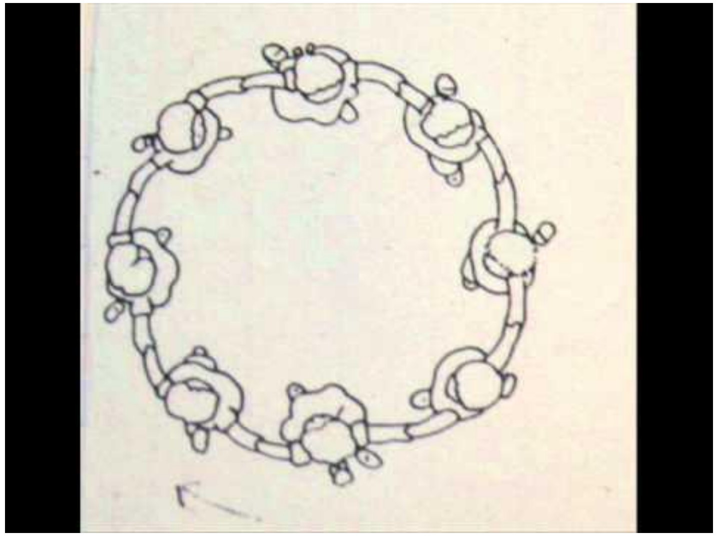
MODELLING OF COVID19 INTERVENCTIONS ON GRAPHS

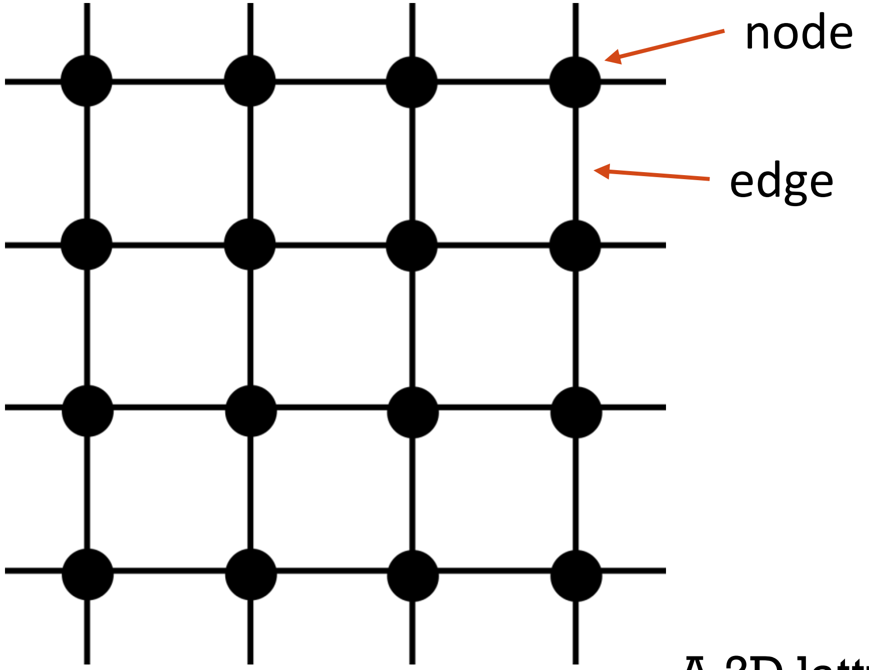
WHY GRAPHS?

- Social structures.
- «Super-spreaders».
- A disease might die out locally but thrive elsewhere.
- Responses (social distancing, travel restrication…) are easy to model.

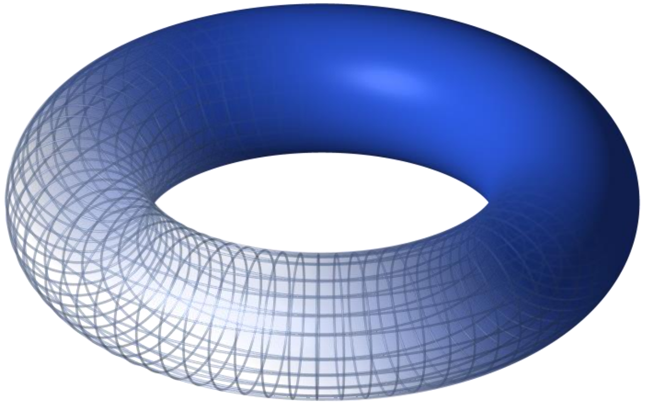
WHAT IS A GRAPH?

- A set of nodes V and a set of edges E connecting various nodes.



