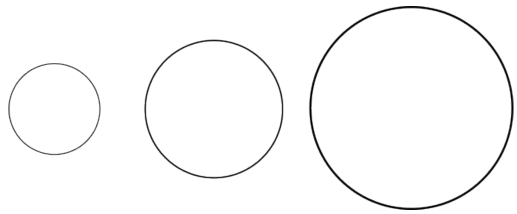


Figure 2: Balloon inflating

Known

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 5$$

Unknown

$$\frac{dr}{dt}$$

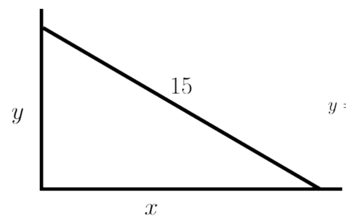
$$\begin{aligned} 4\pi r^2 \frac{dr}{dt} &= \frac{dV}{dt} \\ 4\pi r^2 \frac{dr}{dt} &= 5 \\ \frac{dr}{dt} &= \frac{5}{4\pi r^2} \end{aligned}$$

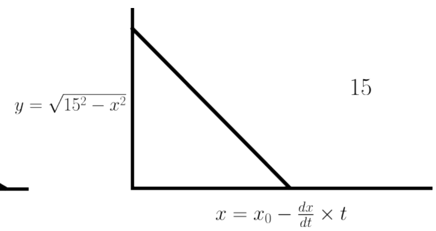
$$D = r + r \therefore r = 10$$

$$\frac{dr}{dt} = \frac{5}{4\pi r^2}$$

$$\frac{dr}{dt} = 3.978\,873\,577 \times 10^{-3}\,\mathrm{ft/min}$$

5





Known

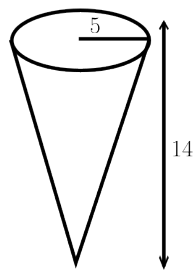
$$\frac{dx}{dt} = \frac{1}{4}$$
$$\text{When } t = 0,\ x = 10$$
$$\therefore t = 0,\ y = \sqrt{15^2 - 10^2}$$
$$y = 5\sqrt{5}$$

$$y^2 + x^2 = 15^2$$
$$2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 0$$
$$\frac{dy}{dt} = \frac{-2x(-\frac{1}{4})}{2y}$$
$$\frac{dy}{dt} = \frac{\frac{1}{2}x}{2y}$$

$$x(t) = x(0) + \frac{dx}{dt} \times t$$
$$x(t) = 10 - \frac{1}{4} \times t$$
$$x(t) = 10 - \frac{t}{4}$$
$$y(t) = \sqrt{15^2 - x^2}$$

$$x = 10 - \frac{t}{4},\quad t = 12,\quad y = \sqrt{15^2 - x^2}$$
$$\frac{dy}{dt} = \frac{\frac{1}{2}(10 - \frac{t}{4})}{2(\sqrt{15^2 - (10 - \frac{t}{4})^2})}$$
$$\frac{dy}{dt} = 0.132\ ft/sec$$

6



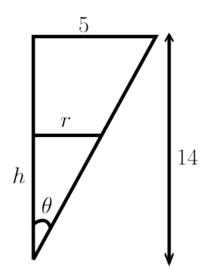
Known

$$\frac{dV}{dt} = -2, \;\; r = 5, \;\; h = 14, \;\; V = \pi r^2 \frac{h}{3}$$

Unknown

$$\frac{dh}{dt}, \;\; \frac{dr}{dt}$$

$$\frac{dV}{dt} = (\pi r^2)(\frac{1}{3}\frac{dh}{dt}) + (2\pi r\frac{dr}{dt})(\frac{h}{3})$$
$$\frac{dV}{dt} = \frac{\pi r^2}{3}\frac{dh}{dt} + \frac{2\pi r h}{3}\frac{dr}{dt}$$



$$\frac{r}{h} = \frac{5}{14}$$
$$r = \frac{5}{15}h$$

$$h = 6$$
$$r = \frac{5(6)}{14}$$
$$r = \frac{15}{7}$$

$$\frac{dr}{dt} = \frac{5}{14}\frac{dh}{dt}$$

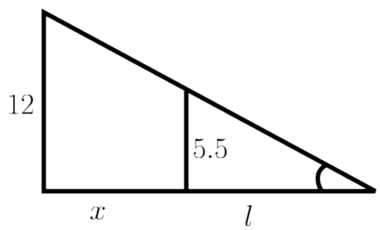
6.1

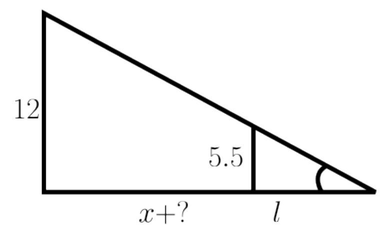
$$-2 = \frac{\pi(\frac{15}{7})^2}{3}\frac{dh}{dt} + \frac{2\pi(\frac{15}{7})(6)}{3}(\frac{5}{14})\frac{dh}{dt}$$
$$-2 = \frac{75}{49}\pi\frac{dh}{dt} + \frac{150}{49}\pi\frac{dh}{dt}$$
$$-2 = \frac{225}{49}\pi\frac{dh}{dt}$$
$$-\frac{2}{\frac{225}{49}\pi} = \frac{dh}{dt}$$
$$\frac{dh}{dt} = -\frac{98}{225}\ ft/hour$$

