



CL-643 Project

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Problem statement

One of the classic problem of stock market is how to construct an investment portfolio that maximises returns while managing risks. This involves in selecting a combination of assets (stocks, bonds, currencies, etc) that provides the best tradeoff between risk and returns. Since the future returns of securities are unknown at the time of the investment decision is made, portfolio selection problem can be categorised as one of the decision-making under risk.



Mathematical Formulation

Following are symbolic representation of general Terms

- n = Number of Assets in Portfolio
- r_i = Expected Returns of asset i
- w_i = Weight(Allocation) of asset i in portfolio. **(Decision Variable)**
- $\text{Cov}(r_i, r_j)$ = Covariance between returns of asset i and j .
- r_p = Expected Return of Portfolio
- σ_p = Standard Deviation of Portfolio (Risk)
- λ = Risk Aversion Parameter



Constraints

Budget Constraint: The sum of the weights should equal 1 to represent a fully invested portfolio.

$$\sum w_i = 1 \text{ (i=1 to n)}$$

Risk Constraint: Control the portfolio's risk (variance) by setting an acceptable level of risk, often measured as portfolio standard deviation (σ_p). This constraint ensures that the portfolio risk is within an acceptable range.

$$\sigma_p \leq$$

$$\sigma_{\text{target}}$$

Asset Allocation Constraint: This is basically the lower bounds and upper bounds to the weights.

$$w_{\min} \leq w_i \leq w_{\max} \text{ (i=1 to n)}$$



Objective Functions

Our goal is to maximise the expected return of portfolio within the targeted risk. This can be expressed as

Maximize: $r_p = \sum (w_i * r_i) \text{ (i=1 to n)}$

Clubbing the above objective function i.e, maximisation of returns with minimisation of risk we get

Mean-Variance Objective function

Maximize $U = r_p * (1 - \lambda) - \lambda * \sigma_p$



Dataset

SOURCE: [Click Here](#)

We use real data for the 10-year period 1990-01-01 to 2000-01-01. Data cover:

23 Italian Stock indices

3 Italian Bond indices (1-3yr, 3-7yr, 5-7yr)

7 international Govt. bond indices

5 Regions Stock Indices: (EMU, Eur-ex-emu, PACIF, EMER, NORAM)

US Corporate Bond Sector Indices (Finance, Energy, Life Ins.)

Exchange rates, ITL to: (FRF, DEM, ESP, GBP, US, YEN, EUR) Also US to EUR.

We choose subset of 8 assets from above as an input dataset



Solution Models

Metaheuristic Model

Solution Model for Metaheuristic Techniques was coded in Matlab using subset of above dataset of about 8 decision variables.

Algorithm Used:

- TLBO
- DE
- PSO

Mathematical Programming Model

Solution model for mathematical programming was developed on GAMS using same subset of above data and also implemented mean-variance optimisation.

Solver Used: Used NLP (i.e, non-linear programming) since standard deviation equation which is risk term in not linear.



Outcomes- Metaheuristic Technique

TLBO	0.1439
DE	0.1444
PSO	No Feasible Solution

Outcomes- Mathematical Programming

MeanVarianceFrontier											
Lambda	z	Variance	ExpReturn	YRS_1_3	EMU	EU_EX	PACIFIC	EMERGT	NOR_AM	CASH_EU	ITMHIST
0.0000	0.2380	0.9355	0.2380	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.1000	0.1396	0.4653	0.2068	0.0000	0.0000	0.0000	0.0000	0.3936	0.6064	0.0000	0.0000
0.2000	0.0982	0.1245	0.1539	0.3452	0.0000	0.0000	0.0000	0.1859	0.4690	0.0000	0.0000
0.3000	0.0762	0.0504	0.1304	0.3560	0.0000	0.0000	0.0000	0.1155	0.3289	0.1996	0.0000
0.4000	0.0601	0.0276	0.1185	0.3583	0.0000	0.0000	0.0000	0.0803	0.2562	0.3052	0.0000
0.5000	0.0464	0.0186	0.1114	0.3597	0.0000	0.0000	0.0000	0.0591	0.2126	0.3686	0.0000
0.6000	0.0338	0.0147	0.1066	0.3606	0.0000	0.0000	0.0000	0.0450	0.1835	0.4108	0.0000
0.7000	0.0220	0.0128	0.1032	0.3613	0.0000	0.0000	0.0000	0.0350	0.1628	0.4410	0.0000
0.8000	0.0106	0.0118	0.1001	0.3561	0.0000	0.0000	0.0000	0.0255	0.1421	0.4690	0.0073
0.9000	0.0000	0.0113	0.0972	0.3478	0.0000	0.0000	0.0000	0.0166	0.1221	0.4949	0.0185
1.0000	-0.011	0.0111	0.0949	0.3411	0.0000	0.0000	0.0000	0.0096	0.1062	0.5156	0.0275

Applications



Applications in Industry

As coming generations enhance their financial literacy, their inclination to invest in assets that are both secure and lucrative has grown. This trend has led many individuals to explore stocks and opt for **mutual funds**. A mutual

fund, overseen by a proficient Fund Manager, functions as a collective investment trust that pools funds from a multitude of investors sharing a common investment objective. The amassed capital is then strategically allocated across a diversified portfolio, encompassing equities, bonds, money market instruments, and various other securities.

This rise in interest have constantly elevated the demand for portfolio management and optimization. As a result, there is a great focus on research and development to devise improved approaches that takes real-life constraints into account.



Current Industry Approach

There are many new mathematical formations which also looks into real life constraint such as transaction costs, management expenses. There are many new algorithm which solves complex business model using **Stochastic Programming**. Algorithms using **Fuzzy-logic** and **Neural Networks (ANN)** are also being devised.



References

1. <https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=dc376e785b58a68cbc5cbe8e4baef6a4ad440c80>(Series of papers were referred from this)
2. <https://www.sciencedirect.com/science/article/pii/S2214716019300399>