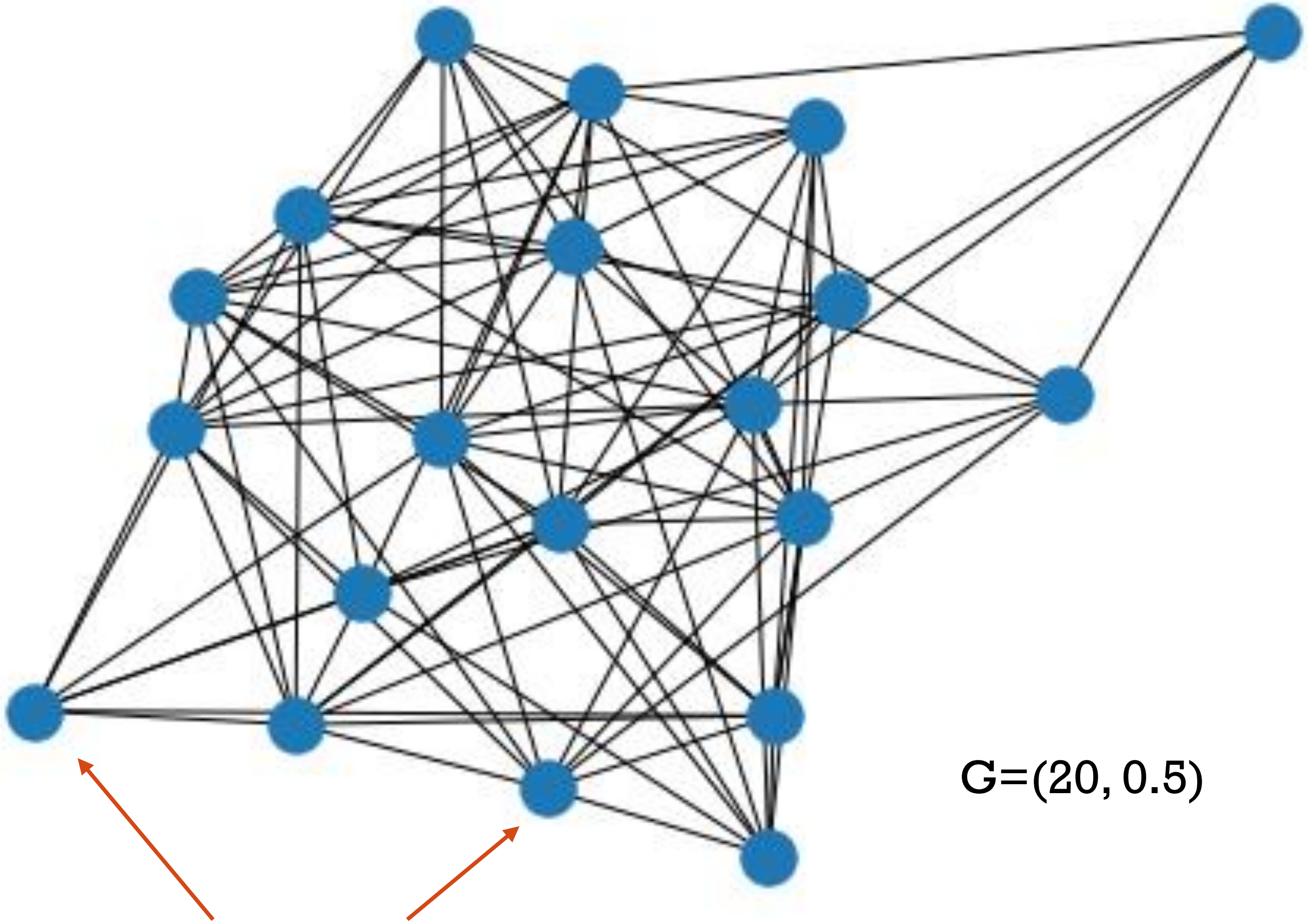


A 2D lattice



A torus

$k_i = 4$

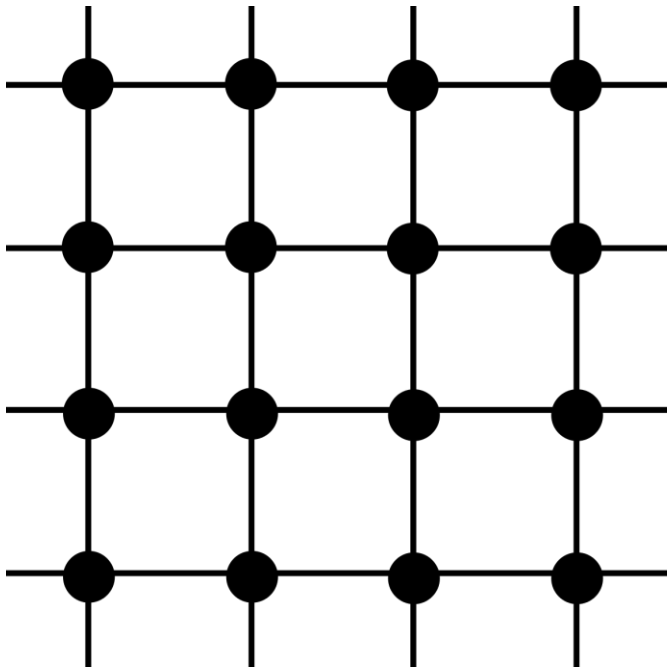


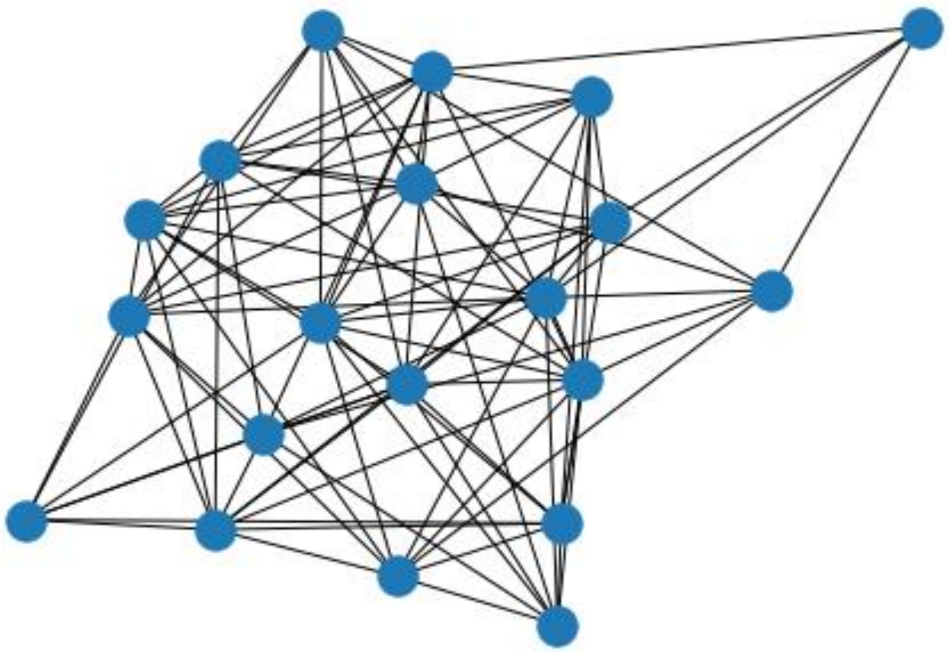
G=(20, 0.5)

The (minimal distance between these nodes is 2.

- Random graph G = (N,p) is defined by N nodes and probability p: each possible edge is assigned with probability p.
- Each node i has a <u>node degree</u> k_i: the number of edges connecting node i to other edges in the graph.
- If u → v_1 → … → v_k exists and is the shortest path between nodes u&v_k, then we say that the <u>(minimal) distance</u> between u and v_k is k.

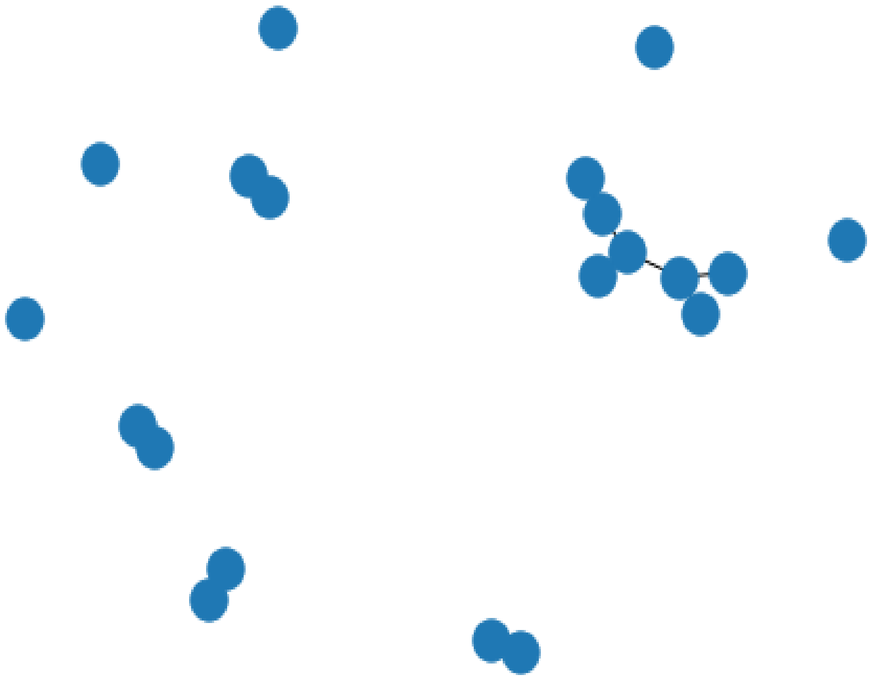
«PART 1: REAL GRAPH PROPERTIES»



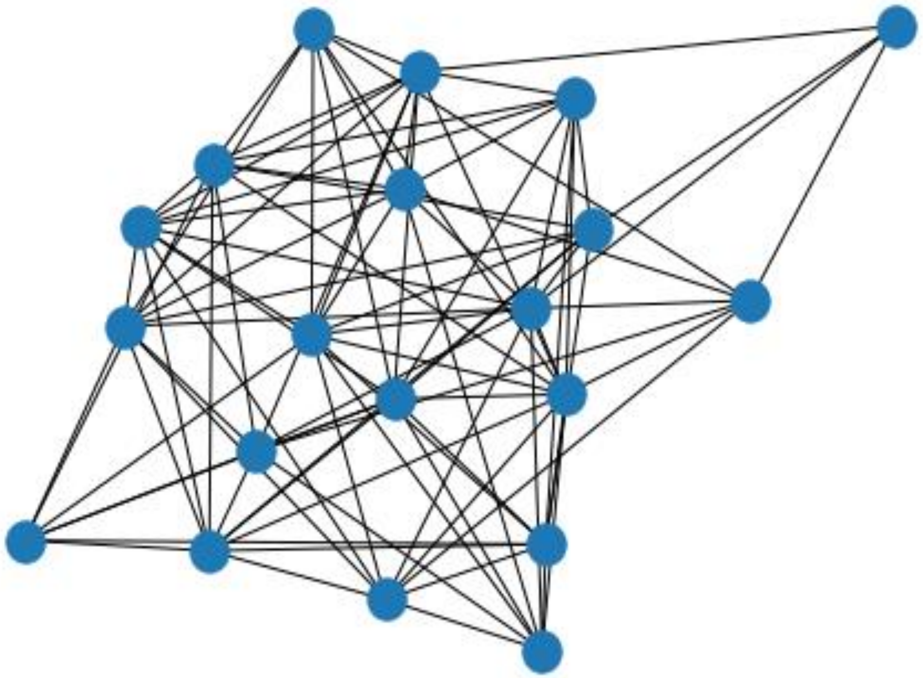


SPARSITY

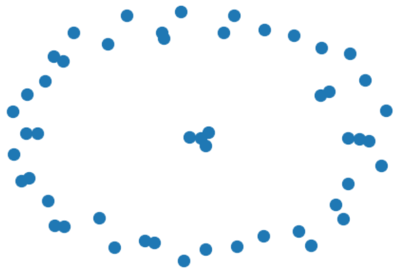
You are likely to be connected to a tiny fraction of the graph.



