Sine

$$f(x) = \sin(x)$$
$$-1 \leq \sin(x) \leq 1$$
$$\sin(0) = 0$$

For all integer multiples of π, $\sin$ attains 0

$$\sin(k\pi) = 0, \text{ Where k is an integer}$$

The graph of sine is periodic with a period of 2π, meaning it repeats itself every interval of 2π

Derivative of $\sin(x)$

If $y = \sin(x)$

$$y' = \cos(x)$$

Proof

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$\frac{\sin(x+h) - \sin(x)}{h}$$
$$\therefore \frac{\sin(x+h) - \sin(x)}{h}$$
$$\frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$
$$\frac{\sin(\cos(h) - 1) + \cos(x)\sin(h)}{h}$$
$$y' = \lim_{h \to 0} \frac{\sin(\cos(h) - 1) + \cos(x)\sin(h)}{h}$$
$$y' = \lim_{h \to 0} \frac{\sin(\cos(h) - 1)}{h} + \frac{\cos(x)\sin(h)}{h}$$
$$= \lim_{h \to 0} \frac{\sin(\cos(h) - 1)}{h} + \lim_{h \to 0} \frac{\cos(x)\sin(h)}{h}$$

$$\because \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = \lim_{x \to a}(f(x) + g(x))$$

$$= \sin(x) \lim_{h \to 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$$

$$\because \lim_{x \to a}(kf(x)) = k \lim_{x \to a} f(x)$$

$$\lim_{h \to 0^-} \frac{\cos(h) - 1}{h} = 0$$
$$\lim_{h \to 0^+} \frac{\cos(h) - 1}{h} = 0$$
$$\therefore \lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$$
$$= \sin(x) \times 0 + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$$
$$= \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$$
$$\lim_{h \to 0^-} \frac{\sin(h)}{h} = 1$$
$$\lim_{h \to 0^+} \frac{\sin(h)}{h} = 1$$

$$\therefore \lim_{h \to 0} \frac{\sin(h)}{h} = 1$$
$$\therefore \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h} = \cos(x) \times 1$$

$$= \cos(x)$$

Cosine

$$f(x) = \cos(x)$$
$$-1 \leq \cos(x) \leq 1$$
$$\cos(0) = 1$$

Tangent

$$f(x) = \tan(x)$$
$$y = \tan(x) = 0$$
$$\sin(x) = 0$$

Vertical Asymptote

> A vertical line $(x, 0)$ where the values of a function rise or fall infinitely.

The line $x = a$ is a vertical asymptote of $f(x)$ if

$$\lim_{x \to a} f(x) = \pm\infty$$

The zero points of $\cos(x)$ create a vertical asymptote in relation to $\tan(x)$

Derivative of $tan(x)$

If $y = \tan(x)$

$$y' = \sec^2(x)$$

Proof

$$y = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$y' = \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2}$$

$$\because y' = \frac{v \times u' - u \times v'}{v^2}$$
$$y' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$
$$y' = \frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)}$$
$$y' = 1 + (\frac{\sin(x)}{\cos(x)})^2$$
$$y' = 1 + \tan^2(x)$$
$$= \sec^2(x)$$

Secant

$$f(x) = \sec(x)$$
$$\sec(0) = 1$$

Derivative of $\sec(x)$

If $y = \sec(x)$

$$y' = \tan(x)\sec(x)$$

Relationships

$$\cos(x - \frac{\pi}{2}) = \sin(x)$$
$$\sin(x + \frac{\pi}{2}) = \cos(x)$$

Identities

Reciprocal Identities

$$\sin(\theta) = \frac{1}{\csc(\theta)} \text{ or } \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \text{ or } \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \text{ or } \cot(\theta) = \frac{1}{\tan(\theta)}$$

Pythagorean Identities

$$\begin{aligned} \sin^2(\theta) + \cos^2(\theta) &= 1 \\ 1 + \tan^2(\theta) &= \sec^2(\theta) \\ \csc^2(\theta) &= 1 + \cot^2(\theta) \end{aligned}$$

Ratio Identities

$$\begin{aligned} \tan(\theta) &= \frac{\sin(\theta)}{\cos(\theta)} \\ \cot(\theta) &= \frac{\cos(\theta)}{\sin(\theta)} \end{aligned}$$

