Assignment 5

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Question 1

1. A proof by contraposition.
2. A proof by contradiction.

Solution:

1. ***Proof:*** The contraposition of the statement ”If $3n + 2$ is even, then n is even”, in the form $p \to q$, where p is ”$3n + 2$ is even” and q is ”n is even”, is $\neg q \to \neg p$, i.e. ”If n is odd, then $3n + 2$ is odd ”.
Assume n is odd.
Then $\exists k \in \mathbb{Z}\,(n = 2k + 1)$.
For $3n + 2$ to be odd
$$\exists t \in \mathbb{Z}\,(3n + 2 = 2t + 1)$$
$$\begin{aligned} 3n + 1 &= 3\,(2k + 1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2\,(3k + 2) + 1 \\ \text{Let } t &= 3k + 2 \\ &= 2t + 1 \end{aligned}$$
Since t is made up of the sum of the product of integers 3, k, and 2, t is an integer.
$\therefore$ If n is odd, then $3n + 2$ is odd.
Hence by contraposition if $3n + 2$ is even, then n is even ☺

2. ***Proof:*** The negation of the statement ”If $3n + 2$ is even, then n is even”, in the form $p \to q$, where p is ”$3n + 2$ is even” and q is ”n is even”, is $p \wedge \neg q$, I.e., ”$3n + 2$ is even and n is odd”.
Assume $3n + 2$ is even and n is odd.
Then $\exists t \in \mathbb{Z}\,(n = 2t + 1)$
$$\begin{aligned} 3n + 2 &= 3\,(2t + 1) + 2 \\ &= 6t + 3 + 2 \\ &= 6t + 4 + 1 \\ &= 2\,(3t + 2) + 1 \\ \text{Let } k &= 3t + 2 \\ &= 2k + 1 \end{aligned}$$
Since k is made up of the sum of the product of integers 3, t, and 2, k is an integer.
Since $3n + 2$ can be expressed in the form $2k + 1$, where $k \in \mathbb{Z}$, $3n + 2$ is odd.
We now have $3n+2$ being odd by our proof and even by our initial assumption, ($p \wedge \neg p$), which is a contradiction.
$\therefore$ $3n + 2$ is even and n is odd is false.
Hence the statement if $3n + 2$ is even, then n is even is true. ☺

Question 2

1. $B \times C \times A$
2. $B \times B \times B$

Solution:

1. $$B \times C \times A = \{(x, 0, a), (x, 0, b), (x, 1, a), (x, 1, b), (y, 0, a), (y, 0, b), (y, 1, a), (y, 1, b)\}$$

2. $$B^3 = \{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, y, y), (y, y, x), (y, x, y), (y, x, x)\}$$

Question 3

1. $\mathcal{U} \in \{\mathcal{U}, \{\mathcal{U}\}\}$
2. $\{\mathcal{U}\} \in \{\mathcal{U}\}$
3. $\{\mathcal{U}\} \subset \{\mathcal{U}, \{\mathcal{U}\}\}$
4. $\{\{\mathcal{U}\}\} \subset \{\mathcal{U}, \{\mathcal{U}\}\}$

Solution:

1. True. $\mathcal{U}$ is an element of the set $\{\mathcal{U}, \{\mathcal{U}\}\}$
2. False. A set cannot be an element of itself.
3. True. All elements found in $\{\mathcal{U}\}$ are also found in $\{\mathcal{U}, \{\mathcal{U}\}\}$ and $\{\mathcal{U}\}$ is not equal to $\{\mathcal{U}, \{\mathcal{U}\}\}$
4. True. All elements found in $\{\{\mathcal{U}\}\}$ are also found in $\{\mathcal{U}, \{\mathcal{U}\}\}$ and $\{\{\mathcal{U}\}\}$ is not equal to $\{\mathcal{U}, \{\mathcal{U}\}\}$