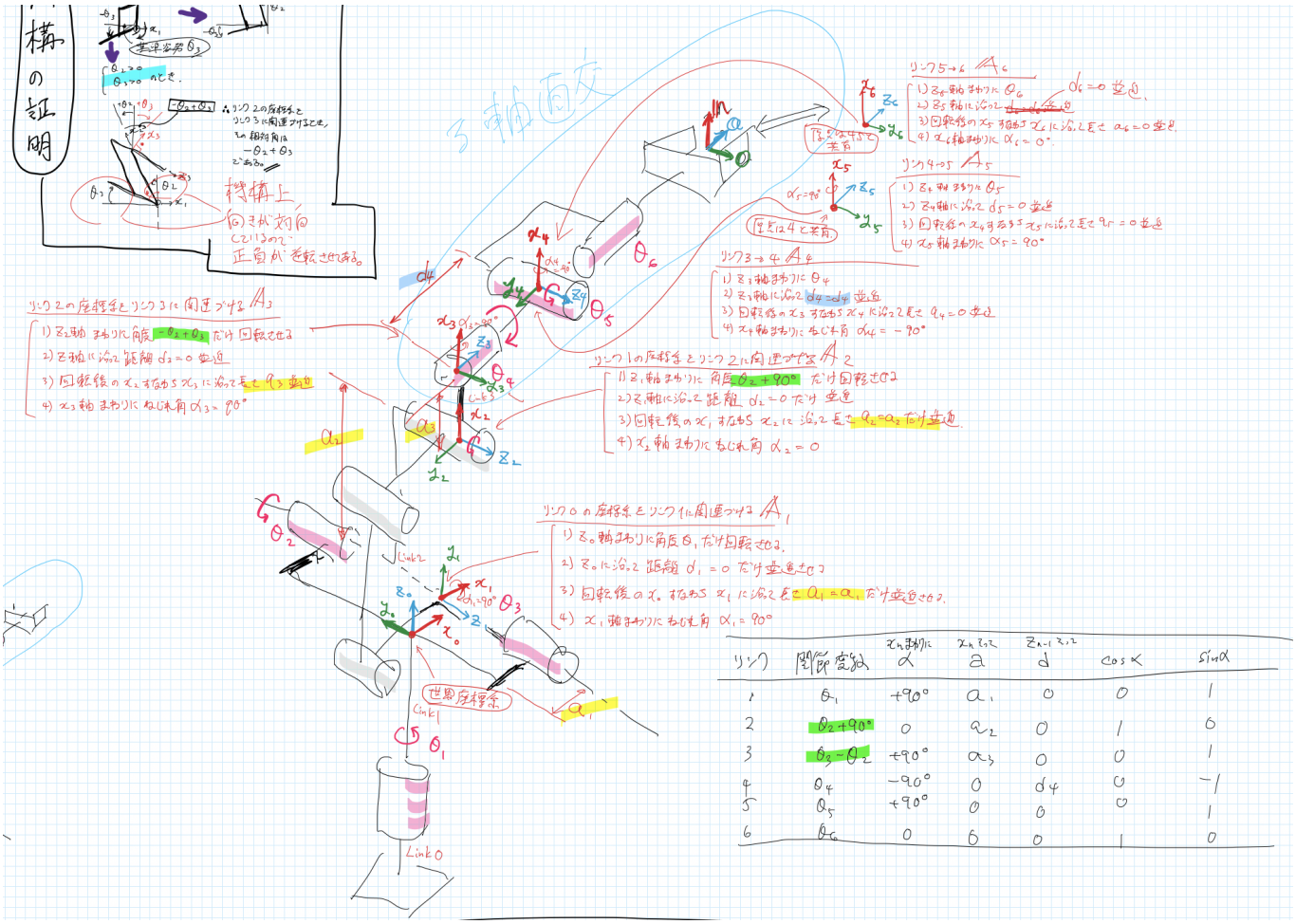
作動関節マニピュレータの逆運動学

Created by Masahiro Furukawa, Aug 18, 2020



リンク	関節変数	z_{n-1}軸まわりに α	x_n 沿って a	z_{n-1} 沿って d	$\cos\alpha$	$\sin\alpha$
1	θ_1	$+90°$	a_1	0	0	1
2	$\theta_2+90°$	0	a_2	0	1	0
3	$\theta_3-\theta_2$	$+90°$	a_3	0	0	1
4	θ_4	$-90°$	0	d_4	0	-1
5	θ_5	$+90°$	0	0	0	1
6	θ_6	0	0	0	1	0

In [1]:

```
#参考URL -> https://qiita.com/tibigame/items/61cecf86fc978628bfee
#参考図書 -> ポールのロボット・マニピュレータ

import numpy as np
import sympy as sym
sym.init_printing()
Pi = sym.S.Pi # 円周率

#sympyの円周率の方を使うことをすすめる(こっちの方が量子化誤差が大きくなる.numpyも同様に大きい)
import math
pi = math.pi

# 角度変数
(J_1,J_2,J_3,J_4,J_5,J_6) = sym.symbols('J_1,J_2,J_3,J_4,J_5,J_6')

# リンクパラメータ
(a_1,a_2,a_3,d_4) = sym.symbols('a_1,a_2,a_3,d_4')

# リンクパラメータ
(j,a,d,alpha) = sym.symbols('j,a,d,alpha')

# T6
(n_x, n_y, n_z, o_x, o_y, o_z, a_x, a_y, a_z, p_x, p_y, p_z) = sym.symbols('n_x, n_y, n_z, o_x, o_
y, o_z, a_x, a_y, a_z, p_x, p_y, p_z')
```

In [2]:

```
#sin.cosの簡易記述用
def S(a):
    return sym.sin(a)

def C(a):
    return sym.cos(a)
```

In [3]:

```
#回転・並進行列
def rotx(a):
    return sym.Matrix([[1, 0, 0, 0], [0, C(a), -S(a), 0], [0, S(a), C(a), 0], [0, 0, 0, 1]])

def roty(a):
    return sym.Matrix([[C(a), 0, S(a), 0], [0, 1, 0, 0], [-S(a), 0, C(a), 0], [0, 0, 0, 1]])

def rotz(a):
    return sym.Matrix([[C(a), -S(a), 0, 0], [S(a), C(a), 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])

def trans(x, y, z):
    return sym.Matrix([[1, 0, 0, x], [0, 1, 0, y], [0, 0, 1, z], [0, 0, 0, 1]])

# DH matrix
def DH(j, alpha, a, d):
    return rotz(j)*trans(a,0,d)*rotx(alpha)

# inverse DH matrix
def DHi(j, alpha, a, d):
    return rotx(-alpha)*trans(-a,0,-d)*rotz(-j)
```

