# Probability

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# Contents

# Chapter 1

# Module 8: Introduction

## 1.1 Introduction To Probability

**Definition 1.1.1: Probability**

A mathematical description of randomness and uncertainty / The likelihood of an event occurring.
The notation for Probability is $\mathbb{P}(X)$ where $X$ is the event.
Probability is always between $0 \leqslant \mathbb{P}(X) \leqslant 1$ or $0\% \leqslant \mathbb{P}(X) \leqslant 100\%$.

There are two ways of determining probability:

- Theoretical / Classical - Determined by the nature of the experiment
- Empirical / Observational - Determined by the results of the experiment

## 1.2 Relative Frequency

**Definition 1.2.1: Relative Frequency**

Relative frequency is the number of times an event occurs divided by the total number of trials.

$$\mathbb{P}(X) = \frac{\text{Number of times event occurs}}{\text{Total number of trials}}$$

**Theorem 1.2.1** The Law of Large Numbers

As the number of trials increases, the relative frequency of an event approaches the theoretical probability of the event.

# Chapter 2

# Module 9: Find the Probability of Events

## 2.1 Sample Spaces and Events

**Definition 2.1.1: Random Experiment**

An experiment whose outcome is determined by chance.

**Definition 2.1.2: Sample Space**

The list of possible outcomes of a random experiment, denoted by $S$.

**Definition 2.1.3: Event**

A statement about the nature of the outcome after the experiment has been conducted, denoted by any capital letter except $S$.

## 2.2 Equally Likely Outcomes

$$\mathbb{P}(A) = \frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S}$$

Where $A$ is an event and $S$ is the sample space.

## 2.3 Probability Rules

### 2.3.1 Rule 1: Probability is a Number Between 0 and 1

For any event $A$, $0 \leq \mathbb{P}(A) \leq 1$.

### 2.3.2 Rule 2: Addition Rule

$\mathbb{P}(S) = 1$, that is the sum of the probabilities of all possible outcomes is 1.

### 2.3.3 Rule 3: Complement Rule

$\mathbb{P}(A') = 1 - \mathbb{P}(A)$, that is the probability of the complement of an event is 1 minus the probability the event occurs.

### 2.3.4 Rule 4: Addition Rule for Mutually Exclusive Events

**Definition 2.3.1: Mutually Exclusive / Disjoint events**

Events that cannot happen at the same time.

$\mathbb{P}(A \text{ or } B) = \mathbb{P}(\text{ event } A \text{ occurs or event } B \text{ occurs or both occur})$

If $A$ and $B$ are mutually exclusive, then $\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B)$

### 2.3.5 Rule 5: Multiplication Rule for Independent Events

$\mathbb{P}(A \text{ and } B) = \mathbb{P}(\text{ event } A \text{ occurs and event } B \text{ occurs })$

**Definition 2.3.2: Independent Events**

Two events $A$ and $B$ are said to be independent if the occurrence of one event does not affect the probability of the other event occurring.

**Definition 2.3.3: Dependent Events**

Two events $A$ and $B$ are said to be dependent if the occurrence of one event affects the probability of the other event occurring.

If $A$ and $B$ are two independent events, then $\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \times \mathbb{P}(B)$

### 2.3.6 Rule 6: General Addition Rule

For any two events $A$ and $B$, $\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \text{ and } B)$. If the event are mutually exclusive, then $\mathbb{P}(A \text{ and } B) = 0$, giving us $\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B)$, i.e. the addition rule for mutually exclusive events.

# Chapter 3

# Module 10: Conditional Probability and Independence

**Definition 3.0.1: Conditional Probability**

The probability an event occurs as a result of another event. I.e. Probability of event $B$, given event event $A$ is,

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \text{ and } B)}{P(A)}$$

## 3.1 Independence

When two events are independent, the probability of one event occurring does not affect the probability of the other event, i.e.

$$\begin{aligned} \mathbb{P}(B \mid A) &= \mathbb{P}(B) \\ \mathbb{P}(A \mid B) &= \mathbb{P}(A) \\ \mathbb{P}(B \mid A) &= \mathbb{P}(B \mid A') \\ \mathbb{P}(A \text{ and } B) &= \mathbb{P}(A) \times \mathbb{P}(B) \end{aligned}$$

## 3.2 The General Multiplication Rule

For any two dependent events $A$ and $B$

$$\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \times \mathbb{P}(B \mid A)$$

## 3.3 Probability Trees

**Definition 3.3.1: Probability Tree**

A diagram that shows the sample space of a random experiment and the probability of each outcome.

