Assignment 6

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Question 1

1. $A \cup \emptyset = A$
2. $A \cap U = A$

Solution:

1.

$$\begin{aligned}
A \cup \emptyset &= \{x \mid x \in A \vee x \in \emptyset\} && \text{Definition of Union}\\
&= \{x \mid x \in A \vee \emptyset\} && \text{Definition of empty set}\\
&= \{x \mid x \in A \cup \emptyset\} && \text{Definition of Union}\\
&= \{x \mid x \in A\} && \text{By Second Identity Law}\\
&= A && \text{Defintion of the set } A
\end{aligned}$$

2.

$$\begin{aligned}
A \cap U &= \{x \mid x \in A \wedge x \in U\} && \text{Defintion of Intersection}\\
&= \{x \mid x \in A \wedge U\} && \text{Definition of Universal Set}\\
&= \{x \mid x \in A \cap U\} && \text{Definition of Intersection}\\
&= \{x|x \in A\} && \text{By First Identity Law}\\
&= A && \text{Definition of the set } A
\end{aligned}$$

Question 2

1. $(A \cup B) \subseteq (A \cup B \cup C)$
2. $(A \cap B \cap C) \subseteq (A \cap B)$

Solution:

1. $(A \cup B) \subseteq (A \cup B \cup C)$ means $\forall x\,(x \in (A \cup B) \rightarrow x \in (A \cup B \cup C))$

	Steps	Reasons
1	$x \in A \cup B$	Premise
2	$x \in A \vee x \in B$	Definition of Union
3	$x \in A \vee x \in B \vee x \in C$	By Addition on 2
4	$x \in A \cup B \cup C$	Definition of Union

$\therefore\ x \in (A \cup B) \rightarrow x \in (A \cup B \cup C)$
Hence $(A \cup B) \subseteq (A \cup B \cup C)$

2. $(A \cap B \cap C) \subseteq (A \cap B)$ means $\forall x\,(x \in (A \cap B \cap C) \rightarrow x \in (A \cap B))$

	Steps	Reasons
1	$x \in A \cap B \cap C$	Premise
2	$x \in A \wedge x \in B \wedge x \in C$	Definition of Intersection
3	$x \in A \wedge x \in B$	By Simplification on 2
4	$x \in A \cap B$	Definition of Intersection

$\therefore\ x \in (A \cap B \cap C) \rightarrow x \in (A \cap B)$
Hence $(A \cap B \cap C) \subseteq (A \cap B)$

Question 3

1. $x \cdot 1 = 0$
2. $x + x = 0$
3. $x \cdot 1 = x$
4. $x \cdot \overline{x} = 1$

Solution:

1. 0
2. 0
3. 0 and 1
4. 0 and 1