Proportional reasoning across formats

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Proportional reasoning across formats

Introduction

Comparing proportions is sometimes very hard! But, even infants seem to be able to do it a little bit. The purpose of this science project was to better understand how well people compare proportions when the proportions are presented in different formats. The purpose of this class assignment is to take the R-code and plots we've been generating over the last several weeks and put it all together into one poster format.

Research Objectives:

1.Does average performance vary across format type? 2.Does average performance vary across numerator congruency status? 3.Does numerator congruency vary across format type?(ie., is there an interaction)

Participants

A total of 99 adults participated in the study.

Methods

First participants were introduced to a story about a magic ball and that the outcome(ie., blue or orange) depended on the proportions. They were then asked to compare the proportions of different images.

In other words, participants were shown two images of the same kind at the same time and asked to decided which had a higher proportion of the shape (or dots) colored in blue.

Conditions There were four different conditions that changed what kinds of images the participants saw:

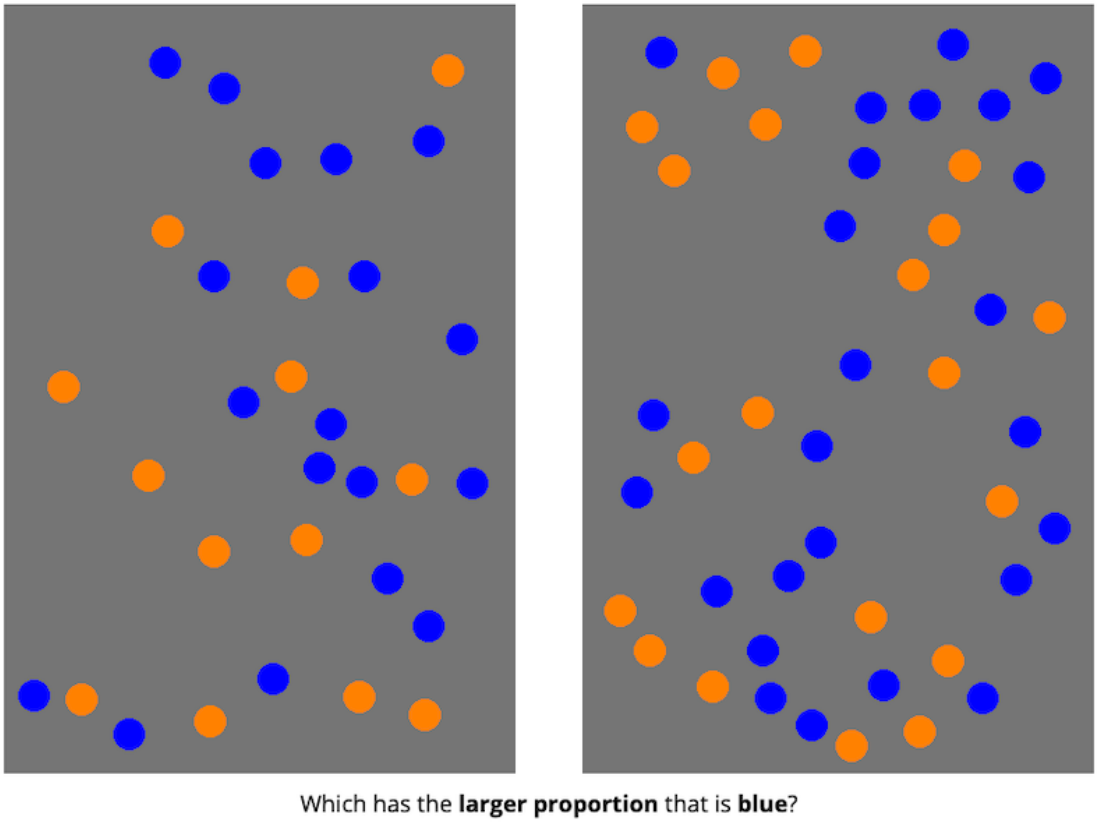

Figure 1. : An example of integrated blobs

- divided blobs: blur and orange were entirely separate.
- integrated blob: one blob, divided to be part blue and part orange.
- separated dots: blue and orange dots were on opposite sides of the image.
- integrated dots: blue and orange dots were intermixed.