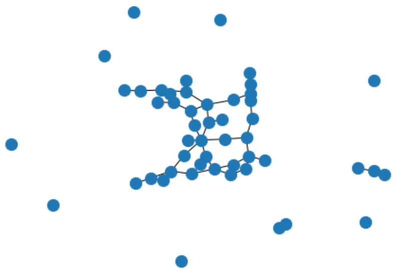
G=(20, 0.05)



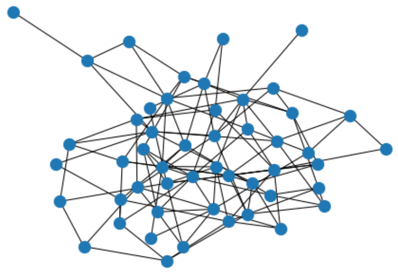
G=(20, 0.5)



Disconnected graph



Giant component



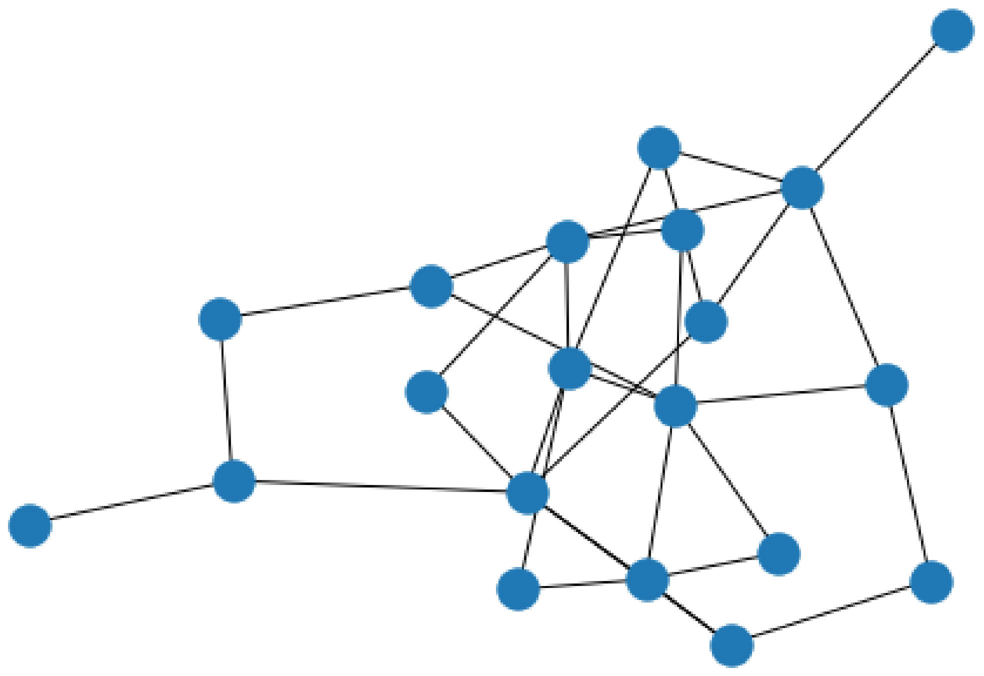
Completely connected graph

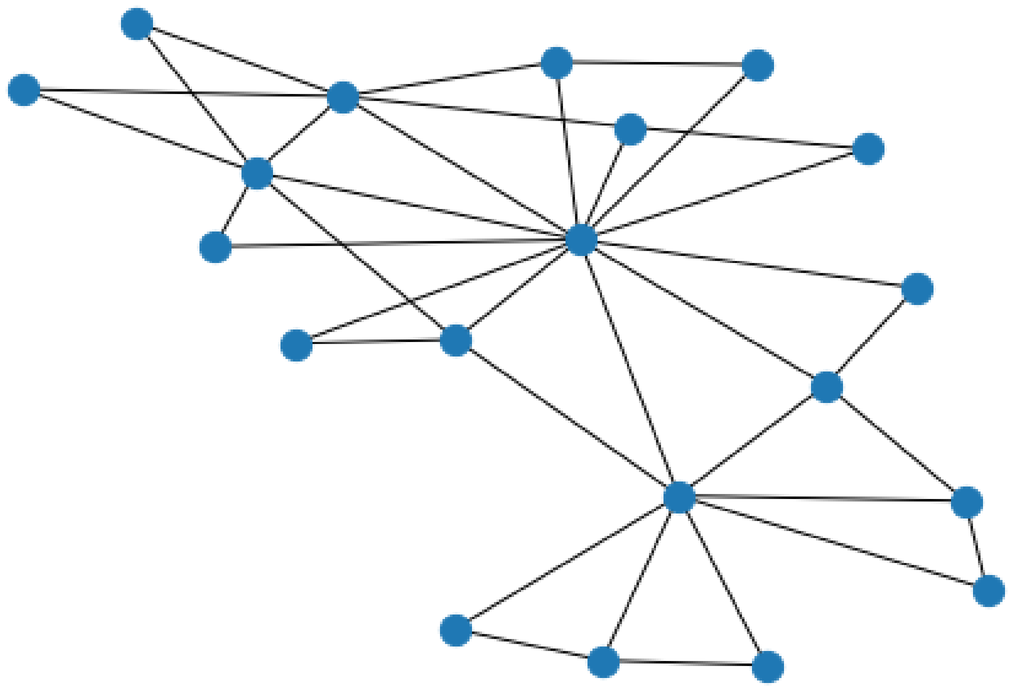
GIANT COMPONENT

From analysis $\langle k \rangle$=1 is the threshold a for finite connected portion and $\langle k \rangle$ = lnN for a fully connected graph.

CLUSTERING

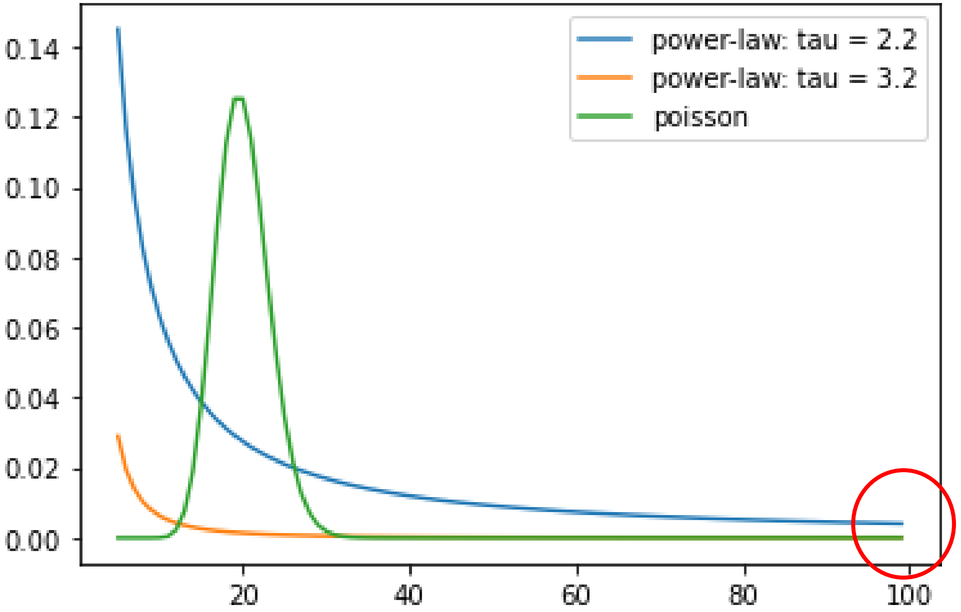
- «The friend of my friend is likely my friend».
- Doesn’t depend on size!





NODE DISTRIBUTION

- Node degrees have a distribution. Which one?
- Power Law: $p(k) = C \cdot k^{-(\tau-1)}$