6.2

$$\frac{dh}{dt} = -\frac{98}{225} \text{ when } h = 6$$
$$\frac{dr}{dt} = \frac{5}{14}(-\frac{98}{225}\pi)$$
$$\frac{dr}{dt} = -\frac{7}{45}\pi\ ft/hour$$

7





Known

$$\frac{dx}{dt} = 2$$

Unknown

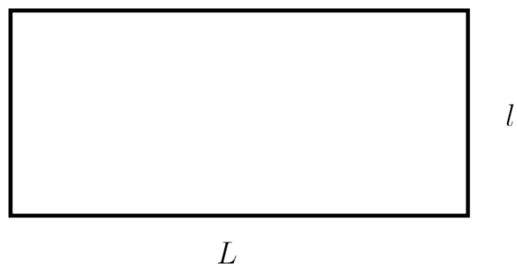
$$\text{Let } L = x + l$$
$$\frac{dL}{dt} \text{ when } x = 25$$

$$\frac{x+l}{12} = \frac{l}{5.5}$$
$$x + l = \frac{24}{11}l$$
$$x = \frac{13}{11}l$$
$$\frac{11}{13}x = l$$
$$\frac{11}{13}\frac{dx}{dt} = \frac{dl}{dt}$$

7.1

$$L = x + lL = x + \frac{11}{13}x$$
$$L = \frac{24}{13}x$$
$$\frac{dL}{dt} = \frac{24}{13}\frac{dx}{dt}$$
$$\frac{dL}{dt} = \frac{48}{13}\ ft/sec$$

8



Known

$$L = 3l, \quad \frac{dl}{dt} = -2$$

Unknown

$$\frac{dL}{dt}$$

8.1

$$\frac{dL}{dt} = 3\frac{dl}{dt}$$

$$\frac{dL}{dt} = -6 \; inches/min$$

8.2

Known

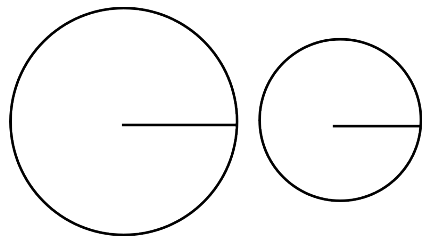
$$A = L \times l, \quad \frac{dl}{dt} = -2, \quad l = 6$$

$$\frac{dA}{dt} = (L)(\frac{dl}{dt}) + (\frac{dL}{dt})(l)$$

$$\frac{dA}{dt} = 3(6)(-2) + 3(-2)(6)$$

$$\frac{dA}{dt} = -72 \; inches^2/min$$

9



Known

$$\frac{dA}{dt} = -0.5, \quad A = 12$$

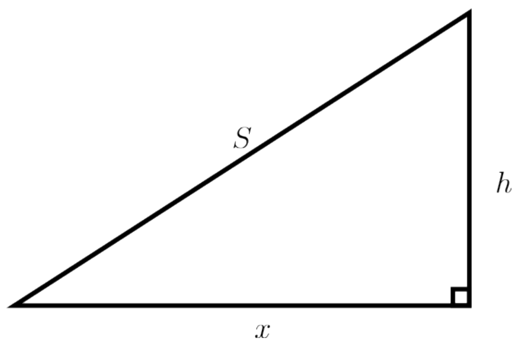
Unknown

$$\frac{dr}{dt}$$

$$A = \pi r^2$$
$$12 = \pi r^2$$
$$\frac{12}{\pi} = r^2$$
$$\sqrt{\frac{12}{\pi}} = r$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$
$$-0.5 = 2\pi(\sqrt{\frac{12}{\pi}})\frac{dr}{dt}$$
$$\frac{-0.5}{2\pi(\sqrt{\frac{12}{\pi}})} = \frac{dr}{dt}$$
$$\frac{dr}{dt} = -0.0407\ m/sec$$

10



Known

$$\frac{dh}{dt} = 15$$

10.1

$$\begin{aligned} 350^2 + h^2 &= S^2 \\ 0 + 2h\frac{dh}{dt} &= 2S\frac{dS}{dt} \end{aligned}$$

$$\begin{aligned} h &= h_0 + \frac{dh}{dt} \times t \\ h &= 0 + 15 \times (20) \\ h &= 300 \end{aligned}$$

$$\begin{aligned} 350^2 + 300^2 &= S^2 \\ 212500 &= S^2 \\ 50\sqrt{85} &= S \\ 2(300)(15) &= 2(50\sqrt{85})\frac{dS}{dt} \\ \frac{2(300(15))}{2(50\sqrt{85})} &= \frac{dS}{dt} \\ \frac{dS}{dt} &= 9.762\ ft/sec \end{aligned}$$

10.2

$$
\begin{aligned}
h &= 15 \times 6 \\
h &= 900
\end{aligned}
$$

$$
\begin{aligned}
350^2 + 900^2 &= S^2 \\
50\sqrt{373} &= S \\
2(900)(15) &= 2(50\sqrt{373})\frac{dS}{dt} \\
&\frac{dS}{dt} = 13.980\ ft/sec
\end{aligned}
$$