

Implicit Application

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1

Height of the wall is $y(t)$

Distance from the wall is $x(t)$

From Pythagoras $y(t)^2 + x(t)^2 = 10^2$

Known

$$\frac{dy}{dx} = 10, \quad x(t) = 6$$

Unknown

$$\frac{dx}{dt}$$

$$y^2 + x^2 = 10^2$$

$$2y \frac{dy}{dt} + 2x \frac{dx}{dt} = 0$$

$$2y(10) + 2x \frac{dx}{dt} = 0$$

$$20y + 2x \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{20y}{2x}$$

$$y^2 = \sqrt{10^2 - x^2}$$

$$\frac{dx}{dt} = -\frac{20(\sqrt{10^2 - (6)^2})}{2(6)}$$

$$\frac{dx}{dt} = -\frac{40}{3} \text{ m/sec}$$

2

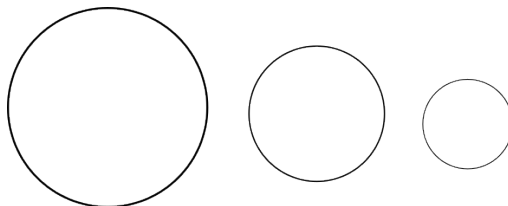


Figure 1: Balloon deflating

Known

$$A = 4\pi r^2, \quad \frac{dV}{dt} = 2, \quad r = 12, \quad V = \frac{4}{3}\pi r^3,$$

Unknown

$$\frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = \frac{dV}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = -2$$

$$\frac{dr}{dt} = -\frac{2}{4\pi r^2}$$

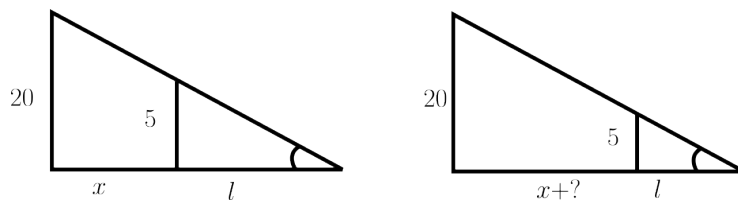
$$4\pi r^2 = A$$

$$8\pi r \frac{dr}{dt} = \frac{dA}{dt}$$

$$8\pi r \left(-\frac{2}{4\pi r^2}\right) = \frac{dA}{dt}$$

$$\frac{dA}{dt} = -\frac{1}{3}ft^2/min$$

3



Known

$$\frac{dx}{dt} = 4$$

Unknown

$$\frac{dl}{dt}$$

$$\frac{x+l}{20} = \frac{l}{5}$$

$$x+l = 4l$$

$$x = 3l$$

$$\frac{x}{3} = l$$

$$\frac{dl}{dx} = \frac{1}{3}$$

$$\frac{dl}{dt} = \frac{dl}{dx} \times \frac{dx}{dt}$$

$$\frac{dl}{dt} = \frac{4}{3}$$

3.1

Let L be the sum of the distance from the lamppost and the length of the boy's shadow.

$$\frac{dL}{dt} = \frac{dl}{dt} + \frac{dx}{dt} \therefore \frac{dL}{dt} = \frac{16}{3} \text{ ft/sec}$$

3.2

$$\frac{dl}{dt} = \frac{4}{3} \text{ ft/sec}$$

4

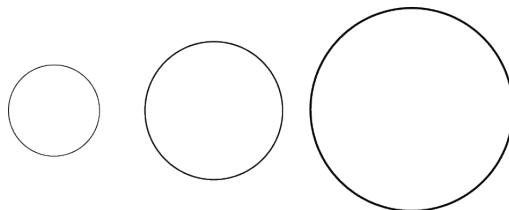


Figure 2: Balloon inflating

Known

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 5$$

Unknown

$$\frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = \frac{dV}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = 5$$

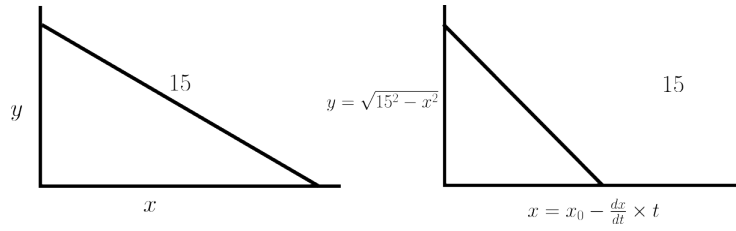
$$\frac{dr}{dt} = \frac{5}{4\pi r^2}$$

$$D = r + r \therefore r = 10$$

$$\frac{dr}{dt} = \frac{5}{4\pi r^2}$$

$$\frac{dr}{dt} = 3.978\,873\,577 \times 10^{-3} \text{ ft/min}$$

5



Known

$$\frac{dx}{dt} = \frac{1}{4}$$

When $t = 0$, $x = 10$

$$\therefore t = 0, y = \sqrt{15^2 - 10^2}$$

$$y = 5\sqrt{5}$$

$$y^2 + x^2 = 15^2$$

$$2y \frac{dy}{dt} + 2x \frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-2x(-\frac{1}{4})}{2y}$$