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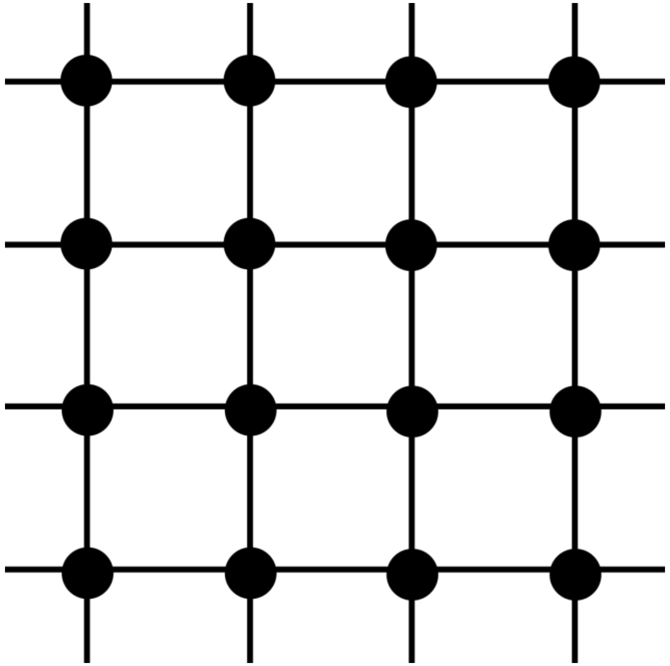
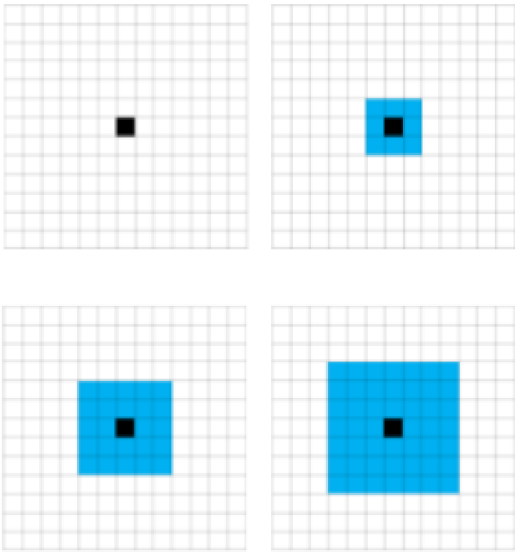
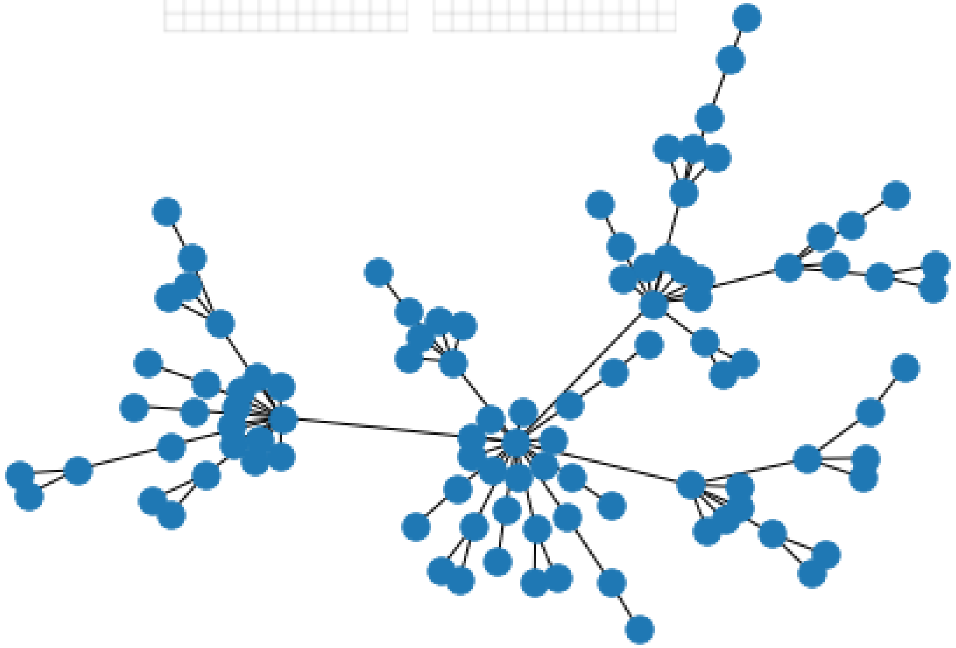


SMALL-WORLD PHENOMENA

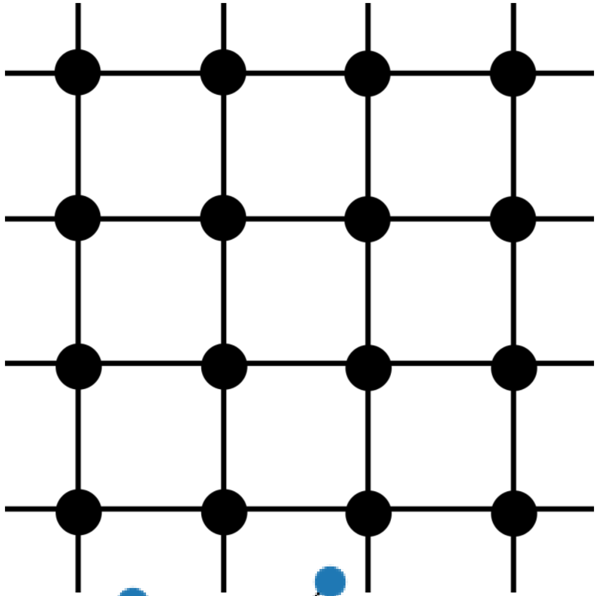


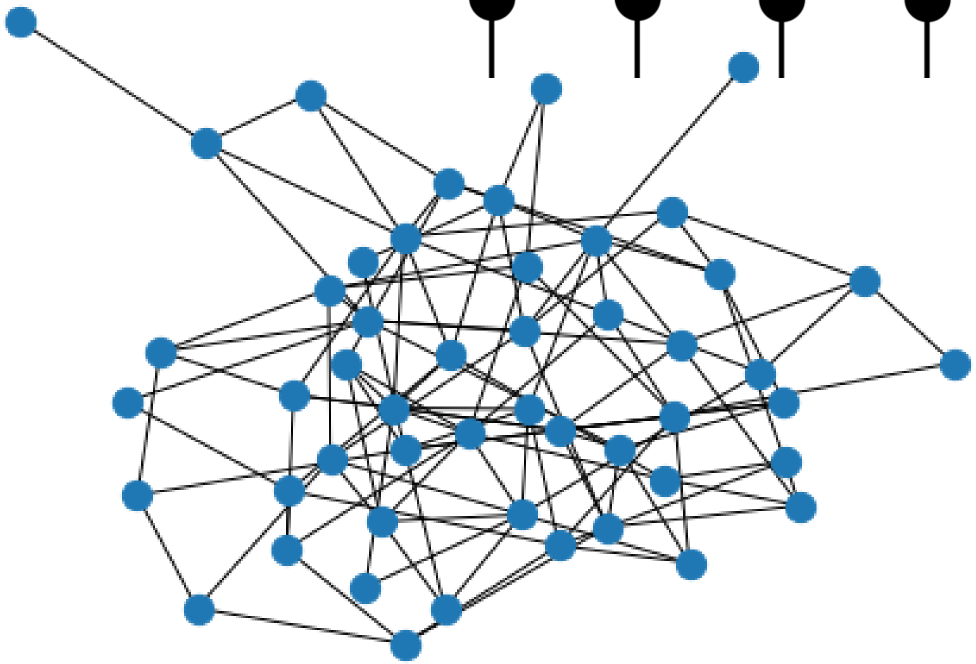
- All k-th moments of node degrees are infinite, for $k > \tau - 1$. Average distances in the graph are small, less than log(N), where N is the graph size.
- If $2 < \tau < 3$, the variance of the node degree is infinite, and the average distance is less than log(log(N)). Such case is called ultra-small world.
- For social network that value is less than 10. That means you can “reach” any person in the world with less than 10 acquaintances of acquaintances.



REAL GRAPHS PROPERTIES:

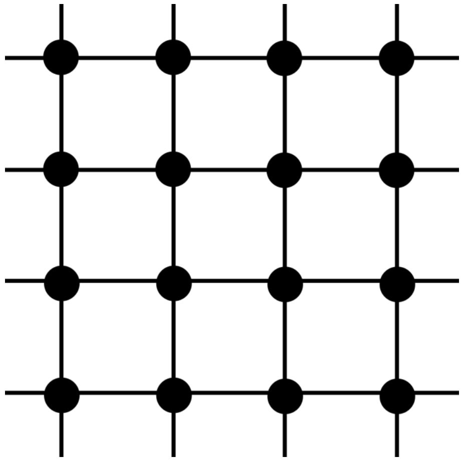
- Sparsity
- Giant component
- High clustering
- Power-law node degree distribution
- Small world phenomena
- Geometry

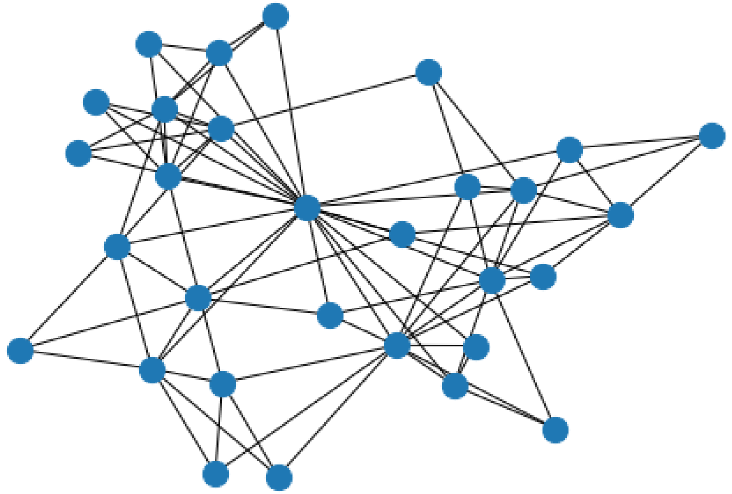




The epidemic is modelled as an ODE model for comparison and on three different types of graphs.

