Dimensional reduced modelling of the heart

A model for the blood flow

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Introduction

- The model accounts for the pressure gradient and the function of the valves.
- 3D model not required to describe the heart: lumped 0D model is simpler and faster to implement.
- Numerical solution with appropriate integrator of ODEs for 14 compartments, to model behavior of pressure, volume, and flow rate.

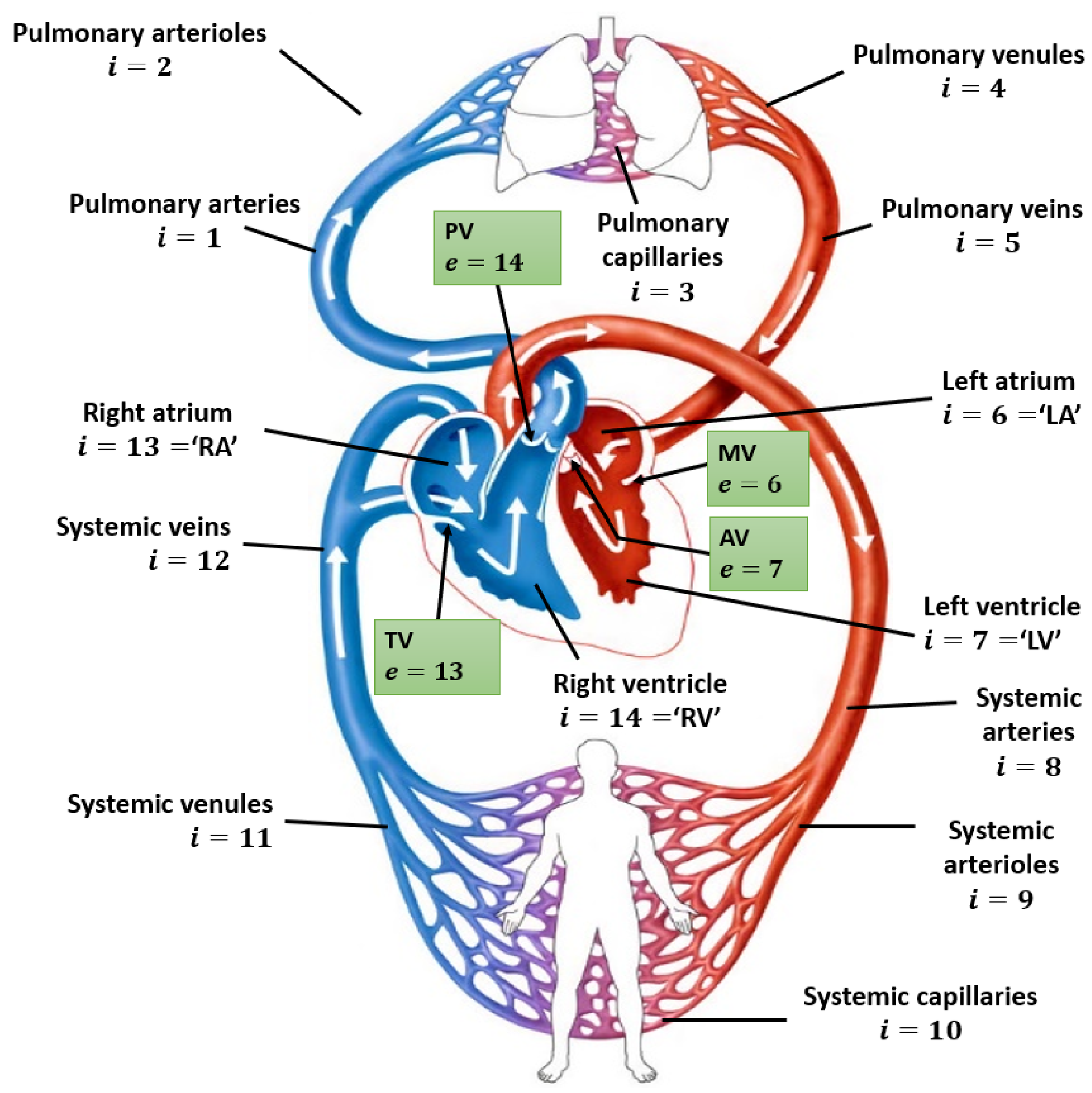

Figure 1: Schema of human cardiovascular system: heart, pulmonary and systemic circulation[5].
In increasing order of edges e, the valves are the mitral valve, aortic valve, tricuspid valve, and pulmonary valve.

