CONTROL AWARE COMMUNICATION DESIGN FOR TIME SENSITIVE WIRELESS SYSTEMS

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ABSTRACT

We consider the problem of allocating radio resources over wireless communication links to control a series of independent low-latency wireless control systems common in industrial settings. Supporting wireless control in time sensitive settings requires fast data rates over wireless links, which comes at the cost of reliability. It is challenging to meet both latency and reliability requirements with an equal or arbitrary allocation of resources. We thus propose a novel control-aware approach to the low-latency scheduling problem in which we incorporate control and channel state information in allocating bandwidth and data rates across the wireless links. Control systems that are in desirable states are given modest requirements on error rates, while systems in undesirable states are given more priority. We derive control-aware packet error rate targets for each system to satisfy stability goals and make scheduling decisions to meet such targets while reducing total transmission time. The resulting control-aware based method is tested in simulation experiments that demonstrate its effectiveness in meeting control-based goals under tight latency constraints relative to control-agnostic scheduling.

Index Terms— wireless control, low-latency, control-aware, scheduling

1. INTRODUCTION

The increasing scale of modern IoT and industrial control systems has motivated the development of wireless control system technology that can achieve reliable performance in these settings [1, 2]. One of the primary challenge of wireless control in industrial settings, however, is the time sensitive nature of the systems, thus requiring low latency wireless transmissions [1]. The noise of the wireless channel makes it difficult to simultaneously maintain high reliability while achieving low latency. This motivates the design of resource allocation and scheduling strategies that can both meet reliability *and* latency requirements of the industrial control system.

In the wireless communications research and industry, many radio resource allocation schemes in the form of wireless scheduling techniques have been proposed to provide reliability, or quality of service (QoS), to users across the network in the form of throughput, fairness and/or latency [3–6]. For time-sensitive applications, delay-aware schedulers such as EDF [7] and WFQ [8] have been developed, while M-LWDF [9] extends these ideas to include channel state information.

Likewise, in the context of wireless control systems, dynamic schedulers have been developed that provide access to the communication medium dynamically at each step. Initial approaches make scheduling decisions based on abstractions of control performance [10, 11]. More recently, "control-aware" scheduling approaches make decisions explicitly based on current control system states , [12–20]. Further work takes into account current wireless channel conditions when attempting to meet target control system reliability requirements [21]. The work here takes a similar channel-based opportunistic approach, while further incorporating current control states with the explicit goal of meeting low-latency requirements. The proposed approach has been developed specifically for the IEEE 802.11ax protocol in [22].

This paper is organized as follows. We formulate the wireless control system in which state information is communicated to the control over a wireless channel as a switched dynamical system (Section 2). We then discuss the communication architecture that determines the speed and error rate of transmissions (Section 2.1). With this formulation, we adapt concepts of control-communication control-aware for low latency settings by using current control states and channel conditions to derive dynamic packet success rates necessary for each user (Section 3). In this manner, control systems with the most critical communication needs are given higher packet delivery rate targets to meet. The scheduling procedure leverages these dynamic, more liberal rate requirements to reduce total latency, incorporating both a selective scheduling procedure (Section 3.1) and an assignment-method based that attempts to further reduce total transmission time (Section 3.2). The performance of the control-aware method is analyzed in a representative low-latency simulation experiment in which its performance is compared against a control-agnostic procedure (Section 4).

2. TIME SENSITIVE WIRELESS CONTROL SYSTEMS

Consider a system of m independent linear control systems, or devices, where each system $i = 1, \ldots, m$ maintains a state variable $\mathbf{x}_i \in \mathbb{R}^p$. The dynamics evolve over a discrete time index k. Applying an input $\mathbf{u}_{i,k} \in \mathbb{R}^q$ causes the state and output to evolve based on the generic state space representation,

$$\mathbf{x}_{i,k+1} = \mathbf{A}_i \mathbf{x}_{i,k} + \mathbf{B}_i \mathbf{u}_{i,k} + \mathbf{w}_k \tag{1}$$

where $\mathbf{A}_i \in \mathbb{R}^{p \times p}$ and $\mathbf{B}_i \in \mathbb{R}^{p \times q}$ are matrices that define the system dynamics, and $\mathbf{w}_k \in \mathbb{R}^p$ is Gaussian noise with co-variance $\mathbf{W}_i$ that captures the noise in the model. We further assume the state transition matrix $\mathbf{A}_i$ is on its own unstable, i.e. has at least one eigenvalue greater than 1. This is to say that, without an input, the dynamics will drive the state $\mathbf{x}_{i,k} \to \infty$ as $k \to \infty$.

