Assignment 7

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$$\overline{(\overline{x} + y)} \left(x + y + \overline{z}\right)$$

Solution:
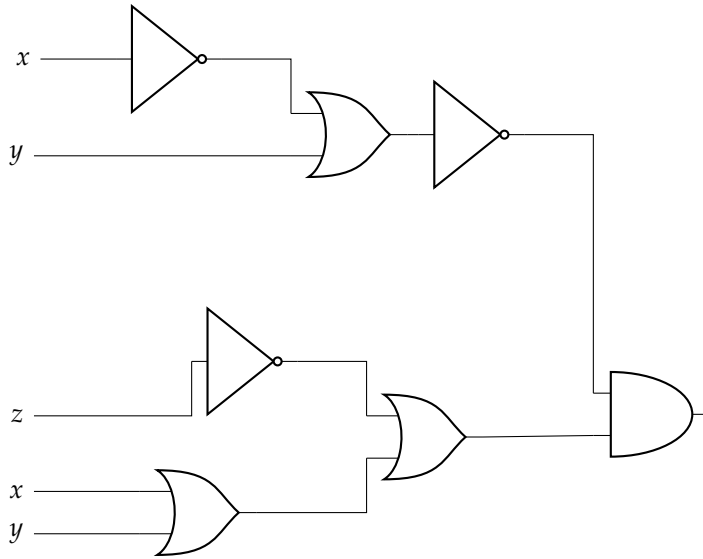
$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n\left(n + 1\right)}{2}\right)^2$$

Whenever n is a positive integer

Proof: Let $P\left(n\right)$ be $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Basis Step

$$P\left(1\right) : 1^3 = \left(\frac{1\left(1 + 1\right)}{2}\right)^2$$

$$1 = \left(\frac{2}{2}\right)^2$$

$$1 = 1$$

Induction Step

To complete this step I must prove $P\left(k\right) \to P\left(k + 1\right)$ for any positive integer k
Assume $P\left(k\right)$ is T for some positive integer k, then

$$P\left(k\right) : 1^3 + 2^3 + \ldots + k^3 = \left(\frac{k^2 + k}{2}\right)^2$$

Then $P(k+1)$ is

$$P(k+1): 1^3 + 2^3 + \ldots + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$$

And can be expressed as $P(k+1): P(k) + (k+1)^3$, therefore:

$$
\begin{aligned}
P(k+1): 1^3 + 2^3 + \ldots + k^3 + (k+1)^3 &= \left(\frac{k^2+k}{2}\right)^2 + (k+1)^3 \\
&= \frac{\left(k^2+k\right)^2}{4} + (k+1)^3 \\
&= \frac{\left(k^2+k\right)^2 + 4(k+1)^3}{4} \\
&= \frac{\left(k^2+k\right)^2 + \left(4k^3 + 12k^2 + 12k + 4\right)}{4} \\
&= \frac{k^4 + 2k^3 + k^2 + 4k^3 + 12k^2 + 12k + 4}{4} \\
&= \frac{k^4 + 6k^3 + 13k^2 + 12k + 4}{4} \\
&= \frac{(k+1)^2 (k+2)^2}{4} \\
&= \left(\frac{(k+1)(k+2)}{2}\right)^2
\end{aligned}
$$

$\therefore P(k+1)$ is T
Hence we can conclude that $P(n)$ is true for all positive integers n

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$$2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^n = \frac{1 - (-7)^{n+1}}{4}$$

Whenever n is a non-negative integer

Proof: Let $P(n)$ be $2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^n = \frac{1-(-7)^{n+1}}{4}$

Basis Step

$$
\begin{aligned}
P(0): 2 &= \frac{1 - (-7)^{0+1}}{4} \\
2 &= \frac{8}{4} \\
2 &= 2
\end{aligned}
$$

Induction Step

To complete this step I must prove $P(k) \rightarrow P(k+1)$ for any non-negative integer k

Assume $P(k)$ is T for some non-negative integer k, then

$$P(k) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^k = \frac{1 - (-7)^{k+1}}{4}$$

Then $P(k+1)$ is

$$P(k+1) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots + 2 \times (-7)^{(k+1)} = \frac{1 - (-7)^{k+2}}{4}$$

And can be expressed as $P(k+1) : P(k) + 2 \times (-7)^{(k+1)}$, therefore:

$$P(k+1) : 2 - 2 \times 7 + 2 \times 7^2 - \ldots - 2 \times (-7)^k + 2 \times (-7)^{(k+1)} = \frac{1 - (-7)^{k+1}}{4} + 2 \times (-7)^{(k+1)}$$

$$= \frac{1 - (-7)^{k+1} + 8 \times (-7)^{(k+1)}}{4}$$

$$= \frac{1 + 8 \times (-7)^{k+1} - (-7)^{k+1}}{4}$$

Let $x = (-7)^{k+1}$

$$= \frac{1 + 8x - x}{4}$$

$$= \frac{1 + 7x}{4}$$

$$= \frac{1 + 7^1 \times (-7)^{k+1}}{4}$$

$$= \frac{1 - (-7)^1 \times (-7)^{k+1}}{4}$$

$$= \frac{1 - (-7)^{k+2}}{4}$$

$\therefore P(k+1)$ is T

Hence we can conclude that $P(n)$ is true for all non-negative integers n

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