Logarithmic Functions and Derivatives

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Chapter 1

1.1 Derivatives of Logarithmic Functions

Definition 1.1.1: Logarithmic Functions

Functions in the form:

$$y = \log_a[f(x)] \quad \text{or} \quad y = \ln[f(x)]$$

1.1.1 Derivate of $\ln(x)$

$$y' = \frac{f'(x)}{f(x)} \quad \text{or} \quad \frac{x'}{x}$$

Proof:

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\text{Where } f(x) = \ln(x) \quad \text{and} \quad f(x+h) = \ln(x+h)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$$
$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$$
$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\ln(\frac{x+h}{x})}{h}$$
$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\ln(1 + \frac{h}{x})}{h}$$
$$\frac{dy}{dx} = \lim_{h \to 0} \frac{1}{h}(\ln(1 + \frac{h}{x}))$$

$$\text{Let } v = \frac{h}{x} \quad \therefore h = vx \quad \Longrightarrow \quad \text{As} \quad h \to 0 \;\; v \to 0$$

$$\frac{dy}{dx} = \lim_{v \to 0} \frac{1}{vx}(\ln(1+v))$$
$$\frac{dy}{dx} = \lim_{v \to 0} \frac{1}{v} \times \frac{1}{x} \ln(1+v)$$
$$\frac{dy}{dx} = \frac{1}{x} \lim_{v \to 0} \frac{1}{v} \ln(1+v)$$
$$\frac{dy}{dx} = \frac{1}{x} \lim_{v \to 0} \ln(1+v)^{\frac{1}{v}}$$

$$\frac{dy}{dx} = \frac{1}{x} \ln[\lim_{v \to 0} (1 + v)^{\frac{1}{v}}]$$

$$\frac{dy}{dx} = \frac{1}{x} \ln[e]$$

$$\frac{dy}{dx} = \frac{1}{x}$$

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