In [4]:

```
# target項を式eqからくくり出すための関数
def obs(eq, target):
    sol = sym.solve(eq, target)
    for i  in range(len(sol)):
        display( sym.Eq(target,  sol[i]))
    print( str(len(sol)) +' equation(s) in total')
    return sol
```

DH法に基づく座標系間の関係表現

座標系 i	Z_i-1軸回りに角度θ_i	X_i軸周りにねじれ角α_iだけ回転	回転後のX_i-1 (=X_i)に沿って長さa_iだけ並進	Z_i-1に沿って距離d_iだけ並進
1	J_1	$\pi/2$	a_1	0
2	$J_2 + \pi/2$	0	a_2	0
3	$J_3 - J_2$	$\pi/2$	a_3	0
4	J_4	$-\pi/2$	0	d_4
5	J_5	$\pi/2$	0	0
6	J_6	0	0	0 $

変換行列 A

In [5]:

```
easy=False
if(easy):
    A1=sym.trigsimp( DH (J_1,          Pi/2, 0, 0))
    A3=sym.trigsimp( DH (J_3 - J_2,  Pi/2, 0, 0))
    A1i=sym.trigsimp( DHi (J_1,          Pi/2, 0, 0))
    A3i=sym.trigsimp( DHi (J_3 - J_2,  Pi/2, 0, 0))
else:
    A1=sym.trigsimp( DH (J_1,          Pi/2, a_1, 0))
    A3=sym.trigsimp( DH (J_3 - J_2,  Pi/2, a_3, 0))
    # inverse matrix
    A1i=sym.trigsimp( DHi (J_1,          Pi/2, a_1, 0))
    A3i=sym.trigsimp( DHi (J_3 - J_2,  Pi/2, a_3, 0))

A2=sym.trigsimp( DH (J_2+ Pi/2,      0, a_2, 0))
A4=sym.trigsimp( DH (J_4,          -Pi/2,     0, d_4))
A5=sym.trigsimp( DH (J_5,          Pi/2,     0, 0))
A6=sym.trigsimp( DH (J_6,                0,     0, 0))

# inverse matrix
A2i=sym.trigsimp( DHi (J_2+ Pi/2,      0, a_2, 0))
A4i=sym.trigsimp( DHi (J_4,          -Pi/2,     0, d_4))
A5i=sym.trigsimp( DHi (J_5,          Pi/2,     0, 0))
A6i=sym.trigsimp( DHi (J_6,                0,     0, 0))
```

In [6]:

```
A3
```

Out[6]:

$$\begin{bmatrix} \cos(J_2 - J_3) & 0 & -\sin(J_2 - J_3) & a_3 \cos(J_2 - J_3) \\ -\sin(J_2 - J_3) & 0 & -\cos(J_2 - J_3) & -a_3 \sin(J_2 - J_3) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

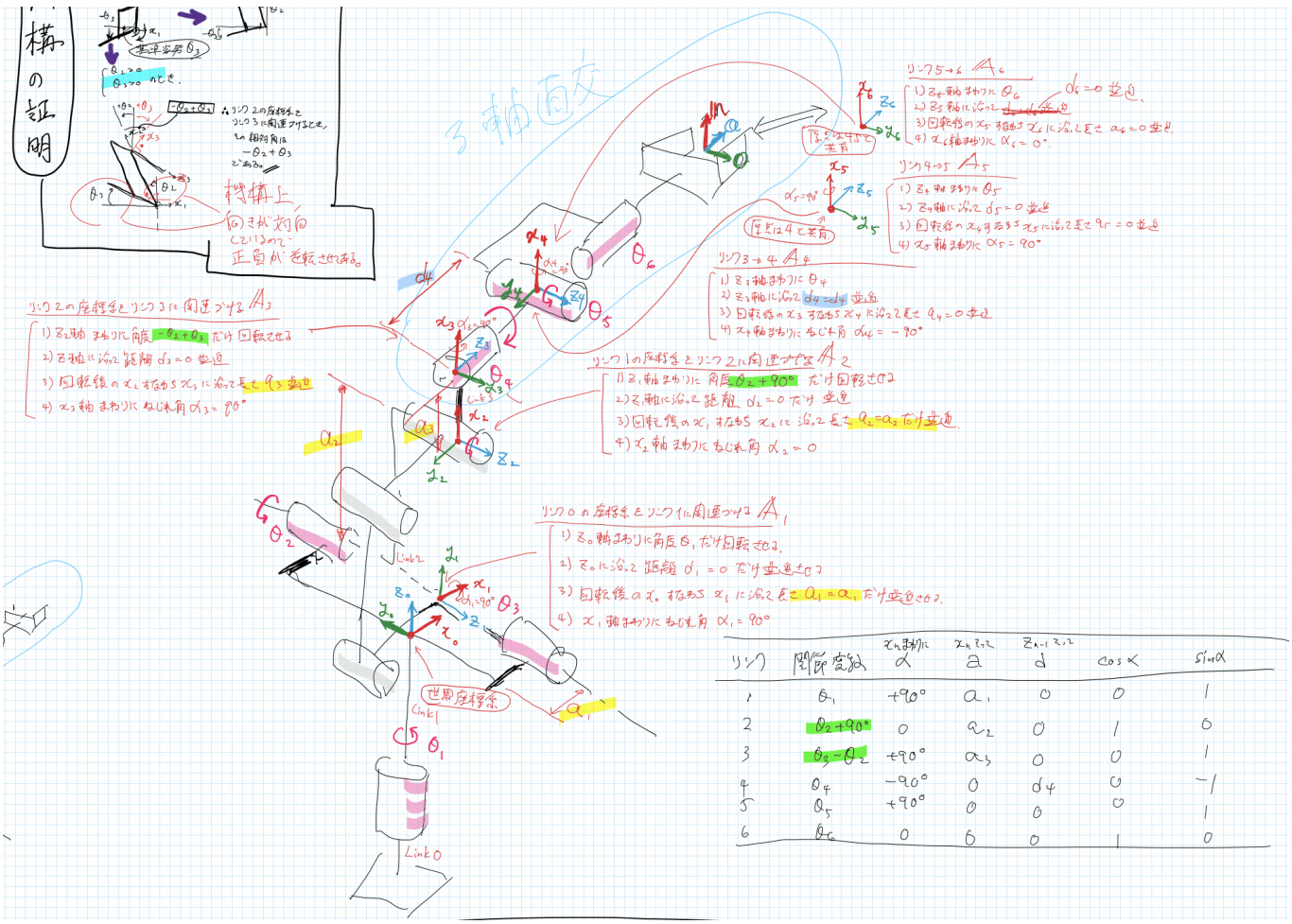
In [7]:

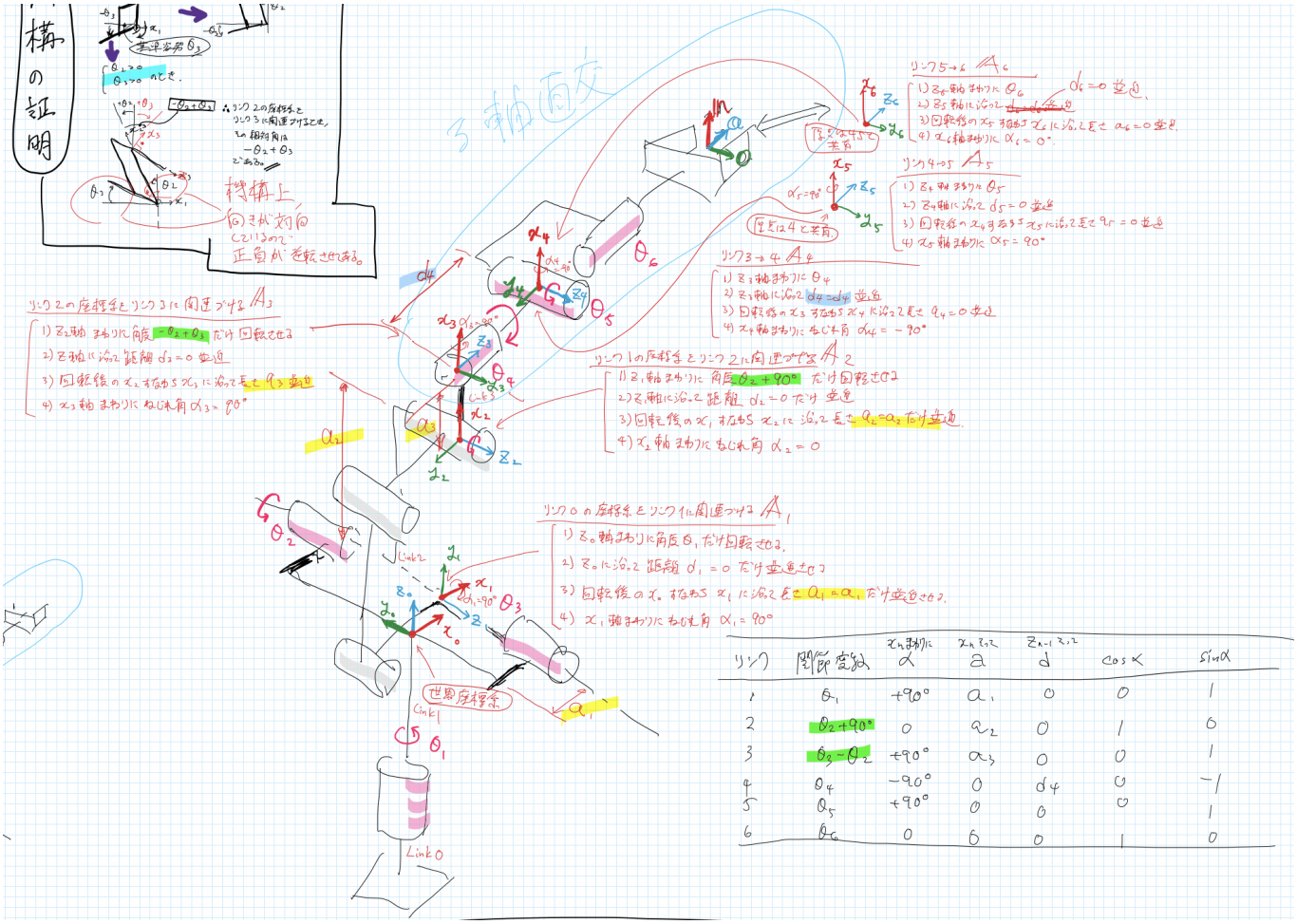
```
# 逆行列をかけると単位行列になることの確認
ret = A2i*A2
sym.trigsimp(ret)
```

Out[7]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

逆運動学





リンク	関節変数	z_nまわりに α	x_nそって a	z_{n-1}そって d	$\cos\alpha$	$\sin\alpha$
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4	θ_4	$-90°$	0	d_4	0	-1
5	θ_5	$+90°$	0	0	0	1
6	θ_6	0	b	0	1	0

In [8]:

```
T6=sym.Matrix([[n_x, o_x, a_x, p_x], [n_y, o_y, a_y, p_y], [n_z, o_z, a_z, p_z], [0, 0, 0, 1]])
T6
```

Out[8]:

$$\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [9]:

```
# forward kinematics
A56 = sym.trigsimp( A5*A6 )
A456 = sym.trigsimp( A4*A5*A6 )
A3456 = sym.trigsimp( A3*A4*A5*A6 )
A23456 = sym.trigsimp( A2*A3*A4*A5*A6 )
T  = sym.trigsimp( A1*A2*A3*A4*A5*A6 )
```

In [10]:

```
T
```

Out[10]:

$$\left[\begin{matrix}((\sin(J_1)\sin(J_4) - \sin(J_3)\cos(J_1)\cos(J_4))\cos(J_5) - \sin(J_5)\cos(J_1)\cos(J_3))\cos(\\ (-(\sin(J_1)\sin(J_3)\cos(J_4) + \sin(J_4)\cos(J_1))\cos(J_5) - \sin(J_1)\sin(J_5)\cos(J_3))\mathrm{c} \\ (J_6) \\ (-\sin(J_3)\sin(J_5) + \cos(J_3)\cos(J_4)\cos(J_5))\cos(J_6 \\ 0\end{matrix}\right.$$