### 3.3.1 Bayes' Theorem

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A) \times P(B \mid A)}{\mathbb{P}(A) \times \mathbb{P}(B \mid A) + \mathbb{P}(A') \times \mathbb{P}(B \mid A')}$$

# Chapter 4

# Module 11: Random Variables

**Definition 4.0.1: Random Variable**

Assigns a unique numerical value to the outcome of a random experiment.

**Definition 4.0.2: Discrete Random Variable**

A random variable that can take on a finite number of values. Discrete random variables are usually counts.

**Definition 4.0.3: Continuous Random Variable**

A random variable that can take on an infinite number of values. Continuous random variables are usually measurements.

## 4.1 Discrete Random Variables

### 4.1.1 Notation

For a given event $X$, the probability of $X$ is denoted by $\mathbb{P}(X)$. For a given value $x$, the probability of $X$ is denoted by $\mathbb{P}(X = x)$, i.e. the probability that $X$ takes on the value $x$.

### 4.1.2 Probability Distribution

**Definition 4.1.1: Probability Distribution**

The list of all possible values of a random variable and their corresponding probabilities.

Any probability distribution must satisfy the following two conditions:

- $0 \leqslant \mathbb{P}(X = x) \leqslant 1$ - The probability of any value of $X$ is between 0 and 1.
- $\Sigma_x \mathbb{P}(X = x) = 1$ - The sum of the probabilities of all possible values of $X$ is 1.

### 4.1.3 Key Words

- At least / No less than - $x \geqslant$
- At most / No more than - $x \leqslant$
- Less than / fewer than - $x <$
- More than / greater than - $x >$

- Exactly - $x =$

### 4.1.4 Mean and Variance of a Discrete Random Variable

#### 4.1.4.1 Mean

**Definition 4.1.2: Mean / Expected value of a Discrete Random Variable**

The average value of a random variable, denoted by $\mu$.

For a given random variable $X$, the mean is given by

$$\mu_X = \Sigma_{i=1}^{n} x_i p_i$$

Where $x_i$ is the value of $X$ and $p_i$ is the probability of $X$ taking on the value $x_i$.

##### 4.1.4.1.1 Applications of the Mean

- The mean of a random variable is the long-term average value of the random variable.
- The mean of a random variable is the centre of the probability distribution of the random variable.

#### 4.1.4.2 Variance

**Definition 4.1.3: Variance**

The average of the squared differences between each value of a random variable and the mean of the random variable, denoted by $\sigma^2$.

For a given random variable $X$, the variance is given by

$$\sigma_X^2 = \Sigma_{i=1}^{n} \left(x_i - \mu_X\right)^2 p_i$$

And standard deviation is given by

$$\sigma_X = \sqrt{\sigma_X^2}$$

Where $x_i$ is the value of $X$ and $p_i$ is the probability of $X$ taking on the value $x_i$.

#### 4.1.4.3 Rules for Mean and Variance of Random Discrete Variables

##### 4.1.4.3.1 Adding or Subtracting a Constant to a Random Variable

If $Y = X + c$, then $\mu_Y = \mu_X + c$, $\sigma_Y^2 = \sigma_X^2$ and $\sigma_Y = \sigma_X$.

If $Y = X - c$, then $\mu_Y = \mu_X - c$, $\sigma_Y^2 = \sigma_X^2$ and $\sigma_Y = \sigma_X$.

##### 4.1.4.3.2 Multiplying a Random Variable by a Constant $> 1$

If $Y = cX$ , $c > 1$, then $\mu_Y = c\mu_X$, $\sigma_Y^2 = c^2\sigma_X^2$ and $\sigma_Y = c\sigma_X$

##### 4.1.4.3.3 Multiplying a Random Variable by a Constant $< 1$

If $Y = cX$ , $c < 1$, then $\mu_Y = c\mu_X$, $\sigma_Y^2 = c^2\sigma_X^2$ and $\sigma_Y = c\sigma_X$

#### 4.1.4.3.4 Linear Transformation of a Random Variable

If $Y = a + bX$, then $\mu_Y = a + b\mu_X$, $\sigma_Y^2 = b^2\sigma_X^2$ and $\sigma_Y = |b|\sigma_X$

#### 4.1.4.3.5 Sum of Two Random Variables

If $Z = X + Y$, then $\mu_Z = \mu_X + \mu_Y$, $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$ and $\sigma_Z = \sqrt{\sigma_X^2 + \sigma_Y^2}$. Only if $X$ and $Y$ are independent.