In the time sensitive wireless control system, each system is closed over a wireless medium through which the sensor located at the control plant sends state information to the controller located at a common wireless base station (BS). Using the state information $\mathbf{x}_{i,k}$ received from device i at time k, the controller determines the input $\mathbf{u}_{i,k}$ to be applied. We stress that, due to the latency constraints of the control system operation, the BS gives each device only a short time window, or transmission opportunity, to finish transmitting its state information. This model restricts its attention to wireless connections in uplink of the control loop, while downlink is assumed to occur over an ideal channel.

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To derive the mathematical model, consider a generic linear control $\mathbf{u}_{i,k} = \mathbf{K}_i \mathbf{x}_{i,k}$ for some matrix $\mathbf{K}_i \in \mathbb{R}^{q \times p}$. Many common control policies indeed can be formulated in such a manner, such as LQR control. This matrix $\mathbf{K}$ is chosen such that the closed loop dynamics $\mathbf{A} + \mathbf{B}\mathbf{K}$ has all eigenvalues less that 1. Due to noise in the channel, there is potential for state information packets to be dropped in the uplink, which is modeled as "open-loop" configuration. Meanwhile, successful transmission are modeled as a "closed-loop" configuration. To account for incomplete state information at the BS due to packet drops, we consider the estimate of state information of device i known to the BS/controller at time k as

$$\hat{\mathbf{x}}_{i,k}^{(l_i)} := (\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_i)^{l_i} \mathbf{x}_{i,k-l_i}, \tag{2}$$

where $k - l_i \geq k - 1$ is the last time instance in which control system i was closed. Observe that in (2) we assume that the BS/controller has knowledge of $\mathbf{A}_i, \mathbf{B}_i$, and $\mathbf{K}_i$, but not the noise $\mathbf{w}_k$ present in (14).

At time k, if the state information is received, the controller applies the input $\mathbf{u}_{i,k} = \mathbf{K}_i \mathbf{x}_{i,k}$ using the exact state, and otherwise applies input $\mathbf{u}_{i,k} = \mathbf{K}_i \hat{\mathbf{x}}_{i,k}$. We obtain then the following switched system dynamics for $\mathbf{x}_{i,k}$ as

$$\mathbf{x}_{i,k+1} = \begin{cases} (\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_i)\mathbf{x}_{i,k} + \mathbf{w}_k, \text{ in closed-loop,} \\ \mathbf{A}_i \mathbf{x}_{i,k} + \mathbf{B}_i \mathbf{K}_i \hat{\mathbf{x}}_{i,k}^{(l_i)} + \mathbf{w}_k, \text{ in open-loop.} \end{cases} \tag{3}$$

The transmission counter l_i is updated at time k as

$$l_i \leftarrow \begin{cases} 1, \text{ in closed-loop,} \\ l_1 + 1, \text{ in open-loop.} \end{cases} \tag{4}$$

Observe that the successive error between the true and estimated state can be written as $\mathbf{e}_{i,k} := \mathbf{x}_{i,k} - \hat{\mathbf{x}}_{i,k}^{(l_i)} = \sum_{j=0}^{l_i-1} \mathbf{A}_i^j \mathbf{w}_{i,k-j-1}$. It is evident that this error grows with the transmission counter l_i. We proceed now to describe the architecture of the wireless communications that determines both the speed and error rate of the state information transmissions over the wireless channel.

2.1. Communication architecture

We consider a standard communication architecture in which, within a transmission window of a single cycle in the control loop, devices are scheduled by the BS across discrete time division (TD) and frequency division (FD) slots, both of which may vary in size. To adapt transmission lengths, each device is given a data rate (DR) parameter. Collectively, the assignment of device to their respective TDs, FDs, and with determined DR fully specifies the scheduling for the given transmission window. This architecture reflects that used scheduling-based multiple access wireless protocols, such as LTE [23] , 5G [24], and next-generation WiFi IEEE 802.11ax [25]. The transmission power is assumed fixed and equal across all devices.

