Bayesian mixed modeling for private passenger air- and landspeeder insurance

(A thought experiment)

A long time ago at an insurance company far, far away . . .

CALRISSIAN HOLDING

. . . has just won a small book of Private Passenger Land- and Airspeeder policies in a card game.

The owner of this holding company has asked you to evaluate this book of business and come up with a segmented rate classification plan for new risks going forward . . .

Episode 1

What are we insuring?

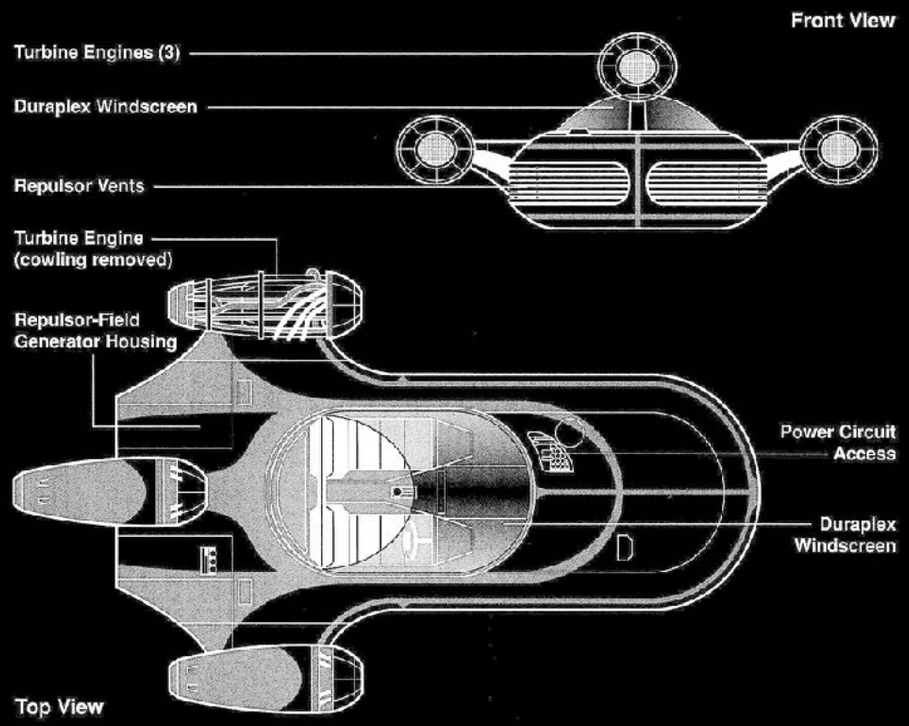
Tatooine

- Desert planet
- Very populated
- Residents from all walks of life
- ~20k speeders insured

Hoth

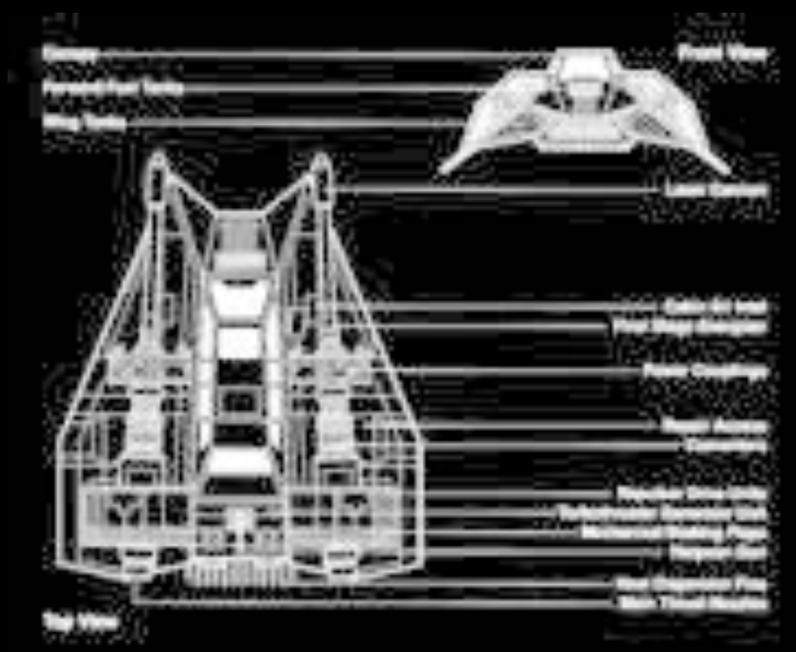
- Ice planet
- Thinly populated
- Population is mainly skilled workers
- ~2k speeders insured

