Assignment 4

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Question 1

$$\begin{array}{rl} & (p \wedge t) \rightarrow (r \vee s) \\ & q \rightarrow (u \wedge t) \\ & u \rightarrow p \\ & \neg s \\ & q \\ \hline \therefore & r \end{array}$$

Solution:

	Steps	Reasons
1	$(p \wedge t) \rightarrow (r \vee s)$	Premise 1
2	$q \rightarrow (u \wedge t)$	Premise 2
3	$u \rightarrow p$	Premise 3
4	$\neg s$	Premise 4
5	q	Premise 5
6	$u \wedge t$	By Modus Ponens from 2 and 5
7	u	By Simplification from 6
8	p	By Modus Ponens from 3 and 7
9	t	By Simplification from 6
10	$p \wedge t$	By Conjunction from 8 and 9
11	$r \vee s$	By Modus Ponens from 1 and 10
12	r	By Disjunctive Syllogism from 11 and 4

Question 2

"Jane is a student in this class. Jane grew up in a family of entrepreneurs. Everyone who grew up in a family of entrepreneurs can build a thriving business. Therefore, someone in this class can build a thriving business."

Solution:

$$\mathbb{U}_x\text{: All people}$$

$$\begin{aligned} S(x)&: x \text{ is a student in this class.} \\ E(x)&: x \text{ grew up in a family of entrepreneurs.} \\ B(x)&: x \text{ can build a thriving business} \end{aligned}$$

$$\begin{array}{rl} & S(\text{Jane}) \\ & E(\text{Jane}) \\ & \forall x\,(E(x) \rightarrow B(x)) \\ \hline \therefore & \exists x\,(S(x) \wedge B(x)) \end{array}$$

	Steps	Reasons
1	$S(\text{Jane})$	Premise 1
2	$E(\text{Jane})$	Premise 2
3	$\forall x\,(E(x) \rightarrow B(x))$	Premise 3
4	$E(\text{Jane}) \rightarrow B(\text{Jane})$	Universal Instantiation from 3.
5	$B(\text{Jane})$	By Modus Ponens from 2 and 4.
6	$S(\text{Jane}) \wedge B(\text{Jane})$	By Conjunction from 1 and 5.
7	$\exists x\,(S(x) \wedge B(x))$	Existential Generalization from 6.

Question 3

The product of two odd numbers is odd

Proof: Using a direct proof I will show that if x and y are odd, then $x \times y$ is odd.
Assume that x and y are odd.
Then $\exists k \in \mathbb{Z}\, x = 2k + 1$ and $\exists t \in \mathbb{Z}\, y = 2t + 1$
For $x \times y$ to be odd

$$x \times y = 2z + 1$$

Where z is an integer

$$\begin{aligned} x \times y &= (2k+1) \times (2t+1) \\ &= 4kt + 2k + 2t + 1 \\ &= 2(2kt + k + t) + 1 \\ \text{Let } z = 2kt + k + t \\ &= 2z + 1 \end{aligned}$$

Since z is the sum of integers it is an integer.
Hence If x and y are odd, then $x \times y$ is odd. 😊