### 4.1.5 Poisson Random Variables

**Definition 4.1.4: Poisson Random Variable**

A random variable that counts the number of events that occur in a fixed interval of time or space, denoted by $X \sim \text{Poisson}(\lambda)$. Where $\lambda$ is the average number of events that occur in the interval.

**Definition 4.1.5: Poisson Experiment**

Random experiments that satisfy the following conditions:

- The number of trials tends to infinity.
- The probability of success tends to zero.
- $np = 1$ is finite

$$\mathbb{P}(X = x) = \frac{\left(e^{-\lambda} \times \lambda^x\right)}{x!}$$

Where $e$ is the base of the natural logarithm, $\lambda$ is the average number of events that occur in the interval and $x$ is the number of events that occur in the interval.

If X is Poisson with parameter $\lambda$, then

$$\mu_X = \lambda$$

And

$$\sigma_X^2 = \mu = \lambda$$
$$\sigma_X = \sqrt{\sigma^2}$$

### 4.1.6 Binomial Random Variables

**Definition 4.1.6: Binomial Random Variable**

A random variable that counts the number of successes in a fixed number of independent trials, denoted by $X \sim \text{Bin}\left(n, p\right)$. Where $n$ is the number of trials and $p$ is the probability of success.

**Definition 4.1.7: Binomial Experiment**

Random experiments that satisfy the following conditions:

- A fixed number of trials, denoted by $n$.
- Each trial is independent of the others.
- There are only two possible outcomes for each trial, success or failure.
- There is a constant probability of success, denoted by $p$, for each trial, which can be expressed as the complement of the probability of failure, $q = 1 - p$.

**Note:-**

The number $(X)$ of success in a sample of size $n$ taken without replacement from a population with proportion $p$ of successes is approximately binomial with $n$ and $p$ as long as the sample size is at most 10% of the population size $(N)$. I.e.

$$n \leqslant 0.1N$$

Or

$$N \geqslant 10n$$

To calculate the probability of a binomial random variable, we use the formula

$$\mathbb{P}(X = x) = \binom{n}{x} p^x q^{n-x}, \text{ where } x = 0, 1, 2, \ldots, n$$

Where $n$ is the number of trials, $x$ is the number of successes, $p$ is the probability of success and $q$ is the probability of failure.

If $X$ is Binomial with parameters $n$ and $p$, then

$$\mu_X = np$$

And

$$\sigma_X^2 = np(1 - p)$$
$$\sigma_X = \sqrt{np(1 - p)}$$

## 4.2 Continuous Random Variables

### 4.2.1 Probability Distribution

For a continuous random variable $X$, the probability distribution is given by the *probability density function*, whose properties are

- $f(x) \geqslant 0$ for all $x$.
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$
- The probability that $X$ takes on a value between $a$ and $b$ is given by

$$\mathbb{P}(a \leqslant X \leqslant b) = \int_a^b f(x)\, dx$$

**Note:-**

- The probability that a continuous random variable takes on a specific value is always 0.
- The strictness of the inequality does not matter, i.e. $\mathbb{P}(X \geqslant a) = \mathbb{P}(X > a)$

### 4.2.2 Normal Random Variables

**Definition 4.2.1: Normal Random Variable**

A random variable that has a bell-shaped probability distribution, denoted by $X \sim N\left(\mu, \sigma^2\right)$. Where $\mu$ is the mean and $\sigma^2$ is the variance.