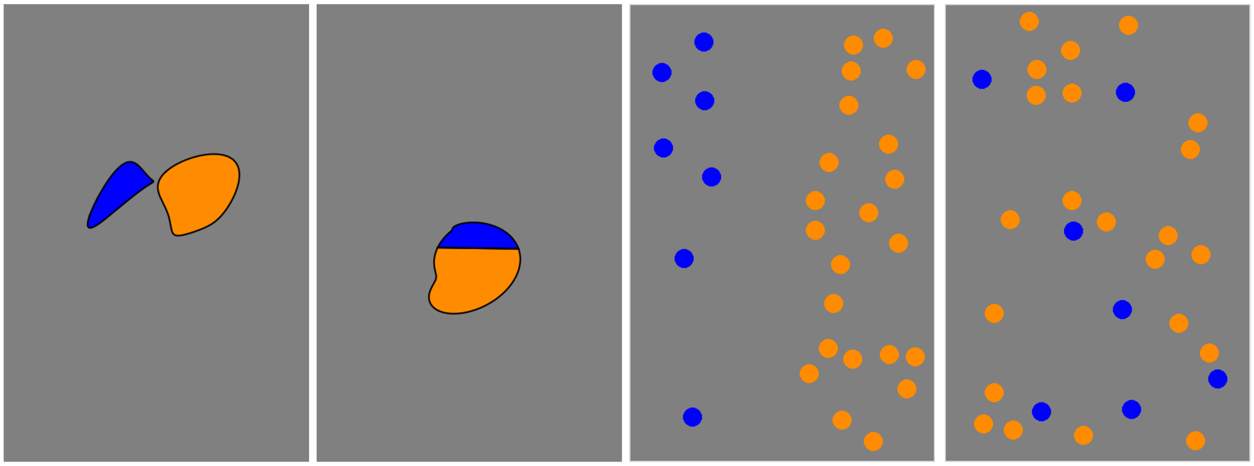

Figure 2. From left to right : divided blobs, integrated blobs, separated dots, integraed dots

Results

1. Does average performance vary across format type, ignoring all other aspects of the stimuli?

Yes, the blue dots in the above plot prove that the average performance varies across formatting types.

list(condition = c("blob_shifted", "blob_stacked", "blob_stacked", "blob_stacked", "blob_stacked",

