Probability

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Contents

Chapter 1

Module 8: Introduction

1.1 Introduction To Probability

Definition 1.1.1: Probability

A mathematical description of randomness and uncertainty / The likelihood of an event occurring.
The notation for Probability is $\mathbb{P}(X)$ where X is the event.
Probability is always between $0 \leqslant \mathbb{P}(X) \leqslant 1$ or $0\% \leqslant \mathbb{P}(X) \leqslant 100\%$.

There are two ways of determining probability:

- Theoretical / Classical - Determined by the nature of the experiment
- Empirical / Observational - Determined by the results of the experiment

1.2 Relative Frequency

Definition 1.2.1: Relative Frequency

Relative frequency is the number of times an event occurs divided by the total number of trials.

$$\mathbb{P}(X) = \frac{\text{Number of times event occurs}}{\text{Total number of trials}}$$

Theorem 1.2.1 The Law of Large Numbers

As the number of trials increases, the relative frequency of an event approaches the theoretical probability of the event.

Chapter 2

Module 9: Find the Probability of Events

2.1 Sample Spaces and Events

Definition 2.1.1: Random Experiment

An experiment whose outcome is determined by chance.

Definition 2.1.2: Sample Space

The list of possible outcomes of a random experiment, denoted by S.

Definition 2.1.3: Event

A statement about the nature of the outcome after the experiment has been conducted, denoted by any capital letter except S.

2.2 Equally Likely Outcomes

$$\mathbb{P}(A) = \frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S}$$

Where A is an event and S is the sample space.

2.3 Probability Rules

2.3.1 Rule 1: Probability is a Number Between 0 and 1

For any event A, $0 \leq \mathbb{P}(A) \leq 1$.

2.3.2 Rule 2: Addition Rule

$\mathbb{P}(S) = 1$, that is the sum of the probabilities of all possible outcomes is 1.

2.3.3 Rule 3: Complement Rule

$\mathbb{P}(A') = 1 - \mathbb{P}(A)$, that is the probability of the complement of an event is 1 minus the probability the event occurs.

2.3.4 Rule 4: Addition Rule for Mutually Exclusive Events

Definition 2.3.1: Mutually Exclusive / Disjoint events

Events that cannot happen at the same time.

$\mathbb{P}(A \text{ or } B) = \mathbb{P}(\text{ event } A \text{ occurs or event } B \text{ occurs or both occur})$

If A and B are mutually exclusive, then $\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B)$

2.3.5 Rule 5: Multiplication Rule for Independent Events

$\mathbb{P}(A \text{ and } B) = \mathbb{P}(\text{ event } A \text{ occurs and event } B \text{ occurs })$

Definition 2.3.2: Independent Events

Two events A and B are said to be independent if the occurrence of one event does not affect the probability of the other event occurring.

Definition 2.3.3: Dependent Events

Two events A and B are said to be dependent if the occurrence of one event affects the probability of the other event occurring.

If A and B are two independent events, then $\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \times \mathbb{P}(B)$

2.3.6 Rule 6: General Addition Rule

For any two events A and B, $\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \text{ and } B)$

Chapter 3

Conditional Probability and Independence

Definition 3.0.1: Conditional Probability

The probability an event occurs as a result of another event. i.e. probability of event B, given event event A is,

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \text{ and } B)}{P(A)}$$

3.1 Independence

When two events are independent, the probability of one event occurring does not affect the probability of the other event, i.e.

$$\begin{aligned} \mathbb{P}(B \mid A) &= \mathbb{P}(B) \\ \mathbb{P}(A \mid B) &= \mathbb{P}(A) \\ \mathbb{P}(B \mid A) &= \mathbb{P}(B \mid A') \\ \mathbb{P}(A \text{ and } B) &= \mathbb{P}(A) \times \mathbb{P}(B) \end{aligned}$$

3.2 The General Multiplication Rule

For any two events A and B

$$\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \times \mathbb{P}(B \mid A)$$

3.3 Probability Trees

Definition 3.3.1: Probability Tree

A diagram that shows the sample space of a random experiment and the probability of each outcome.