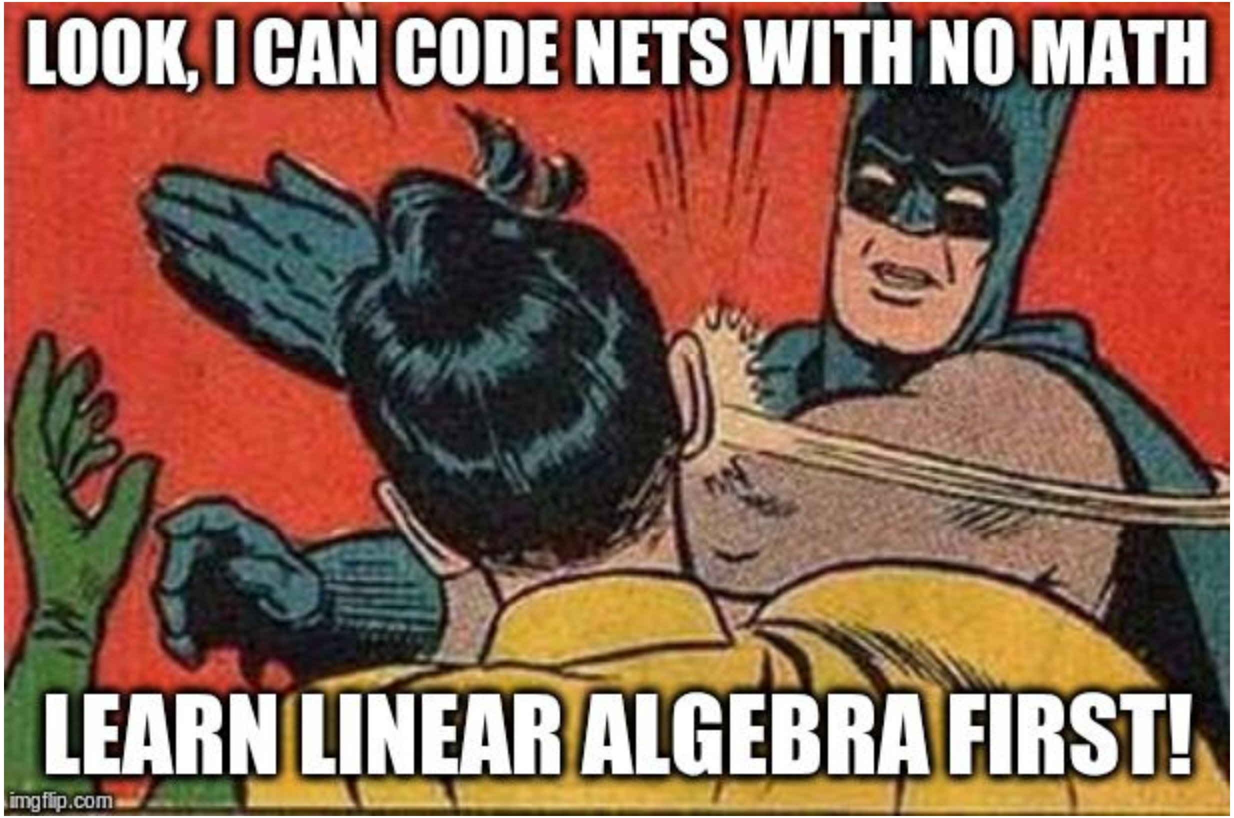
LOOK, I CAN CODE NETS WITH NO MATH
LEARN LINEAR ALGEBRA FIRST!
imgflip.com

Vector and matrix operations

Valerio Velardo

Vector

- Array of numbers

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Scalar operations

- Involve a vector and a number
- Addition/subtraction/multiplication/division

Scalar operations

$$\mathbf{a} + n = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + n = \begin{bmatrix} a_1 + n \\ a_2 + n \\ a_3 + n \end{bmatrix}$$

Scalar operations

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$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + 1 = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}}$$

Vector addition/subtraction

- Two vectors must have same dimension
- Element-wise operation

$$\mathbf{a} + \mathbf{b} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{bmatrix}$$

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$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 + 3 \\ 2 + 5 \\ 3 + 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}}$$

Dot product

- Two vectors involved
- Result is a scalar

$$\mathbf{a} \cdot \mathbf{b} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