$$\frac{dy}{dt} = \frac{\frac{1}{2}x}{2y}$$

$$x(t) = x(0) + \frac{dx}{dt} \times t$$

$$x(t) = 10 - \frac{1}{4} \times t$$

$$x(t) = 10 - \frac{t}{4}$$

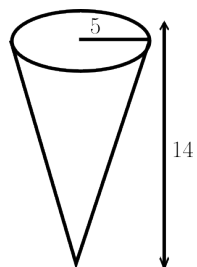
$$y(t) = \sqrt{15^2 - x^2}$$

$$x = 10 - \frac{t}{4}, \quad t = 12, \quad y = \sqrt{15^2 - x^2}$$

$$\frac{dy}{dt} = \frac{\frac{1}{2}(10 - \frac{t}{4})}{2(\sqrt{15^2 - (10 - \frac{t}{4})^2})}$$

$$\frac{dy}{dt} = 0.132 \text{ ft/sec}$$

6



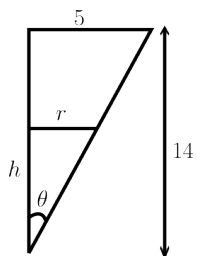
Known

$$\frac{dV}{dt} = -2, \quad r = 5, \quad h = 14, \quad V = \pi r^2 \frac{h}{3}$$

Unknown

$$\frac{dh}{dt}, \quad \frac{dr}{dt}$$

$$\begin{aligned} \frac{dV}{dt} &= (\pi r^2) \left(\frac{1}{3} \frac{dh}{dt} \right) + (2\pi r \frac{dr}{dt}) \left(\frac{h}{3} \right) \\ \frac{dV}{dt} &= \frac{\pi r^2}{3} \frac{dh}{dt} + \frac{2\pi r h}{3} \frac{dr}{dt} \end{aligned}$$



$$\frac{r}{h} = \frac{5}{14}$$

$$r = \frac{5}{15}h$$

$$h = 6$$

$$r = \frac{5(6)}{14}$$

$$r = \frac{15}{7}$$

$$\frac{dr}{dt} = \frac{5}{14} \frac{dh}{dt}$$

6.1

$$-2 = \frac{\pi(\frac{15}{7})^2}{3} \frac{dh}{dt} + \frac{2\pi(\frac{15}{7})(6)}{3} \left(\frac{5}{14}\right) \frac{dh}{dt}$$

$$-2 = \frac{75}{49}\pi \frac{dh}{dt} + \frac{150}{49}\pi \frac{dh}{dt}$$

$$-2 = \frac{225}{49}\pi \frac{dh}{dt}$$

$$-\frac{2}{\frac{225}{49}\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{98}{225} \text{ ft/hour}$$

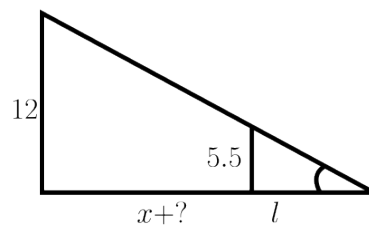
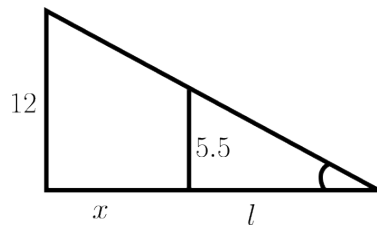
6.2

