

# Limits

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# Chapter 1

## 1.1 The Limit of a Function

Reporting about the behaviour of function within the range of its dangerous values

$$f(x) = x^2 + \frac{1}{x}$$

Input variable =  $x$

Output variable =  $f(x)$

Name of function =  $f$

”Acceptable”/Permissible input values of  $x$  - All real numbers except zero

$$\begin{aligned} x, f(x) & \quad x+h, f(x+h) \\ f(x+h) - \frac{f(x)}{x+h-x} \\ f(x+h) - \frac{f(x)}{h} \end{aligned}$$

### 1.1.1 Proof

$$y = -16t^2 + 100t + 6$$

Points used:  $(0, 6)$   $(1, 90)$   $(3, 162)$

When  $t = 0$  and  $y = 6$

$$\begin{aligned} y &= at^2 + bt + c \\ 6 &= a(0)^2 + b(0) + c \\ c &= 6 \end{aligned}$$

When  $t = 1$  and  $y = 90$

$$\begin{aligned} 90 &= a(1)^2 + b + 6 \\ 90 &= a + b + 6 \\ 84 &= a + b \\ 84 - b &= a \end{aligned}$$

When  $t = 3$  and  $y = 162$

$$\begin{aligned}
 162 &= a(3)^2 + b + 6 \\
 162 &= 9a + 3b + 6 \\
 162 &= 9(84 - b) + 3b + 6 \\
 162 &= 756 - 9b + 3b + 6 \\
 -594 &= -6b + 6 \\
 -600 &= -6b \\
 b &= 100
 \end{aligned}$$

$$\therefore b = 100$$

$$\begin{aligned}
 84 - 100 &= a \\
 a &= -16
 \end{aligned}$$

Therefore  $a = 16$ ,  $b = 100$  and  $c = 6$

### Question 1

Given  $f(x) = x^2$  find the Limit of  $f(x)$  at  $x = 3$

**Solution:**

$$\begin{aligned}
 \text{As } x \rightarrow 3^-, f(x) &\rightarrow 9 \\
 \text{As } x \rightarrow 3^+, f(x) &\rightarrow 9
 \end{aligned}$$

Or

$$\begin{aligned}
 \lim_{x \rightarrow 3^-} f(x) &= 9 \\
 \lim_{x \rightarrow 3^+} f(x) &= 9
 \end{aligned}$$

The first 9 is known as the left limit of  $f(x)$  and the other 9 is known as the right limit of  $f(x)$

Therefore the limit of  $f(x)$  at  $x = 3$  is:

$$\lim_{x \rightarrow 3} f(x) = 9$$

This is because the left limit and right limit converge.

In the case where:

$$\begin{aligned}
 \lim_{x \rightarrow 1^-} f(x) &= 5 \\
 \lim_{x \rightarrow 1^+} f(x) &= 4
 \end{aligned}$$

The left and right limits do not converge so there is no limit of  $f(x)$  for  $x = 1$  and is written as:

$$\lim_{x \rightarrow 1} f(x) = \text{No such unique number.}$$

In the case where one limit does not exist, i.e. Increasing without bounds:

$$\begin{aligned}\lim_{x \rightarrow 1^-} f(x) &\rightarrow \infty \\ \lim_{x \rightarrow 1^+} f(x) &\rightarrow 4\end{aligned}$$

The limit does not exist because the left limit does not exist. This is written as:

$$\lim_{x \rightarrow 1} f(x) = \text{does not exist}$$