For a normally distributed random variable $X$:

- There is a 68% chance that $X$ takes on a value within one standard deviation of the mean, i.e. $0.68 = \mathbb{P}\left(\mu - \sigma < X < \mu + \sigma\right)$
- There is a 95% chance that $X$ takes on a value within two standard deviations of the mean, i.e. $0.95 = \mathbb{P}\left(\mu - 2\sigma < X < \mu + 2\sigma\right)$
- There is a 99.7% chance that $X$ takes on a value within three standard deviations of the mean, i.e. $0.997 = \mathbb{P}\left(\mu - 3\sigma < X < \mu + 3\sigma\right)$

#### 4.2.2.1 Finding Probabilities for Normal Random Variables

##### 4.2.2.1.1 Standardizing Values

**Definition 4.2.2: $z$-score**

The number of standard deviations a value is from the mean of a normal random variable, denoted by $z$.

To standardize a normal random variable $X$, we must find its $z$-score, given by

$$z = \frac{x - \mu}{\sigma}$$

##### 4.2.2.1.2 Finding Probabilities with the $z$-score

**Definition 4.2.3: Normal Table**

A table that shows the probability that a standard normal random variable takes on a value less than a given $z$-score.

Using the $z$-score we can find the probability that a normal random variable takes on a value less than a given value $x$, by tracing the $z$-score to the normal table.

$$\mathbb{P}(X < x) = \mathbb{P}(Z < z)$$

On a standard normal table $z$-score are written to two decimal places as row headers and for additional precision the column headers are the first two decimal places of the $z$-score.

### 4.2.3 Uniform Distribution

**Definition 4.2.4: Uniform Distribution**

,denoted by $X \sim U(a, b)$

For a random variable $X$, if is uniformly distributed over the interval $a$ and $b$ then its *probability distribution density function* is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, \; a < x < b \\ 0, \text{ otherwise} \end{cases}$$

The mean, variance, and standard deviation of a uniformly distributed random variable is given by

$$\mu_X = \frac{a+b}{2}$$

$$\sigma_X^2 = \frac{(b-a)^2}{12}$$

$$\sigma_X = \sqrt{\frac{(b-a)^2}{12}}$$

# Chapter 5

# Module 12: Sampling Distributions

## 5.1 Parameters vs. Statistics

**Definition 5.1.1: Sampling Distribution**

The probability distribution of a statistic that is obtained from a sample.

**Definition 5.1.2: Parameter**

A numerical value that describes a characteristic of a population, denoted by a Greek letter, e.g. $\mu, \sigma^2$.

**Definition 5.1.3: Statistic**

A numerical value that describes a characteristic of a sample, denoted by a Roman letter, e.g $\bar{x}, s^2$.

**Definition 5.1.4: Proportion**

A statistic that estimates the proportion of a population or sample that has a certain characteristic, denoted by $p$ for a population and $\hat{p}$ for a sample.

**Definition 5.1.5: Sampling Variability**

The variability of a statistic from one sample to another.

## 5.2 Behaviour of Sample Proportion $\hat{p}$

### 5.2.1 Centre

The mean of the sample proportion is the same as the population proportion, i.e.

$$\mu_{\hat{p}} = p$$

As it is reasonable to expect all the sample proportions in repeated samples to average out to the underlying population.

### 5.2.2 Spread

The sample size has an effect on the spread of the distribution of the sample proportion, i.e. the **larger the sample size, the less spread out the distribution** of the sample proportion and **more spread for smaller**

**sample sizes**. We can describe the spread of the distribution of the sample proportion more precisely by finding the actual standard deviation of the sample proportion. i.e.

$$\sigma_{\hat{p}} = \sqrt{\frac{p\left(1-p\right)}{n}}$$

Where $p$ is the population proportion and $n$ is the sample size.

### 5.2.3 Shape

The shape of the distribution of the sample proportion is approximately normal if the sample size is large enough. I.e. if

$$np \geqslant 10 \text{ and } n\left(1-p\right) \geqslant 10$$

Therefore

$$\hat{p} \sim N\left(p, \frac{p\left(1-p\right)}{n}\right)$$

**Definition 5.2.1: Sampling of Distribution of $\hat{p}$**

The distribution of the values of the sample proportions $\hat{p}$ in repeated samples.

### 5.2.4 Standard Error of Sample Proportion

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$$

## 5.3 Behaviour of Sample Mean $\overline{X}$

### 5.3.1 Centre

The mean of the sample mean is the same as the population mean, i.e.

$$\mu_{\overline{X}} = \mu$$

### 5.3.2 Spread

The sample size has an effect on the spread of the distribution of the sample mean, i.e. the **larger the sample size, the less spread out the distribution** of the sample mean and **more spread for smaller sample sizes**. We can describe the spread of the distribution of the sample mean more precisely by finding the actual standard deviation of the sample mean. i.e.