We state this model formally with the following variable definitions. Consider that the total allowable bandwidth is divided into b discrete bands of equal size, the FD slot of device i is specified by a binary vector $\varsigma_i \in \{0, 1\}^b$, where the j element $\varsigma_i(j) = 1$ if device i transmits in the jth frequency band. Note that a device may transmit in multiple *adjacent* bands simultaneously to indicate a FD slot of larger bandwidth. Because not all binary vectors of length b define a possible FD, we define the set $\mathcal{S} \subset \{0, 1\}^b$ to collect such a set of definable FD vectors. The FD assignment $\mathbf{0} \in \mathcal{S}$ is defined to reflect that a device does not transmit.

We further define for device i a positive integer value $\alpha_i \in \{1, 2, \ldots, S\}$ that denotes its TD slot and a real-valued $\mu_i \geq \mu_0$ to denote its DR, where μ_0 is a minimum allowable rate. Finally, we define $\mathbf{h}_{i,k} \in \mathbb{R}_+^b$ to be the set of fading channel states experienced by device i at cycle k, where $\mathbf{h}_{i,k}(j)$ is the fading channel gain in frequency band j. We assume that channel conditions do not change across different TDs within a single cycle/transmission window k.

The variables $\{\varsigma_i, \alpha_i, \mu_i\}$ define the scheduling specification for user i and, with the channel conditions $\mathbf{h}_{i,k}$, determine the communication performance obtained with such a scheduling decision. We first define a function $q(\mathbf{h}, \mu, \varsigma)$ which returns the probability of successful transmission/closing loop, otherwise called packet delivery rate (PDR)—given the channel conditions, DR and FD selections (this is independent of TD selection). Likewise, define by $\tau(\mu, \varsigma)$ a function that, given an DR μ and FD ς, returns the maximum time taken for a single transmission attempt (this is independent of TD selection and channel conditions). Both of these functions play a critical role in determining scheduling decisions in time-sensitive wireless control system settings. We are, in particular, interested in exploring the trade-off between PDR and transmission time that comes from varying μ. Generally speaking, the functions $q(\mathbf{h}, \mu, \varsigma)$ and $\tau(\mu, \varsigma)$ relate to μ by

$$\mu' > \mu \implies q(\mathbf{h}, \mu', \varsigma) \leq q(\mathbf{h}, \mu, \varsigma), \quad \tau(\mu', \varsigma) \leq \tau(\mu, \varsigma). \tag{5}$$

The goal then is to determine schedule $\{\varsigma_i, \alpha_i, \mu_i\}_{i=1}^m$ at every cycle k that keeps all control systems in a region of desirable performance, while keeping the total transmission time across all TDs small to meet latency requirements.

3. CONTROL-COMMUNICATION CO-DESIGN

We develop a control-communication control-aware approach towards scheduling in time-sensitive settings. Due to the tight latency constraints placed on the communications, we leverage knowledge of the control state and dynamics to determine a more principled and opportunistic method of scheduling devices. In particular, we use control information to identify maximum data rates we can achieve while maintaining strong control performance so as to meet latency targets. We first derive a manner in which we can evaluate control performance. Consider a quadratic Lyapunov function $L(\mathbf{x}) := \mathbf{x}^T \mathbf{P} \mathbf{x}$ for some positive definite $\mathbf{P} \in \mathbb{R}^{p \times p}$ that measures the performance of a system as a function of the state. For the system to both remain stable and over time be driver to zero, it is necessary for value of $L(\mathbf{x}_{k+1})$ to decrease relative to $L(\mathbf{x}_k)$ at all times k. We cannot guarantee this occurs deterministically, but instead consider a condition on the estimated future Lyapunov cost, i.e.

