
Valuation Concepts

The value of money, its purchasing power, tends to be different at different time due to inflation/deflation. The nominal value of cash flows should be adjusted accordingly to obtain the real value of cash flows.

Methods of Interest Computation

Interest

The cost of borrowing money or the return on investment is often expressed as interest. There are two primary methods of calculating interest: simple interest and compound interest. Interest is needed because:

- The opportunity cost of money - To compensate them for the opportunity cost of forgoing the possible returns from alternative investments.
- The risk of losing money - To compensate them for risks they bear due to future uncertainties.

Determination of Market Interest Rate The nominal/quoted interest rate on a security is determined as:

$$k^* + IP + DRP + LP + MRP = NIR$$

Where: - k^* - Real risk-free rate of interest - IP - Inflation Premium, i.e. compensation for reduced purchasing power due to inflation - DRP - Default Risk Premium, i.e. compensation for risk of default by borrower - LP - Liquidity Premium, i.e. offset the inconvenience of not being able to sell a security for cash at maturity - MRP - Maturity Risk Premium, i.e. longer duration security should pay higher rate. - NIR - Nominal Interest Rate

Simple Interest

$$SI = P_0 \times i \times t$$

Where: - SI - Simple Interest in currency - P_0 - Principal amount (initial investment) - i - Annual interest rate - t - Time period

The future amount is the sum of the Principal and Simple Interest:

$$FV = P_0 + SI = P_0(1 + it)$$

Or

$$FV_t = P_0[1 + it]$$

The present value of a future receipt / payment is calculated using the formula:

$$PV_0 = P_0 = \frac{FV_t}{1 + it}$$

Compound Interest

Amount Payable The amount payable at the end of a specified future period is the aggregate of the principal and interest accrued.

With Periodic Compounding

$$A_n = P_0(1 + i)^n$$

Where: - A_n - Amount after n periods - P_0 - Principal amount (initial investment) - i - Interest rate per period - n - Number of compounding periods

With continuous compounding

$$A_n = P_0 e^{in}$$

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Interest Accrued

With Periodic Compounding

$$CI = P_0((1 + i)^n - 1)$$

Or

$$CI = P_0(1 + i)^n - P_0$$

Or

$$CI = A_n - P_0$$

And the future value can be expressed as:

$$FV = P_0 + CI$$

With Continuous Compounding

$$CI = P_0(e^{in} - 1)$$

Or

$$CI = P_0e^{in} - P_0$$

Multiple Compounding

Periodic Compounding When interest is accrued for fixed periods such as monthly, quarterly, semi-annually, etc.:

$$A_n = P_0 \left(1 + \frac{i}{m}\right)^{mn}$$

Where: - A_n - Amount after n years - P_0 - Principal amount (initial investment) - i - Annual nominal interest rate - m - Number of compounding periods per year - n - Number of years

Continuous Compounding When interest is compounded continuously, the formula is:

$$A_n = P_0 e^{in}$$

Where: - A_n - Amount after n years - P_0 - Principal amount (initial investment) - i - Annual nominal interest rate - n - Number of years

Equivalent Periodic Rate (EPR) The equivalent periodic rate (EPR) is the interest rate per compounding period that is equivalent to the nominal annual interest rate when compounding occurs multiple times per year. It can be calculated as:

$$EPR = \frac{i}{m}$$

Nominal Rate vs. Effective Rate

Nominal Interest Rate (NIR) - The face value cost of a loan, without taking compounding into account

Effective Interest Rate (EIR) - The true cost of a loan after accounting for compounding within the year

Effective Annual Rate (EAR)

$$EAR = (1 + HPR)^m - 1$$

Where HPR is the holding period rate:

$$HPR = \frac{i}{m}$$

For example: Jane is considering two investment options: - Option A requires a deposit of GHS 10,000 now. Interest will be paid on the deposit at an interest rate of 15%