Main Objectives

1. Formulation of mathematical model
2. Efficient and stable MATLAB simulation
3. Graphical User Interface to allow variation of parameters

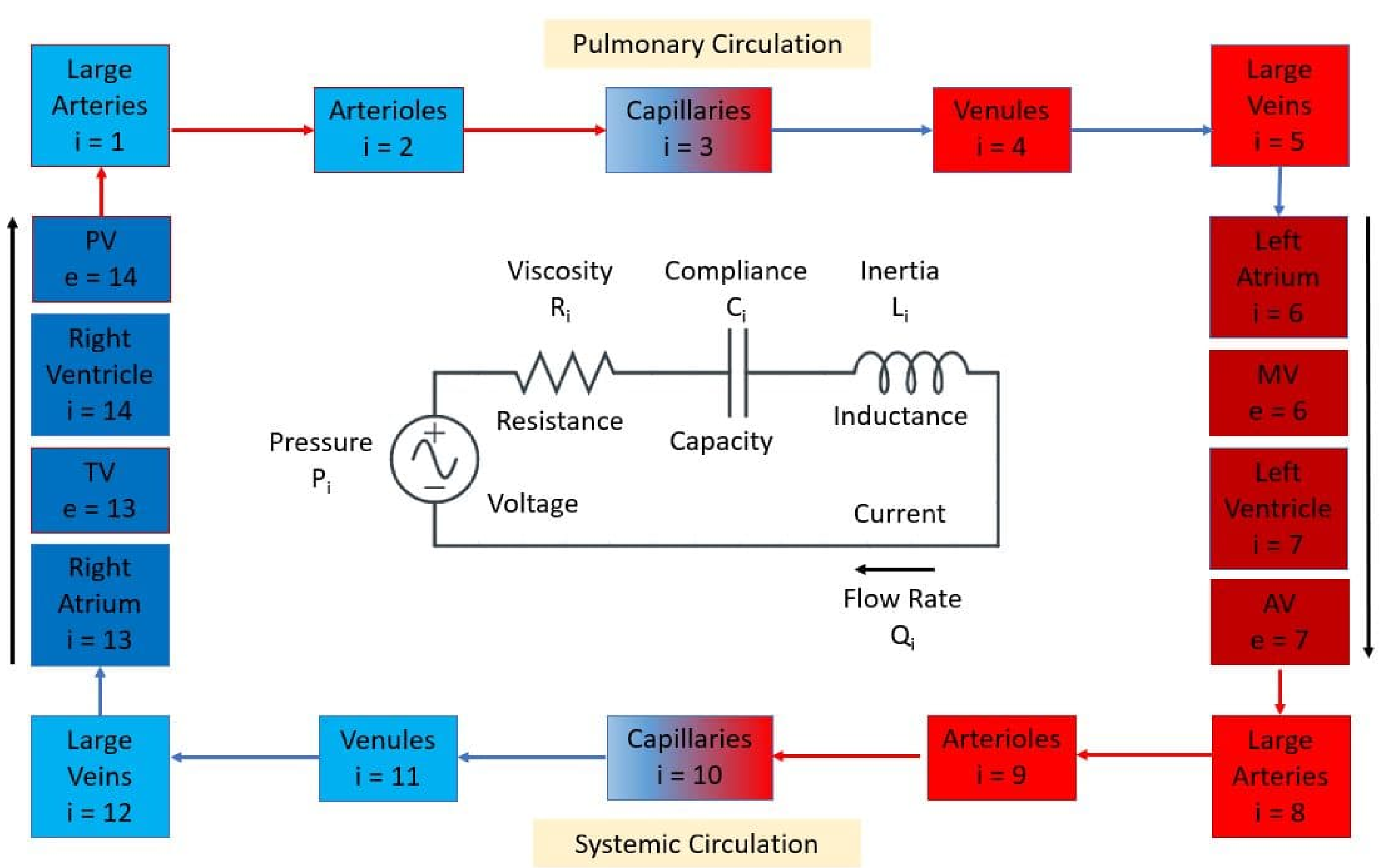

Figure 2: Comparison between model parameters and electrical circuits

Mathematical Section

- Index $i \in \{1, \ldots, 14\}$ represents the compartments, while $e \in \{1, \ldots, 14\}$ the edges (input/output) of respective compartment.
- $V_i(t)$ represents the volume of compartment i, $P_i(t)$ the pressure, and $Q_e(t)$ the flow rate at edge e.
- Opening coefficient for the valve in edges $e \in \{6, 7, 13, 14\}$ inside the heart, denoted by $O_e(t)$.

Each volume and flow rate follow an Ordinary differential equation:

$$\dot{V}_i(t) = Q_{in}(t) - Q_{out}(t) \qquad \text{for each compartment } i \text{ and its adjacent edges} \quad (1)$$

$$L_e\dot{Q}_e(t) = P_i(t) - P_{i+1}(t) - R_eQ_e(t) - B_eQ_e(t)|Q_e(t)| \qquad \text{for each edge } e \text{ adjacent to the heart} \quad (2)$$

$$L_e\dot{Q}_e(t) = P_i(t) - P_{i+1}(t) - R_eQ_e(t) \qquad \text{for edges } e \text{ not connected to any heart chamber} \quad (3)$$

