Assignment 3

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Question 1

1. $\exists x P(x, 4)$
2. $\forall y \neg P(1, y)$
3. $\exists y \forall x P(x, y)$
4. $\forall x \exists y P(x, y)$

Solution:

1.
$$\begin{aligned} \exists x P(x, 4) &\equiv P(x, 4) \vee P(x, 4) \vee P(x, 4) \\ &\equiv P(1, 4) \vee P(2, 4) \vee P(3, 4) \end{aligned}$$

2.
$$\begin{aligned} \forall y \neg P(1, y) &\equiv P(1, y) \wedge P(1, y) \\ &\equiv P(1, 3) \wedge P(1, 4) \end{aligned}$$

3.
$$\begin{aligned} \exists y \forall x P(x, y) &\equiv \forall x P(x, 3) \vee \forall x P(x, 4) \\ &\equiv [P(1, 3) \wedge P(2, 3) \wedge P(3, 3)] \vee [P(1, 4) \wedge P(2, 4) \wedge P(3, 4)] \end{aligned}$$

4.
$$\begin{aligned} \forall x \exists y P(x, y) &\equiv \exists y P(1, y) \wedge \exists y P(2, y) \wedge \exists y P(3, y) \\ &\equiv [P(1, 3) \vee P(1, 4)] \wedge [P(2, 3) \vee P(2, 4)] \wedge [P(3, 3) \vee P(3, 4)] \end{aligned}$$

Question 2

1. All clear explanations are satisfactory.
2. Some excuses are not satisfactory.
3. Some excuses are not clear explanations.

Solution:

1. $\forall x (P(x) \rightarrow Q(x))$
2. $\exists x (R(x) \wedge \neg Q(x))$
3. $\exists x (R(x) \wedge \neg P(x))$

Question 3

1. $\exists x S(x, \text{open})$
2. $\forall x (S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$
3. $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$
4. $\exists x \neg S(x, \text{available})$

Solution:

1. There exists a system that is in state open.
2. All systems are either malfunctioning or in diagnostic state.
3. There exists a system that is in state open or there exists a system that is in state diagnostic.
4. There exists a system that is not in state available.

Question 4

1. Everybody can fool Fred.
2. Evelyn can fool Everybody.
3. Everybody can fool somebody.
4. No one can fool everybody.
5. Everyone can be fooled by somebody.
6. No one can fool himself or herself.

Solution:

1. $\forall x F\left(x, \text{Fred}\right)$
2. $\forall y F\left(\text{Evelyn}, y\right)$
3. $\forall x \exists y F\left(x, y\right)$
4. $\neg \exists x \forall y F\left(x, y\right)$
5. $\exists x \forall y F\left(x, y\right)$
6. $\neg \exists x F\left(x, x\right)$