Sum and Difference of Angles

$$\begin{aligned} \sin(\alpha + \beta) &= \sin(\alpha) \times \cos(\beta) + \cos(\alpha) \times \cos(\beta) \\ \sin(\alpha - \beta) &= \sin(\alpha) \times \cos(\beta) - \cos(\alpha) \times \sin(\beta) \\ \cos(\alpha + \beta) &= \cos(\alpha) \times \cos(\beta) - \sin(\alpha) \times \sin(\beta) \\ \cos(\alpha - \beta) &= \cos(\alpha) \times \cos(\beta) + \sin(\alpha) \times \sin(\beta) \\ \tan(\alpha + \beta) &= \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \times \tan(\beta)} \\ \tan(\alpha - \beta) &= \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \times \tan(\beta)} \end{aligned}$$

Double Angles

$$\begin{aligned}
\sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\
\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\
&= 2\cos^2(\theta) - 1 \\
&= 1 - 2\sin^2(\theta) \\
\tan(2\theta) &= \frac{2\tan(\theta)}{1 - \tan^2(\theta)}
\end{aligned}$$

Questions

$y = (\sin x)^2$

$$y' = 2\cos(\sin x)$$

$y = \cos(5x + 4)$

$$y' = -5\sin(5x + 4)$$

$$g(x) = 3\sec(x) - 10\tan(x)$$

$$h(x) = 3w^{-4} - w^2\tan(w)$$

$$y = 5\sin(x)\cos(x) + 9\sec(x)$$

$$y = \frac{\sin(t)}{3 - 2\cos(t)}$$

$$y = \sin(10x)$$

$$f(w) = \tan(w)\sec(w)$$

$$y = 2\sin(3x + \tan(x))$$

$$h(z) = \sin(z^6) + \sin^6(2)$$

$$f(t) = \sin(2t) + \cos(4t)$$

$$f(x) = [\sqrt[3]{2x} + \sin^2(3x)]^{-\frac{1}{2}}$$

$$y = \frac{4\sin(x^2)}{\cos(x^2)}$$

$$h(x) = x^2\cos(x^3)$$

$$y = \sqrt{5z + \tan(4z)}$$

Q1. $g(x) = 3\sec(x) - 10\tan(x)$

$$\begin{aligned} g'(x) &= 3(\tan(x)\sec(x)) - 10\sec(x) \\ &= 3\tan(x)\sec(x) - 10\sec^2(x) \\ &= \sec(x)(3\tan(x) - 10\sec(x)) \end{aligned}$$

$$g'(x) = \sec(x)(3\tan(x) - 10\sec(x))$$

Q2. $h(w) = 3w^{-4} - w^2\tan(w)$

$$h'(w) = -12w^{-5} - (2w)(\tan(w)) + (w^2)(\sec^2(w))$$

$$h'(w) = -12w^{-5} - 2w\tan(w) - w^2\sec^2(w)$$

Q3. $y = 5\sin(x)\cos(x) + 4\sec(x)$

$$y' = 5[\cos(x) \times \cos(x) - \sin(x) \times \sin(x)] + 4\tan(x)\sec(x)$$

$$y' = 5(\cos^2(x) - \sin^2(x)) + 4\tan(x)\sec(x)$$

$$y' = 5\cos(2x) + 4\tan(x)\sec(x)$$

Q4. $y = \frac{\sin(t)}{3-2\cos(t)}$

$$y' = \frac{(\cos(t)(3 - 2\cos(t))) - (2\sin(t))(\sin(t))}{(3 - 2\cos(t))^2}$$

$$y' = \frac{3\cos(t) - 2\cos^2(t) - 2\sin^2(t)}{(3 - 2\cos(t))^2}$$

$$y' = \frac{3\cos(t) - 2(\cos^2(t) + \sin^2(t))}{(3 - 2\cos(t))^2}$$

$$y' = \frac{3\cos(t) - 2}{(3 - 2\cos(t))^2}$$

Q5. $y = \frac{\sin(10z)}{z}$

$$y' = \frac{(10\cos(10z))(z) - (1)(\sin(10z))}{z^2}$$

$$y' = \frac{10z\cos(10z) - \sin(10z)}{z^2}$$

Q6. $f(w) = \tan(w)\sec(w)$

$$f'(w) = (\sec^2(w))(\sec(w)) + (\sec(w)\tan(w))(\tan(w))$$
$$= \sec^3(w) + \sec(w)\tan^2(w)$$
$$f'(w) = \sec(w)(\sec^2(w) + \tan^2(w))$$

Q14. $y = \sqrt{5z + \tan(4z)}$

$$y = (5z + \tan(4z))^{\frac{1}{2}}$$
$$y' = (\frac{1}{2})(5 + 4\sec^2(4z))(5z + \tan(4z))^{-\frac{1}{2}}$$
$$y' = \frac{\frac{5}{2} + 2\sec^2(4z)}{\sqrt{5z + \tan(4z)}}$$