Where L_e is the inertia and R_e, the viscosity at edge e. The turbulence produced by the heart valves is denoted by B_e. Observe that the turbulence term introduces a point of non-differentiability in our model.

Pressure follows an algebraic equation:

$$[1 - 0.0005(Q_{e-1}(t) - Q_e(t))]\, P_i(t) = E_i(t)(V_i(t) - V_{0,i}) \qquad \text{for each heart chamber } i \text{ and edges} \quad (4)$$

$$C_iP_i(t) = V_i(t) - V_{0,i} \qquad \text{for each compartment } i \text{ outside heart} \quad (5)$$

Where C_i is the compliance, $V_{0,i}$ the dead volume and $E_i(t)$ the elasticity function of compartment i. The opening coefficients follow an Ordinary differential equation:

$$C_{d,e}\dot{O}_e(t) = 0.5\left[1 + \tanh\left(C_{v,e}(P_i(t) - P_{i+1}(t))\right)\right] - O_e(t) \qquad \text{for each heart valve in edge } e \quad (6)$$

Lastly, a post-processing for the valve edges takes place:

$$Q_e(t) = O_e(t)Q_e(t) \qquad \text{for each heart valve in edge } e \quad (7)$$

Method

- System solved using Runge-Kutta-Fehlberg 4(5) method: lower computational cost
- When flow is zero, non differentiability of ODE: classical RK4 plus approximation of solution as an alternative to Fehlberg's adaptive stepsize computation
- Coexistence of adaptive stepsize with root detection: RK method applied only when the ODE is differentiable