$$\mathbb{E}[L(\mathbf{x}_{i,k+1}) \mid \hat{\mathbf{x}}_{i,k}^{(l_i)}, \mathbf{h}_{i,k}, \mu_i, \varsigma_i) \leq \rho \mathbb{E}[L(\mathbf{x}_{i,k}) \mid \hat{\mathbf{x}}_{i,k}^{(l_i)}] + c_i, \tag{6}$$

for some $\rho \in (0, 1)$. The condition in (6) specifies that the expected Lyapunov cost for system i should decrease by a factor of ρ from cycle k to $k + 1$ (up to constant c_i). Observe that this expectation is conditioned upon the estimated state $\hat{\mathbf{x}}_{i,k}^{(l_i)}$, channel conditions $\mathbf{h}_{i,k}$, as well as a scheduling decision $\{\mu_i, \varsigma_i\}$. The scheduling decision impacts this expected value through the resulting PDR $q(\mathbf{h}_i, \mu_i, \varsigma_i)$ which determines the probability of closing the control loop and thus diminishing this Lyapunov cost. We may derive an explicit or equivalent condition on $q(\mathbf{h}_i, \mu_i, \varsigma_i)$ to satisfy the condition in (6), which we present in the following proposition. The proof can be found in [22].

Proposition 1 *Consider the switched dynamics in* (3). *Define the closed-loop state transition matrix* $\mathbf{A}_i^c := \mathbf{A}_i + \mathbf{B}_i \mathbf{K}_i$ *and j-accumulated noise* $\omega_i^j := Tr[(\mathbf{A}_i^T \mathbf{P}^{1/j} \mathbf{A}_i)^j \mathbf{W}_i]$. *The control constraint in* (6) *is satisfied for device i if and only if the following condition on PDR* $q(\mathbf{h}_{i,k}, \mu_i, \varsigma_i)$ *holds,*

$$q(\mathbf{h}_{i,k}, \mu_i, \varsigma_i) \geq \tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)}) := \tag{7}$$

$$\frac{1}{\Delta_i} \left[\left\| (\mathbf{A}_i^c - \rho_i \mathbf{I}) \hat{\mathbf{x}}_{i,k}^{(l_i)} \right\|_{\mathbf{P}^{\frac{1}{2}}}^2 + (1 - \rho_i) \sum_{j=0}^{l_i - 1} \omega_i^j + \omega_i^{l_i} - c_i \right],$$

where we have further defined the constant

$$\Delta_i := \sum_{j=0}^{l_i-1} [\omega_i^{j+1} - Tr(\mathbf{A}_i^{cT} (\mathbf{A}_i^T \mathbf{P}^{1/j} \mathbf{A}_i)^j \mathbf{A}_i^c \mathbf{W}_i)]. \tag{8}$$

Proposition 1 formally establishes lower bound $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})$ on the PDR of device i such that its Lyapunov condition in (6) is met in expectation. This bound is determined based upon the current estimated state, overall system dynamics, and transmission history and effectively sets a constraint on the scheduling bandwidth of FD ς_i and DR μ_i. We point out the relevant components of the expression in (7). First, note that the first term on the right hand will grow larger as the state gets larger, or closer to instability. Likewise, the latter two terms on the right hand side together grow larger as the estimating noise increases due to successive dropped packets, due to both the noise variance $\mathbf{W}_i$ and last-update counter l_i. In this manner, it is both the current control state and transmission history, as they relate to the dynamics of the system, that set a delivery rate requirement for each device.

The PDR condition in (7) is valuable in the time-constrained settings because it allows us to dynamically adapt the data rate needs of each device relative to their control state. While the latency requirement effectively constrains the total resources available, the ability to properly identify the users to be given scheduling slots is and important consideration in maintaining overall reliable performance. It is worth pointing out, that depending upon the system dynamics of a particular control system, the PDRs derived in (7) may often be in practice significantly lower than the standard, fixed PDR requirements used in high-reliability systems, e.g. ≥ 0.99. In such cases, the identification of PDR requirements necessary for proper operation of the control system can reduce a large amount of resource constraints, as is later seen in the simulations in Section 4. We proceed now to discuss the ways in which the dynamic and more lenient PDR targets in (7) may be leveraged by the scheduling to further reduce the total transmission times.

