$$\lim_{t \to 4} t^2 + 5t + 1$$

$$\lim_{t \to 2} \frac{t^2 - 2}{t^2 + t - 6}$$

$$\lim_{x \to 5} \frac{x^2 - x}{x^2 + 2x - 3}$$

$$\lim_{h\to 0}\frac{(1+h)^2-1}{h}$$

$$\lim_{t\to 0}\frac{(t+4)^2-16}{t}$$

$$\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$$

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$$

$$\lim_{x \to -1} \frac{x}{(x+1)^2}$$

$$\lim_{x \to -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2}$$

$$\lim_{x \to -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2}$$

$$\lim_{x \to \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1}$$

$$\lim_{x\to -\infty}\frac{x^3+\sqrt{4x^6+4}}{5x^3+2x}$$

$$\text{For the function} f(x) = \begin{cases} x^3 - 2, & \text{if } x \geq 2 \\ 1 + x^2, & \text{if } x < 2 \end{cases} \quad \text{find the } \lim_{x \to 1} f(x)$$

Q1.

$$\lim_{t \to 4} t^2 + 5t + 1$$

$$= (4)^2 + 5(4) + 1$$

$$= 16 + 20 + 1$$

$$\lim_{t \to 4} t^2 + 5t + 1 = 37$$

Q2.

$$\begin{split} \lim_{t \to 2} \frac{t^2 - 2}{t^2 + t - 6} \\ &= \frac{t^2 - 2}{(t - 2)(t + 3)} \\ & \therefore \lim_{t \to 2} \frac{t^2 - 2}{t^2 + t - 6} = \text{Does not exist} \end{split}$$

Q3.

$$\begin{split} \lim_{x \to 5} \frac{x^2 - x}{x^2 + 2x - 3} \\ &= \frac{5^2 - 5}{5^2 + 2(5) - 3} \\ &= \frac{25 - 5}{25 + 10 - 3} \\ &= \frac{20}{32} \\ \lim_{x \to 5} \frac{x^2 - x}{x^2 + 2x - 3} = \frac{5}{8} \end{split}$$

Q4.

$$\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$$

$$= \frac{h^2 + 2h}{h}$$

$$= h + 2$$

$$\lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = 2$$

Q5.

$$\lim_{t \to 0} \frac{(t+4)^2 - 16}{t}$$

$$= \lim_{t \to 0} \frac{t^2 + 8t}{t}$$

$$= \lim_{t \to 0} (t+8)$$

$$= (0+8)$$

$$\lim_{t \to 0} \frac{(t+4)^2 - 16}{t} = 8$$

Q6.

$$\begin{split} \lim_{x \to 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} \\ &= \frac{\sqrt{x+2} - \sqrt{2}}{x} \times \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}} \\ &= \frac{(\sqrt{x+2} - \sqrt{2})(\sqrt{x+2} + \sqrt{2})}{x(\sqrt{x+2} + \sqrt{2})} \\ &= \frac{x+2-2}{x(\sqrt{x+2} + \sqrt{2})} \\ &= \frac{1}{\sqrt{x+2} + \sqrt{2}} \\ &= \frac{1}{\sqrt{0+2} + \sqrt{2}} \\ \lim_{x \to 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} = \frac{\sqrt{2}}{4} \end{split}$$

Q7.

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2}$$

$$= \frac{x-1}{\sqrt{x^2+3}-3} \times \frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$

$$= \frac{(x-1)(\sqrt{x^2+3}+2)}{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)}$$

$$= \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2+3-4}$$

$$= \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2-1}$$

$$= \frac{(x-1)(\sqrt{x^2+3}+2)}{(x-1)(x+1)}$$

$$= \frac{\sqrt{x^2+3}+2}{x+1}$$

$$= \frac{\sqrt{1^2+3}+2}{1+1}$$

$$= \frac{4}{2}$$

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2} = 2$$

Q8.

$$\begin{split} \lim_{x \to -1} \frac{x}{(x+1)^2} \\ &= \frac{x}{x^2 + 2x + 1} \\ &= \frac{x}{x(x+2+\frac{1}{x})} \\ &= \frac{-1}{-1(-1+2-1)} \\ &= \frac{-1}{0} \\ &\lim_{x \to -1} \frac{x}{(x+1)^2} = -\infty \end{split}$$

Q9.

$$\begin{split} \lim_{x \to -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2} \\ &= \frac{\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^4}}{1 - \frac{2}{x^4}} \\ &= \frac{-\frac{1}{\infty} - \frac{2}{\infty} + \frac{1}{\infty}}{1 - \frac{2}{\infty}} \\ &= \frac{0 - 0 + 0}{1 - 0} \\ &\lim_{x \to -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2} = 0 \end{split}$$

Q10.

$$\begin{split} \lim_{x \to -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2} \\ &= \frac{x^3(-x^2 - 1 + \frac{x}{x^3} - \frac{3}{x^3})}{x^3(2 + \frac{3x}{x^3} - \frac{2}{x^3})} \\ &= \frac{x^2 - 1 + \frac{1}{x^2} - \frac{3}{x^3}}{2 + \frac{3}{x^2} - \frac{2}{x^2}} \\ &= \frac{-\infty - 1 + 0 + 0}{2 + 0 + 0} \\ &= \frac{-\infty - 1}{2} \\ &\lim_{x \to -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2} = -\infty \end{split}$$

Q11.

$$\begin{split} \lim_{x \to \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1} \\ &= \frac{x^4 (1 - \frac{5x^2}{x^4} + \frac{x}{x^4} - \frac{1}{x^4})}{x^4 (3 + \frac{x}{x^2} - \frac{1}{x^4})} \\ &= \frac{1 - \frac{5}{x^2} + \frac{1}{x^3} - \frac{1}{x^4}}{3 + \frac{1}{x^2} - \frac{1}{x^4}} \\ &= \frac{1 - \frac{5}{\infty} + \frac{1}{\infty} - \frac{1}{\infty}}{3 + \frac{1}{\infty} - \frac{1}{\infty}} \\ &= \frac{1 - 0 + 0 - 0}{3 + 0 - 0} \\ &\lim_{x \to \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1} = \frac{1}{3} \end{split}$$

Q12.

$$\begin{split} \lim_{x \to -\infty} \frac{x^3 + \sqrt{4x^6 + 4}}{5x^3 + 2x} &= \\ &= \frac{x^3 + 2\sqrt{x^6 + 1}}{5x^3 + 2x} \\ &= \frac{x^3(1 - 2\sqrt{1 + \frac{1}{x^6}})}{x^3(5 + \frac{2}{x^2})} \\ &= \frac{1 - 2\sqrt{1 + \frac{1}{x^6}}}{5 + \frac{2}{x^2}} \\ &= \frac{1 - 2\sqrt{1 + 0}}{5 + 0} \\ \lim_{x \to -\infty} \frac{x^3 + \sqrt{4x^6 + 4}}{5x^3 + 2x} &= -\frac{1}{5} \end{split}$$

Q13.

$$\lim_{x\to 1} f(x)$$