# Assignment 7

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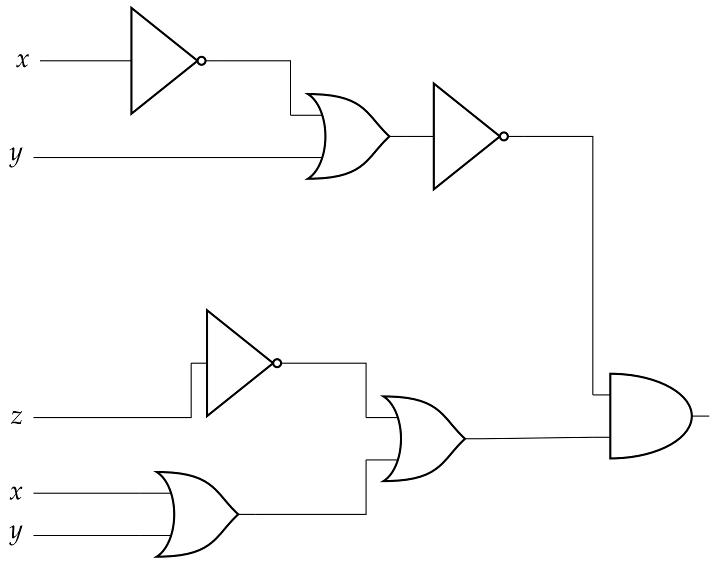
**Question 1**

$$\overline{(\overline{x} + y)}\,(x + y + \overline{z})$$

***Solution:***




**Question 2**

$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Whenever $n$ is a positive integer

***Proof:*** Let $P(n)$ be $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Basis Step

$$\begin{aligned} P(1) : 1^3 &= \left(\frac{1(1+1)}{2}\right)^2 \\ 1 &= \left(\frac{2}{2}\right)^2 \\ 1 &= 1 \end{aligned}$$

Induction Step

To complete this step I must prove $P(k) \rightarrow P(k+1)$ for any positive integer $k$
Assume $P(k)$ is $T$ for some positive integer $k$, then

$$P(k) : 1^3 + 2^3 + \ldots + k^3 = \left(\frac{k^2 + k}{2}\right)^2$$

Then $P(k+1)$ is

$$P(k+1) : 1^3 + 2^3 + \ldots + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$$

And can be expressed as $P(k+1) : P(k) + (k+1)^3$, therefore:

$$\begin{aligned}
P(k+1) : 1^3 + 2^3 + \ldots + k^3 + (k+1)^3 &= \left(\frac{k^2+k}{2}\right)^2 + (k+1)^3 \\
&= \frac{\left(k^2+k\right)^2}{4} + (k+1)^3 \\
&= \frac{\left(k^2+k\right)^2 + 4(k+1)^3}{4} \\
&= \frac{\left(k^2+k\right)^2 + \left(4k^3+12k^2+12k+4\right)}{4} \\
&= \frac{k^4+2k^3+k^2+4k^3+12k^2+12k+4}{4} \\
&= \frac{k^4+6k^3+13k^2+12k+4}{4} \\
&= \frac{(k+1)^2(k+2)^2}{4} \\
&= \left(\frac{(k+1)(k+2)}{2}\right)^2
\end{aligned}$$

$\therefore P(k+1)$ is $T$
Hence we can conclude that $P(n)$ is true for all positive integers $n$

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**Question 3**

$$2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^n = \frac{1-(-7)^{n+1}}{4}$$

Whenever $n$ is a non-negative integer

***Proof:*** Let $P(n)$ be $2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^n = \frac{1-(-7)^{n+1}}{4}$

Basis Step

$$\begin{aligned}
P(0) : 2 &= \frac{1-(-7)^{0+1}}{4} \\
2 &= \frac{8}{4} \\
2 &= 2
\end{aligned}$$

Induction Step

To complete this step I must prove $P(k) \rightarrow P(k+1)$ for any non-negative integer $k$

Assume $P(k)$ is $T$ for some non-negative integer $k$, then

$$P(k) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^k = \frac{1 - (-7)^{k+1}}{4}$$

Then $P(k+1)$ is

$$P(k+1) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^{(k+1)} = \frac{1 - (-7)^{k+2}}{4}$$

And can be expressed as $P(k+1) : P(k) + 2 \times (-7)^{(k+1)}$, therefore:

$$\begin{aligned}
P(k+1) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots - 2 \times (-7)^k + 2 \times (-7)^{(k+1)} &= \frac{1 - (-7)^{k+1}}{4} + 2 \times (-7)^{(k+1)} \\
&= \frac{1 - (-7)^{k+1} + 8 \times (-7)^{(k+1)}}{4} \\
&= \frac{1 + 8 \times (-7)^{k+1} - (-7)^{k+1}}{4} \\
\text{Let } x = (-7)^{k+1} & \\
&= \frac{1 + 8x - x}{4} \\
&= \frac{1 + 7x}{4} \\
&= \frac{1 + 7^1 \times (-7)^{k+1}}{4} \\
&= \frac{1 - (-7)^1 \times (-7)^{k+1}}{4} \\
&= \frac{1 - (-7)^{k+2}}{4}
\end{aligned}$$

$\therefore P(k+1)$ is $T$

Hence we can conclude that $P(n)$ is true for all non-negative integers $n$

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