3.1. Selective scheduling

We first consider a stochastically *selective scheduling* protocol, whereby we do not attempt to schedule every device at each transmission cycle, but instead select a subset to schedule a principled random manner. Define by $\nu_{i,k} \in [0,1]$ the probability that device i is included in the transmission schedule at time k and further recall by $q(\mathbf{h}_{i,k}, \mu_i, \varsigma_i)$ to be the packet delivery rate with which it transmits. Then, we may consider the *effective* packet delivery rate $\hat{q}$ as

$$\hat{q}(\mathbf{h}_{i,k}, \mu_i, \varsigma_i) = \nu_{i,k} q(\mathbf{h}_{i,k}, \mu_i, \varsigma_i) \tag{9}$$

Observe that in order to meet the PDR target defined in (9), device i would need to meet a *modified* PDR target $q(\mathbf{h}_{i,k}, \mu_i, \varsigma_i) \geq \tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})/\nu_{i,k}$. While imposing a tighter PDR requirement will indeed require longer transmission times, this added time cost is generally less than the transmission overhead of scheduling all users. In particular, the scheduling probability of device i is defined relative to its PDR requirement $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})$ as

$$\nu_{i,k} := e^{\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})-1}. \tag{10}$$

Notice that, when a transmission is necessary, i.e. $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)}) = 1$, then device i is included in the scheduling with probability $\nu_{i,k} = 1$.

3.2. Assignment-based scheduling

Given the set of devices selectively scheduled via (**??**), we proceed to discuss an assignment-based formulation that can be employeed to select a low-latency schedule. Define the set of m_k devices to selected be scheduled as $\mathcal{I}_k \subseteq \{1, 2, \ldots, m\}$ where $|\mathcal{I}_k| = m_k$ and device $j \in \mathcal{I}_k$ with probability $\nu_{i,k}$. Further define $\mathcal{S}_{(n)} \subset \mathcal{S}$ to be an arbitrary set of n FDs that do not intersect over any frequency bands, i.e. $\sum_{j \in \mathcal{S}_{(n)}} \varsigma_j \leq \mathbf{1}$. To accommodate the m_k devices to be scheduled, we consider a set of S such sets $\{\mathcal{S}_{(n_s)}^s\}_{s=1}^S$ with size n_s, whose combined elements total $\sum_{s=1}^S n_s = m_k$. We define this full set of assignable FDs at cycle k as

$$\mathcal{S}_k' := \mathcal{S}_{(n_1)}^1 \cup \mathcal{S}_{(n_2)}^1 \cup \ldots \cup \mathcal{S}_{(n_{S_k})}^{S_k}. \tag{11}$$

Observe that an FD ς is further superindexed by its TD slot s y. In this way (11) defines a complete set of combinations of frequency-allocated FD and *time*-allocated TDs to assign users during this cycle. An example of a possible $\mathcal{S}_k'$ for scheduling $m_k = 12$ devices is shown in Table 1.

For all $i \in \mathcal{I}_k$ and FD $\varsigma \in \mathcal{S}_k'$, define the largest affordable DR given the *modified* PDR requirement $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})/\nu_{i,k}$ by

$$\mu_{i,k}(\varsigma) := \begin{cases} \max\{\mu \mid q(\mathbf{h}_{i,k}, \mu, \varsigma) \geq \tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})/\nu_{i,k}\} \\ \mu_0, \quad \text{if } q(\mathbf{h}_{i,k}, \mu, \varsigma) < \tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})/\nu_{i,k} \ \forall \mu \end{cases} \tag{12}$$

Observe in (12) that, when no DR achieves the desired PDR in a particular FD, this value is set to $\mu = \mu_0$ by default. The DR defined in (12) subsequently incurs a time cost $\tau(\mu_{i,k}(\varsigma), \varsigma)$ for assigning device i to FD ς. Define an *assignment* $\mathcal{V} = \{v_{ij}^s\}$ that assigns each device $i \in \mathcal{I}_k$ to an FD/TD pair (j, s) corresponding to an element in $\mathcal{S}_k'$. For time sensitive applications, the goal is to minimize the total transmission time across all TDs. The transmission time of a single TD is limited by the slowest device (i.e. a TD cannot finish until all devices finish transmitting). Thus, we can write the total transmission time as

$$T(\mathcal{V}) = \sum_{s=1}^S \max_j \left[v_{ij}^s \tau(\mu_{i,k}(\varsigma_j^s), \varsigma_j^s) \right]. \tag{13}$$

The problem of minimizing $T(\mathcal{V})$ is a particular version of a non-linear assignment problem, where the goal is to choose an assignment—or schedule—that minimizes transmission time while meeting the control-aware PDR targets in (7). These problems are combinatorial in nature and challenging to solve exactly. We may approximate this problem by applying, e.g., the Hungarian method [26], a well-known method for solving linear-cost assignment problems. Other heuristic assignment approaches may be designed to approximate the solution to (13). For

