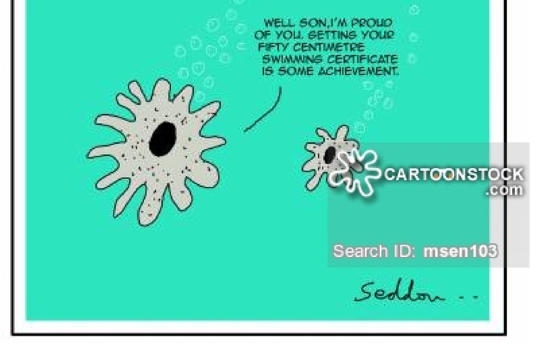

Microscopic Life in Moving Fluids

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Abstract

Microorganisms live in a strange world, a world at low Reynolds numbers. Reynholds numbers can be defined as the ratio of viscous forces and inertial forces in a fluid.

$$Re = \frac{\rho\, U L}{\eta}$$

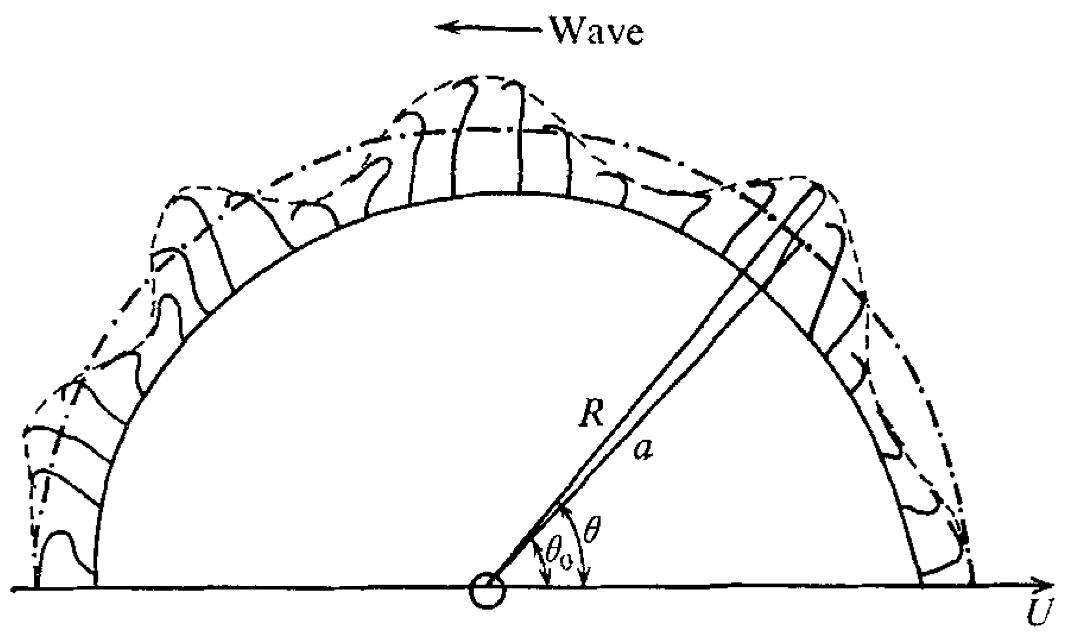
where ρ is the fluid density, η is the viscosity and U and L are characteristic velocity and length scales of the flow, respectively. At small scales, movement relies exclusively on viscous damping rather than inertia.

Hence propulsion is produced by a cyclic and asymmetrical deformation (eg. using Cilia) of the microorganism's body.