順運動学計算用Cソースコードを得る

In [11]:

```
# Masahiro Furukawa
# Aug, 17, 2020
#
# refernce : https://qiita.com/JmpM/items/4bea4997aaf406cca3b4

# Cソースを得る
for ii in range(4):
    for jj in range(4):
        idx = jj*4+ii
        code = sym.ccode(T[idx],assign_to=('Trans['+str(jj)+']['+str(ii)+']'), standard='C89')
        print(code)
    print()
```

```
Trans[0][0] = ((sin(J_1)*sin(J_4) - sin(J_3)*cos(J_1)*cos(J_4))*cos(J_5) - sin(J_
5)*cos(J_1)*cos(J_3))*cos(J_6) + (sin(J_1)*cos(J_4) + sin(J_3)*sin(J_4)*cos(J_
1))*sin(J_6);
Trans[1][0] = (-(sin(J_1)*sin(J_3)*cos(J_4) + sin(J_4)*cos(J_1))*cos(J_5) - sin(J_
1)*sin(J_5)*cos(J_3))*cos(J_6) + (sin(J_1)*sin(J_3)*sin(J_4) - cos(J_1)*cos(J_4))
*sin(J_6);
Trans[2][0] = (-sin(J_3)*sin(J_5) + cos(J_3)*cos(J_4)*cos(J_5))*cos(J_6) - sin(J_
4)*sin(J_6)*cos(J_3);
Trans[3][0] = 0;
()
Trans[0][1] = -((sin(J_1)*sin(J_4) - sin(J_3)*cos(J_1)*cos(J_4))*cos(J_5) - sin(J_
5)*cos(J_1)*cos(J_3))*sin(J_6) + (sin(J_1)*cos(J_4) + sin(J_3)*sin(J_4)*cos(J_
1))*cos(J_6);
Trans[1][1] = -(-(sin(J_1)*sin(J_3)*cos(J_4) + sin(J_4)*cos(J_1))*cos(J_5) - sin(J
_1)*sin(J_5)*cos(J_3))*sin(J_6) + (sin(J_1)*sin(J_3)*sin(J_4) - cos(J_1)*cos(J_
4))*cos(J_6);
Trans[2][1] = -(-sin(J_3)*sin(J_5) + cos(J_3)*cos(J_4)*cos(J_5))*sin(J_6) - sin(J_
4)*cos(J_3)*cos(J_6);
Trans[3][1] = 0;
()
Trans[0][2] = (sin(J_1)*sin(J_4) - sin(J_3)*cos(J_1)*cos(J_4))*sin(J_5) + cos(J_
1)*cos(J_3)*cos(J_5);
Trans[1][2] = -(sin(J_1)*sin(J_3)*cos(J_4) + sin(J_4)*cos(J_1))*sin(J_5) + sin(J_
1)*cos(J_3)*cos(J_5);
Trans[2][2] = sin(J_3)*cos(J_5) + sin(J_5)*cos(J_3)*cos(J_4);
Trans[3][2] = 0;
()
Trans[0][3] = (a_1 - a_2*sin(J_2) - a_3*sin(J_3) + d_4*cos(J_3))*cos(J_1);
Trans[1][3] = (a_1 - a_2*sin(J_2) - a_3*sin(J_3) + d_4*cos(J_3))*sin(J_1);
Trans[2][3] = a_2*cos(J_2) + a_3*cos(J_3) + d_4*sin(J_3);
Trans[3][3] = 1;
()
```

inverse kinematics

$$A_1{}^{-1}T_6 = A_2A_3A_4A_5A_6 \qquad (3.75)$$

$$A_1{}^{-1}T_6 = {}^1T_6 \qquad (3.76)$$

In [12]:

```
T16 = sym.trigsimp(                A1i*T6 ) #  eq(3.70)
T26 = sym.trigsimp(            A2i*A1i*T6 ) #  eq(3.71)
T36 = sym.trigsimp(        A3i*A2i*A1i*T6 ) #  eq(3.72)
T46 = sym.trigsimp(    A4i*A3i*A2i*A1i*T6 ) #  eq(3.73)
T56 = sym.trigsimp( A5i*A4i*A3i*A2i*A1i*T6 ) #  eq(3.74)
```

In [13]:

```
# Left hand  of (3.76)
A1iT6 = T16
A1iT6
```

Out[13]:

$$\left[\begin{matrix} n_x \cos(J_1) + n_y \sin(J_1) & o_x \cos(J_1) + o_y \sin(J_1) & a_x \cos(J_1) + a_y \sin(J_1) & -a_1 + \\ n_z & o_z & a_z & \\ n_x \sin(J_1) - n_y \cos(J_1) & o_x \sin(J_1) - o_y \cos(J_1) & a_x \sin(J_1) - a_y \cos(J_1) & p_x \\ 0 & 0 & 0 & \end{matrix}\right.$$

In [14]:

```
# Right hand of (3.76)
A23456
```

Out[14]:

$$\left[\begin{matrix} -(\sin(J_3)\cos(J_4)\cos(J_5) + \sin(J_5)\cos(J_3))\cos(J_6) + \sin(J_3)\sin(J_4)\sin(J_6) \\ (-\sin(J_3)\sin(J_5) + \cos(J_3)\cos(J_4)\cos(J_5))\cos(J_6) - \sin(J_4)\sin(J_6)\cos(J_3) \quad - \\ \sin(J_4)\cos(J_5)\cos(J_6) + \sin(J_6)\cos(J_4) \\ 0 \end{matrix}\right.$$

$$A_{1i}T_6 = A_{23456}$$

より以下の等式群を得る

In [15]:

```
for idx in range(12):
    display(sym.simplify ( sym.expand( sym.Eq( A1iT6[idx], A23456[idx])) ) )
```

$$n_x \cos(J_1) + n_y \sin(J_1) = \sin(J_3)\sin(J_4)\sin(J_6) - \sin(J_3)\cos(J_4)\cos(J_5)\cos(J_6) -$$

$$o_x \cos(J_1) + o_y \sin(J_1) = \sin(J_3)\sin(J_4)\cos(J_6) + \sin(J_3)\sin(J_6)\cos(J_4)\cos(J_5) +$$

$$a_x \cos(J_1) + a_y \sin(J_1) = -\sin(J_3)\sin(J_5)\cos(J_4) + \cos(J_3)\cos(J_5)$$

$$-a_1 + p_x \cos(J_1) + p_y \sin(J_1) = -a_2 \sin(J_2) - a_3 \sin(J_3) + d_4 \cos(J_3)$$

$$n_z = -\sin(J_3)\sin(J_5)\cos(J_6) - \sin(J_4)\sin(J_6)\cos(J_3) + \cos(J_3)\cos(J_4)\cos(J_5)\,\mathrm{cc}$$

