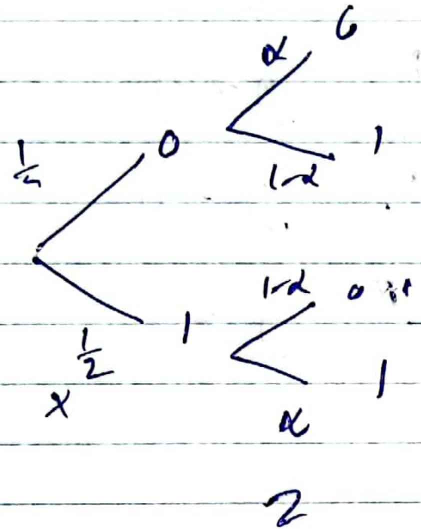
EN 2074 Quiz 1

Q1) (i) $Pr(x=0) = Pr\{x=1\} = \frac{1}{2}$

$Pr\{z=0 / x=0\} = Pr\{z=1 \mid x=1\} = \alpha$

$Pr\{N=1\} = p$



(i) $Pr\{z=0\} = Pr\{X=0\}\, Pr\{z=0 / x=0\} + Pr\{x=1\}\, Pr\{z=0 \mid x=1\}$

$= \frac{1}{2}(\alpha) + \frac{1}{2}(1-\alpha)$

$= \frac{1}{2}$

$\boxed{\therefore Pr\{z=0\} = \frac{1}{2}}$

$Pr\{z=1\} = 1 - \frac{1}{2}$

$= \frac{1}{2}$

(ii) $Pr(Y=0) = Pr(z=0 \cap N=0) + Pr(z=1 \cap N=0) + Pr(z=0 \cap N=1)$

Since z & N are independent $= Pr(z=0).Pr(N=0) + Pr(z=1).Pr(N=0) + Pr(z=0).Pr(N=1)$

$= \frac{1}{2}\left[1-p + p + 1-p\right]$

$\boxed{P(Y=0) = 1 - \frac{p}{2}}$

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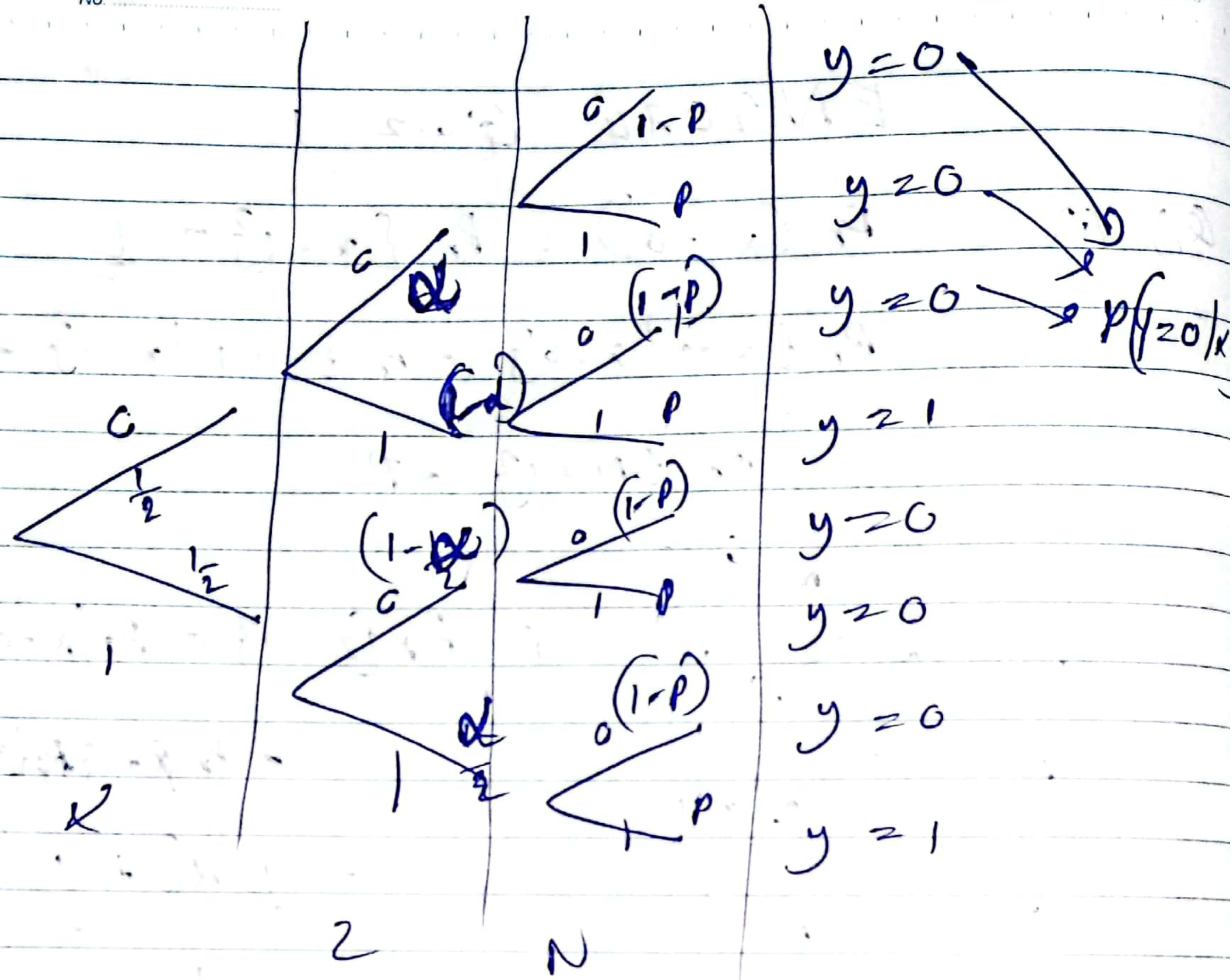
(iii)