SoroSuub X-34



- Very agile
- Light cargo
- Useless in weather

Incomm T-47



- Modified for weather
- Not very agile in populated areas

Jedi pilots

Experienced pilots

Inexperienced pilots

Episode 2

The ‘Galaxywide’ model

Features

Independent variable

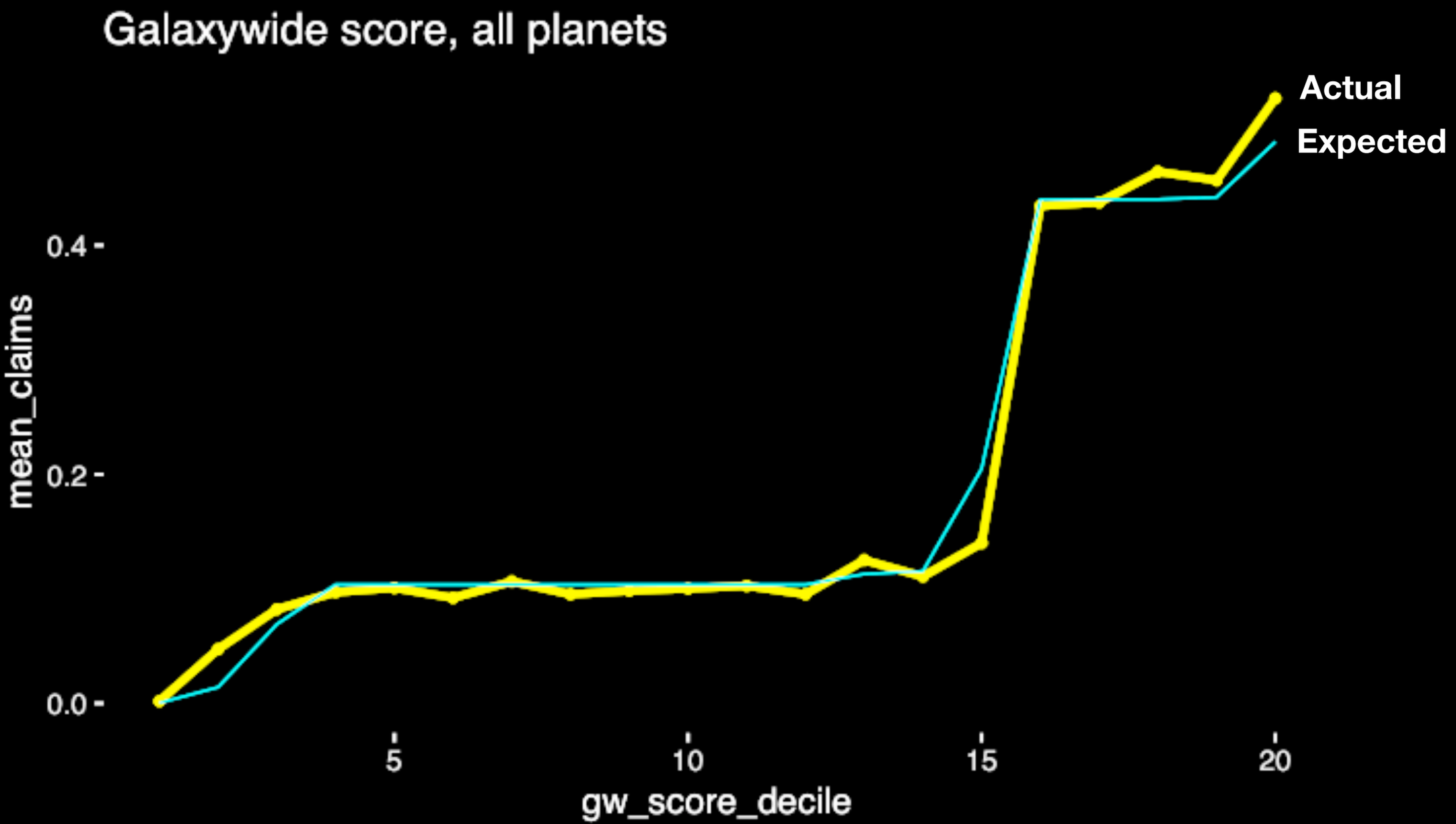
pilot_id	planet	vehicle	pilot	total_claims
00023	Tatooine	X-34	Inexperienced	0
00023	Tatooine	X-34	Inexperienced	0
00023	Tatooine	X-34	Inexperienced	0
00024	Tatooine	T-37	Experienced	1
00024	Tatooine	T-37	Experienced	0

One row per policy configuration

$$(Y \mid \mathbf{x}) \sim \text{Pois}(\exp(\beta_0 + \beta' \mathbf{x}))$$

(All configurations have a period of exposure of one galactic year)

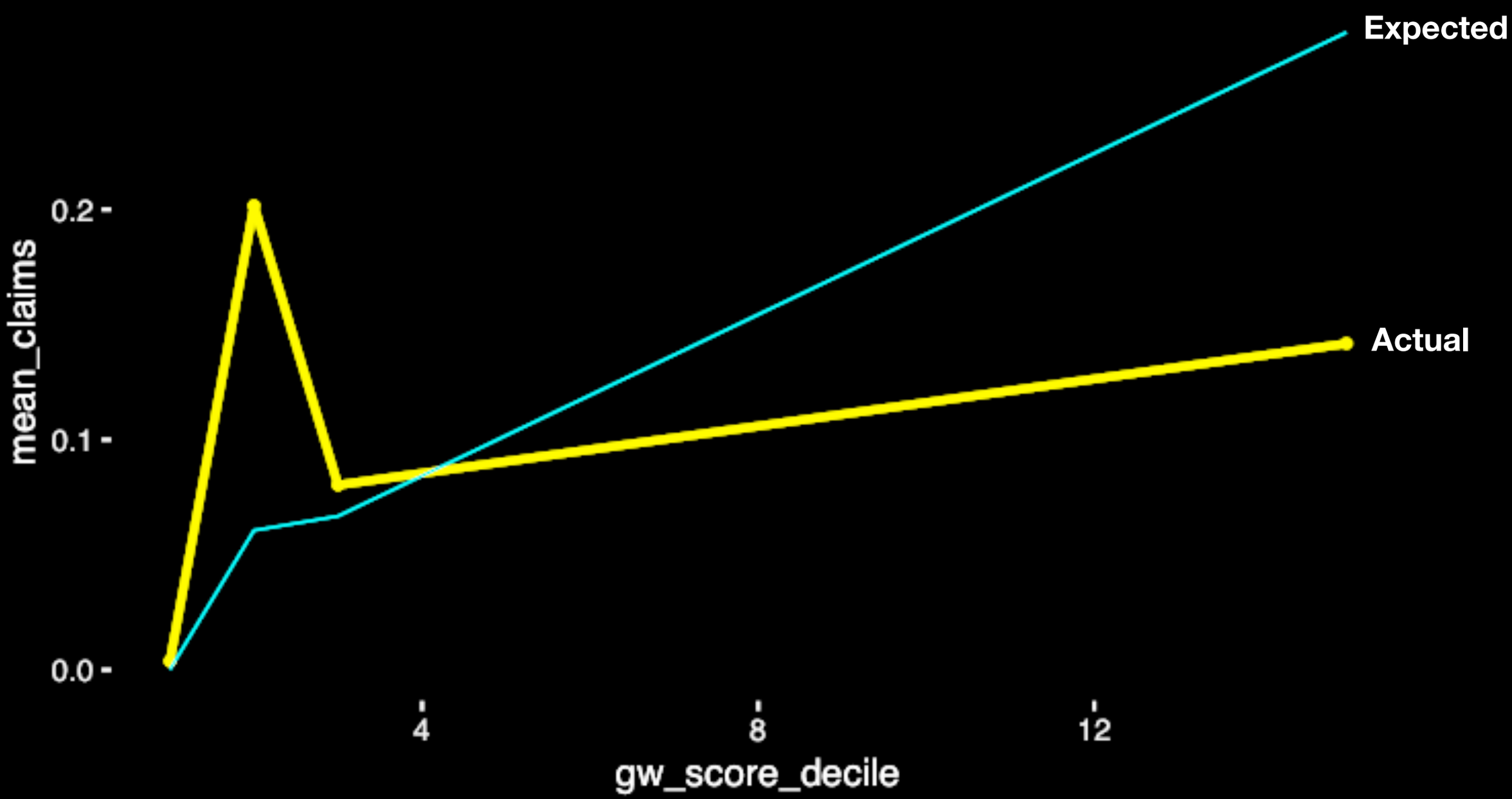
```
Call:
glm(formula = total_claims ~ planet + pilot + vehicle, family = poisson(link = "log"),
    data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.9903  -0.4806  -0.4554  -0.0277   4.1876

Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)      -2.26651    0.02329 -97.327  < 2e-16 ***
planetb.) Hoth   -0.53729    0.05891  -9.121  < 2e-16 ***
pilotb.) Youthful  1.44592    0.02723  53.096  < 2e-16 ***
pilotc.) Jedi     -5.70908    0.99954  -5.712 1.12e-08 ***
vehicleb.) T-47    0.10795    0.03124   3.455  0.00055 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 22559  on 32037  degrees of freedom
Residual deviance: 18303  on 32033  degrees of freedom
AIC: 29025

Number of Fisher Scoring iterations: 9
```