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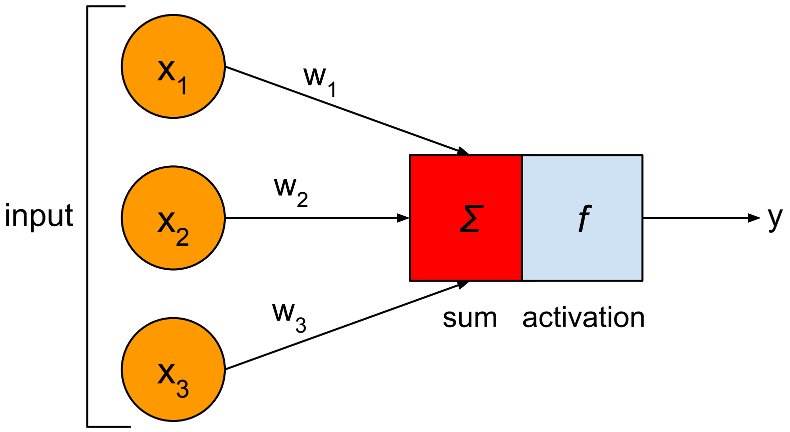
$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} = 1 \cdot 4 + 2 \cdot (-2) + 3 \cdot 1 = 3}$$

Revisiting the notation of the artificial neuron



$$h = \sum_i x_i w_i = x_1 w_1 + x_2 w_2 + x_3 w_3$$

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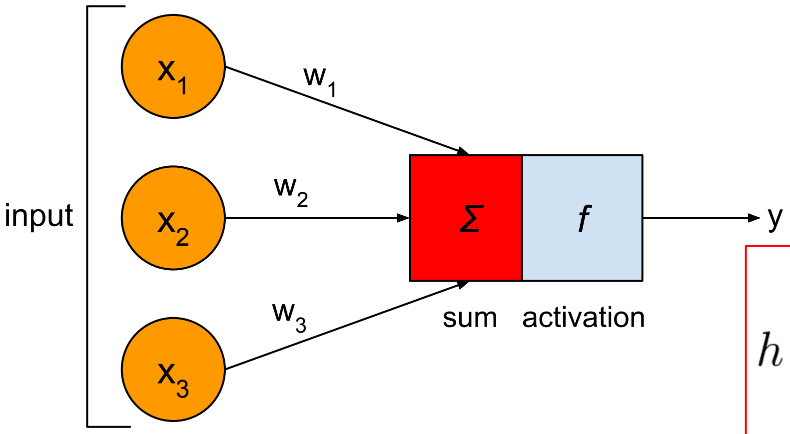

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Matrices

- Rectangular grid of numbers (like a spreadsheet)

$$A_{i,j} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

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- Dimension indicated by # rows and columns

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$$\begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 0.5 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 3 \\ 1.2 & -3 & 2 \end{bmatrix}$$

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(3,2) (2,3)

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$$\boxed{A^T_{ij} = A_{ji}}$$

Scalar operations

- Addition/subtraction/multiplication/division of matrix with a number

$$nA = n \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} na_{11} & na_{12} \\ na_{21} & na_{22} \\ na_{31} & na_{32} \end{bmatrix}$$

Matrix addition/subtraction

- Matrices must have same dimension
- Element-wise operation

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+1 & b+2 \\ c+3 & d+4 \end{bmatrix}$$

Matrix multiplication

- # of columns of the 1st matrix must be equal # of rows of the 2nd
- Product of an (m,n) matrix and an (n,k) matrix is an (m,k) matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1a + 2c & 1b + 2d \\ 3a + 4c & 3b + 4d \\ 5a + 6c & 5b + 6d \end{bmatrix}$$

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$$\boxed{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix} = 1a + 2c}$$

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Matrix multiplication

$$\begin{array}{c c c} & & \begin{array}{cc} \mathbf{b}_1 & \mathbf{b}_2 \end{array} \\ \begin{array}{c} \mathbf{a}_1 \\ \mathbf{a}_2 \end{array} & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & \begin{bmatrix} 2 & -2 \\ 1 & 5 \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{a}_1 \cdot \mathbf{b}_1 & \mathbf{a}_1 \cdot \mathbf{b}_2 \\ \mathbf{a}_2 \cdot \mathbf{b}_1 & \mathbf{a}_2 \cdot \mathbf{b}_2 \end{bmatrix}$$

What’s up next?