Matrix Algebra

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Contents

Chapter 1

Matrix Operations

If A is a $n \times m$ matrix then the scalar entry in the ith row and the jth column of A is denoted by a_{ij}, and is called the (i, j)-entry. Each column of A is a list of m real numbers in the $\mathbb{R}^m$ vector space. Therefore the columns of A can be represented as vectors in $\mathbb{R}^m$:

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \ldots \mathbf{a}_n \end{bmatrix}$$

Definition 1.0.1: Diagonals

The diagonal entries of a matrix A of dimension $n \times m$, are the entries a_{ij}, where $i = j$. This is called the **main diagonal** of the matrix A. A **diagonal matrix** is a square matrix $n \times n$ whose non-diagonal entries are all zero.

1.1 Sums and Scalar Multiples

Definition 1.1.1: Equality of Matrices

Two matrices A and B, are equal if:

- The are of the same size i.e, $m \times x$
- The corresponding entries are equal i.e, $A_{ij} = B_{ij}$

Theorem 1.1.1 Axioms of Matrix Addition

Let A, B and C be matrices of the same size, and let r and s be scalars. Then the following axioms hold:
Communtativity $A + B = B + A$
Associativity $(A + B) + C = A + (B + C)$
Additive Identity $A + 0 = A$
Distruibutivity 1 $r (A + B) = rA + rB$
Distruibutivity 2 $(r + s) A = rA + sA$
Compatibility with Scalar Multiplication $r (sA) = (rs) A$

1.2 Matrix Multiplication

When a matrix B multiples a vector $\mathbf{x}$, it transforms $\mathbf{x}$ into the vector $B\mathbf{x}$. If this vector is then multiples by another matrix A, the result is the vector $A (B\mathbf{x})$. Thus $A (B\mathbf{x})$ is produced by a composition of mappings / linear transformations. This can be also expressed as:

$$A (B\mathbf{x}) = (AB) \mathbf{x}$$

Because, if A is $m \times n$, B is $n \times p$ and $\mathbf{x}$ is in $\mathbb{R}^p$, can denote the columns of B, by $\mathbf{b}_1, \ldots, \mathbf{b}_p$ and the entries of $\mathbf{x}$ by, $x_1, \ldots, x_p$. Then

$$B\mathbf{x} = x_1\mathbf{b}_1 + \ldots + x_p\mathbf{b}_p$$

By the linearity of matrix multiplication, we have:

$$\begin{aligned} A\left(B\mathbf{x}\right) &= A\left(x_1\mathbf{b}_1\right) + \ldots + A\left(x_p\mathbf{b}_p\right) \\ &= x_1\left(A\mathbf{b}_1\right) + \ldots + x_p\left(A\mathbf{b}_p\right) \end{aligned}$$

The vector $A\left(B\mathbf{x}\right)$ is then a linear combination of the vectors $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$, using the entries of $\mathbf{x}$ as weights. This can be expressed in matrix notation as:

$$A\left(B\mathbf{x}\right) = \begin{bmatrix} A\mathbf{b}_1 & A\mathbf{b}_2 & \ldots & A\mathbf{b}_p \end{bmatrix} \mathbf{x}$$

Theorem 1.2.1

If A is an $m \times n$ matrix, and if B is an $n \times p$ matrix with columns $\mathbf{b}_1, \ldots, \mathbf{b}_p$, then the product AB is the $m \times p$ matrix whose columns are $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$.That is:

$$A\left(B\mathbf{x}\right) = \begin{bmatrix} A\mathbf{b}_1 & A\mathbf{b}_2 & \ldots & A\mathbf{b}_p \end{bmatrix} \mathbf{x}$$

Example 1.2.1

Question 1

Compute AB where $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$, and $B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$

Solution:

$$\begin{aligned} A\mathbf{b}_1 &= \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 8+3 \\ 4+-5 \end{bmatrix} \\ &= \begin{bmatrix} 11 \\ -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A\mathbf{b}_2 &= \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 6-6 \\ 3+10 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 13 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A\mathbf{b}_3 &= \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 21 \\ -9 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & -9 \end{bmatrix}$$

Theorem 1.2.2 Row-Column Rule

If the product AB is defined, them the entry in row i and column j of AB is the sum of the products of corresponding entries of the row i of A and column j of B. If $(AB)_{ij}$ denotes the (i, j)-entry in AB, and if A is an $m \times n$, then

$$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$$

Example 1.2.2

Use the row–column rule to compute two of the entries in AB for the matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$$

An inspection of the numbers involved will make it clear how the two methods for calculating AB produce the same matrix.