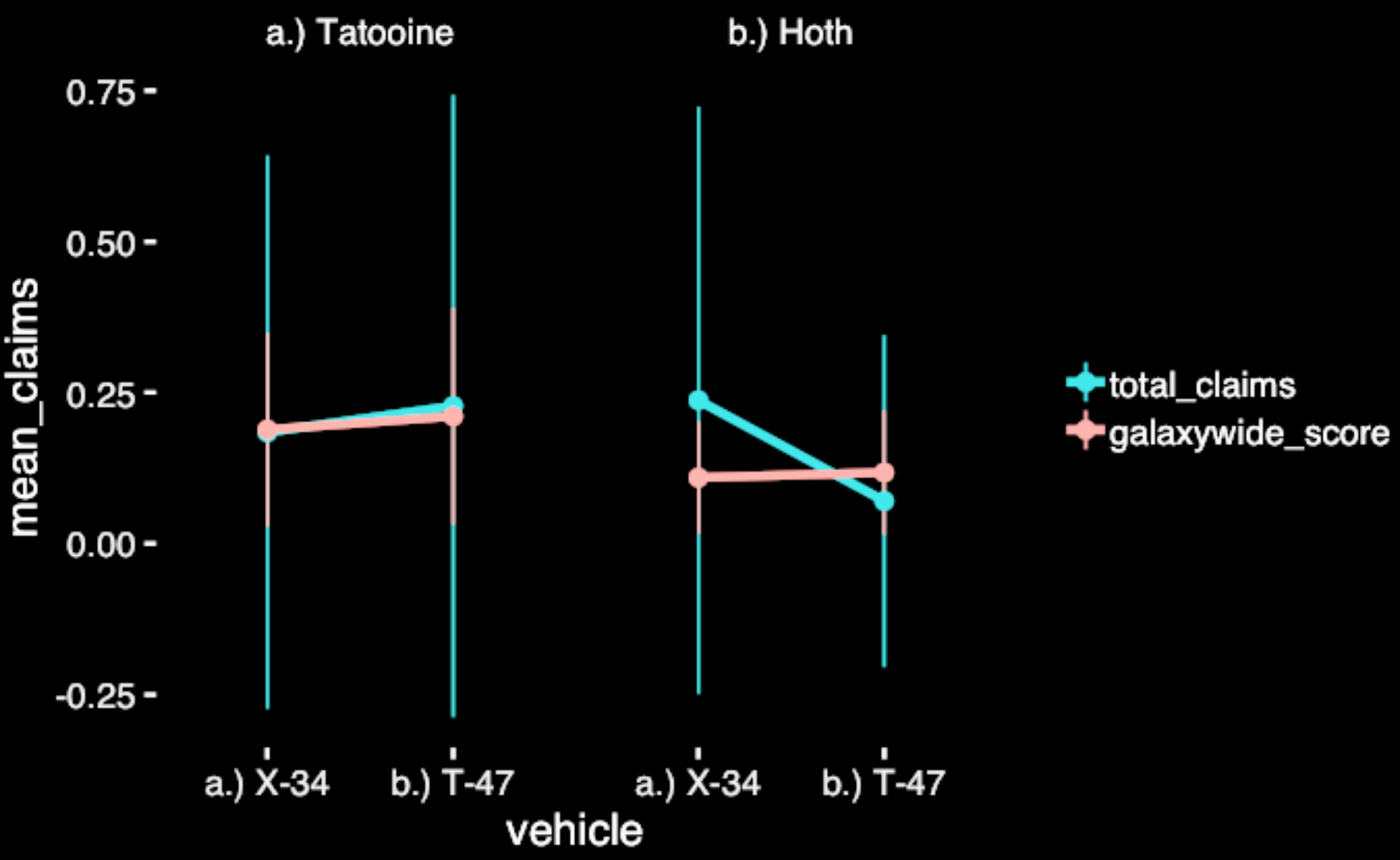

Galaxywide score, all planets
Actual
Expected
0.4
0.2
0.0
mean_claims
5
10
15
20
gw_score_decile