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

### 5.3.3 Shape

The shape of the distribution of the sample mean is approximately normal if the sample size is large enough. I.e. if

$$n \geqslant 30$$

Therefore

$$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

**Definition 5.3.1: Sampling of Distribution of $\overline{X}$**

The distribution of the values of the sample mean $\overline{X}$ in repeated samples.

#### 5.3.3.1 Standard Error of Sample Mean

$$SE_{\overline{x}} = \frac{s}{\sqrt{n}}$$

# Chapter 6

# Exercises

**Question 1**

Three cards are drawn with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces drawn. Also, find the mean and variance of the distribution.

***Solution:***

| $x$ | $P(x)$ |
|---|---|
| 0 | $\binom{3}{0}\left(\frac{4}{52}\right)^0\left(1-\frac{4}{52}\right)^{3-0} = 0.7865$ |
| 1 | $\binom{3}{1}\left(\frac{4}{52}\right)^1\left(1-\frac{4}{52}\right)^{3-1} = 0.1966$ |
| 2 | $\binom{3}{2}\left(\frac{4}{52}\right)^2\left(1-\frac{4}{52}\right)^{3-2} = 0.0164$ |
| 3 | $\binom{3}{3}\left(\frac{4}{52}\right)^3\left(1-\frac{4}{52}\right)^{3-3} = 0.0005$ |

$$\begin{aligned}
\mu &= \sum_{i=1}^{4} x_i \times p_i \\
&= (0\times 0.7865) + (1\times 0.1966) + (2\times 0.0164) + (3\times 0.0005) \\
&= 0.2309 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= \sum_{i=1}^{4} \left(x_i-\mu\right)^2 p_i \\
&= \left((0-0.2309)^2\times 0.7865\right) + \left((1-0.2309)^2\times 0.1966\right) + \left((2-0.2309)^2\times 0.0164\right) + \left((3-0.2309)^2\times 0.0005\right) \\
&= 0.21338519 \\
&= 0.2134
\end{aligned}$$

**Question 2**

Paper clips are produced in a variety of colours The proportion of red paper clips produced is 0.20, Determine the probability that, in a random sample of 50 coloured paper clips, the number of red clips is:

1. Fewer than 10
2. At least 8 but at most 12

***Solution:***

$$X \sim B\left(50, 0.20\right)$$

1.

$$\begin{aligned} P\left(X < 10\right) &= \sum_{i=0}^{9} \binom{50}{i} \left(0.2\right)^{i} \left(1 - 0.20\right)^{50-i} \\ &= 0.4437 \end{aligned}$$

2.

$$\begin{aligned} P\left(8 \leqslant X \leqslant 12\right) &= P\left(X \leqslant 12\right) - P\left(X \leqslant 8\right) \\ &= \sum_{i=8}^{12} \binom{50}{i} \left(0.20\right)^{i} \left(1 - 0.20\right)^{50-i} \\ &= 0.6235 \end{aligned}$$

**Question 3**

A recent large-scale survey established that 15 percent of cars have fully functioning brake lights

1. Calculate the probability that, in a random sample of 18 cars, exactly 2 cars have faulty brake lights.
2. Determine the probability that, in a random sample of 50 cars, more than 5 cars but fewer than 10 cars have faulty brake lights.

***Solution:***

1.

$$\begin{aligned} &X \sim B\left(18, 0.15\right) \\ &P\left(X = 2\right) = \binom{18}{2} \left(0.15\right)^{2} \left(1 - 0.15\right)^{18-2} \\ &= 0.2556 \end{aligned}$$