Algorithm 1 Control-aware scheduling for low-latency at cycle k

Parameters: Lyapunov decrease rate ρ
Input: Channel conditions $\mathbf{h}_{i,k}$ and estimated states $\hat{\mathbf{x}}_{i,k}^{(l_i)}$ for all i
Compute target PDR $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})$ for each device i [cf. (7)].
Determine selection probabilities $\nu_{i,k}$ for each device [cf. (10)].
Select devices $\mathcal{I}_k$ with probs. $\{\nu_{1,k}, \ldots, \nu_{m,k}\}$
Determine set of FDs/TDs $\mathcal{S}_k'$ [cf. (11)].
Determine max. DR for each device/FD assignment [cf. (12)].
Schedule selected devices via assignment method [26].
Return: Scheduling variables $\{\varsigma_i, \mu_i, \alpha_i\}_{i=1}^m$

<table>
<tr><th>TD 1</th><th>TD 2</th><th>TD 3</th></tr>
<tr><td>FD 1</td><td rowspan="2">FD 8</td><td rowspan="4">FD 11</td></tr>
<tr><td>FD 2</td></tr>
<tr><td>FD 3</td><td rowspan="2">FD 9</td></tr>
<tr><td>FD 4</td></tr>
<tr><td>FD 5</td><td rowspan="3">FD 10</td><td rowspan="3">FD 12</td></tr>
<tr><td>FD 6</td></tr>
<tr><td>FD 7</td></tr>
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Table 1: Example of FD selection with $m_k = 12$ devices. There are a total of $S_k = 3$ TDs, given $n_1 = 9$, $n_2 = 3$, $n_3 = 2$ FDs, respectively.

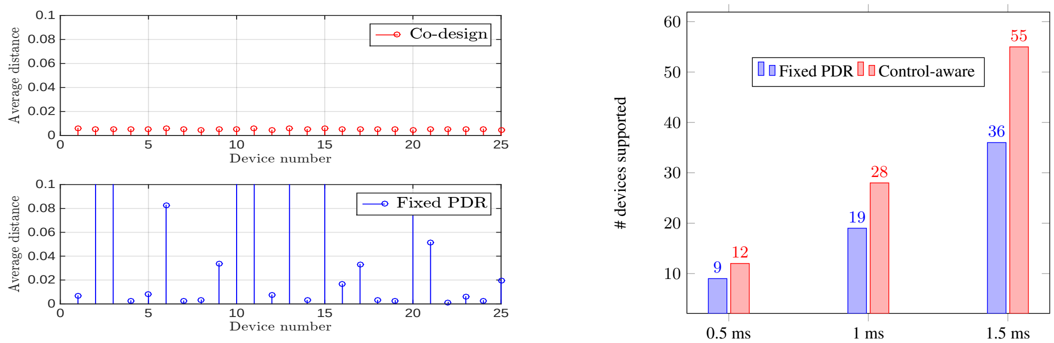

Fig. 1: Simulation results for a series of inverted pendulums controlled over shared wireless channel. (left) The average distances from center vertical for $m = 25$ pendulums. The control-aware, or "co-design", scheduler keeps all pendulums close, unlike the fixed PDR scheduler. (right) For different latency bounds, the control-aware can support more pendulums than the fixed PDR scheduler.

the simulations performed later in this paper, we apply such a heuristic method, the details of which are left out for proprietary reasons.

The complete control-aware scheduling procedure for low-latency settings is present in Algorithm 1. At each cycle k, the BS uses current channel states $\mathbf{h}_{i,k}$ (obtained via pilot signals) and the current estimated control states $\hat{\mathbf{x}}_{i,k}^{(l_i)}$ (obtained via (2) for each device i) to compute control-aware target PDRs $\tilde{q}_i(\hat{\mathbf{x}}_{i,k}^{(l_i)})$ for each device via (7) in Step 3. In Step 4, the target PDRs are used to establish selection probabilities $\nu_{i,k}$ for each agent with (10). After randomly selecting devices $\mathcal{I}_k$ in Step 5, an appropriate set of FDs and TDs $\mathcal{S}'_k$ are selected in Step 6. In Step 7, the associated DR values are determined for each possible assignment of device to FDs via (12). Finally, in Step 8 the assignment is performed using, e.g., the Hungarian method or other user-designed heuristic assignment method. The resulting assignment determines the scheduling parameters $\{\varsigma_i, \mu_i, \alpha_i\}$ for all devices i in the current cycle.

