

Reporting about the behaviour of a function within the range of its dangerous values.

$$f(x) = x^2 + \frac{1}{x}$$

Input variable - x

Output - $f(x)$

Name of function - f

“Acceptable”/Permissible input values of x - All real numbers except zero

$$\begin{aligned} &(x, f(x)), (x+h, f(x+h)) \\ &\frac{f(x+h) - f(x)}{x+h-x} \\ &\frac{f(x+h) - f(x)}{h} \end{aligned}$$

Proof Points used: $(0, 6), (1, 90), (3, 162)$

When $t = 0$ and $y = 6$

$$\begin{aligned} y &= at^2 + bt + c \\ 6 &= a(0)^2 + b(0) + c \\ c &= 6 \end{aligned}$$

When $t = 1$ and $y = 90$

$$\begin{aligned} 90 &= a(1)^2 + b + 6 \\ 90 &= a + b + 6 \\ 84 &= a + b \\ 84 - b &= a \end{aligned}$$

When $t = 3$ and $y = 162$

$$\begin{aligned}162 &= a(3)^2 + 3b + 6 \\162 &= 9a + 3b + 6 \\162 &= 9(84 - b) + 3b + 6 \\162 &= 756 - 9b + 3b + 6 \\-594 &= -6b + 6 \\-600 &= -6b \\b &= 100\end{aligned}$$

$$\therefore b = 100$$

$$\begin{aligned}84 - 100 &= a \\a &= -16\end{aligned}$$

Therefore $a = -16$, $b = 100$, and $c = 6$

Given $f(x) = x^2$, find the Limit of $f(x)$ at $x = 3$

$$\text{As } x \rightarrow 3^-, f(x) \rightarrow 9$$

$$\text{As } x \rightarrow 3^+, f(x) \rightarrow 9$$

Or

$$\begin{aligned}\lim_{x \rightarrow 3^-} f(x) &= 9 \\ \lim_{x \rightarrow 3^+} f(x) &= 9\end{aligned}$$

The first 9 is known as the left limit of $f(x)$ and the other 9 is known as the right limit of $f(x)$

The Limit of $f(x)$ at $x = 3$

$$\lim_{x \rightarrow 3} f(x) = 9$$

This is because the left limit and right limit converge.

In the case where:

$$\lim_{x \rightarrow 1^-} f(x) = 5$$

$$\lim_{x \rightarrow 1^+} f(x) = 4$$

The left and right limits do not converge so there is no limit of $f(x)$ for $x = 1$

$$\lim_{x \rightarrow 1} f(x) = \text{No such unique number}$$

\therefore The limit of $f(x)$ at $x = 1$ does not exist

In the case where the one limit does not exist (increasing without bounds):

$$\lim_{x \rightarrow 1^-} f(x) \rightarrow \infty$$

$$\lim_{x \rightarrow 1^+} f(x) \rightarrow 4$$

The limit does not exist because the left limit does not exist.

$$\lim_{x \rightarrow 1} f(x) = \text{does not exist}$$

\therefore the left limit does not exist

Graphical

Given $f(x) = x^2$, evaluate the Limit of $f(x)$ at $x = 3$ using the graphical approach.