$$\cancel{P[Y=0 / X=0) = \frac{1}{2}\left(\frac{1}{2}(1-p)\alpha\right) + \frac{1}{2}}$$

$$P(Y=0/X=0) = \frac{1}{2}\times\alpha\times(1-p) + \frac{1}{2}\times\alpha\times p + \frac{1}{2}\times(1-\alpha)\times(1-p)$$

$$= \frac{1}{2}\left[\alpha + (1-\alpha)(1-p)\right]$$

$$= \frac{1}{2}(\alpha + 1 - \alpha - p - \alpha p)$$

$$= \frac{1}{2}(1 - p - \alpha p)$$

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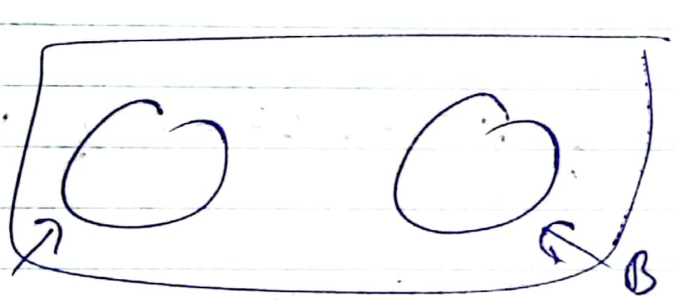
$$P(X=0/Y=0) = \frac{P(X=0)\; P(Y=0/X=0)}{P(Y=0)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}(1-p-\alpha p)}{\frac{(2-p)}{2}}$$

$$= \frac{(1-p-\alpha p)}{2(2-p)}$$

Q.2).



A closed

B closed.

$$Pr(X > 1 / A_{\text{closed}}) = Pr\left[Y > \frac{1-(-1)}{1}\right]$$

$$= Pr(Y > 2)$$

$$= 1 - Pr(y < 2)$$

$$= 1 - 0.97724$$

$$= 0.02276$$

$$Pr(X > 1 / B_{\text{close}}) = Pr\left[Y > \frac{1-1}{1}\right]$$

$$= Pr(Y > 0)$$

$$= 0.5$$

Since $Pr(A \text{ open} \cap B \text{ open}) = 0 \Rightarrow$ they are mutually exclusive

$Pr\{A\ Closed\} + Pr\{B\ Closed\} = 1$

$2\times Pr\{B\ Closed\} + Pr(B\ Closed) = 1$

$Pr\{B\ Closed\} = \frac{1}{3}$

$Pr\{A\ Closed\} = \frac{2}{3}$

$\therefore Pr\{X > 1\} = \frac{2}{3} \times 0.02276 + \frac{1}{3} \times 0.5$

$= 0.18184$

(ii) $Pr\{A_{Closed} \mid X > 1\} = \frac{Pr(A\ Close \cap X > 1)}{Pr(X > 1)}$

$= \frac{\frac{2}{3} \times 0.02276}{0.18184}$

$= 0.0834$

$Pr(B\ Closed \mid X > 1) = \frac{Pr\{B\ Closed \cap X > 1\}}{Pr\{X > 1\}}$

$= \frac{\frac{1}{3} \times 0.5}{0.18184} = 0.91656$

$\therefore$ Since $Pr\{B_{close} \mid X > 1\} > Pr\{A_{close} \mid X > 1\}$

B is the most likely source.