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$$q\left(x_{t+1} \mid x_t, x_0\right)$$

$$\begin{aligned}
q\left(x_{t-1} \mid x_t, x_0\right) &= \frac{q\left(x_t, x_{t-1}, x_0\right)}{q\left(x_t, x_0\right)} \\
&= \frac{q\left(x_t \mid x_{t-1}, x_0\right) q\left(x_{t-1}, x_0\right)}{q\left(x_t, x_0\right)} \\
&= \frac{q\left(x_t \mid x_{t-1}, x_0\right) q\left(x_{t-1}\right) q\left(x_0\right)}{q\left(x_t\right) q\left(x_0\right)} \\
&= \frac{q\left(x_t \mid x_{t-1}, x_0\right) q\left(x_{t-1}\right)}{q\left(x_t\right)} \\
&= \frac{q\left(x_t \mid x_{t-1}\right) q\left(x_{t-1}\right)}{q\left(x_t\right)}
\end{aligned}$$

$$q\left(x_t \mid x_{t-1}\right) = N\left(x_t; \sqrt{\alpha_t} x_{t-1}, (1-\alpha_t) I\right)$$

$$q\left(x_t \mid x_0\right) = N\left(x_t; \sqrt{\alpha_t} x_0, (1-\alpha_t) I\right)$$

$$q\left(x_{t+1} \mid x_t, x_0\right) = \frac{N\left(x_{t+1}; \sqrt{\alpha_{t+1}} x_0, (1-\alpha_{t+1}) I\right) N\left(x_t; \sqrt{\alpha_t} x_0, (1-\alpha_t) I\right)}{N\left(x_t; \sqrt{\alpha_t} x_0, (1-\alpha_t) I\right)}$$

$$\propto \exp\left\{-\frac{1}{2}\left(\frac{\|x_t - \sqrt{\alpha_t} x_{t-1}\|^2}{1-\alpha_t} + \frac{\|x_{t+1} - \sqrt{\alpha_{t+1}} x_0\|^2}{1-\alpha_{t+1}}\right)\right\}$$

$$= \exp\left\{-\frac{1}{2}\left(\frac{\alpha_t}{1-\alpha_t} + \frac{1}{1-\alpha_{t+1}}\right)\|x_{t+1}\|^2\right\}$$

$$= \exp\left\{-\frac{1}{2}\left(\frac{2\sqrt{\alpha_t}}{1-\alpha_t}\right) x_{t+1} x_t + C\left(x_t, x_0\right)\right\}$$

$$= \exp\left\{-\frac{1}{2}\frac{\|x_{t+1} - \mu_{q(x_{t+1}|x_t,x_0)}\|^2}{\sigma^2_{q(x_{t+1}|x_t,x_0)}}\right\}$$

$$
\begin{aligned}
\hat{\sigma}^2_{q(x_{t+1})} &= \left(\frac{\alpha_t}{1-\alpha_t} + \frac{1}{1-\alpha_{t+1}} \right)^{-1} \\
&= \frac{\alpha_t(1-\alpha_{t+1}) + (1-\alpha_t)}{(1-\alpha_t)(1-\alpha_{t+1})} \\
&= \frac{(1-\alpha_t)(1-\alpha_{t+1})}{1-\alpha_t}
\end{aligned}
$$