The dimensions of the resultant matrix is 2×3, therefore the entries of AB are:

$$\begin{aligned} AB &= \begin{bmatrix} 2(4) + 3(1) & 2(3) + 3(-2) & 2(6) + 3(3) \\ 1(4) - 5(1) & 1(3) - 5(-2) & 1(6) - 5(3) \end{bmatrix} \\ &= \begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & 9 \end{bmatrix} \end{aligned}$$

Example 1.2.3

Question 2

Find the entries in the second row of AB where,

$$A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$$

Solution:

$$\begin{aligned} \begin{bmatrix} -1 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix} \\ \begin{bmatrix} -4 + 21 - 12 & 6 + 3 - 8 \end{bmatrix} \\ \begin{bmatrix} 5 & 1 \end{bmatrix} \end{aligned}$$

Theorem 1.2.3 Axioms of Matrix Multiplication

Let A be an $m \times n$ matrix and let B and C have sizes for which the indicated sums and products are defined:
Associativity $A(BC) = (AB)C$
Left Distruibutivity $A(B + C) = AB + AC$
Right Distruibutivity $(B + C)A = BA + CA$
Scalar Associativity $r(AB) = (rA)B = A(rB), \forall r, r \in \mathbb{F}$
Mutliplicative Identitiy $I_m A = A = AI_n$

Example 1.2.4

Question 3

Let $A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$. Show that these matrices do not commute, I.e, verify $AB \neq BA$

Solution:

$$\begin{aligned} AB &= \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 3 \\ -2 & -6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 2 \\ 29 & -2 \end{bmatrix} \end{aligned}$$

$$\therefore AB \neq BA$$

1.2.1 Powers of a Matrix

Definition 1.2.1: Powers of a Matrix

If A is an $n \times n$ matrix and if k is a positive integer, then A^k denotes the product of k copies of A:

$$A^k = A_1 \dots A_k$$

Where $A_1 = A_2 \wedge A_2 = A_3 \wedge \dots \wedge A_{k-1} = A_k$

If A is non-zero and if $\mathbf{x}$ is in $\mathbb{R}^n$, then $A^k\mathbf{x}$ is the result of left-multiplying $\mathbf{x}$ by A repeatedly k times.

If $k = 0$, then $A^0\mathbf{x}$ is $\mathbf{x}$. Thus A^0 is interpreted as the Identity matrix.

1.3 The Transpose of a Matrix

Definition 1.3.1: The Transpose of a Matrix

Given a matrix A, its *transpose*, denoted by A^T, is defined by transforming the rows of A into columns. For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Therefore formally, the transpose of a matrix $A_{m,n}$ is defined as:

$$A^T_{m,n} = A_{n,m}$$

Therefore, let A and B denote matrices whose sizes are appropriate for the following sums and products:

1. $\left(A^T\right)^T = A$
2. $(A + B)^T = A^T + B^T$

3. $\forall r \in \mathbb{F}, (rA)^T = rA^T$
4. $(AB)^T = B^T A^T$

Usually $(AB)^T$ is not equal $A^T B^T$, even when A and B have dimensions such that $A^T B^T$ is defined. The generalization of axiom 4 to products more than two factors is as follows:

> **Theorem 1.3.1**
> The transpose of a product of matrices equals the product of their transpose in the reverse order.

Chapter 2

The Inverse Of A Matrix

2.1 Invertibility

Definition 2.1.1: Invertibility

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad - bc \neq 0$, then A is invertible and

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, then A is not invertible. Where $ad - bc$ is known as the *determinant* and denoted by

$$\det A = ad - bc$$

Theorem 2.1.1

If A is an invertible $n \times n$ matrix, then for each $\mathbf{b}$ in $\mathbb{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has the unique solution:

$$\mathbf{x} = A^{-1}\mathbf{b}$$

Theorem 2.1.2

1. If A is an invertible matrix, then A^{-1} is invertible and

$$\left(A^{-1}\right)^{-1} = A$$

2. If A and B are $n \times n$ invertible matrices, then so AB, and the inverse of AB is the product of the inverses of A and B in the reverse order:

$$(AB)^{-1} = B^{-1}A^{-1}$$

3. If A is an invertible matrix, then so is A^T and the inverse of A^T is the transpose of A^{-1}:

$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$$

2.2 Elementary Matrices

Definition 2.2.1: Elementary Matrix

A matrix obtained by performing a single elementary row operation on an identity matrix.