- A lattice (2D grid on a torus, so that there’s no boundary behaviour). It is geometric, but all nodes have the same degree.
- The configuration model (a random graph, which is constructed such that the nodes have a power law degree distribution). It has clusters, the small world (or ultra-small world) property, but it has no geometry.
- The Geometric inhomogeneous random graph (GIRG), a mixture between the previous two. It has all the properties we require.

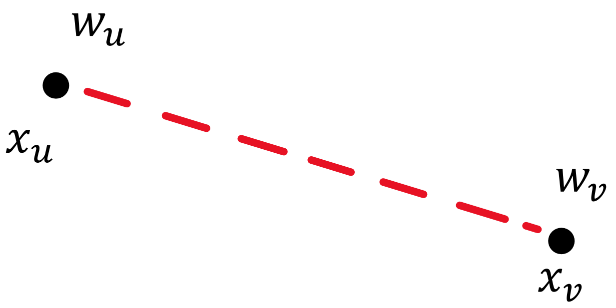


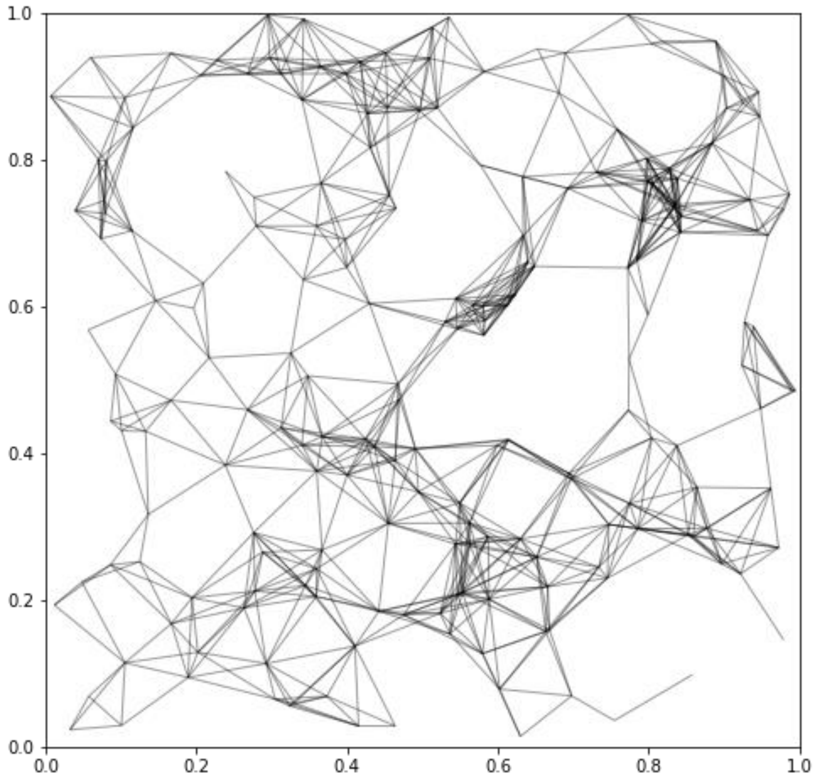


GEOMETRIC INHOMOGENOUS RANDOM GRAPHS (GIRG): CONSTRUCTION

- Given N nodes, each node u is assigned a weight w_u according to a power-law distribution, and a uniformly random position x_u. Given nodes u, v, values w_u, x_u, x_v, w_v and parameter, the likelihood that an edge is assigned between u and v is:

$$Prob(edge\ between\ u\ and\ v)\ = \left(\frac{w_u \cdot w_v}{dist(x_u,\ \ x_v)^2}\right)^{\alpha}$$





Increasing α we decrease the probability of having long edges.

PART 2: MODELLING

- On graphs, each node can be in three states: susceptible, infective and temporary immune.
- Each (discrete) time step each infective node can infect each susceptible neighbour with probability β, it can itself heal with probability γ; temporary immune nodes can lose their immunity with probability η.

$$\begin{cases} s' = ? \\ i' = ? \\ t' = ? \end{cases}$$

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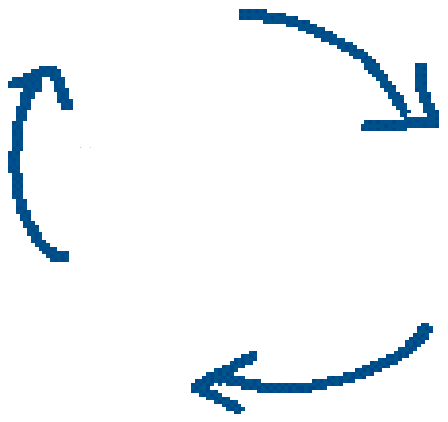

$$\begin{cases} s' = -\beta \cdot s\, i + \eta \cdot \mathrm{t} \\ i' = \beta \cdot s\, i - \gamma \cdot i \\ t' = \gamma \cdot i - \eta \cdot \mathrm{t} \end{cases}$$

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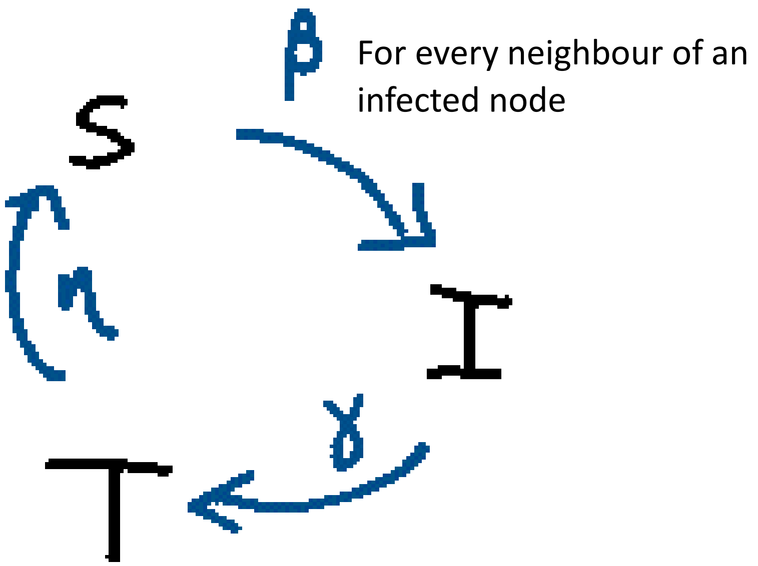

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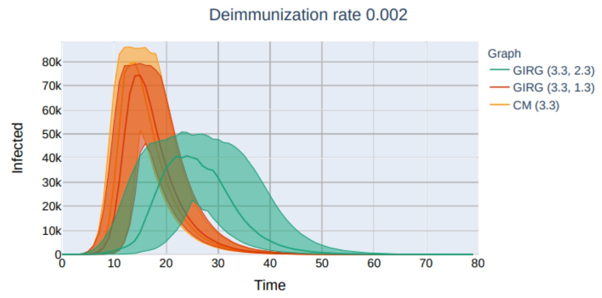
SIMULATION WITHOUT RESTRICTIONS

- In the ode model, the disease either dies out if $R_0<1$, or it has an initial exponential increase, followed by damped oscillations until it reaches stability.
- Stochastic effect: the disease may die out with non-zero probability also for $R_0>1$. See Galton Watson process studied (for those who took the course) in mathematical biology 1.
- Extra possible outcome: if η (loss-of-immunity parameter) is big enough there's one peak and then the disease dies out (SIR situation).
- There is a sharp transition of parameters between the three possible phases.