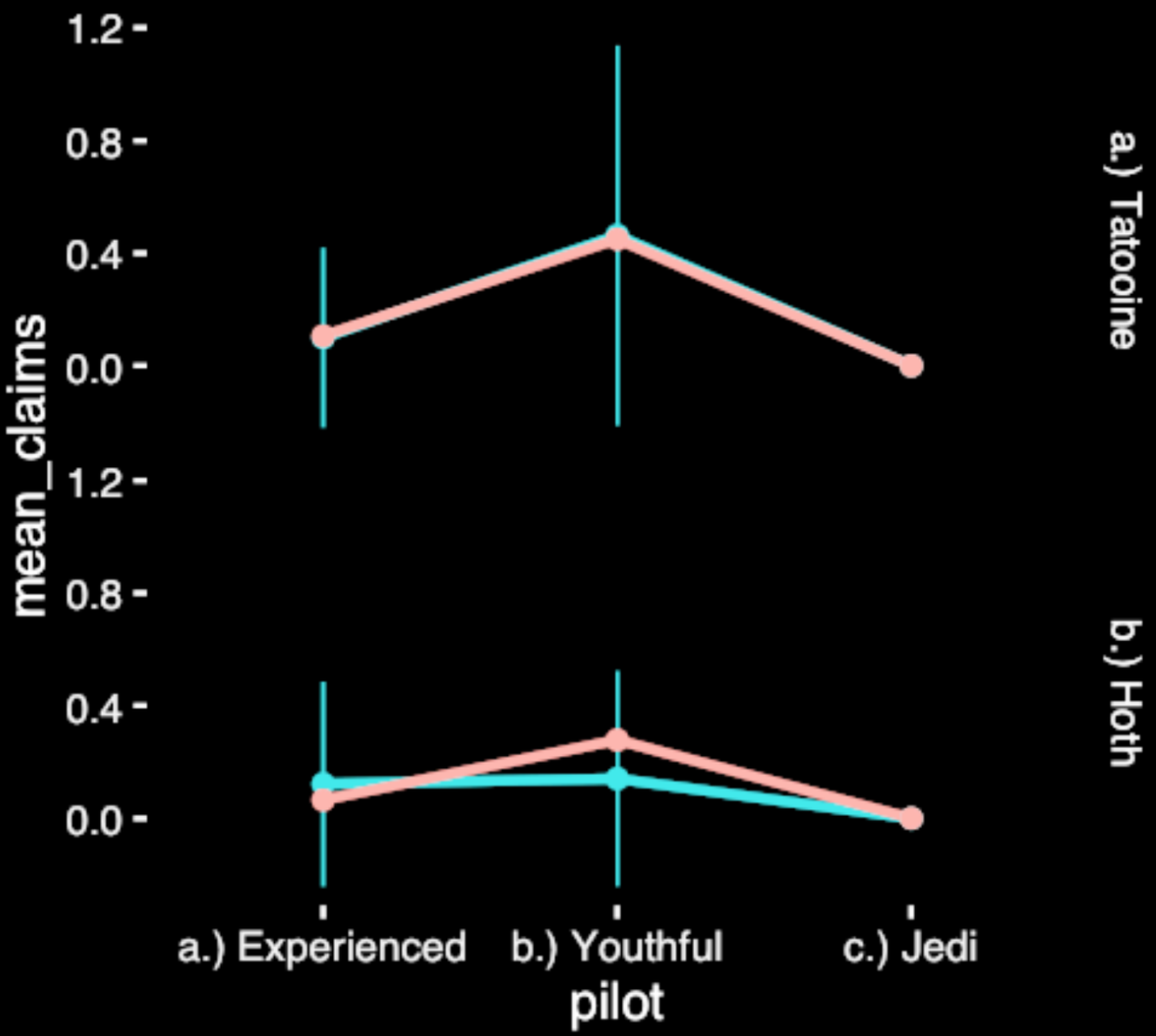
Episode 3

For some reason, you start selling a lot of X-34 policies on Hoth

Galaxywide score, Hoth




a.) Tatooine
b.) Hoth
0.75
0.50
0.25
0.00
-0.25
mean_claims
a.) X-34
b.) T-47
a.) X-34
b.) T-47
vehicle
total_claims
galaxywide_score


1.2
0.8
0.4
0.0
1.2
0.8
0.4
0.0
mean_claims
a.) Experienced
b.) Youthful
c.) Jedi
pilot
a.) Tatooine
b.) Hoth
total_claims
galaxywide_score

You might try fitting a Hoth-only model

```
Call:
glm(formula = total_claims ~ pilot + vehicle, family = poisson(link = "log"),
    data = train[train$planet == "b.) Hoth", ])

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.7535  -0.4132  -0.3890  -0.3890   3.0617

Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)       -1.3799     0.0835 -16.525   <2e-16 ***
pilotb.) Youthful  0.1207     0.1175   1.027    0.304
pilotc.) Jedi    -16.0633   321.2075  -0.050    0.960
vehicleb.) T-47   -1.2015     0.1106 -10.867   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 1513.4  on 2866  degrees of freedom
Residual deviance: 1327.5  on 2863  degrees of freedom
AIC: 1955.7

Number of Fisher Scoring iterations: 16
```

This doesn’t look too good . . .

Episode 4

Bayesian mixed modeling

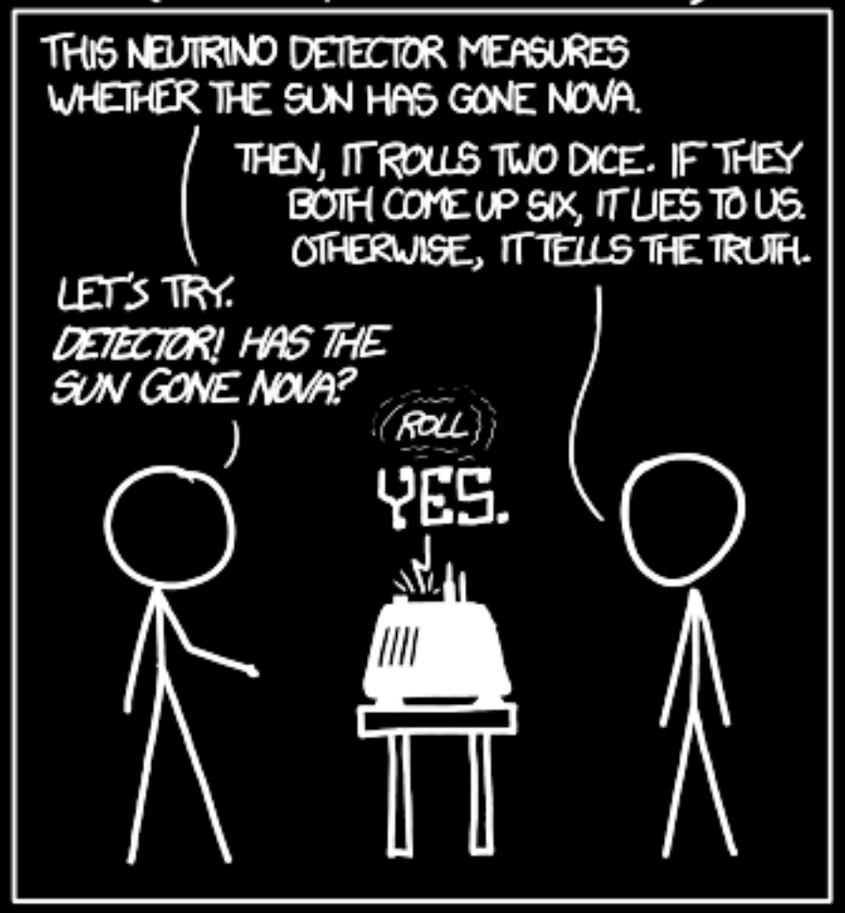
Why Bayesian?

