Logarithmic Functions and Derivatives

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Chapter 1

1.1 Derivatives of Logarithmic Functions

Definition 1.1.1: Logarithmic Functions

Functions in the form:

$$y = \log_a[f(x)] \quad \text{or} \quad y = \ln[f(x)]$$

1.1.1 Derivate of $\ln(x)$

$$y' = \frac{f'(x)}{f(x)} \quad \text{or} \quad \frac{x'}{x}$$

Where $y = \log_a[f(x)]$

$$y' = \frac{f'(x)}{x\ln(a)}$$

Proof:

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\text{Where } f(x) = \ln(x) \quad \text{and} \quad f(x+h) = \ln(x+h)$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\ln(\frac{x+h}{x})}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\ln(1+\frac{h}{x})}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{1}{h}(\ln(1+\frac{h}{x}))$$

$$\text{Let } v = \frac{h}{x} \quad \therefore h = vx \quad \Longrightarrow \quad \text{As} \quad h \to 0 \;\; v \to 0$$

$$\frac{dy}{dx} = \lim_{v\to 0} \frac{1}{vx}(\ln(1+v))$$

$$\frac{dy}{dx} = \lim_{v \to 0} \frac{1}{v} \times \frac{1}{x} \ln(1+v)$$
$$\frac{dy}{dx} = \frac{1}{x} \lim_{v \to 0} \frac{1}{v} \ln(1+v)$$
$$\frac{dy}{dx} = \frac{1}{x} \lim_{v \to 0} \ln(1+v)^{\frac{1}{v}}$$
$$\frac{dy}{dx} = \frac{1}{x} \ln[\lim_{v \to 0} (1+v)^{\frac{1}{v}}]$$
$$\frac{dy}{dx} = \frac{1}{x} \ln[e]$$
$$\frac{dy}{dx} = \frac{1}{x}$$

😄

Questions

Question 1

$$y = 4^x - 5\log_7 x$$

Solution:

$$y' = 4^x \ln(4) - \frac{5}{\ln(7)x}$$

Question 2

$$f(t) = 4\log_3(t) - \ln(t)$$

Solution:

$$f'(t) = \frac{4}{\ln(3)t} - \frac{1}{t}$$

Question 3

$$y = 3^x \log(x)$$

Solution:

$$y' = (3^x \ln(3))(\log(x)) + (3^x)(\frac{1}{\ln(10)x})$$
$$y' = 3^x \ln(3)\log(x) + \frac{3^x}{\ln(10)x}$$