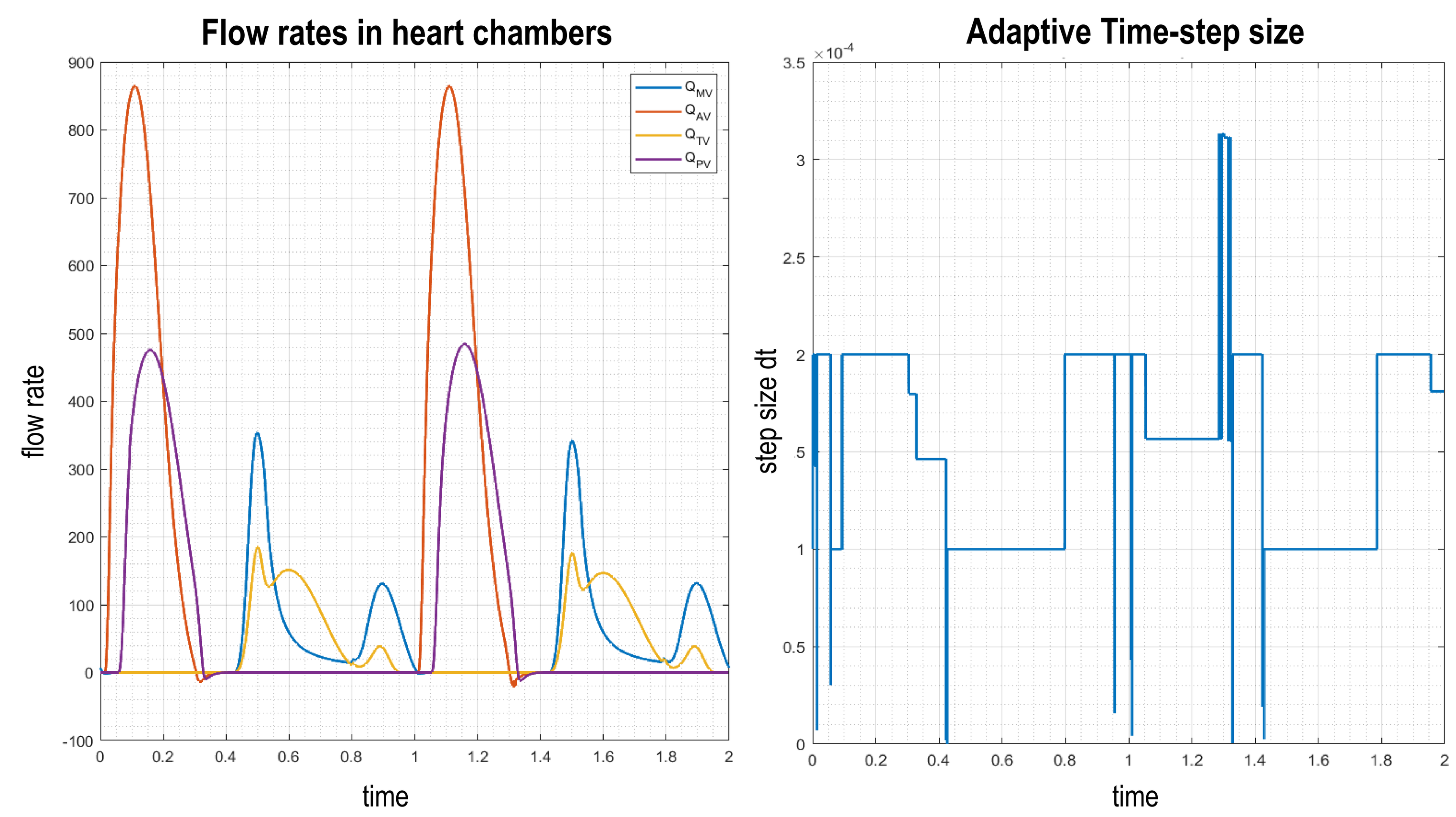

Figure 3: Left: Flow rate in heart cavities. Right: Variation of step size along time.

Results

- Variation of heart rate and elasticity affects the other parameters
- Solved the problem of the computation at points with no differentiability.