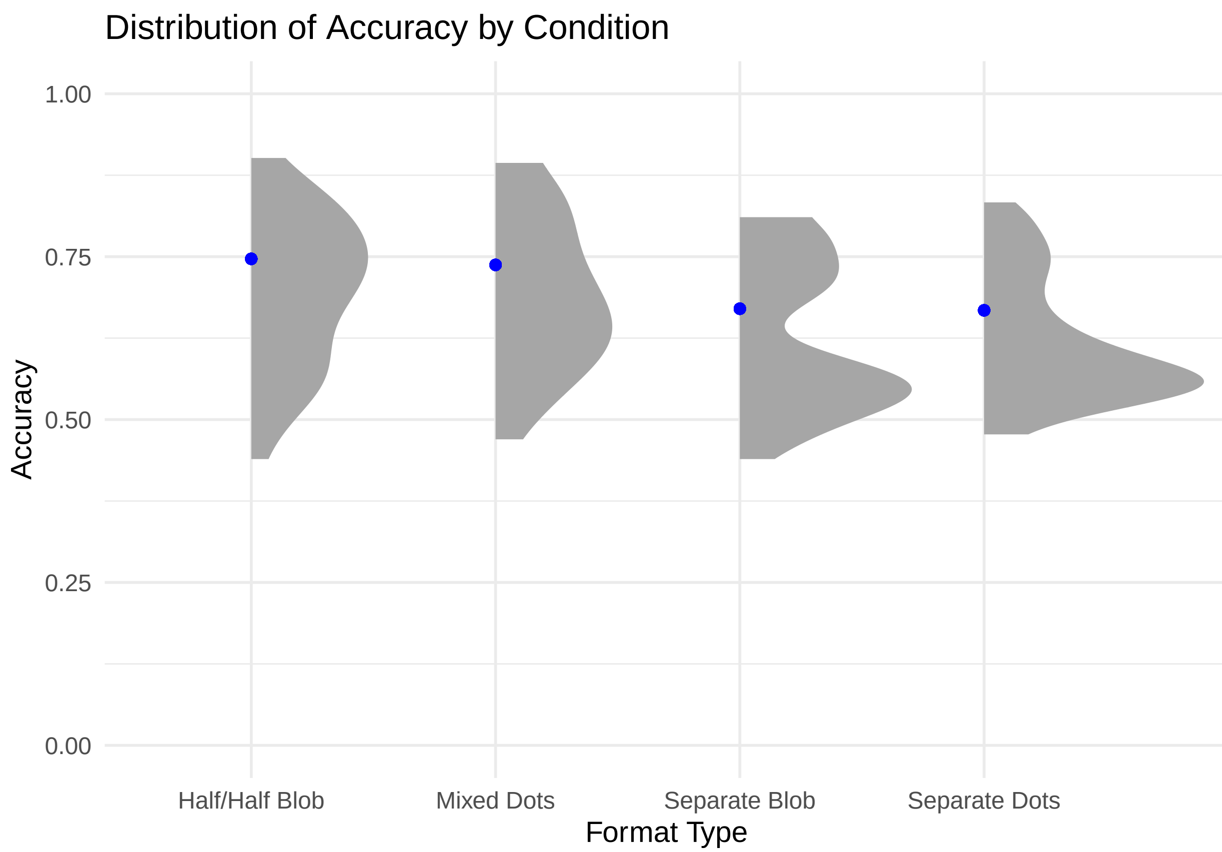

Figure 3. Plot of Distribution of Accuracy by Condition

“blob_stacked”, “dots_EqSizeRand”, “dots_EqSizeRand”, “dots_EqSizeRand”, “dots_EqSizeRand”, “dots_EqSizeRand”, “dots_EqSizeRand”, “dots_EqSizeRand”,

“dots_EqSizeRand”, “dots_EqSizeSep”, “dots_EqSizeSep”), mean_prop_corr = c(0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515, 0.620151515151515,

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Blob”, “Half/Half Blob”, “Mixed Dots”, “Separate Dots”, “Separate

Dots", "Separate Dots", "Separate Dots", "Separate Dots", "Separate Dots")), list(,), , , list(x = ~condition_name, y = ~prop_corr), list(line = list(colour = "black", linewidth = 0.5, linetype = 1, lineend = "butt", arrow = FALSE, inherit.blank = TRUE), rect = list(fill = "white", colour = "black", linewidth = 0.5, linetype = 1, inherit.blank = TRUE), text = list(family = " ", face ="plain", colour = "black", size = 11, hjust = 0.5, vjust = 0.5, angle = 0, lineheight = 0.9, margin = c(0, 0, 0, 0), debug = FALSE, inherit.blank = TRUE), title = NULL, aspect.ratio = NULL, axis.title = NULL, axis.title.x = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 1, angle = NULL, lineheight = NULL, margin = c(2.75, 0, 0, 0), debug = NULL, inherit.blank = TRUE), axis.title.x.top = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 0, angle = NULL, lineheight = NULL, margin = c(0, 0, 2.75, 0), debug = NULL, inherit.blank = TRUE), axis.title.x.bottom = NULL, axis.title.y = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 1, angle = 90, lineheight = NULL, margin = c(0, 2.75, 0, 0), debug = NULL, inherit.blank = TRUE), axis.title.y.left = NULL, axis.title.y.right = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 1, angle = -90, lineheight = NULL, margin = c(0, 0, 0, 2.75), debug = NULL, inherit.blank = TRUE), axis.text = list(family = NULL, face = NULL, colour = "grey30", size = 0.8, hjust = NULL, vjust = NULL, angle = NULL, lineheight = NULL, margin = NULL, debug = NULL, inherit.blank = TRUE), axis.text.x = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 1, angle = NULL, lineheight = NULL, margin = c(2.2, 0, 0, 0), debug = NULL, inherit.blank = TRUE), axis.text.x.top = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = 0, angle = NULL, lineheight = NULL, margin = c(0, 0, 2.2, 0), debug = NULL, inherit.blank = TRUE), axis.text.x.bottom = NULL, axis.text.y = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = 1, vjust = NULL, angle = NULL, lineheight = NULL, margin = c(0, 2.2, 0, 0), debug = NULL, inherit.blank = TRUE), axis.text.y.left = NULL, axis.text.y.right = list(family = NULL,