Example 2.2.1

Question 4

Let

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}, A \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Compute E_1A, E_2A, E_3A, and describe how these products can be obtained by elementary row operations on A.
Solution:

$$\begin{aligned} E_1A &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\ &= \begin{bmatrix} a & b & c \\ d & e & f \\ -4a+g & -4b+h & -4c+1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} E_2A &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\ &= \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix} \end{aligned}$$

$$\begin{aligned} E_3A &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\ &= \begin{bmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{bmatrix} \end{aligned}$$

- E_1A could be obtained by the elementary row operation $-4R_1 + R_3 \longrightarrow R_3$
- E_2A could be obtained by the elementary row operation $R_1 \leftrightarrow R_2$
- E_3A could be obtained by the elementary row operation $5R_3 \longrightarrow R_3$

Corollary 2.2.1

If an elementary row operation is performed on an $m \times n$ matrix A, the resulting matrix can be expressed as EA, where E is the $m \times m$ matrix created by performing the same row operation on I_m

Since row operations are reversible, all elementary matrices are invertible. Therefore there exists an elementary matrix F such that

$$FE = I$$

And since E and F correspond to reverse operations $EF = I$, also.

Example 2.2.2

Question 5

Find the inverse of $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

Solution: To transform this matrix into I_3 we must get rid of the -4 entry in the third row. This can be done by the row operation $4R_1 + R_3 \longrightarrow R_3$, which corresponds to the elementary matrix:

$$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

Checking our answer:

$$\begin{aligned} E_1 E_1^{-1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

This is indeed the identity matrix I_m

Theorem 2.2.1

An $n \times n$ matrix A is invertible if and only if A is row equivalent to I_n, and in this case, any sequence of elementary row operations that reduces A to I_n also transforms I_n into A^{-1}

2.2.1 Finding A^{-1}

To find the inverse of a matrix A, we can augment A with the $n \times n$ identity matrix I_n and then row reduce. If A is row equivalent to I_n then $\begin{bmatrix} A & I \end{bmatrix}$ is row equivalent to $\begin{bmatrix} I & A^{-1} \end{bmatrix}$. Otherwise, A does not have an inverse.

Example 2.2.3

Question 6

Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$

Solution:

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3$$

$$\begin{bmatrix} 4 & -3 & 8 & 0 & 0 & 1 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{4}R_1 - R_2 \rightarrow R_2$$

$$\begin{bmatrix} 4 & -3 & 8 & 0 & 0 & 1 \\ 0 & \frac{-3}{4} & -1 & 0 & -1 & \frac{1}{4} \\ 0 & 1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\frac{-4}{3}R_2 - R_3 \rightarrow R_3$$

$$\begin{bmatrix} 4 & -3 & 8 & 0 & 0 & 1 \\ 0 & \frac{-3}{4} & -1 & 0 & -1 & \frac{1}{4} \\ 0 & 0 & \frac{-2}{3} & -1 & \frac{4}{3} & \frac{-1}{3} \end{bmatrix}$$

$$4R_2 - R_1 \rightarrow R_1$$

$$\begin{bmatrix} -4 & 0 & -12 & 0 & -4 & 0 \\ 0 & \frac{-3}{4} & -1 & 0 & -1 & \frac{1}{4} \\ 0 & 0 & \frac{-2}{3} & -1 & \frac{4}{3} & \frac{-1}{3} \end{bmatrix}$$

$$\frac{3}{2}R_3 - R_2 \rightarrow R_2$$

$$\begin{bmatrix} -4 & 0 & -12 & 0 & -4 & 0 \\ 0 & \frac{3}{4} & 0 & \frac{-3}{2} & 3 & \frac{-3}{4} \\ 0 & 0 & \frac{-2}{3} & -1 & \frac{4}{3} & \frac{-1}{3} \end{bmatrix}$$