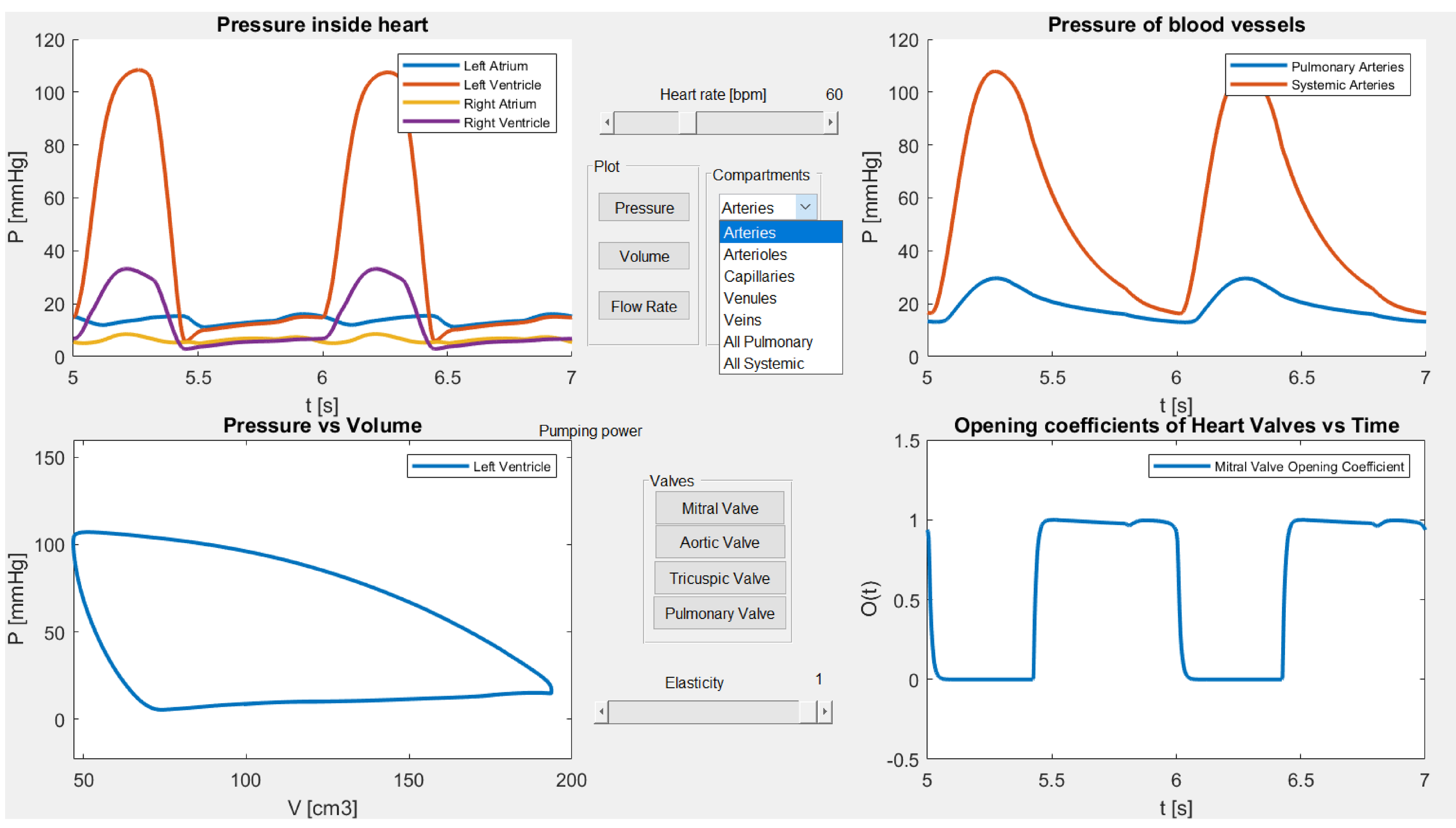

Figure 4: Preliminary Graphical User Interface to show the dependence of Pressure, Volume and Flux on Time, Heart Rate and Elasticity

Forthcoming Research

- Numerical solver: more efficient and accurate
- GUI: variation of heartbeat and elasticity
- Valves: more accurate equations
- Present research on website of partner institution:
  - High Performance Computing Center Stuttgart (HRLS)

References and Acknowledgements

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