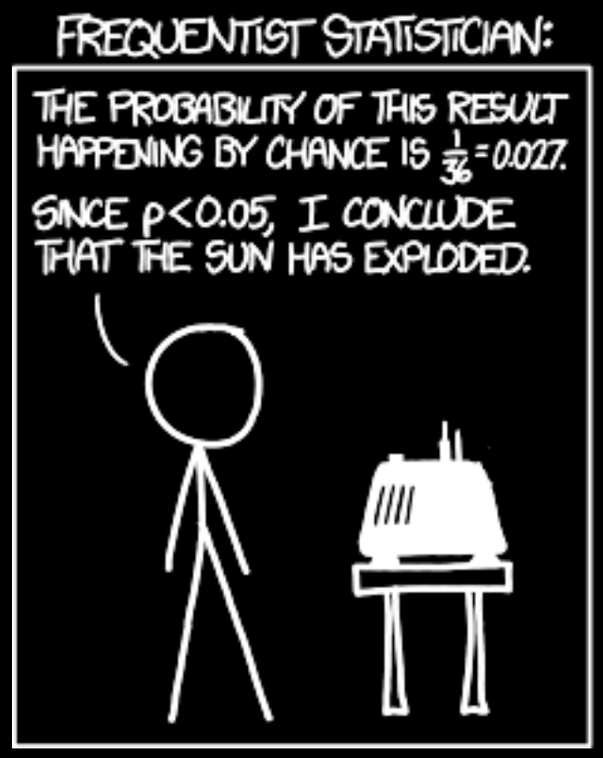
It's perfectly reasonable to attempt to fit Generalized Linear Mixed Models through maximizing likelihood (or AIC), but:

- **The number of parameters to estimate grows very quickly, and you don't have to add too many random effects before you need tons of data**
- **There isn't an analytic solution, and numerical optimization is difficult.**
  - **I had a hard time with `lme4` on simulated data for this thought experiment**

Markov chain Monte Carlo (MCMC) is also very computationally complex, but with the right parameterization it is possible to get very good estimates.

DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)





What's really going on here?

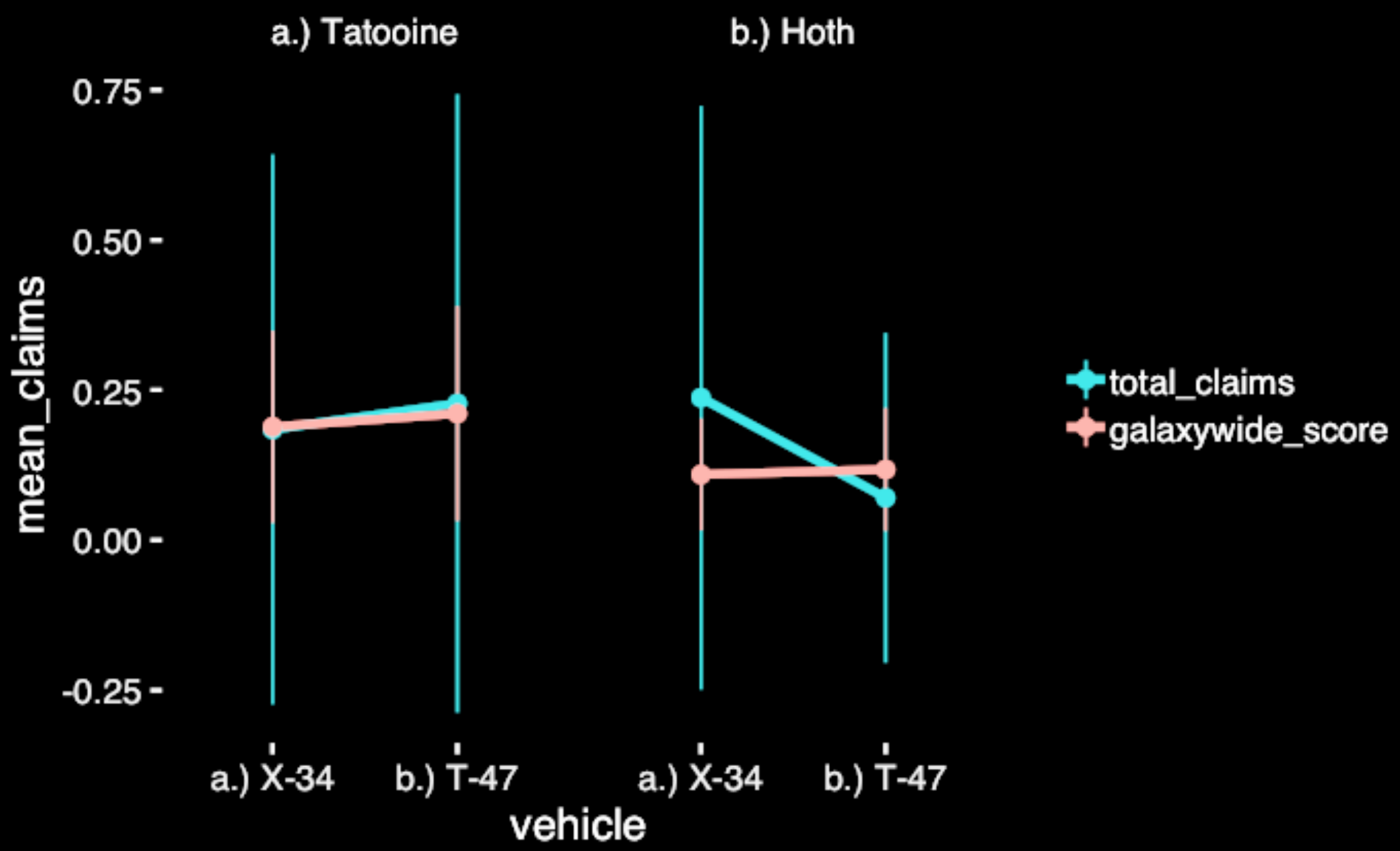
- **The winter-ready speeders fare a lot better in a wintery environment, and worse in densely populated areas**
- **Inexperienced drivers on Hoth are likely skilled workers with training**
- **Very imbalanced exposure makes good estimates of different parameters for each planet difficult**