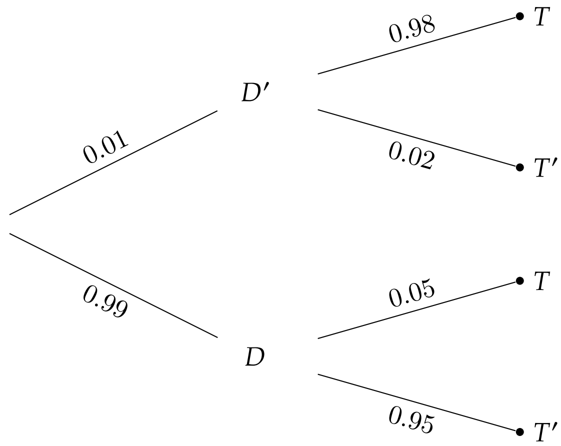
2.

$$\begin{aligned} &X \sim B\left(50, 0.20\right) \\ &P\left(5 < X < 10\right) = P\left(X < 10\right) - P\left(X < 5\right) \\ &= \sum_{i=6}^{9} \binom{50}{i} \left(0.15\right)^{i} \left(1 - 0.15\right)^{50-i} \\ &= 0.5717 \end{aligned}$$

## Question 4

You are diagnosed with an uncommon disease. You know that there only is a 1% chance. Use the letter $D$ for the event "you have a disease" and $T$ for "the test says so". It is known that the test is perfect. $P(T \mid D) = 0.98$ and $P(T' \mid D') = 0.95$

1. Given that you test positive, what is the probability that you really have the disease?
2. You obtain a second opinion: in an independent repetition of the test. You test positive again. Given this, what is the probability that you really have the disease.




***Solution:***

1.

$$
\begin{aligned}
P(D \mid T) &= \frac{P(D \cap T)}{P(T)} \\
&= \frac{P(D) \times P(T \mid D)}{P(D) \times P(T \mid D) + P(D') \times P(T \mid D')} \\
&= \frac{0.01 \times 0.98}{(0.01 \times 0.98) + (0.99 \times 0.05)} \\
&= 0.1653
\end{aligned}
$$

2.

$$
\begin{aligned}
P((D \mid T) \cap (D \mid T)) &= 0.1653 \times 0.1653 \\
&= 0.0273
\end{aligned}
$$

## Question 5

Selorm, arriving at a bus stop, just misses the bus. Suppose that he decides to walk if the (next) bus takes longer than 5 minutes to arrive. Suppose also that the time in minutes between the arrivals of buses at the bus stop is a continuous random variable with a $U(4,6)$. Let $X$ be the time Selorm will wait.

1. What is the probability that $X$ is less than $4\frac{1}{2}$ minutes
2. What is the probability that $X$ equals 5 minutes?
3. Is $X$ a discrete random variable or a continuous random variable?

***Solution:***

1.

$$
\begin{aligned}
f(x) &= \frac{1}{6-4} \\
f(x) &= 0.5 \\
P\left(X < \frac{9}{2}\right) &= (4.5-4) \times 0.5 \\
&= 0.25
\end{aligned}
$$

2.

$$P(X = 5) = 0$$

**Question 6**

If random variable $X$ follows a Poisson distribution with mean 3.4. Find $P(X = 6)$

***Solution:***

$$
\begin{aligned}
X &\sim \text{Poisson}(3.4) \\
P(X = 6) &= \frac{e^{-3.4} \times 3.4^6}{6!} \\
&= 0.0716
\end{aligned}
$$

**Question 7**

Cretan Airlines services which arrive late to Athens Airport on a typical week can be modelled by a Poisson distribution with mean of 4.5

1. Determine the probability that on a given week there will be
   (a) four late arrivals
   (b) less than four late arrivals
   (c) at least seven late arrivals
2. Determine the probability that on a given two week period there will be between eight and thirteen (inclusive) late arrivals.

***Solution:*** $X \sim \text{Poisson}(4.5)$

1. (a)

$$
\begin{aligned}
P(X = 4) &= \frac{e^{-4.5} \times 4.5^4}{4!} \\
&= 0.18980
\end{aligned}
$$

(b)

$$
\begin{aligned}
P(X < 4) &= \sum_{i=0}^{3} \frac{e^{-4.5} \times 4.5^i}{i!} \\
&= 0.3423
\end{aligned}
$$

(c)

$$
\begin{aligned}
P(X \geqslant 7) &= 1 - P(X < 7) \\
&= 1 - \sum_{i=0}^{6} \frac{e^{-4.5} \times 4.5^{i}}{i!} \\
&= 1 - 0.8311 \\
&= 0.1689
\end{aligned}
$$