$$o_z = \sin(J_3)\sin(J_5)\sin(J_6) - \sin(J_4)\cos(J_3)\cos(J_6) - \sin(J_6)\cos(J_3)\cos(J_4)\cos($$

$$a_z = \sin(J_3)\cos(J_5) + \sin(J_5)\cos(J_3)\cos(J_4)$$

$$p_z = a_2 \cos(J_2) + a_3 \cos(J_3) + d_4 \sin(J_3)$$

$$n_x \sin(J_1) - n_y \cos(J_1) = \sin(J_4)\cos(J_5)\cos(J_6) + \sin(J_6)\cos(J_4)$$

$$o_x \sin(J_1) - o_y \cos(J_1) = -\sin(J_4)\sin(J_6)\cos(J_5) + \cos(J_4)\cos(J_6)$$

$$\sin(J_4)\sin(J_5) = a_x \sin(J_1) - a_y \cos(J_1)$$

$$p_x \sin(J_1) - p_y \cos(J_1) = 0$$

J1

In [16]:

```
# このうち位置に関する項のみを抽出すると
for idx in [3,7,11]:
    display(sym.simplify ( sym.expand( sym.Eq( A1iT6[idx], A23456[idx])) ) )
```

$$-a_1 + p_x \cos(J_1) + p_y \sin(J_1) = -a_2 \sin(J_2) - a_3 \sin(J_3) + d_4 \cos(J_3)$$

$$p_z = a_2 \cos(J_2) + a_3 \cos(J_3) + d_4 \sin(J_3)$$

$$p_x \sin(J_1) - p_y \cos(J_1) = 0$$

In [17]:

```
# の3元連立方程式を得る. 上記第3式は,
sym.Eq(A1iT6[11] , A23456[11])
```

Out[17]:

$$p_x \sin(J_1) - p_y \cos(J_1) = 0$$

p_x = 0 なら直ちに

In [18]:

```
sol = sym.solve(p_y * C(J_1), J_1)

for i in range(len(sol)):
    display( sym.simplify(sym.Eq(J_1, sol[i])))
print( str(len(sol)) +' equation(s) in total')
```

$$J_1 = \frac{\pi}{2}$$

$$J_1 = \frac{3\pi}{2}$$

2 equation(s) in total

In [19]:

```
# しかし値域(-PI < J1 < PI)より, p_x = 0 なら直ちに
sol = sym.solve(p_y * C(J_1), J_1)

for i in range(len(sol)):
    if -Pi < sol[0] and sol[i] < Pi:
        display( sym.simplify(sym.Eq(J_1, sol[i])), 'where p_x = 0')
```

$$J_1 = \frac{\pi}{2}$$

'where p_x = 0'

p_y = 0 なら直ちに

In [20]:

```
sol = sym.solve(p_x * S(J_1), J_1)

for i in range(len(sol)):
    display( sym.simplify(sym.Eq(J_1, sol[i])))
print( str(len(sol)) +' equation(s) in total')
```

$$J_1 = 0$$

$$J_1 = \pi$$

2 equation(s) in total

In [21]:

```
# であるが, 同様に値域(-PI < J1 < PI)より, p_y = 0 なら直ちに
sol = sym.solve(p_x * S(J_1), J_1)

for i in range(len(sol)):
    if -Pi < sol[0] and sol[i] < Pi:
        display( sym.simplify(sym.Eq(J_1, sol[i])) , 'where p_y = 0')
```

$$J_1 = 0$$

'where p_y = 0'

p_x != 0 であることが確定したならば,

In [22]:

```
# 両辺をp_xで割ることができ,
# p_x != 0 ならば 前述の通りC(J1) != 0であることが必然的に求まるためC(J1)で両辺を割っていいため,
eq =sym.simplify((A1iT6[11] - A23456[11])/p_x/C(J_1))
display(eq)
```

$$\tan(J_1) - \frac{p_y}{p_x}$$

In [23]:

```
# この方程式をJ1について解くと,
sol = sym.solve(eq, J_1)
for i  in range(len(sol)):
    display( sym.simplify(sym.Eq(J_1,  sol[i])) ,  'where p_x != 0')
sol_J1 = sol
```

$$J_1 = \operatorname{atan}\left(\frac{p_y}{p_x}\right)$$

'where p_x != 0'

がJ1に関する解析解である.

J2

In [24]:

```
# 次に上記第2式に着目すると
sym.Eq(A1iT6[7] , A23456[7])
```

Out[24]:

$$p_z = a_2 \cos(J_2) + a_3 \cos(J_3) + d_4 \sin(J_3)$$

In [25]:

```
# であるから,上記第1式に S(J2) を代入すべく C(J2)を求めると
eq =A1iT6[7] - A23456[7]
target = C(J_2)
CJ2 = obs(eq, target)[0]
```

$$\cos(J_2) = \frac{-a_3 \cos(J_3) - d_4 \sin(J_3) + p_z}{a_2}$$

1 equation(s) in total

In [26]:

```
# 従って,S(J2)を求めるためには,C(J2)**2 + S(J2)**2 = 1 を展開し
# S(J2) = sqrt(1-C(J2)**2) を行いたいので,【→これは誤り.後述のとおり平方根を開いてはいけない】
# まずは平方根を取る前に平方根内部の正負を確認する

# 前記右辺を S(J2)**2 = 1- C(J2)**2 に代入し,これをKとおく
K = 1 - sol[0]**2
display(K)
```

$$1 - \operatorname{atan}^2\left(\frac{p_y}{p_x}\right)$$

In [27]:

```
# もしKが負(K<0)であればJ3に対する解は存在しない.従ってJ2も求まらない.しかしJ3が未定なため,このままでは判別式を評価し得ない.
# もし K=0 であれば,直ちにJ3が求まり,
sol = sym.solve( sym.trigsimp(K), J_3  )
for i  in range(len(sol)):
   display( sym.Eq(J_3,  sol[i]))
print( str(len(sol)) +' equation(s) in total')
```

0 equation(s) in total

In [28]:

```
# しかしやはり現時点ではKの正負を評価し得ない.そこでK>0と仮定してS(J2)を導出することにする.
# 再度標的となる式を再掲すると,
eq = sym.Eq(A1iT6[3] , A23456[3])
display(eq)
```

$$-a_1 + p_x \cos(J_1) + p_y \sin(J_1) = -a_2 \sin(J_2) - a_3 \sin(J_3) + d_4 \cos(J_3)$$