$$\frac{dh}{dt} = -\frac{98}{225} \text{ when } h = 6$$

$$\frac{dr}{dt} = \frac{5}{14} \left(-\frac{98}{225}\pi\right)$$

$$\frac{dr}{dt} = -\frac{7}{45}\pi \text{ ft/hour}$$

7



Known

$$\frac{dx}{dt} = 2$$

Unknown

$$\text{Let } L = x + l$$

$$\frac{dL}{dt} \text{ when } x = 25$$

$$\frac{x + l}{12} = \frac{l}{5.5}$$

$$x + l = \frac{24}{11}l$$

$$x = \frac{13}{11}l$$

$$\frac{11}{13}x = l$$

$$\frac{11}{13} \frac{dx}{dt} = \frac{dl}{dt}$$

7.1

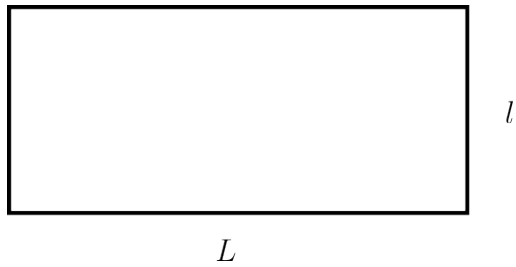
$$L = x + l \quad L = x + \frac{11}{13}x$$

$$L = \frac{24}{13}x$$

$$\frac{dL}{dt} = \frac{24}{13} \frac{dx}{dt}$$

$$\frac{dL}{dt} = \frac{48}{13} \text{ ft/sec}$$

8



Known

$$L = 3l, \quad \frac{dl}{dt} = -2$$

Unknown

$$\frac{dL}{dt}$$

8.1

$$\begin{aligned} \frac{dL}{dt} &= 3 \frac{dl}{dt} \\ \frac{dL}{dt} &= -6 \text{ inches/min} \end{aligned}$$

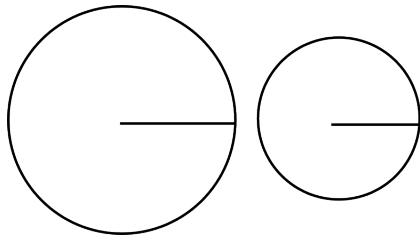
8.2

Known

$$A = L \times l, \quad \frac{dl}{dt} = -2, \quad l = 6$$

$$\begin{aligned} \frac{dA}{dt} &= (L) \left(\frac{dl}{dt} \right) + \left(\frac{dL}{dt} \right) (l) \\ \frac{dA}{dt} &= 3(6)(-2) + 3(-2)(6) \\ \frac{dA}{dt} &= -72 \text{ inches}^2/\text{min} \end{aligned}$$

9



Known

$$\frac{dA}{dt} = -0.5, \quad A = 12$$

Unknown

$$\frac{dr}{dt}$$

$$A = \pi r^2$$

$$12 = \pi r^2$$

$$\frac{12}{\pi} = r^2$$

$$\sqrt{\frac{12}{\pi}} = r$$

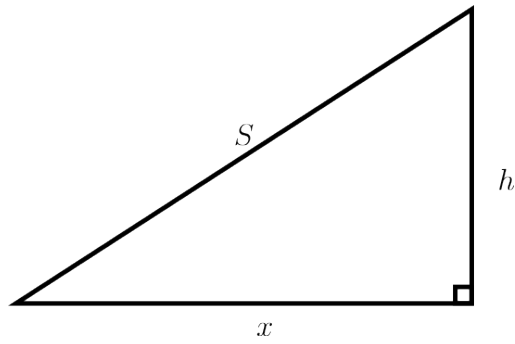
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$-0.5 = 2\pi \left(\sqrt{\frac{12}{\pi}} \right) \frac{dr}{dt}$$

$$\frac{-0.5}{2\pi \left(\sqrt{\frac{12}{\pi}} \right)} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = -0.0407 \text{ m/sec}$$

10



Known

$$\frac{dh}{dt} = 15$$

10.1

$$\begin{aligned} 350^2 + h^2 &= S^2 \\ 0 + 2h \frac{dh}{dt} &= 2S \frac{dS}{dt} \end{aligned}$$

$$\begin{aligned} h &= h_0 + \frac{dh}{dt} \times t \\ h &= 0 + 15 \times (20) \\ h &= 300 \end{aligned}$$

$$\begin{aligned} 350^2 + 300^2 &= S^2 \\ 212500 &= S^2 \\ 50\sqrt{85} &= S \\ 2(300)(15) &= 2(50\sqrt{85}) \frac{dS}{dt} \\ \frac{2(300)(15)}{2(50\sqrt{85})} &= \frac{dS}{dt} \\ \frac{dS}{dt} &= 9.762 \text{ ft/sec} \end{aligned}$$

10.2

$$h = 15 \times 6$$

$$h = 900$$

$$350^2 + 900^2 = S^2$$

$$50\sqrt{373} = S$$

$$2(900)(15) = 2(50\sqrt{373})\frac{dS}{dt}$$

$$\frac{dS}{dt} = 13.980 \text{ ft/sec}$$