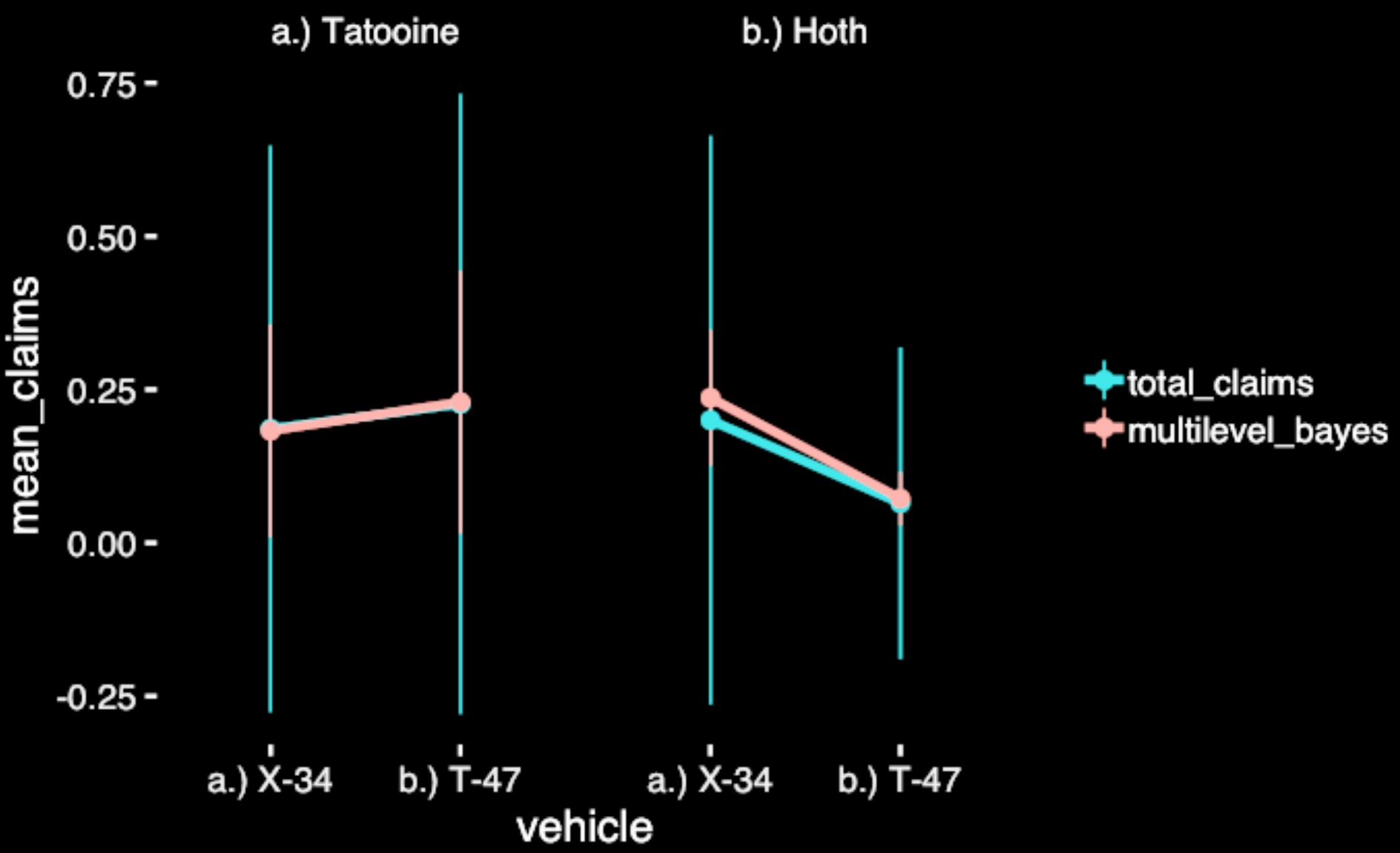
```
total_claims ~ (1 | planet) +
               pilot + vehicle +
               (0 + pilot + vehicle | planet)
```