$$18R_3 - R_1 \rightarrow R_1$$

$$\begin{bmatrix} 4 & 0 & 0 & -18 & 28 & \frac{-6}{1} \\ 0 & \frac{3}{4} & 0 & \frac{-3}{2} & 3 & \frac{-3}{4} \\ 0 & 0 & \frac{-2}{3} & -1 & \frac{4}{3} & \frac{-1}{3} \end{bmatrix}$$

$$-R_1 \rightarrow R_1$$

$$\begin{bmatrix} -4 & 0 & 0 & 18 & -28 & \frac{6}{1} \\ 0 & \frac{3}{4} & 0 & \frac{-3}{2} & 3 & \frac{-3}{4} \\ 0 & 0 & \frac{-2}{3} & -1 & \frac{4}{3} & \frac{-1}{3} \end{bmatrix}$$

$$\frac{-1}{4}R_1 \rightarrow R_1$$

$$\frac{4}{3}R_2 \rightarrow R_2$$

$$\frac{-3}{2}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-9}{2} & 7 & \frac{-3}{2} \\ 0 & 1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix}$$

Since $A \sim I$, A is invertible and

$$A^{-1} = \begin{bmatrix} \frac{-9}{2} & 7 & \frac{-3}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix}$$

Checking our answer:

$$\begin{aligned} AA^{-1} &= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix} \begin{bmatrix} \frac{-9}{2} & 7 & \frac{-3}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Chapter 3

Determinants

Chapter 4

Exercises

Question 7

Compute the product AB using:

- The definition where Ab_1, Ab_2 are computed separately.
- The row-column rule.

$$A = \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix}, B \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$$

Solution:

1.

$$\begin{aligned} Ab_1 &= \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} -3 - 4 \\ 15 - 8 \\ 6 + 6 \end{bmatrix} \\ &= \begin{bmatrix} -7 \\ 7 \\ 12 \end{bmatrix} \\ Ab_2 &= \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ -6 \\ -7 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} -7 & 4 \\ 7 & -6 \\ 12 & -7 \end{bmatrix}$$

2.

$$
\begin{aligned}
AB &= \begin{bmatrix} -1\times 3+2\times -2 & -1\times -2+2\times 1 \\ 5\times 3+4\times -2 & 5\times -2+4\times 1 \\ 2\times 3+-3\times -2 & -2\times 2+-3\times 1 \end{bmatrix} \\
&= \begin{bmatrix} -7 & 4 \\ 7 & -6 \\ 12 & -7 \end{bmatrix}
\end{aligned}
$$

Question 8

Suppose the last column of AB is entirely zero but B itself has no column of zeros. What can you say about the columns of A?

Solution: If the last column of AB is entirely zero, then the last column of A must be a linear combination of the columns of B. Therefore the columns of A are linearly dependent.

Question 9

Find the inverses of the following matrices:

1.
$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$$

2.
$$\begin{bmatrix} 3 & -4 \\ 7 & -8 \end{bmatrix}$$

Solution:

1.

$$
\begin{aligned}
\det(A) &= 32-30 \\
&= 2
\end{aligned}
$$

$$
\begin{aligned}
A^{-1} &= \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} \\
&= \begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix}
\end{aligned}
$$

2.

$$
\begin{aligned}
\det(A) &= -24+28 \\
&= 4 \\
A^{-1} &= \frac{1}{4}\begin{bmatrix} -8 & 4 \\ -7 & 3 \end{bmatrix} \\
&= \begin{bmatrix} -4 & 1 \\ -\frac{7}{4} & \frac{3}{4} \end{bmatrix}
\end{aligned}
$$

Question 10

Use the inverse found in 6 1 to solve the system:

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

Solution:

$$\begin{aligned} \mathbf{b} &= \begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ \mathbf{x} &= A^{-1}\mathbf{b} \end{aligned}$$

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 7 \\ -9 \end{bmatrix} \end{aligned}$$

Question 11

Find the inverse of the following matrix if it exists:

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$$

$$4R_1 - R_2 \longrightarrow R_2$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -2 & 6 & -4 \end{bmatrix}$$

$$-2R_1 - R_3 \longrightarrow R_3$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$2R_2 - R_3 \longrightarrow R_3$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \det(A) &= 1 \times -1 \times 0 \\ &= 0 \end{aligned}$$

$\therefore$ the matrix does not have an inverse