4. SIMULATION RESULTS

We perform a series of experimets on the canonical latency-constrained control problem of controlling a series of inverted pendulums on a horizontal cart. While conceptually simple, the highly unstable dynamics of the inverted pendulum make it a representative example of control system that requires fast control cycles, and subsequently low-latency communications when being controlled over a wireless medium. Consider a series of m identical inverted pendulums—where each pendulum is attached at one end to a cart that can move along a single, horizontal axis—using the modeling of the inverted pendulum as provided by Quanser [27]. The state is $p = 4$ dimensional vector that maintains the position and velocity of the cart along the horizontal axis, and the angular position and velocity of the pendulum, i.e. $\mathbf{x}_{i,k} := [x_{i,k}, \dot{x}_{i,k}, \theta_{i,k}, \dot{\theta}_{i,k}]$. The system input $u_{i,k}$ reflects a horizontal force placed upon ith pendulum. By applying a zeroth order hold on the continuous dynamics with a state sampling rate of 0.01 seconds and linearizing, we obtained the following discrete linear dynamic matrices of the pendulum system

$$\mathbf{A}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.055 & -0.722 & 4.828 \\ 0 & 0.023 & 0.91 & 0.037 \\ 0 & 0.677 & -0.453 & 2.055 \end{bmatrix}, \mathbf{B}_i = \begin{bmatrix} 0.034 \\ 0.168 \\ 0.019 \\ 0.105 \end{bmatrix}. \tag{14}$$

Because the state $\mathbf{x}_{i,k}$ measures the angle of the ith pendulum at time k, the goal is to keep this close to zero, signifying that the pendulum remains upright. The input matrix $\mathbf{K}$ is computed to be a standard LQR-controller. For the definition of FDs, TDs, and data rates, we use the parameters of IEEE 802.11ax Wi-Fi protocol [25], referred to as resource units, PPDUs, and MCS, respectivley.

We perform a set of experiments with the inverted pendulum varying both the latency threshold and number of devices m. We perform the scheduling using the proposed control-aware method for control-aware low latency scheduling and, as a point of comparison, consider scheduling using a standard fixed PDR of 0.99 for all devices. Each simulation is run for a total of 1000 seconds and is deemed "successful" if all pendulums remain upright for the entire run. We perform 100 such simulations for each combination of latency threshold and number of devices to determine how many devices we can support at each latency threshold using both the control-aware and fixed-PDR methods for scheduling.

In the left of Figure 1 we show the results of a representative simulation of the control of $m = 25$ pendulum systems with a latency bound of $\tau_{\max} = 10^{-3}$ seconds. In both graphs we show the average distance from the center vertical of each pendulum over the course of 1000 seconds. In the top figure, we see by using the control-aware control-aware method we are able to keep each of the 25 pendulums close to the vertical for the whole simulation. Meanwhile, using the standard fixed PDR, we are unable to meet the scheduling limitations imposed by the latency threshold, and many of the pendulums swing are unable to be kept upright, as signified by the large deviations from the origin. This is due to the fact that certain pendulums were not scheduled when most critical, and they subsequently became unstable, while the control-aware adapted successfully to the needs of each system.

We present in the right of Figure 1 the maximum number of uses each scheduler could support under different latency requirements without any instability. Each device must be kept with a $|\theta_{i,k}| \leq 0.05$ error region for 100 independent simulations. We observe that the control-aware approach is able to increase the number of devices supported in each case, with up to 1.5 factor increase over the fixed PDR scheduling.

5. CONCLUSION

In this paper we develop a control-communication control-aware approach towards scheduling for low-latency, or time sensitive, wireless control systems. Because many control systems in industrial settings require very low latency transmission to operate effectively, there is an intrinsic challenge in trading off the data rates necessary to achieve low latency with the packet error rates necessary for high reliability. We demonstrate that packet delivery rates required to meet control-specific goals changes dynamically with the state of the systems. This allows to develop a scheduling method that smartly allocates bandwidth and data rates among many independent systems based upon their own current PDR requirements. We demonstrate the improvements relative to control-agnostic scheduling procedures in simulation experiments.

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