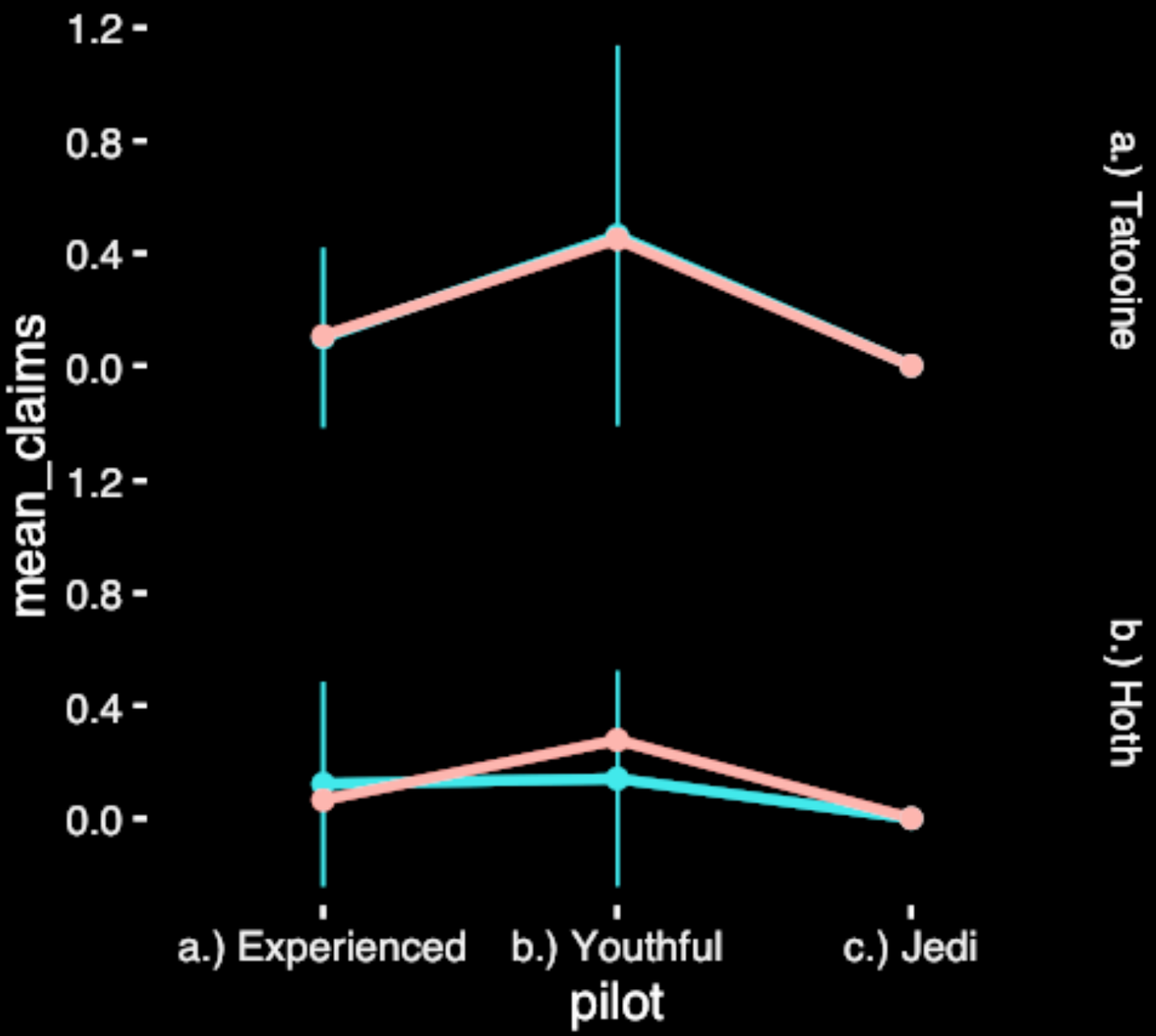
Each planet is a group with its own intercept (base rates are determined in aggregate state by . . . er, ‘planet by planet’ anyhow).

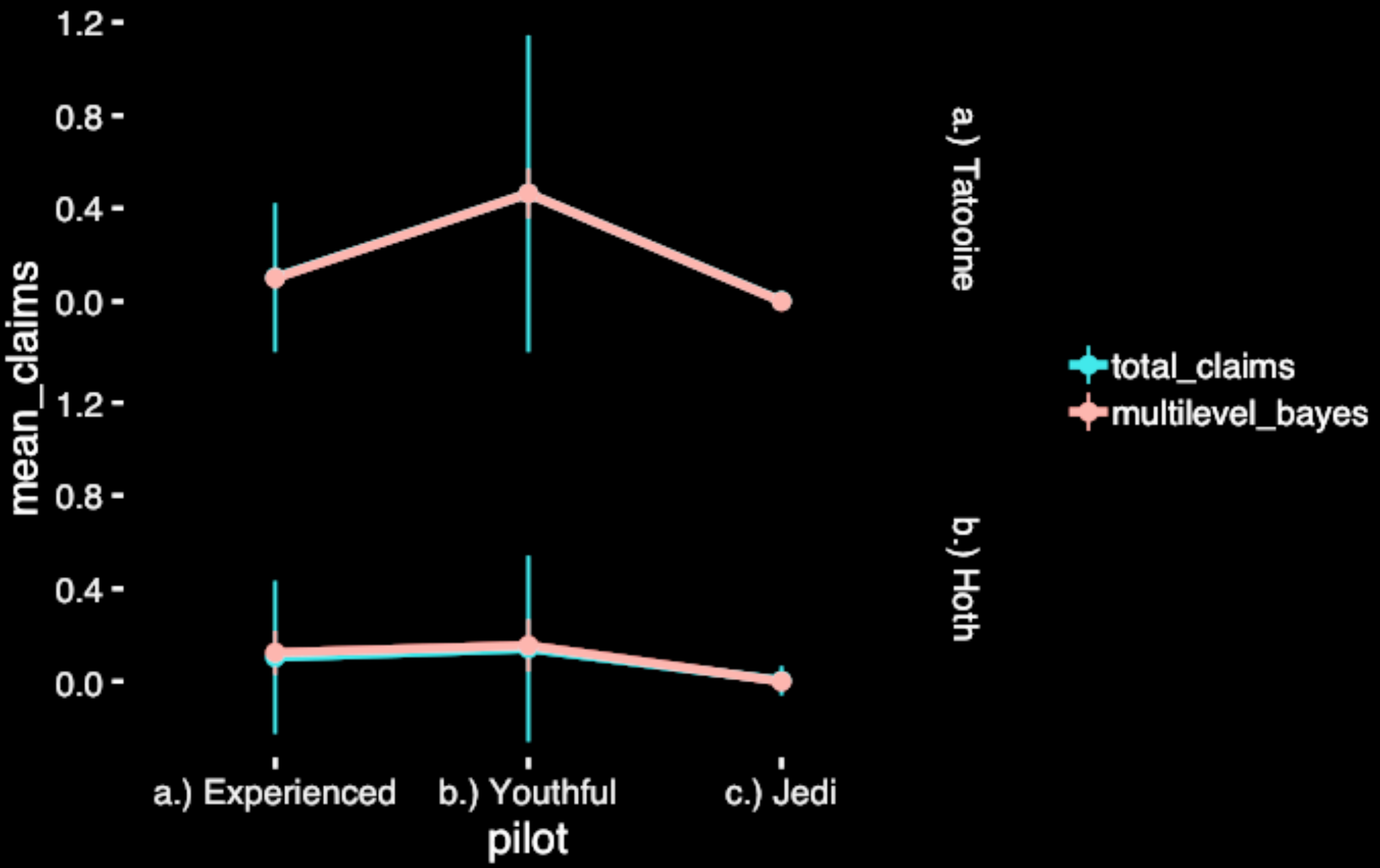
Driver and vehicle characteristics have population level effects, but these effects may differ by state (i.e., the risk characteristics might mean something different in a different environment).