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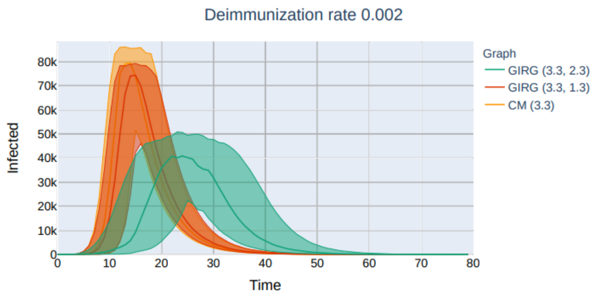

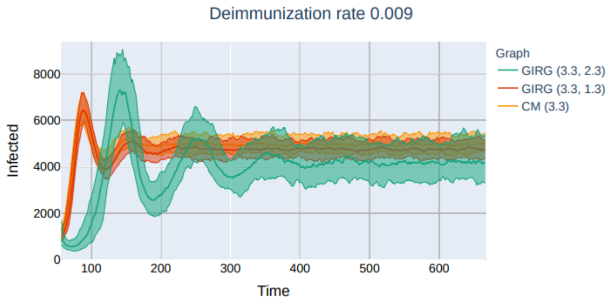

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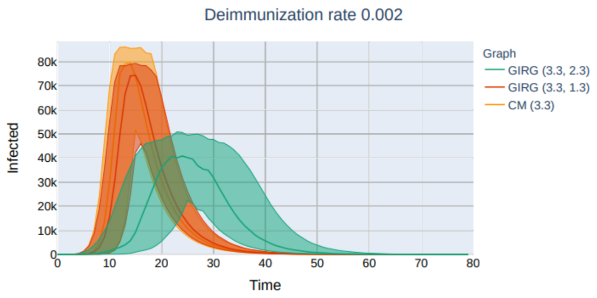
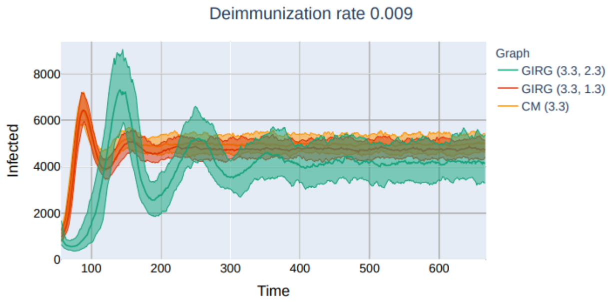
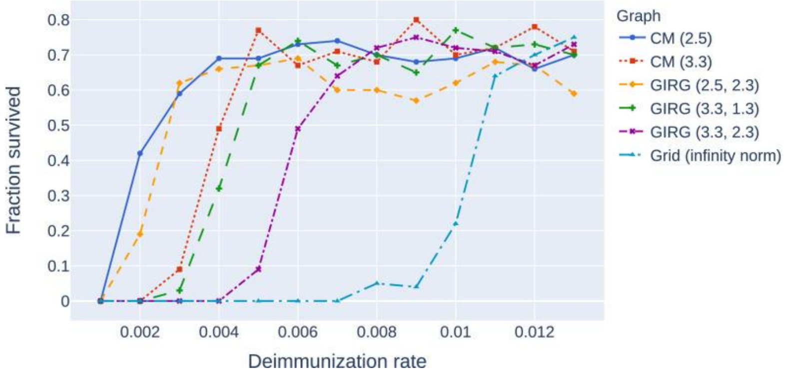
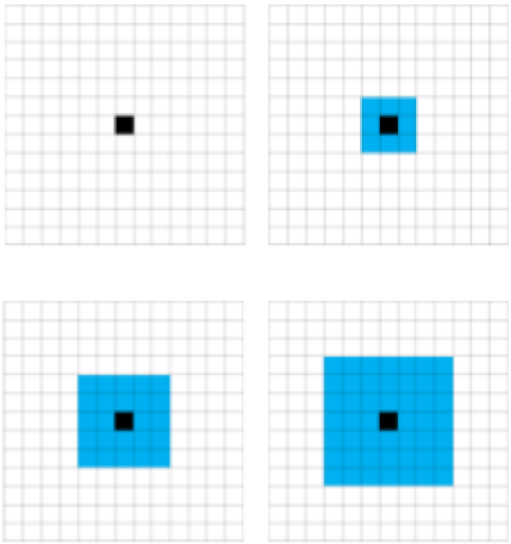


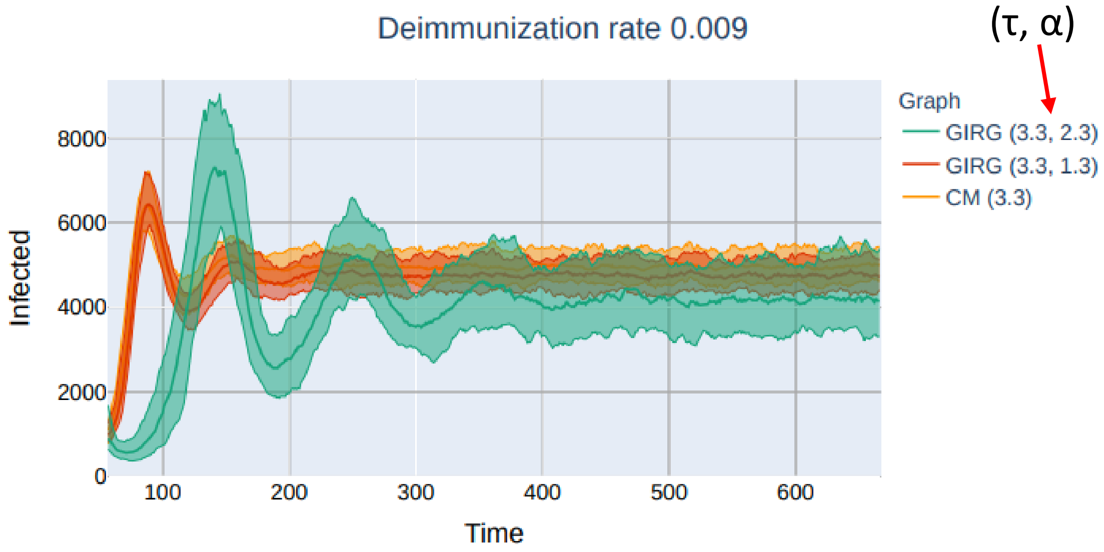


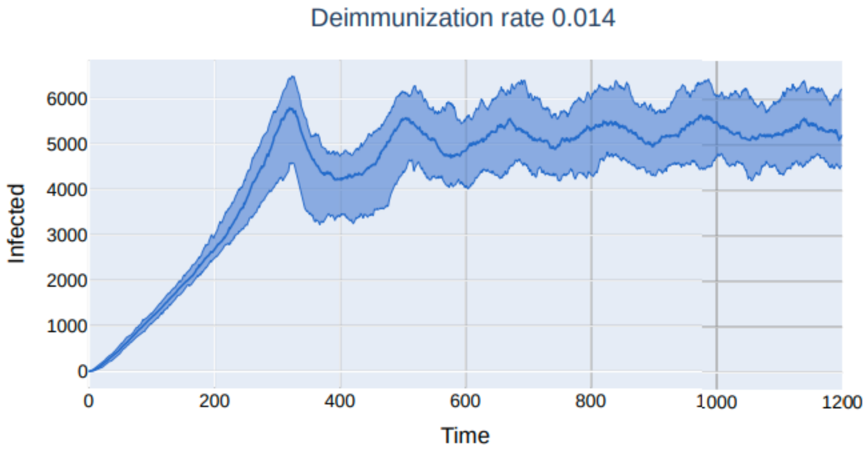


DIFFERENCES BETWEEN THE 3 GRAPHS

- Note: ODE, Configuration model & GIRG (with many hubs and long connections) have all initial exponential increase, when R_0>1. The lattice & GIRG (without many hubs and long connections) it's linear.