In [29]:

```
# を得る. J1が既知のため上式の左辺をLとおく
sub = A1iT6[3]
L = sym.symbols('L')
eq = eq.subs(sub, L)
display(sym.trigsimp(eq))
```

$$L = -a_2 \sin(J_2) - a_3 \sin(J_3) + d_4 \cos(J_3)$$

In [30]:

```
# S(J2)について整理すると,
target = S(J_2)
SJ2 = obs(eq, target)[0]
```

$$\sin(J_2) = \frac{-L - a_3 \sin(J_3) + d_4 \cos(J_3)}{a_2}$$

1 equation(s) in total

In [31]:

```
# ここでS(J2)**2 + C(J2)**2 = 1 より
eq = sym.trigsimp(sym.Eq((SJ2**2+CJ2**2),1))
display(eq)
display(sym.collect(sym.trigsimp(eq.expand()), [C(J_3),S(J_3)]))
# 方程式をJ3について解くと,
target = J_3
sol_J3 = obs(eq, target)
```

$$\frac{(L + a_3 \sin(J_3) - d_4 \cos(J_3))^2}{a_2^2} + \frac{(-a_3 \cos(J_3) - d_4 \sin(J_3) + p_z)^2}{a_2^2} = 1$$

$$\frac{L^2 + 2La_3 \sin(J_3) - 2Ld_4 \cos(J_3) + a_3^2 - 2a_3 p_z \cos(J_3) + d_4^2 - 2d_4 p_z \sin(J_3) + p_z^2}{a_2^2}$$

$$J_3 = 2 \operatorname{atan}\left(\frac{-2La_3 + 2d_4 p_z + \sqrt{-L^4 + 2L^2 a_2^2 + 2L^2 a_3^2 + 2L^2 d_4^2 - 2L^2 p_z^2 - a_2^4 + 2a}}{L^2 + 2Ld_4 - a_2^2 + a_3^2 + 2}\right.$$

$$J_3 = -2 \operatorname{atan}\left(\frac{2La_3 - 2d_4 p_z + \sqrt{-L^4 + 2L^2 a_2^2 + 2L^2 a_3^2 + 2L^2 d_4^2 - 2L^2 p_z^2 - a_2^4 + 2a}}{L^2 + 2Ld_4 - a_2^2 + a_3^2 +}\right.$$

2 equation(s) in total

In [33]:

```
# Masahiro Furukawa
# Aug, 21, 2020
#
# refernce : https://qiita.com/JmpM/items/4bea4997aaf406cca3b4

# J3 に対するCソースを得る
code = sym.ccode(sub, assign_to=('L'), standard='C89')
print("// constant L \n"  + code + "\n")

for ii in range(len(sol_J3)):
    code = sym.ccode(sol_J3[ii], assign_to=('J3'), standard='C89')
    print("// Solusion #"+ str(ii) +"\n"  + code + "\n")
```

```
// constant L
L = -a_1 + p_x*cos(J_1) + p_y*sin(J_1);

// Solusion #0
J3 = 2*atan((-2*L*a_3 + 2*d_4*p_z + sqrt(-pow(L, 4) + 2*pow(L, 2)*pow(a_2, 2) +
2*pow(L, 2)*pow(a_3, 2) + 2*pow(L, 2)*pow(d_4, 2) - 2*pow(L, 2)*pow(p_z, 2) - p
ow(a_2, 4) + 2*pow(a_2, 2)*pow(a_3, 2) + 2*pow(a_2, 2)*pow(d_4, 2) + 2*pow(a
_2, 2)*pow(p_z, 2) - pow(a_3, 4) - 2*pow(a_3, 2)*pow(d_4, 2) + 2*pow(a_3, 2)*po
w(p_z, 2) - pow(d_4, 4) + 2*pow(d_4, 2)*pow(p_z, 2) - pow(p_z, 4)))/(pow(L, 2) +
2*L*d_4 - pow(a_2, 2) + pow(a_3, 2) + 2*a_3*p_z + pow(d_4, 2) + pow(p_z, 2)));

// Solusion #1
J3 = -2*atan((2*L*a_3 - 2*d_4*p_z + sqrt(-pow(L, 4) + 2*pow(L, 2)*pow(a_2, 2) +
2*pow(L, 2)*pow(a_3, 2) + 2*pow(L, 2)*pow(d_4, 2) - 2*pow(L, 2)*pow(p_z, 2) - p
ow(a_2, 4) + 2*pow(a_2, 2)*pow(a_3, 2) + 2*pow(a_2, 2)*pow(d_4, 2) + 2*pow(a
_2, 2)*pow(p_z, 2) - pow(a_3, 4) - 2*pow(a_3, 2)*pow(d_4, 2) + 2*pow(a_3, 2)*po
w(p_z, 2) - pow(d_4, 4) + 2*pow(d_4, 2)*pow(p_z, 2) - pow(p_z, 4)))/(pow(L, 2) +
2*L*d_4 - pow(a_2, 2) + pow(a_3, 2) + 2*a_3*p_z + pow(d_4, 2) + pow(p_z, 2)));
```

J2

In [34]:

```
# 次に,J2について解く.C(J2)について,CJ2として得られているからこれらの方程式から以下を得る.
eq = sym.Eq(C(J_2), CJ2)
display(eq)
sol_J2 = obs(eq, J_2)
```

$$\cos\left(J_2\right) = \frac{-a_3\cos\left(J_3\right) - d_4\sin\left(J_3\right) + p_z}{a_2}$$

$$J_2 = -\operatorname{acos}\left(-\frac{a_3\cos\left(J_3\right) + d_4\sin\left(J_3\right) - p_z}{a_2}\right) + 2\pi$$

$$J_2 = \operatorname{acos}\left(\frac{-a_3\cos\left(J_3\right) - d_4\sin\left(J_3\right) + p_z}{a_2}\right)$$

2 equation(s) in total

In [35]:

```
# J2 に対するCソースを得る
for ii in range(len(sol_J3)):
    code = sym.ccode(sol_J2[ii], assign_to=('J2'), standard='C89')
    print("// Solusion #"+ str(ii) +"\n"  + code + "\n")
```

```
// Solusion #0
J2 = -acos(-(a_3*cos(J_3) + d_4*sin(J_3) - p_z)/a_2) + 2*M_PI;

// Solusion #1
J2 = acos((-a_3*cos(J_3) - d_4*sin(J_3) + p_z)/a_2);
```

以上の導出から J1, J2, J3 が求まった.

