Reporting about the behaviour of a function within the range of its dangerous values.

$$f(x) = x^2 + \frac{1}{x}$$

Input variable - x

Output - $f(x)$

Name of function - f

"Acceptable"/Permissible input values of x - All real numbers except zero

$$(x, f(x)), (x + h, f(x + h))$$
$$\frac{f(x + h) - f(x)}{x + h - x}$$
$$\frac{f(x + h) - f(x)}{h}$$

Proof

$$y = -16t^2 + 100t + 6$$

Points used: $(0, 6), (1, 90), (3, 162)$

When $t = 0$ and $y = 6$

$$y = at^2 + bt + c$$
$$6 = a(0)^2 + b(0) + c$$
$$c = 6$$

When $t = 1$ and $y = 90$

$$90 = a(1)^2 + b + 6$$
$$90 = a + b + 6$$
$$84 = a + b$$
$$84 - b = a$$

When $t = 3$ and $y = 162$

$$\begin{aligned}162 &= a(3)^2 + 3b + 6\\ 162 &= 9a + 3b + 6\\ 162 &= 9(84 - b) + 3b + 6\\ 162 &= 756 - 9b + 3b + 6\\ -594 &= -6b + 6\\ -600 &= -6b\\ b &= 100\end{aligned}$$

$\therefore b = 100$

$$\begin{aligned}84 - 100 &= a\\ a &= -16\end{aligned}$$

Therefore $a = -16$, $b = 100$, and $c = 6$

Given $f(x) = x^2$, find the Limit of $f(x)$ at $x = 3$

As x $\to 3^-,\ f(x)$->9
As x $\to 3^+,\ f(x)$->9

Or

$$\lim_{x \to 3^-} f(x) = 9$$
$$\lim_{x \to 3^+} f(x) = 9$$

The first 9 is known as the left limit of $f(x)$ and the other 9 is known as the right limit of $f(x)$

The Limit of $f(x)$ at $x = 3$

$$\lim_{x \to 3} f(x) = 9$$

This is because the left limit and right limit converge.

In the case where:

$$\lim_{x \to 1^-} f(x) = 5$$
$$\lim_{x \to 1^+} f(x) = 4$$

The left and right limits do not converge so there is no limit of $f(x)$ for $x = 1$

$$\lim_{x \to 1} f(x) = \text{No such unique number}$$
$$\therefore \text{The limit of } f(x) \text{ at } x = 1 \text{ does not exist}$$

In the case where the one limit does not exist (increasing without bounds):

$$\lim_{x \to 1^-} f(x) \to \infty$$
$$\lim_{x \to 1^+} f(x) \to 4$$

The limit does not exist because the left limit does not exist.

$$\lim_{x \to 1} f(x) = \text{does not exist}$$
$$\because \text{ the left limit does not exist}$$

Graphical

Given $f(x) = x^2$, evaluate the Limit of $f(x)$ at $x = 3$ using the graphical approach.