

Implicit Differentiation

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Chapter 1

Implicit

1.1 Explicit vs Implicit

$$y = x^2 + x + 1$$

The dependent variable y is separate from the independent variable x , making this function explicitly written.

$$x^2 + y^2 = 1$$

The dependent variable y is not separate from the independent variable x , making this function implicitly written.

1.2 Implicit Differentiation

Given an implicitly written function, $2x^3 + 2y^3 = 9xy$. We can find the derivative by doing the following:

$$\begin{aligned} 2x^3 + 2y^3 &= 9xy \\ \frac{d}{dx}(2x^3 + 2y^3) &= \frac{d}{dx}(9xy) \\ 6x^2 + 6y^2 \frac{dy}{dx} &= (9)(y) + (9x) \frac{dy}{dx} \\ 6x^2 &= 9y = 9x \frac{dy}{dx} - 6y^2 \frac{dy}{dx} \\ 6x^2 &= 9y = (9x - 6y^2) \frac{dy}{dx} \\ &= \frac{6x^2 - 9y}{9x - 6y^2} = \frac{dy}{dx} \end{aligned}$$