
$$\lim_{t \rightarrow 4} t^2 + 5t + 1$$

$$\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^2 + t - 6}$$

$$\lim_{x \rightarrow 5} \frac{x^2 - x}{x^2 + 2x - 3}$$

$$\lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1}{h}$$

$$\lim_{t \rightarrow 0} \frac{(t + 4)^2 - 16}{t}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$$

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$$

$$\lim_{x \rightarrow -1} \frac{x}{(x + 1)^2}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2}$$

$$\lim_{x \rightarrow -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2}$$

$$\lim_{x \rightarrow \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3 + \sqrt{4x^6 + 4}}{5x^3 + 2x}$$

$$\text{For the function } f(x) = \begin{cases} x^3 - 2, & \text{if } x \geq 2 \\ 1 + x^2, & \text{if } x < 2 \end{cases} \text{ find the } \lim_{x \rightarrow 1} f(x)$$

Q1.

$$\begin{aligned}\lim_{t \rightarrow 4} t^2 + 5t + 1 \\&= (4)^2 + 5(4) + 1 \\&= 16 + 20 + 1 \\ \lim_{t \rightarrow 4} t^2 + 5t + 1 &= 37\end{aligned}$$

Q2.

$$\begin{aligned}\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^2 + t - 6} \\&= \frac{t^2 - 2}{(t - 2)(t + 3)} \\ \therefore \lim_{t \rightarrow 2} \frac{t^2 - 2}{t^2 + t - 6} &= \text{Does not exist}\end{aligned}$$

Q3.

$$\begin{aligned}\lim_{x \rightarrow 5} \frac{x^2 - x}{x^2 + 2x - 3} \\&= \frac{5^2 - 5}{5^2 + 2(5) - 3} \\&= \frac{25 - 5}{25 + 10 - 3} \\&= \frac{20}{32} \\ \lim_{x \rightarrow 5} \frac{x^2 - x}{x^2 + 2x - 3} &= \frac{5}{8}\end{aligned}$$

Q4.

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1}{h} \\&= \frac{h^2 + 2h}{h} \\&= h + 2 \\ \lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1}{h} &= 2\end{aligned}$$

Q5.

$$\begin{aligned}
 \lim_{t \rightarrow 0} \frac{(t+4)^2 - 16}{t} &= \lim_{t \rightarrow 0} \frac{t^2 + 8t}{t} \\
 &= \lim_{t \rightarrow 0} (t + 8) \\
 &= (0 + 8) \\
 \lim_{t \rightarrow 0} \frac{(t+4)^2 - 16}{t} &= 8
 \end{aligned}$$

Q6.

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} &= \frac{\sqrt{x+2} - \sqrt{2}}{x} \times \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}} \\
 &= \frac{(\sqrt{x+2} - \sqrt{2})(\sqrt{x+2} + \sqrt{2})}{x(\sqrt{x+2} + \sqrt{2})} \\
 &= \frac{x + 2 - 2}{x(\sqrt{x+2} + \sqrt{2})} \\
 &= \frac{1}{\sqrt{x+2} + \sqrt{2}} \\
 &= \frac{1}{\sqrt{0+2} + \sqrt{2}} \\
 \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} &= \frac{\sqrt{2}}{4}
 \end{aligned}$$

Q7.

$$\begin{aligned}
\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2} &= \frac{x-1}{\sqrt{x^2+3}-2} \times \frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2} \\
&= \frac{(x-1)(\sqrt{x^2+3}+2)}{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)} \\
&= \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2+3-4} \\
&= \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2-1} \\
&= \frac{(x-1)(\sqrt{x^2+3}+2)}{(x-1)(x+1)} \\
&= \frac{\sqrt{x^2+3}+2}{x+1} \\
&= \frac{\sqrt{1^2+3}+2}{1+1} \\
&= \frac{4}{2} \\
\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2} &= 2
\end{aligned}$$

Q8.

$$\begin{aligned}
\lim_{x \rightarrow -1} \frac{x}{(x+1)^2} &= \frac{x}{x^2+2x+1} \\
&= \frac{x}{x(x+2+\frac{1}{x})} \\
&= \frac{-1}{-1(-1+2-1)} \\
&= \frac{-1}{0} \\
\lim_{x \rightarrow -1} \frac{x}{(x+1)^2} &= -\infty
\end{aligned}$$

Q9.

$$\begin{aligned}
\lim_{x \rightarrow -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2} &= \frac{\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^4}}{1 - \frac{2}{x^4}} \\
&= \frac{-\frac{1}{\infty} - \frac{2}{\infty} + \frac{1}{\infty}}{1 - \frac{2}{\infty}} \\
&= \frac{0 - 0 + 0}{1 - 0} \\
\lim_{x \rightarrow -\infty} \frac{x^3 - 2x^2 + 1}{x^4 - 2} &= 0
\end{aligned}$$

Q10.

$$\begin{aligned}
\lim_{x \rightarrow -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2} &= \frac{x^3(-x^2 - 1 + \frac{x}{x^3} - \frac{3}{x^3})}{x^3(2 + \frac{3x}{x^3} - \frac{2}{x^3})} \\
&= \frac{x^2 - 1 + \frac{1}{x^2} - \frac{3}{x^3}}{2 + \frac{3}{x^2} - \frac{2}{x^2}} \\
&= \frac{-\infty - 1 + 0 + 0}{2 + 0 + 0} \\
&= \frac{-\infty - 1}{2} \\
\lim_{x \rightarrow -\infty} \frac{-x^5 - x^3 + x - 3}{2x^3 + 3x - 2} &= -\infty
\end{aligned}$$

Q11.

$$\begin{aligned}
\lim_{x \rightarrow \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1} &= \frac{x^4(1 - \frac{5x^2}{x^4} + \frac{x}{x^4} - \frac{1}{x^4})}{x^4(3 + \frac{x}{x^2} - \frac{1}{x^4})} \\
&= \frac{1 - \frac{5}{x^2} + \frac{1}{x^3} - \frac{1}{x^4}}{3 + \frac{1}{x^2} - \frac{1}{x^4}} \\
&= \frac{1 - \frac{5}{\infty} + \frac{1}{\infty} - \frac{1}{\infty}}{3 + \frac{1}{\infty} - \frac{1}{\infty}} \\
&= \frac{1 - 0 + 0 - 0}{3 + 0 - 0} \\
\lim_{x \rightarrow \infty} \frac{x^4 - 5x^2 + x - 1}{3x^4 + x - 1} &= \frac{1}{3}
\end{aligned}$$

Q12.

$$\begin{aligned}\lim_{x \rightarrow -\infty} \frac{x^3 + \sqrt{4x^6 + 4}}{5x^3 + 2x} &= \\&= \frac{x^3 + 2\sqrt{x^6 + 1}}{5x^3 + 2x} \\&= \frac{x^3(1 - 2\sqrt{1 + \frac{1}{x^6}})}{x^3(5 + \frac{2}{x^2})} \\&= \frac{1 - 2\sqrt{1 + \frac{1}{x^6}}}{5 + \frac{2}{x^2}} \\&= \frac{1 - 2\sqrt{1 + 0}}{5 + 0} \\&= \lim_{x \rightarrow -\infty} \frac{x^3 + \sqrt{4x^6 + 4}}{5x^3 + 2x} = -\frac{1}{5}\end{aligned}$$

Q13.

$$\begin{aligned}\lim_{x \rightarrow 1} f(x) \\&= \end{aligned}$$