J5

In [36]:

```
A3iA2iA1iT6 = A3i*A2i*A1i*T6
for idx in range(12):
    display(sym.simplify ( sym.expand( sym.Eq( A3iA2iA1iT6[idx], A456[idx])) ) )
```

$$\sin(J_4)\sin(J_6) - \cos(J_4)\cos(J_5)\cos(J_6) = -\frac{n_x \sin(J_1 - J_3)}{2} + \frac{n_x \sin(J_1 + J_3)}{2} +$$

$$\sin(J_4)\cos(J_6) + \sin(J_6)\cos(J_4)\cos(J_5) = -\frac{o_x \sin(J_1 - J_3)}{2} + \frac{o_x \sin(J_1 + J_3)}{2} +$$

$$\sin(J_5)\cos(J_4) = \frac{a_x \sin(J_1 - J_3)}{2} - \frac{a_x \sin(J_1 + J_3)}{2} - \frac{a_y \cos(J_1 - J_3)}{2} + \frac{a_y \cos(J_1}{2}$$

$$-a_1 \sin(J_3) + a_2 \cos(J_2 - J_3) + a_3 + p_x \sin(J_3)\cos(J_1) + p_y \sin(J_1)\sin(J_3) - p_z \cos$$

$$n_x \sin(J_1) - n_y \cos(J_1) = \sin(J_4)\cos(J_5)\cos(J_6) + \sin(J_6)\cos(J_4)$$

$$o_x \sin(J_1) - o_y \cos(J_1) = -\sin(J_4)\sin(J_6)\cos(J_5) + \cos(J_4)\cos(J_6)$$

$$\sin(J_4)\sin(J_5) = a_x \sin(J_1) - a_y \cos(J_1)$$

$$p_x \sin(J_1) - p_y \cos(J_1) = 0$$

$$\sin(J_5)\cos(J_6) = -\frac{n_x \cos(J_1 - J_3)}{2} - \frac{n_x \cos(J_1 + J_3)}{2} - \frac{n_y \sin(J_1 - J_3)}{2} - \frac{n_y \sin(}{}$$

$$\sin(J_5)\sin(J_6) = \frac{o_x \cos(J_1 - J_3)}{2} + \frac{o_x \cos(J_1 + J_3)}{2} + \frac{o_y \sin(J_1 - J_3)}{2} + \frac{o_y \sin(J_1}{2}$$

$$\cos(J_5) = \frac{a_x \cos(J_1 - J_3)}{2} + \frac{a_x \cos(J_1 + J_3)}{2} + \frac{a_y \sin(J_1 - J_3)}{2} + \frac{a_y \sin(J_1 + J_3)}{2}$$

$$d_4 = -a_1 \cos(J_3) + a_2 \sin(J_2 - J_3) + p_x \cos(J_1)\cos(J_3) + p_y \sin(J_1)\cos(J_3) + p_z \sin$$

In [37]:

```
# 上記式の第11式は,
idx=10
eq = sym.simplify ( sym.Eq( A3iA2iA1iT6[idx], A456[idx]))
display(eq)
```

$$\cos(J_5) = a_x \cos(J_1)\cos(J_3) + a_y \sin(J_1)\cos(J_3) + a_z \sin(J_3)$$

In [38]:

```
sol_J5 = obs(eq, J_5)
```

$$J_5 = -\operatorname{acos}\left(a_x \cos(J_1)\cos(J_3) + a_y \sin(J_1)\cos(J_3) + a_z \sin(J_3)\right) + 2\pi$$

$$J_5 = \operatorname{acos}\left(a_x \cos(J_1)\cos(J_3) + a_y \sin(J_1)\cos(J_3) + a_z \sin(J_3)\right)$$

2 equation(s) in total

In [39]:

```
# J5 に対するCソースを得る
for ii in range(len(sol_J5)):
    code = sym.ccode(sol_J5[ii], assign_to=('J5'), standard='C89')
    print("// Solusion #"+ str(ii) +"\n"  + code + "\n")
```

```
// Solusion #0
J5 = -acos(a_x*cos(J_1)*cos(J_3) + a_y*sin(J_1)*cos(J_3) + a_z*sin(J_3)) + 2*M_
PI;

// Solusion #1
J5 = acos(a_x*cos(J_1)*cos(J_3) + a_y*sin(J_1)*cos(J_3) + a_z*sin(J_3));
```

J4

In [40]:

```
# 上記式の第7式は,
idx=6
eq = sym.simplify ( sym.Eq( A3iA2iA1iT6[idx], A456[idx]))
display(eq)
```

$$\sin(J_4)\sin(J_5) = a_x \sin(J_1) - a_y \cos(J_1)$$

In [41]:

```
sol_J4 = obs(eq, J_4)
```

$$J_4 = \pi - \operatorname{asin}\left(\frac{a_x \sin(J_1) - a_y \cos(J_1)}{\sin(J_5)}\right)$$

$$J_4 = \operatorname{asin}\left(\frac{a_x \sin(J_1) - a_y \cos(J_1)}{\sin(J_5)}\right)$$

2 equation(s) in total

In [42]:

```
# J4 に対するCソースを得る
for ii in range(len(sol_J4)):
    code = sym.ccode(sol_J4[ii], assign_to=('J4'), standard='C89')
    print("// Solusion #"+ str(ii) +"\n"  + code + "\n")
```

```
// Solusion #0
J4 = M_PI - asin((a_x*sin(J_1) - a_y*cos(J_1))/sin(J_5));

// Solusion #1
J4 = asin((a_x*sin(J_1) - a_y*cos(J_1))/sin(J_5));
```

J6

In [43]:

```
A5iA4iA3iA2iA1iT6 = A5i*A4i*A3i*A2i*A1i*T6
for idx in range(12):
    display(sym.simplify ( sym.expand( sym.Eq( A5iA4iA3iA2iA1iT6[idx], A6[idx])) ) )
```

$\cos(J_6) = n_x \sin(J_1)\sin(J_4)\cos(J_5) - n_x \sin(J_3)\cos(J_1)\cos(J_4)\cos(J_5) - n_x \sin(J_5$
$- n_y \sin(J_1)\sin(J_5)\cos(J_3) - n_y \sin(J_4)\cos(J_1)\cos(J_5) - n_z \sin(J_3)\sin(J_5) + n_z \mathrm{co}$

