Implicit Differentiation

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Chapter 1

Implicity

1.1 Explicit vs Implicit

$$y = x^2 + x + 1$$

The dependent variable y is separate from the independent variable x, making this function explicitly written.

$$x^2 + y^2 = 1$$

The dependent variable y is not separate from the independent variable x, making this function implicitly written.

1.2 Implicit Differentiation

Given an implicitly written function, $2x^3 + 2y^3 = 9xy$. We can find the derivative by doing the following:

$$\begin{aligned}
2x^3 + 2y^3 &= 9xy \\
\frac{d}{dx}(2x^3 + 2^3) &= \frac{d}{dx}(9xy) \\
6x^2 + 6y^2\frac{dy}{dx} &= (9)(y) + (9x)\frac{dy}{dx} \\
6x^2 = 9y &= 9x\frac{dy}{dx} - 6y^2\frac{dy}{dx} \\
6x^2 = 9y &= (9x - 6y^2)\frac{dy}{dx} \\
&= \frac{6x^2 - 9y}{9x - 6y^2} = \frac{dy}{dx}
\end{aligned}$$