The model shown above allows for independent group-level adjustments from population-level parameters (`vehicle` and `pilot`).


a.) Tatooine
b.) Hoth
0.75
0.50
0.25
0.00
-0.25
mean_claims
a.) X-34
b.) T-47
a.) X-34
b.) T-47
vehicle
total_claims
galaxywide_score


a.) Tatooine
b.) Hoth
0.75
0.50
0.25
0.00
-0.25
mean_claims
a.) X-34
b.) T-47
a.) X-34
b.) T-47
vehicle
total_claims
multilevel_bayes


1.2
0.8
0.4
0.0
1.2
0.8
0.4
0.0
mean_claims
a.) Experienced
b.) Youthful
c.) Jedi
pilot
a.) Tatooine
b.) Hoth
total_claims
galaxywide_score


1.2
0.8
0.4
0.0
1.2
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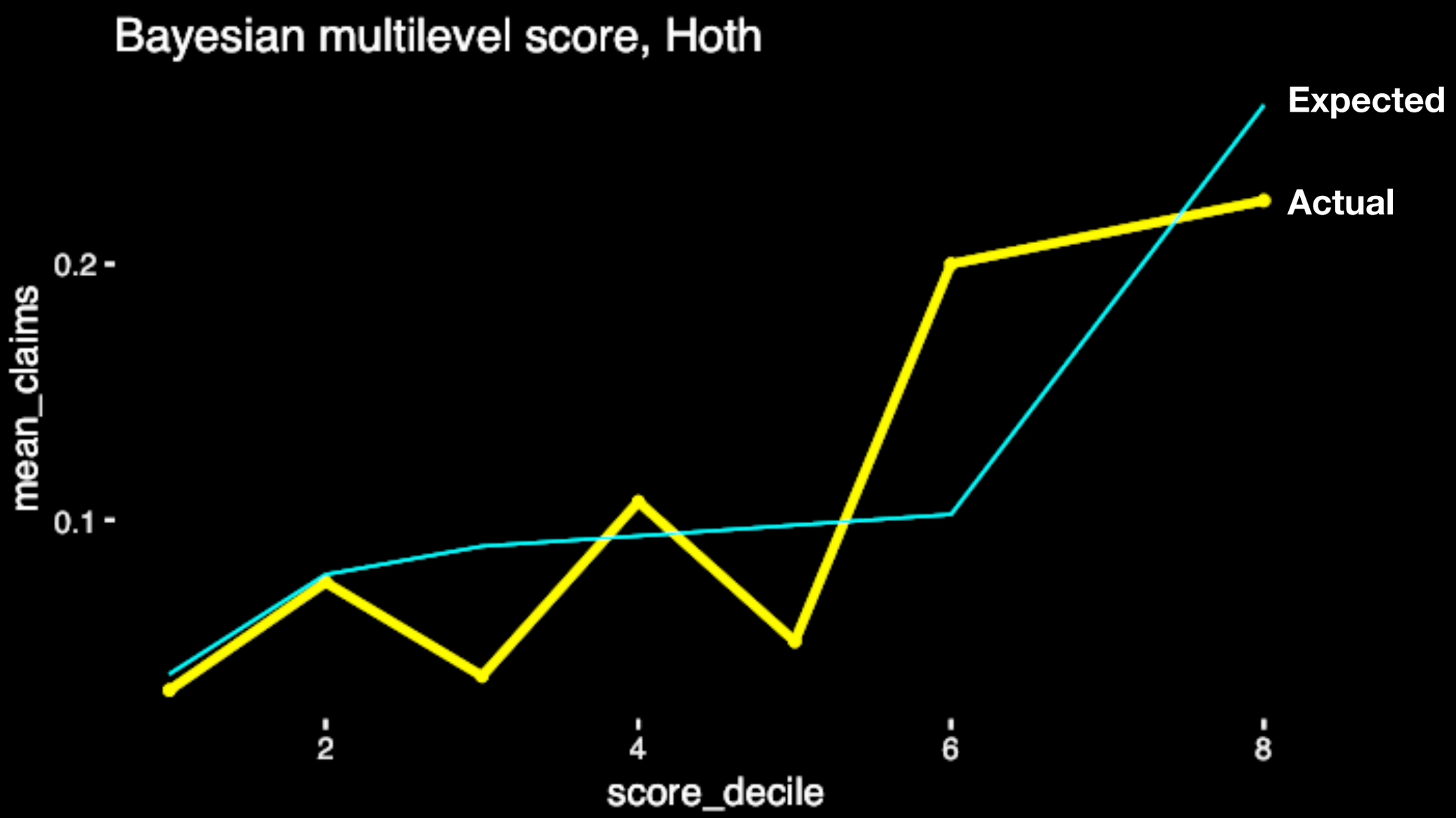
0.4
0.0
mean_claims
a.) Experienced
b.) Youthful
c.) Jedi
pilot
a.) Tatooine
b.) Hoth
total_claims
multilevel_bayes



2 chains, 1000 iterations

This still doesn’t look great, but recall there are only 2,000 speeders on Hoth; there are 310 claims in the holdout set

One more thing . . .

These are the same person

pilot_id	planet	vehicle	pilot	total_claims
00023	Tatooine	X-34	Inexperienced	0
00023	Tatooine	X-34	Inexperienced	0
00023	Tatooine	X-34	Inexperienced	0
00024	Tatooine	T-37	Experienced	1
00024	Tatooine	T-37	Experienced	0

GLMs assume each observation is independent of the rest, but at many carriers it is an accepted practice to model each configuration and term of a policy as if it were independent, even though it may be the same individual, and even the same vehicle.

Mixed effects models are a standard way to model repeated measurements of the same subject.

THANK YOU

All code is available at
https://github.com/madwsa/sw-bayesian