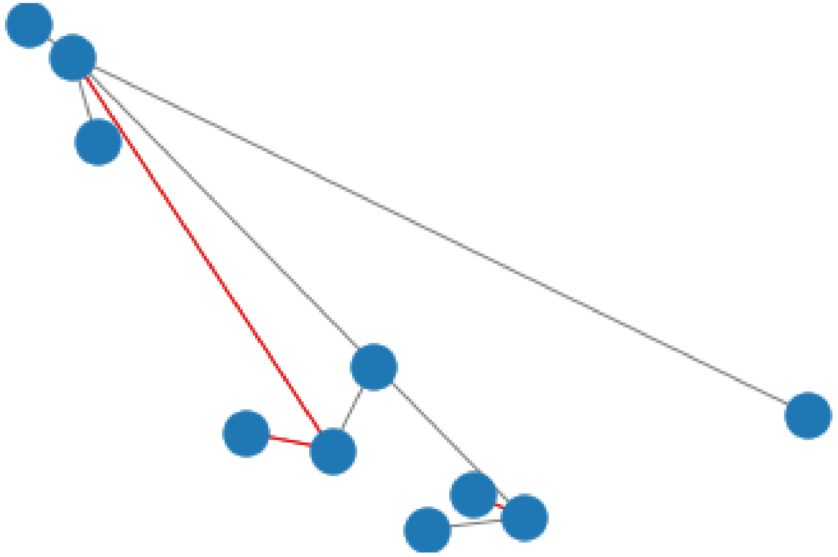


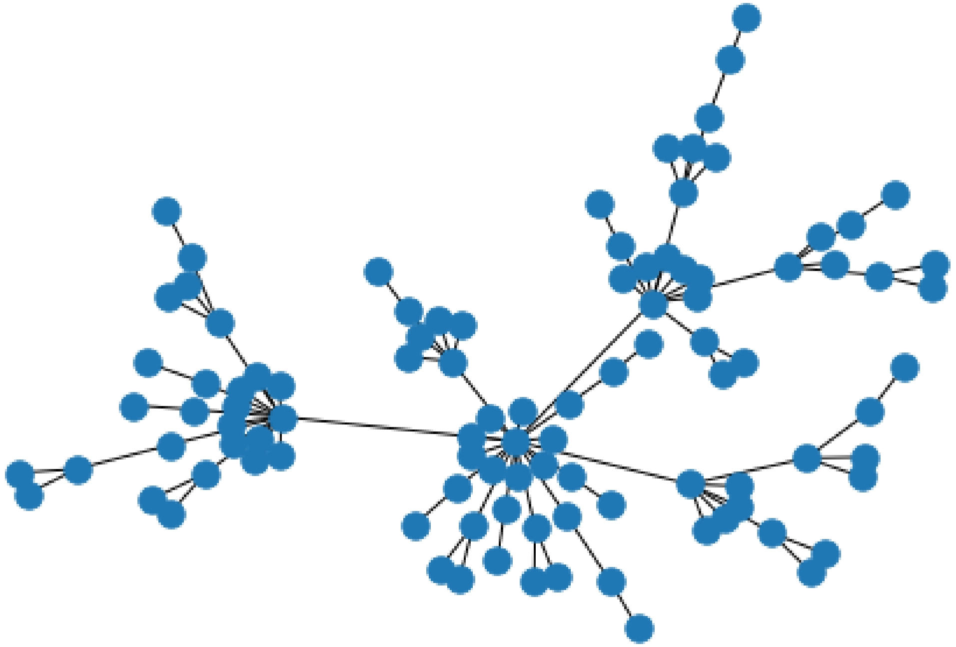




MODELLING OF RESTRICTIONS: PHYSICAL DISTANCE

- Randomly remove edges from the graph.





MODELLING OF RESTRICTIONS: TRAVEL RESTRICTIONS

$$P = \left(\frac{w_u \cdot w_v}{dist(x_u,\ x_v)^2}\right)^{\alpha}$$

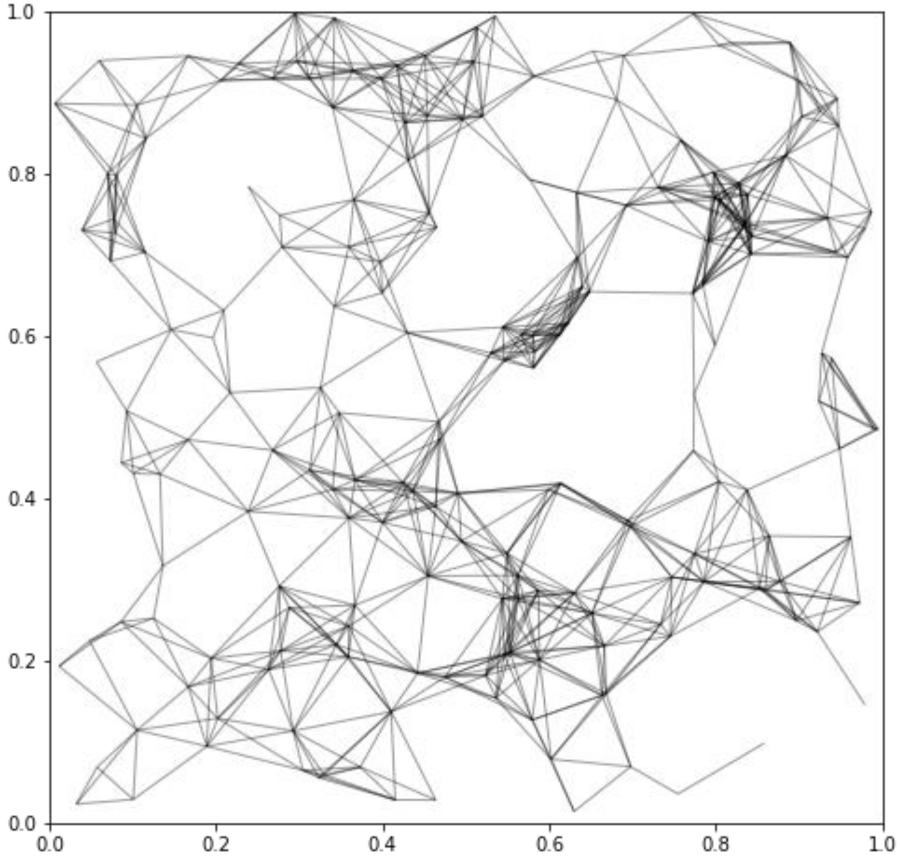
- Two ideas for removing long edges from the GIRG are:

1. Increasing parameter α.

It works for $\tau > 3$, but it doesn't for $2 < \tau < 3$.

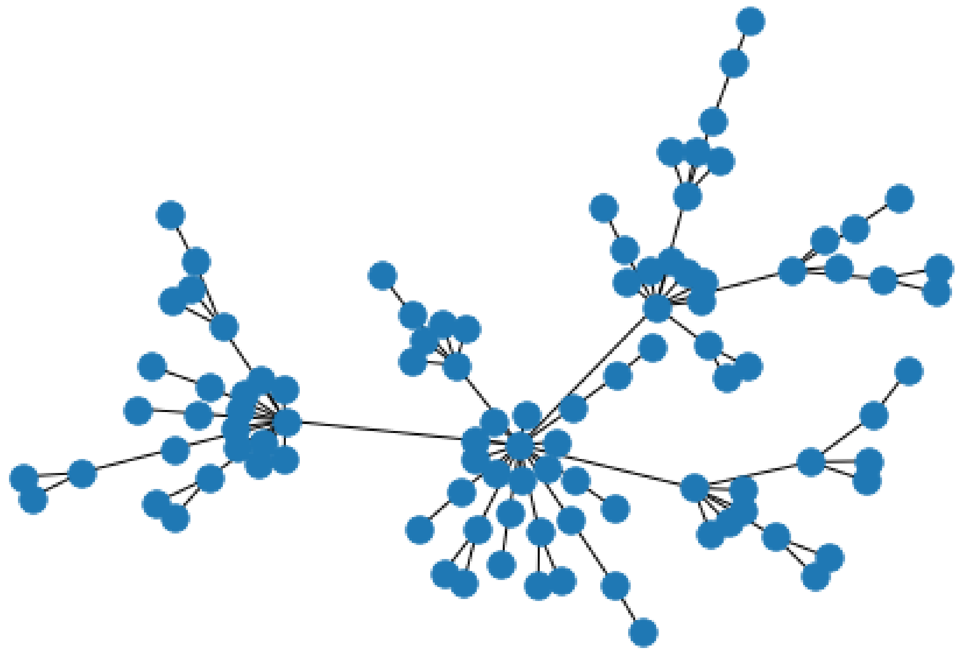
Why? Remember: the node degree variance is infinite, so increasing parameter α is not enough to "destroy" the hub structure of the graph.

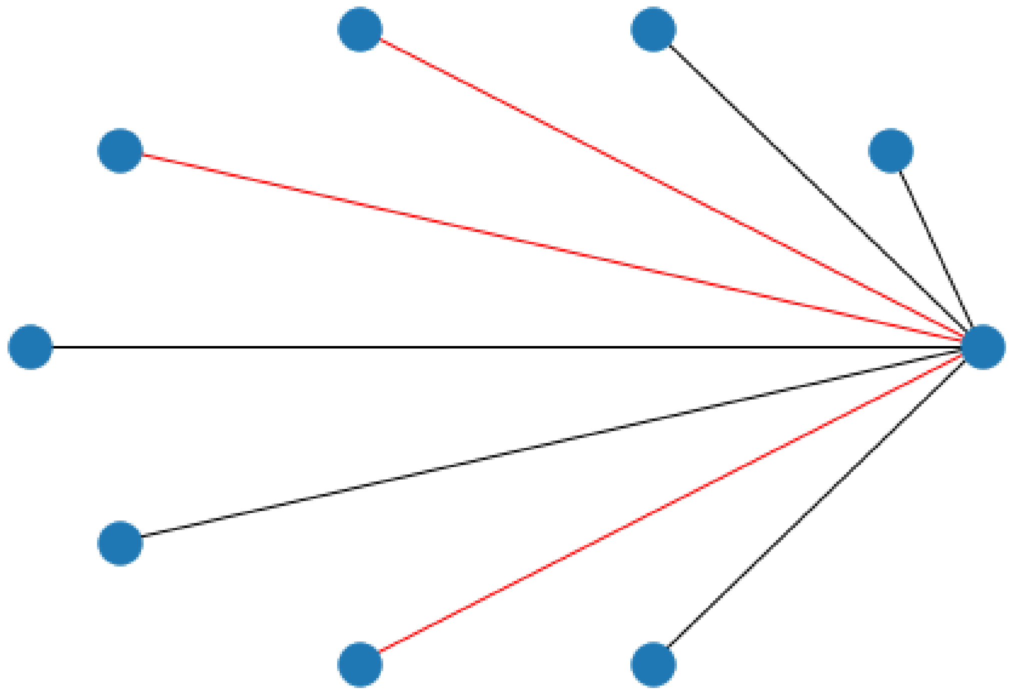
2. For each edge between nodes u, v, set a cut-off value L and if $L < d(x_u, x_v)$ then we delete the edge.



