

Optimization

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Chapter 1

Maximum and Minimum

Note:-

Local - Subsection of range

Global - Whole range

1.1 Maximize

$$R(x) = 45 - \frac{x^2}{3}, \quad 0 \leq x \leq 1$$

Find all local maximum values

Find the global maximum value

1.2 Minimize

- Local Minimum - Minimum in specified range
- Global Minimum - Overall minimum

1.3 Local/Relative Maximum/Minimum (Optimum)

- Find the critical values of the function:

Stationary points, i.e. $f'(\cdot) = 0$

Undefined points, i.e. $f'(\cdot) = \emptyset$

- Assess them for potential local maximum/minimum:

Find the first derivative, input values from the left and right of the critical points and check the change in signs:

+ to -: Maximum

- to +: Minimum

Find second derivative, input the critical values and check the sign:

-: Minimum

+: Maximum

1.4 Global/Absolute Maximum/Minimum (Optima)

To find the Absolute Optima of a function whose domain is unrestricted:

$$\lim_{x \rightarrow \infty} f(x) \quad \lim_{x \rightarrow -\infty} f(x)$$

1.4.1 Conditions for finding the Absolute Optima easily

1. Closed Domain, i.e. $[x_1, x_2]$
2. Function is continuous for the duration of the closed domain

Theorem 1.4.1 Extreme Value Theorem

If a real valued function f is continuous on the closed interval $[a, b]$, the f must attain a maximum and minimum at least once.

$$\begin{aligned} f(c) &\geq f(x) \geq F(d) \\ \forall x &\in [a, b] \end{aligned}$$

Where $f(c)$ is the function's minimum value and $F(d)$ is the function's maximum value.

Example 1.4.1

$$f(x) = x^3 \text{ on } [-1, 10]$$

- $f(x)$ is continuous due to it being a polynomial
- The function's domain is closed due to the end values being included in the domain

By EVT(1.4.1) $f(x)$ must attain absolute maximum and minimum at least once on the interval. Possibly at:

1. End points of the domain
2. Critical values of $f(x)$

$$\begin{aligned} f(x) &= x^3 \\ f'(x) &= 3x^2 \\ 0 &= 3x^2 \\ \frac{0}{3} &= x^2 \\ 0 &= x \end{aligned}$$

$$\begin{aligned} f(-1) &= -1 \\ f(10) &= 1000 \end{aligned}$$

\therefore Absolute Maximum is 1000
Absolute Minimum is -1

Chapter 2

Concavity

Let f be a function that is differentiable over an open interval I

- If f' is increasing over I , we say f is concave up over I , i.e. $f'' > 0$
- If f' is decreasing over I , we say f is concave down over I , i.e. $f'' < 0$

2.1 Inflection

A point where a function switches concavity, i.e:

$$\begin{aligned} f''(x^-) = +\text{ve} \text{ to } f''(x^+) = -\text{ve} \\ \text{or} \\ f''(x^-) = -\text{ve} \text{ to } f''(x^+) = +\text{ve} \end{aligned}$$

2.2 Curvature

2.2.1 Concave Up

The cave is facing up

2.2.2 Concave Down

The cave is facing down