Ciliar Motion

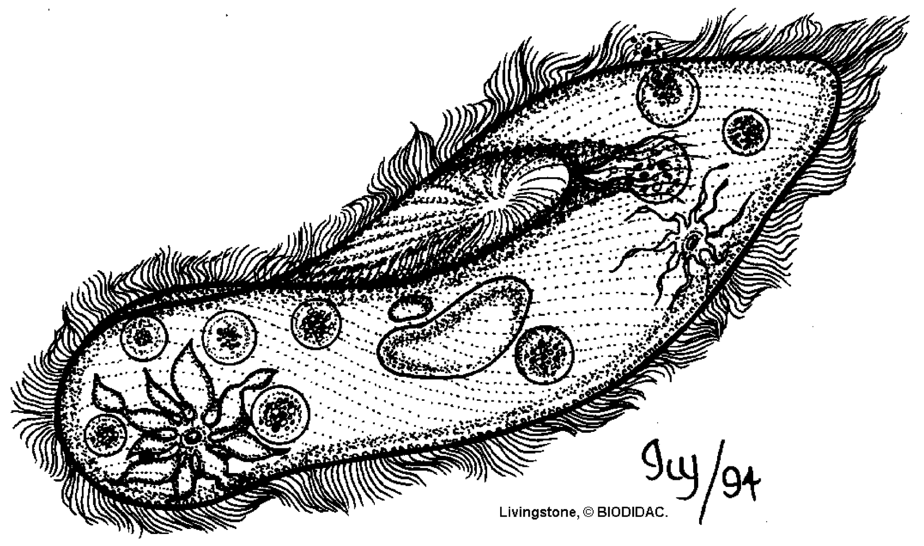
Metachronal Wave

Cilia are small and short appendages on microorganisms such as paramecium. The deformation of the body occurs through cilia who are synchronized to move in a wave around their body. This leads to propulsion.



Paramecium

Paramecium, one of the microorganisms using cilia by 'beating' them in a coordinated way in order to to flee from predators.



Theory

Stokes Equation

Fluid motion can be explained through Navier-Stokes equations. If we apply some restraints, such as **low Reynolds Numbers,newtonian fluid** and **incompressible fluid motion**, then we can cancel out some terms and get a special case: the Stokes equation.

$$-\nabla\, p + \mu\nabla^2 u = 0 = \nabla \cdot \sigma \quad and \quad \nabla \cdot u = 0 \qquad (1)$$

where u and p are the fluid velocity and pressure, respectively, and σ is stress tensor.

An extension to Stokes equation is Lorentz Reciprocal theorem.

Lorentz Reciprocal Theorem

Let $(\vec{u}_1, \vec{f}_1)$ (a) and $(\vec{u}_2, \vec{f}_2)$ (b) be solution pairs to (1) where $\vec{u}_i$ are the velocities and $\vec{f}_i$ are the tractions: force per unit surface. For (a) there is no net force or torque on the swimming body while for (b) the body moves at a velocity $U(t)$ due to an external force $F(t)$. The reciprocal theorem states that the two solution are related by

$$\int_{S(t)} \vec{f}_2 \cdot \vec{u}_1\, dS \;=\; \int_{S(t)} \vec{f}_1 \cdot \vec{u}_2\, dS \qquad (2)$$

where S(t) is the instantaneous boundary of the swimming object.

Further Assumptions

We assume for simplicity the body to be a **rigid sphere**. Solution pair (a): there is no external force, so $\int_{S(t)} \vec{f}_1\, dS = 0$ and we can write $\vec{u}_1$ as $\vec{u}_1 = \tilde{u} + U(t)\ \hat{k}$ where U(t) is the swimmer speed, $\tilde{u}$ is the slip velocity and $\hat{k}$ is the unit vector in the direction of the speed. Solution pair (b): the force $\vec{F}(t)$ to which the body is subjected is $\vec{F_2} = \int_{S(t)} \vec{f_2}\, dS = \hat{k}$. From Stokes equations we can get an expressions for $\vec{u_2}$ and $\vec{f_2}$:

$\vec{u_2} = \frac{\hat{k}}{6\pi R\eta} =$ costant, $\quad \vec{f_2} = \frac{\hat{k}}{4\pi R^2}$.

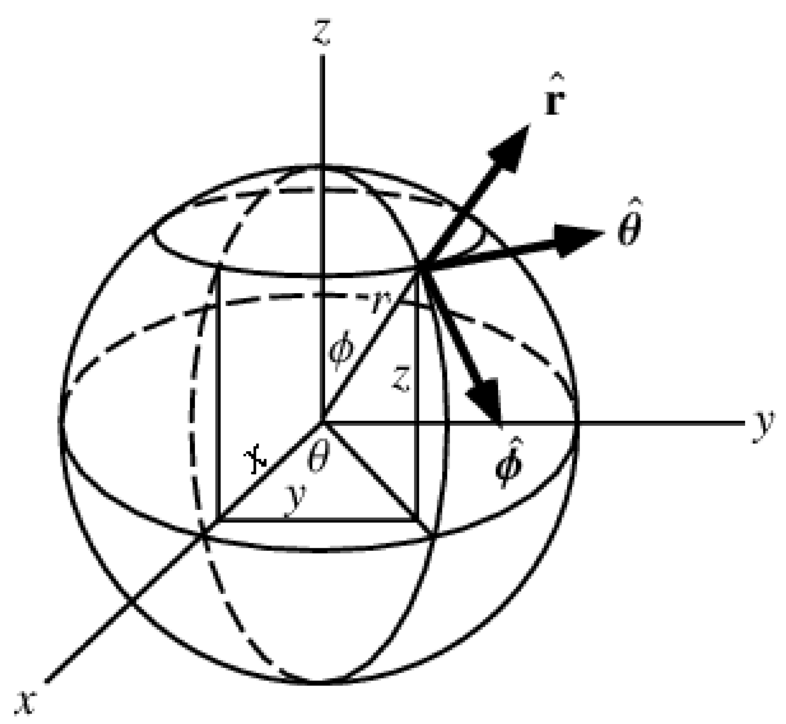
$\int_{S(t)} \vec{f_1} \cdot \vec{u_2}\, dS \ = 0$ and