MODELLING OF RESTRICTIONS: MAXIMUM NUMBER OF CONTACTS PER PERSON

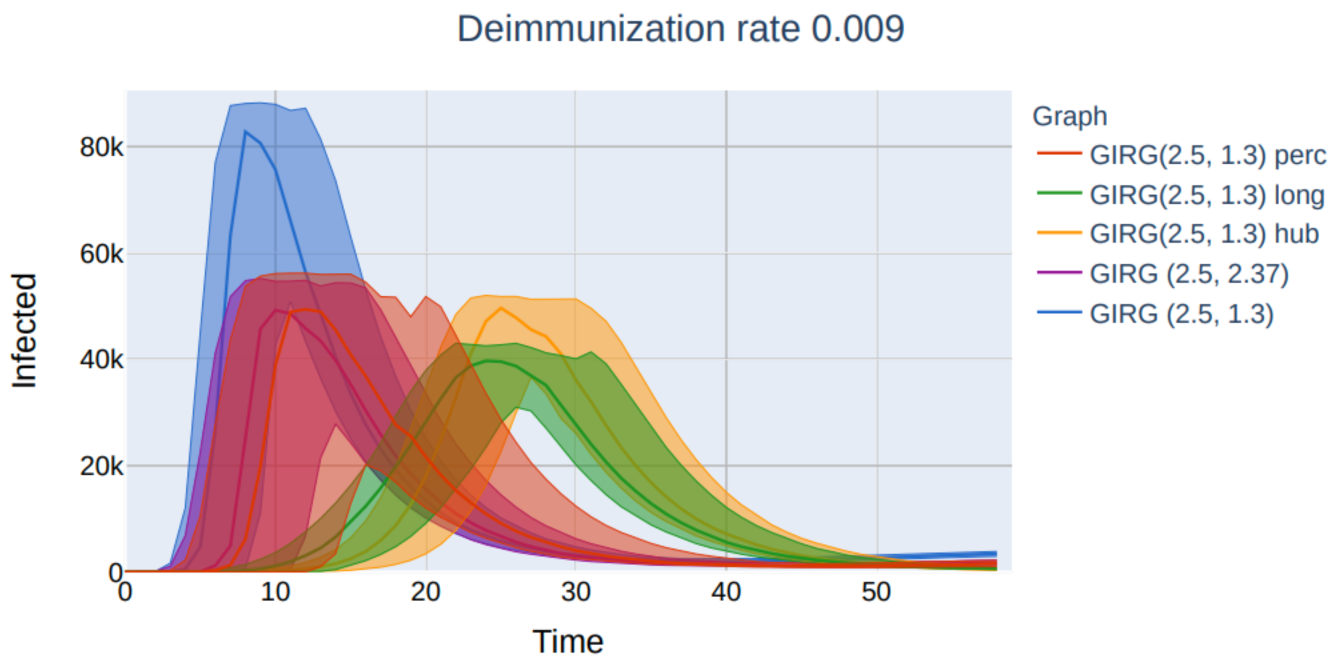
- Set a maximal node degree M for all nodes v that have a higher node degree we randomly delete edges connected to v until its degree is within M.





OTHER RESULTS

- All interventions decrease the height of the first peak. On the other hand, they extend the time period of the peak. The critical η is shortened, so it is possible that intervention cause the disease to survive instead of dying out after the first peak. Travel restrictions (2) are the most effective in lowering the first peak.



LIMITATIONS OF THE MODEL?

SOURCES

- https://networksciencebook.com/
- Jorritsma J, Hulshof T, Komjáthy J. Not all interventions are equal for the height of the second peak. Chaos Solitons Fractals. 2020 Oct;139:109965. doi: 10.1016/j.chaos.2020.109965. Epub 2020 Aug 25. PMID: 32863609; PMCID: PMC7445132.
- Bringmann, Karl & Keusch, Ralph & Lengler, Johannes. (2015). Geometric Inhomogeneous Random Graphs. Theoretical Computer Science. 760. 10.1016/j.tcs.2018.08.014.

OTHER SOURCES

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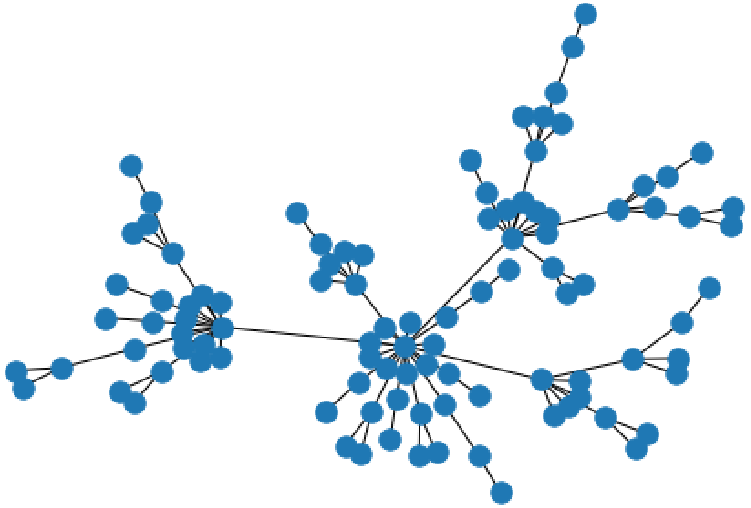
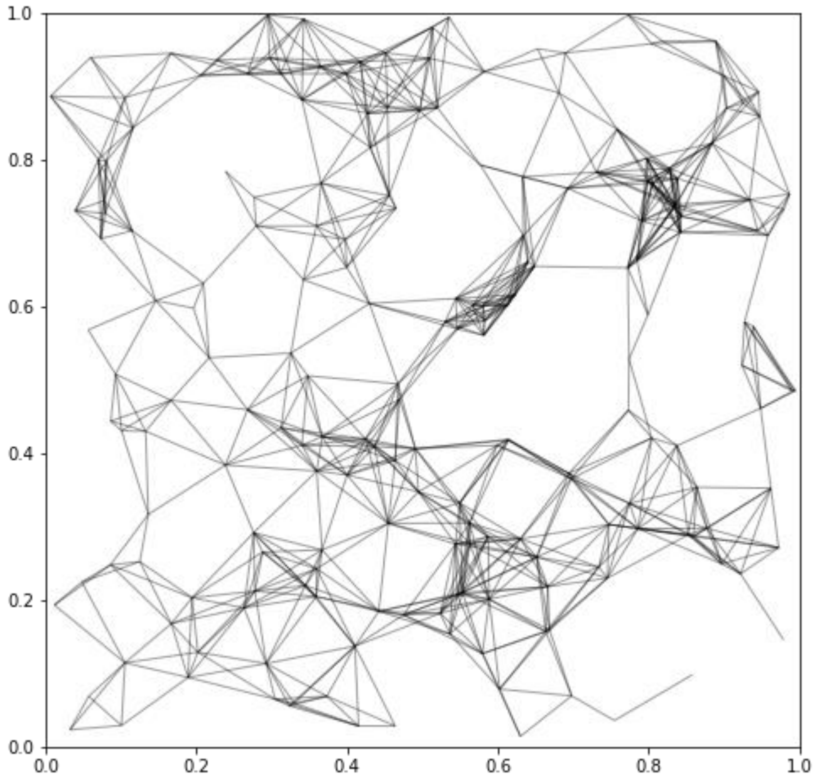
- https://networkx.org/documentation/networkx-1.7/examples/drawing/random_geometric_graph.html

THANKS TO …

- Prof. Kuttler, for providing the opportunity of this seminar
- You, for the attention





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