

Assignment 5

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Question 1

1. A proof by contraposition.
2. A proof by contradiction.

Solution:

1. **Proof:** The contraposition of the statement "If $3n + 2$ is even, then n is even", in the form $p \rightarrow q$, where p is " $3n + 2$ is even" and q is " n is even", is $\neg q \rightarrow \neg p$, i.e. "If n is odd, then $3n + 2$ is odd". Assume n is odd.
Then $\exists k \in \mathbb{Z} (n = 2k + 1)$.
For $3n + 2$ to be odd

$$\exists t \in \mathbb{Z} (3n + 2 = 2t + 1)$$

$$\begin{aligned} 3n + 1 &= 3(2k + 1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2(3k + 2) + 1 \end{aligned}$$

$$\begin{aligned} \text{Let } t &= 3k + 2 \\ &= 2t + 1 \end{aligned}$$

Since t is made up of the sum of the product of integers 3, k , and 2, t is an integer.

\therefore If n is odd, then $3n + 2$ is odd.

Hence by contraposition if $3n + 2$ is even, then n is even ⊖

2. **Proof:** The negation of the statement "If $3n + 2$ is even, then n is even", in the form $p \rightarrow q$, where p is " $3n + 2$ is even" and q is " n is even", is $p \wedge \neg q$, i.e., " $3n + 2$ is even and n is odd". Assume $3n + 2$ is even and n is odd.
Then $\exists t \in \mathbb{Z} (n = 2t + 1)$

$$\begin{aligned} 3n + 2 &= 3(2t + 1) + 2 \\ &= 6t + 3 + 2 \\ &= 6t + 4 + 1 \\ &= 2(3t + 2) + 1 \end{aligned}$$

$$\begin{aligned} \text{Let } k &= 3t + 2 \\ &= 2k + 1 \end{aligned}$$

Since k is made up of the sum of the product of integers 3, t , and 2, k is an integer.

Since $3n + 2$ can be expressed in the form $2k + 1$, where $k \in \mathbb{Z}$, $3n + 2$ is odd.

We now have $3n + 2$ being odd by our proof and even by our initial assumption, ($p \wedge \neg p$), which is a contradiction.

$\therefore 3n + 2$ is even and n is odd is false.

Hence the statement if $3n + 2$ is even, then n is even is true. ⊕

Question 2

1. $B \times C \times A$
2. $B \times B \times B$

Solution:

1.

$$B \times C \times A = \{(x, 0, a), (x, 0, b), (x, 1, a), (x, 1, b), (y, 0, a), (y, 0, b), (y, 1, a), (y, 1, b)\}$$

2.

$$B^3 = \{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, y, y), (y, y, x), (y, x, y), (y, x, x)\}$$

Question 3

1. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
2. $\{\emptyset\} \in \{\emptyset\}$
3. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
4. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

Solution:

1. True. \emptyset is an element of the set $\{\emptyset, \{\emptyset\}\}$
2. False. A set cannot be an element of itself.
3. True. All elements found in $\{\emptyset\}$ are also found in $\{\emptyset, \{\emptyset\}\}$ and $\{\emptyset\}$ is not equal to $\{\emptyset, \{\emptyset\}\}$
4. True. All elements found in $\{\{\emptyset\}\}$ are also found in $\{\emptyset, \{\emptyset\}\}$ and $\{\{\emptyset\}\}$ is not equal to $\{\emptyset, \{\emptyset\}\}$