Instantaneous rate of change

In the case where f is a function of x $f'(x)$ measures the instantaneous rate of change of f with respect to x.

Example

> The term widget is an economic term for a generic unit of manufacturing output. Suppose a company produces widgets and knows that the market supports a price of $10 per widget. Let $P(n)$ give the profit, in dollars, earned by manufacturing and selling n widgets. The company likely cannot make a (positive) profit making just one widget; the start–up costs will likely exceed $10. Mathematically, we would write this as $P(1) < 0$.
>
> What do $P(1000) = 500$ and $P'(1000) = 0.25$ mean? Approximate $P(1100)$.
>
> The equation $P(1000) = 500$ means that selling 1,000 widgets returns a profit of $500. We interpret $P'(1000) = 0.25$ as meaning that the profit is increasing at rate of $0.25 per widget (the units are "dollars per widget."). Since we have no other information to use, our best approximation for P(1100) is:
>
> $$\begin{aligned} P(1100) &\approx P(1000) + P'(1000) \times 100 \\ &= P(1000) + P'(1000) \times 100 \\ &= 500 + 0.25 \times 100 \\ &= 525 \end{aligned}$$
>
> We approximate that selling 1,100 widgets returns a profit of $525.

The Slope of the Tangent Line

We can measure the instantaneous rate of change at a given x value c of a non-linear function by computing $f'(c)$. We can determine the behaviour of the function f by observing the slopes of its tangent lines.

Increasing Functions

$f(x)$ is increasing whenever $x_1 < x_2$ and $f(x_1) < f(x_2)$, I.e as you go up the x axis the y or function values increase.

$f(x)$ is increasing if the slope on any point on it's graph is positive throughout the function's entire domain.

Decreasing Functions

$f(x)$ is decreasing whenever $x_1 < x_2$ and $f(x_1) > f(x_2)$, .I.e as you go up the x axis the y or function values decrease

$f(x)$ is increasing if the slope on any point on it's graph is negative throughout the function's entire domain.

Critical Points

- Points where the gradient is equal 0, i.e. $f'(x) = 0$
- Points where the gradient does not exist, i.e. $f'(x) = \emptyset$

Examples

$t\sqrt[3]{t^2 - 4}$

$$g(t) = t\sqrt[3]{t^2-4}$$
$$g(t) = t(t^2-4)^{\frac{1}{3}}$$
$$g'(t) = (1)(t^2-4)^{\frac{1}{3}} + (\frac{1}{3})(2t)(t^2-4) \times (t)$$
$$g'(t) = (t^2-4)^{\frac{1}{3}} + \frac{2}{3}t^2(t^2-4)^{-\frac{2}{3}}$$
$$g'(t) = (t^2-4)^{\frac{1}{3}} + \frac{2t^2}{3(t^2-4)^{\frac{2}{3}}}$$
$$g'(t) = \frac{(t^2-4)^{\frac{1}{3}}}{1} + \frac{2t^2}{3(t^2-4)^{\frac{2}{3}}}$$
$$g'(t) = \frac{3(t^2-4)+t^2}{3(t^2-4)^{\frac{2}{3}}}$$
$$0 = \frac{3(t^2-4)+t^2}{3(t^2-4)^{\frac{2}{3}}}$$
$$0 = 3t^2 - 12 + 2t^2$$
$$0 = 5t^2 - 12$$
$$12 = 5t^2$$
$$\frac{12}{5} = t^2$$
$$\pm\sqrt{\frac{12}{5}} = t$$
$$3(t^2-4)^{\frac{2}{3}} = 0$$
$$(t^2-4)^{\frac{2}{3}} = 0$$
$$t = \pm 2$$

Interval	Test Value	Slope$_{g'(x)}$
$x < -2$	-3	$+$
$-2 < x < -\sqrt{\frac{12}{5}}$	-1.7	$+$
$-\sqrt{\frac{12}{5}} < x < \sqrt{\frac{12}{5}}$	0	$-$
$\sqrt{\frac{12}{5}} < x < 2$	2	$+$
$x > 2$	7	$+$

$\therefore$ When $g'(x) = 0,\ x = -\sqrt{\frac{12}{5}},\ x = \sqrt{\frac{12}{5}}$

$\therefore$ Increasing $(-\infty, -2),\ (2, \infty),\ (-2, -\sqrt{\frac{12}{5}}),\ (\sqrt{\frac{12}{5}}, 2)$

Decreasing $(-\sqrt{\frac{12}{5}}, \sqrt{\frac{12}{5}})$