Assignment 5

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Question 1

1. A proof by contraposition.
2. A proof by contradiction.

Solution:

1. ***Proof:*** The contraposition of the statement "If $3n+2$ is even, then n is even", in the form $p \longrightarrow q$, where p is "$3n+2$ is even" and q is "n is even", is $\neg q \longrightarrow \neg p$, i.e. "If n is odd, then $3n+2$ is odd ".
   Assume n is odd.
   Then $\exists k \in \mathbb{Z}\,(n = 2k+1)$.
   For $3n+2$ to be odd
   $$\exists t \in \mathbb{Z}\,(3n+2 = 2t+1)$$
   $$\begin{aligned} 3n+1 &= 3\,(2k+1)+2 \\ &= 6k+3+2 \\ &= 6k+4+1 \\ &= 2\,(3k+2)+1 \\ \text{Let } t &= 3k+2 \\ &= 2t+1 \end{aligned}$$
   Since t is made up of the sum of the product of integers 3, k, and 2, t is an integer.
   $\therefore$ If n is odd, then $3n+2$ is odd.
   Hence by contraposition if $3n+2$ is even, then n is even ☺
2. ***Proof:*** The negation of the statement "If $3n+2$ is even, then n is even", in the form $p \longrightarrow q$, where p is "$3n+2$ is even" and q is "n is even", is $p \wedge \neg q$, I.e., "$3n+2$ is even and n is odd".
   Assume $3n+2$ is even and n is odd.
   Then $\exists t \in \mathbb{Z}\,(n = 2t+1)$
   $$\begin{aligned} 3n+2 &= 3\,(2t+1)+2 \\ &= 6t+3+2 \\ &= 6t+4+1 \\ &= 2\,(3t+2)+1 \\ \text{Let } k &= 3t+2 \\ &= 2k+1 \end{aligned}$$
   Since k is made up of the sum of the product of integers 3, t, and 2, k is an integer.
   Since $3n+2$ can be expressed in the form $2k+1$, where $k \in \mathbb{Z}$, $3n+2$ is odd.
   We now have $3n+2$ being odd by our proof and even by our initial assumption, ($p \wedge \neg p$), which is a contradiction.
   $\therefore$ $3n+2$ is even and n is odd is false.
   Hence the statement if $3n+2$ is even, then n is even is true. ☺

Question 2

1. $B \times C \times A$
2. $B \times B \times B$

Solution:

1. $$B \times C \times A = \{(x,0,a),(x,0,b),(x,1,a),(x,1,b),(y,0,a),(y,0,b),(y,1,a),(y,1,b)\}$$
2. $$B^3 = \{(x,x,x),(x,x,y),(x,y,x),(x,y,y),(y,y,y),(y,y,x),(y,x,y),(y,x,x)\}$$

Question 3

1. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
2. $\{\emptyset\} \in \{\emptyset\}$
3. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
4. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

Solution:

1. True. $\emptyset$ is an element of the set $\{\emptyset, \{\emptyset\}\}$
2. False. A set cannot be an element of itself.
3. True. All elements found in $\{\emptyset\}$ are also found in $\{\emptyset, \{\emptyset\}\}$ and $\{\emptyset\}$ is not equal to $\{\emptyset, \{\emptyset\}\}$
4. True. All elements found in $\{\{\emptyset\}\}$ are also found in $\{\emptyset, \{\emptyset\}\}$ and $\{\{\emptyset\}\}$ is not equal to $\{\emptyset, \{\emptyset\}\}$