Applying these values to (2) we get that $\int_{S(t)} \vec{f_2} \cdot \vec{u_1}\, dS = U(t) + \frac{1}{4\pi R^2}\int_{S(t)} \tilde{u} \cdot \hat{k}$.

Hence we derive an expression for U(t):

$$U(t) = -\frac{1}{4\pi R^2}\int_{S(t)} \tilde{u} \cdot \hat{k}\ dS \qquad (3)$$

Spherical Coordinates & Final Expression for U(t)



Defining spherical coordinates as:

$$x = r\cos\phi\sin\theta \qquad (0 \leq \quad r < \infty)$$
$$y = r\sin\phi\sin\theta \qquad (0 \leq \quad \theta < \ \pi)$$
$$z = r\cos\theta \qquad (0 \leq \quad \phi < 2\pi)$$

Hence, we get that $dS = R^2\ sin\theta\ d\theta\ d\phi$

Also, we assume the flow to be **axisymmetric**, then we find the slip velocity $\tilde{u} \ = \ g(\theta) \cdot \hat{\theta}$ on S for some function $g(\theta)$ where $\hat{\theta} = cos\theta\ cos\phi\ \hat{\imath} + cos\theta\ sin\phi\ \hat{\jmath} - sin\theta\ \hat{k}$

Using these spherical coordinates we get the expression for the speed of the body:

$$U(t) = -\frac{1}{4\pi R^2}\int_0^{\pi}\int_0^{2\pi} R^2\ g(\theta)\ sin^2\theta\ d\phi\, d\theta = \ \frac{1}{2}\int_0^{2\pi} g(\theta)\ sin^2\theta\ d\theta \qquad (4)$$

The speed depends uniquely on this function $g(\theta)$.

Changing $g(\theta)$

If we take $g(\theta)$ to be constant, from (3) we find that $U(t) = \frac{\pi}{4}\ g(\theta)$.

Non-costant $g(\theta)$

For some non-costant g the value of U could be anything, but one can observe that the graph of $sin(\theta)^2$ is always above the x axis.

Hence, even though $sin(\theta)^2$ does have an impact on the solution of U(t), we can look whether $\int_0^{2\pi} g(\theta) d\theta$ is big or small, higly negative or highly positive; also the value of U(t) will be so.

Applications

Medicine Applications

For this project, I have made numerous assumptions such as **low Reynolds numbers, incompressible and axisymmetric fluid motion and a rigid spherical body.** However, every model has to start by taking some assumptions, which they can eventually get rid of once there is better understanding of the topic. One field of interest where studying the way microorganisms propel themselves might be important is Medicine.

In the future we might want bacteria and other microorganisms to deliver drugs to targeted areas of the body via veins in order to be more effective and at the same time reduce collateral effects. Another use might be to study the spread of Pathogens through human fluids in order to prevent them from swimming and consequently curing diseases.

References

[1] WolframMathWorld. Spherical Coordinates. Available from: http://mathworld.wolfram.com/SphericalCoordinates.html [Accessed June 2017]

[2] Eric Keaveny. 2017. *Life in moving fluids.* [Lecture] Imperial College London, 31st May.

[3] Stone,H. and Samuel, A. (1996) Propulsion of Microorganisms by Surface Distortions. *Physical Review Letters.* 77. 19. 4102-4104 Available from: https://bb.imperial.ac.uk/bbcswebdav/pid-1109133-dt-content-rid-3767817_1/courses/DSS-M1R-16_17/StonePRL1996.pdf [June 2017]

[4] Blake, J. (1971) A spherical envelope approach to ciliary propulsion. *Journal of Fluid Mechanics.* 46. 199-208 Available from: https://doi.org/10.1017/S002211207100048X [June 2017]

[5] Lauge, E. and Powers, T. (2009) The hydrodynamics of swimming microorganisms. *Reports on Progress in Physics.* 72. 1-36 Available from: stacks.iop.org/RoPP/72/096601 [June 2017]

[6] sluh. *Cell Structure.* Available from: http://www2.sluh.org/bioweb/apbio/studysheets/ss_cell_structure.htm [June 2017]