face = NULL, colour = NULL, size = NULL, hjust = 0, vjust = NULL, angle = NULL, lineheight = NULL, margin = c(0, 0, 0, 2.2), debug = NULL, inherit.blank = TRUE), axis.text.theta = NULL, axis.text.r = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = 0.5, vjust = NULL, angle = NULL, lineheight = NULL, margin = c(0, 2.2, 0, 2.2), debug = NULL, inherit.blank = TRUE), axis.ticks = list(), axis.ticks.x = NULL, axis.ticks.x.top = NULL, axis.ticks.x.bottom = NULL, axis.ticks.y = NULL, axis.ticks.y.left = NULL, axis.ticks.y.right = NULL, axis.ticks.theta = NULL, axis.ticks.r = NULL, axis.minor.ticks.x.top = NULL, axis.minor.ticks.x.bottom = NULL, axis.minor.ticks.y.left = NULL, axis.minor.ticks.y.right = NULL, axis.minor.ticks.theta = NULL, axis.minor.ticks.r = NULL, axis.ticks.length = 2.75, axis.ticks.length.x = NULL, axis.ticks.length.x.top = NULL, axis.ticks.length.x.bottom = NULL, axis.ticks.length.y = NULL, axis.ticks.length.y.left = NULL, axis.ticks.length.y.right = NULL, axis.ticks.length.theta = NULL, axis.ticks.length.r = NULL, axis.minor.ticks.length = 0.75, axis.minor.ticks.length.x = NULL, axis.minor.ticks.length.x.top = NULL, axis.minor.ticks.length.x.bottom = NULL, axis.minor.ticks.length.y = NULL, axis.minor.ticks.length.y.left = NULL, axis.minor.ticks.length.y.right = NULL, axis.minor.ticks.length.theta = NULL, axis.minor.ticks.length.r = NULL, axis.line = list(), axis.line.x = NULL, axis.line.x.top = NULL, axis.line.x.bottom = NULL, axis.line.y = NULL, axis.line.y.left = NULL, axis.line.y.right = NULL, axis.line.theta = NULL, axis.line.r = NULL, legend.background = list(), legend.margin = c(5.5, 5.5, 5.5, 5.5), legend.spacing = 11, legend.spacing.x = NULL, legend.spacing.y = NULL, legend.key = list(), legend.key.size = 1.2, legend.key.height = NULL, legend.key.width = NULL, legend.key.spacing = 5.5, legend.key.spacing.x = NULL, legend.key.spacing.y = NULL, legend.frame = NULL, legend.ticks = NULL, legend.ticks.length = 0.2, legend.axis.line = NULL, legend.text = list(family = NULL, face = NULL, colour = NULL, size = 0.8, hjust = NULL, vjust = NULL, angle = NULL, lineheight = NULL, margin = NULL, debug = NULL, inherit.blank = TRUE), legend.text.position = NULL, legend.title = list(family = NULL, face = NULL, colour =

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NULL, colour = “grey10”, size = 0.8, hjust = NULL, vjust = NULL, angle = NULL, lineheight = NULL, margin = c(4.4, 4.4, 4.4, 4.4), debug = NULL, inherit.blank = TRUE), strip.text.x = NULL, strip.text.x.bottom = NULL, strip.text.x.top = NULL, strip.text.y = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = NULL, angle = -90, lineheight = NULL, margin = NULL, debug = NULL, inherit.blank = TRUE), strip.text.y.left = list(family = NULL, face = NULL, colour = NULL, size = NULL, hjust = NULL, vjust = NULL, angle = 90, lineheight = NULL, margin = NULL, debug = NULL, inherit.blank = TRUE), strip.text.y.right = NULL, strip.switch.pad.grid = 2.75, strip.switch.pad.wrap = 2.75), , , , , list(x = “Format Type”, y = “Accuracy”, title = “Distribution of Accuracy by Condition”, xdist = “xdist”, ydist = “ydist”, dist = “dist”, args = “args”, arg1 = “arg1”, arg2 = “arg2”, arg3 = “arg3”, arg4 = “arg4”, arg5 = “arg5”, arg6 = “arg6”, arg7 = “arg7”, arg8 = “arg8”, arg9 = “arg9”, weight = “weight”, datatype = “datatype”, thickness = “f”)

2.

Data analysis

We used R (Version 4.4.1; R Core Team, 2024) and the R-packages *dplyr* (Version 1.1.4; Wickham, François, Henry, Müller, & Vaughan, 2023), *forcats* (Version 1.0.0; Wickham, 2023a), *ggdist* (Version 3.3.2; Kay, 2024), *ggplot2* (Version 3.5.1; Wickham, 2016), *lubridate* (Version 1.9.3; Grolemund & Wickham, 2011), *papaja* (Version 0.1.3; Aust & Barth, 2024), *purrr* (Version 1.0.2; Wickham & Henry, 2023), *readr* (Version 2.1.5; Wickham, Hester, & Bryan, 2024), *stringr* (Version 1.5.1; Wickham, 2023b), *tibble* (Version 3.2.1; Müller & Wickham, 2023), *tidyr* (Version 1.3.1; Wickham, Vaughan, & Girlich, 2024), *tidyverse* (Version 2.0.0; Wickham et al., 2019) and *tinylabels* (Version 0.2.4; Barth, 2023) for all our analyses.

Discussion

References

use r_refs(file = “references.bib”)

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