$\sin(J_6) = -o_x \sin(J_1)\sin(J_4)\cos(J_5) + o_x \sin(J_3)\cos(J_1)\cos(J_4)\cos(J_5) + o_x \sin(J$
$+ o_y \sin(J_1)\sin(J_5)\cos(J_3) + o_y \sin(J_4)\cos(J_1)\cos(J_5) + o_z \sin(J_3)\sin(J_5) - o_z \mathrm{co}$

$-a_x \sin(J_1)\sin(J_4)\cos(J_5) + a_x \sin(J_3)\cos(J_1)\cos(J_4)\cos(J_5) + a_x \sin(J_5)\cos(J_1)$
$(J_1)\sin(J_5)\cos(J_3) + a_y \sin(J_4)\cos(J_1)\cos(J_5) + a_z \sin(J_3)\sin(J_5) - a_z \cos(J_3)\mathrm{co}$

$-a_1 \sin(J_3)\cos(J_4)\cos(J_5) - a_1 \sin(J_5)\cos(J_3) + a_2 \sin(J_5)\sin(J_2 - J_3) + a_2 \cos(.$
$(J_5) - p_x \sin(J_1)\sin(J_4)\cos(J_5) + p_x \sin(J_3)\cos(J_1)\cos(J_4)\cos(J_5) + p_x \sin(J_5)\mathrm{co}$
$+ p_y \sin(J_1)\sin(J_5)\cos(J_3) + p_y \sin(J_4)\cos(J_1)\cos(J_5) + p_z \sin(J_3)\sin(J_5) - p_z \mathrm{cos}$

$\sin(J_6) = n_x \sin(J_1)\cos(J_4) + n_x \sin(J_3)\sin(J_4)\cos(J_1) + n_y \sin(J_1)\sin(J_3)\sin(J_4)$

$\cos(J_6) = o_x \sin(J_1)\cos(J_4) + o_x \sin(J_3)\sin(J_4)\cos(J_1) + o_y \sin(J_1)\sin(J_3)\sin(J_4)$

$a_x \sin(J_1)\cos(J_4) + a_x \sin(J_3)\sin(J_4)\cos(J_1) + a_y \sin(J_1)\sin(J_3)\sin(J_4) - a_y \cos(.$

$-a_1 \sin(J_3)\sin(J_4) + a_2 \sin(J_4)\cos(J_2 - J_3) + a_3 \sin(J_4) + p_x \sin(J_1)\cos(J_4) + p_x$
$- p_y \cos(J_1)\cos(J_4) - p_z \sin(J_4)\cos(J_3) = 0$

$n_x \sin(J_1)\sin(J_4)\sin(J_5) - n_x \sin(J_3)\sin(J_5)\cos(J_1)\cos(J_4) + n_x \cos(J_1)\cos(J_3)\mathrm{c}$
$(J_1)\cos(J_3)\cos(J_5) - n_y \sin(J_4)\sin(J_5)\cos(J_1) + n_z \sin(J_3)\cos(J_5) + n_z \sin(J_5)\mathrm{co}$

$o_x \sin(J_1)\sin(J_4)\sin(J_5) - o_x \sin(J_3)\sin(J_5)\cos(J_1)\cos(J_4) + o_x \cos(J_1)\cos(J_3)\mathrm{c}$
$(J_1)\cos(J_3)\cos(J_5) - o_y \sin(J_4)\sin(J_5)\cos(J_1) + o_z \sin(J_3)\cos(J_5) + o_z \sin(J_5)\mathrm{cos}$

$a_x \sin(J_1)\sin(J_4)\sin(J_5) - a_x \sin(J_3)\sin(J_5)\cos(J_1)\cos(J_4) + a_x \cos(J_1)\cos(J_3)\mathrm{c}$
$(J_1)\cos(J_3)\cos(J_5) - a_y \sin(J_4)\sin(J_5)\cos(J_1) + a_z \sin(J_3)\cos(J_5) + a_z \sin(J_5)\mathrm{co}$

$a_1 \sin(J_3)\sin(J_5)\cos(J_4) - a_1 \cos(J_3)\cos(J_5) - a_2 \sin(J_5)\cos(J_4)\cos(J_2 - J_3) + a$
$(J_5) + p_x \sin(J_1)\sin(J_4)\sin(J_5) - p_x \sin(J_3)\sin(J_5)\cos(J_1)\cos(J_4) + p_x \cos(J_1)\mathrm{cos}$
$+ p_y \sin(J_1)\cos(J_3)\cos(J_5) - p_y \sin(J_4)\sin(J_5)\cos(J_1) + p_z \sin(J_3)\cos(J_5) + p_z \mathrm{sin}$

In [44]:

```
# 上記式の第6式は,
idx=5
eq = sym.simplify ( sym.Eq( A5iA4iA3iA2iA1iT6 [idx], A6[idx]))
display(eq)
```

$\cos(J_6) = o_x (\sin(J_1)\cos(J_4) + \sin(J_3)\sin(J_4)\cos(J_1)) + o_y (\sin(J_1)\sin(J_3)\sin(J_4)$

In [45]:

```
sol_J6 = obs(eq, J_6)
```

$$J_6 = -\operatorname{acos}\left(o_x \sin(J_1)\cos(J_4) + o_x \sin(J_3)\sin(J_4)\cos(J_1) + o_y \sin(J_1)\sin(J_3)\sin(\right.$$

$$J_6 = \operatorname{acos}\left(o_x \sin(J_1)\cos(J_4) + o_x \sin(J_3)\sin(J_4)\cos(J_1) + o_y \sin(J_1)\sin(J_3)\sin(J_4\right.$$

2 equation(s) in total

In [46]:

```
# J6 に対するCソースを得る
for ii in range(len(sol_J6)):
    code = sym.ccode(sol_J6[ii], assign_to=('J6'), standard='C89')
    print("// Solusion #"+ str(ii) +"\n"  + code + "\n")
```

```
// Solusion #0
J6 = -acos(o_x*sin(J_1)*cos(J_4) + o_x*sin(J_3)*sin(J_4)*cos(J_1) + o_y*sin(J_1)
*sin(J_3)*sin(J_4) - o_y*cos(J_1)*cos(J_4) - o_z*sin(J_4)*cos(J_3)) + 2*M_PI;

// Solusion #1
J6 = acos(o_x*sin(J_1)*cos(J_4) + o_x*sin(J_3)*sin(J_4)*cos(J_1) + o_y*sin(J_1)
*sin(J_3)*sin(J_4) - o_y*cos(J_1)*cos(J_4) - o_z*sin(J_4)*cos(J_3));
```

以上から J4, J5, J6を得る.