Assignment 8

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Question 1

1. do not contain the base T.
2. contain the sequence ACG.
3. contain all four bases A, T, C, and G.
4. contain exactly three of the four bases A, T, C, and G.

Solution:

1. Using the restrictions given we are limited to the bases A, G and C for each position. This means there are three options for each position and since there are four positions we can split the task choosing a base into four steps, resulting in:

   $$3^4 = 81$$

   81 possible sequences.

2. The sequence ACG is taken as a unit filling the first three positions from the left, allowing the remaining position to be filled with any if the four bases. Using the product rule this results in 4 possible ways.

   But the unit ACG can also fill the first three positions from the right, which again only leaves one position left, using sum rule, this result in the total number of ways being:

   $$4 + 4 = 8$$

   8 possible ways

3. The sequence must contain all for bases, this means that each of the four bases should appear once in the four element sequence, accounting for the order of appearance of all the bases being counted as a different sequence, we can use the permutation formula to calculate the total number of ways:

   $$\begin{aligned} P(4,4) &= \frac{4!}{(4-4)!} \\ &= 24 \end{aligned}$$

   24 possible ways

4. The sequence can contain exactly three of the four bases A, T, C, and G. This means that one of the bases must be left out, This leaves us with 3 possible bases to choose from for three of the four positions, and 3 ways to choose the base to leave out. Using the product rule, we can calculate the total number of ways to choose a letter for each positions:

   $$\begin{aligned} C(4,3) \times C(3,1) &= \frac{4!}{(4-3)! \times 3!} \times \frac{3!}{(3-1)!1!} \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

   Now there are 4 possible positions to occupy with one of the bases as we have to choose exactly three out of four bases means exactly one base will be repeated out of the three bases chosen. This means that there are three possible positions to place the repeated base, giving us exactly 12 possible ways to choose the sequence. Using the product rule, then calculate the total number of ways adhere to the restrictions:

   $$\begin{aligned} \text{Total number of ways to choose a letter} \times \text{Number of ways to choose a sequence} &= 12 \times 12 \\ &= 144 \end{aligned}$$

Question 2

What is the minimum number of students, each of whom comes from one of the 54 African countries, who must be enrolled at Ashesi university to guarantee that there are at least 5 who come from the same country? Explain.

Solution: Using the generalized pigeon-hole principle, the number of boxes k is the number of African countries, 54, and can represent the number of students needed to guarantee that there are at least 5 students from the same country, by N. Therefore:

$$\left\lceil \frac{N}{54} \right\rceil = 5$$
$$4 < \frac{N}{54} \leqslant 5$$

$$\frac{N}{54} > 4$$
$$\frac{N}{54} \leqslant 5$$

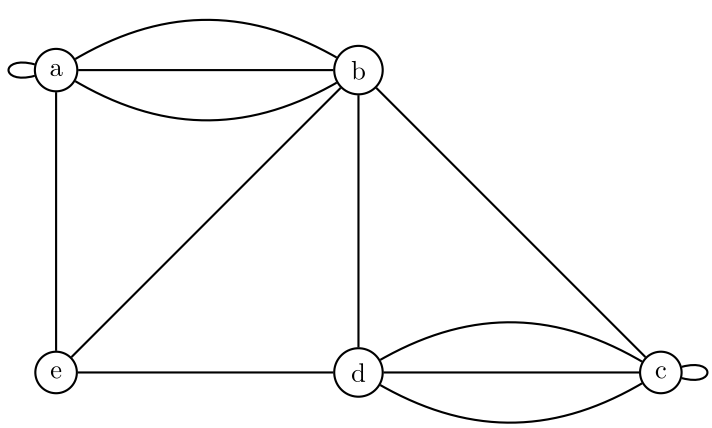
$$\frac{N}{54} > 4$$
$$N > 216$$

$$\frac{N}{54} \leqslant 5$$
$$N \leqslant 270$$

$$216 < N \leqslant 270$$

$\therefore N = 217$

Question 3



Solution:

1. There are 5 vertices, and 13 edges in the graph, with each vertex having a degree of:
    - deg(a) - 6

- deg(b) - 6
- deg(c) - 6
- deg(d) - 5
- deg(e) - 3

2. Let m be the number of edges, where $m = 13$.

$$2m = \sum_{v \in V} \deg(v)$$

$$\begin{aligned} 2m &= 26 \\ \sum_{v \in V} \deg(v) &= 3(6) + 5 + 3 \\ &= 26 \\ \frac{26}{2} &= 13 \end{aligned}$$