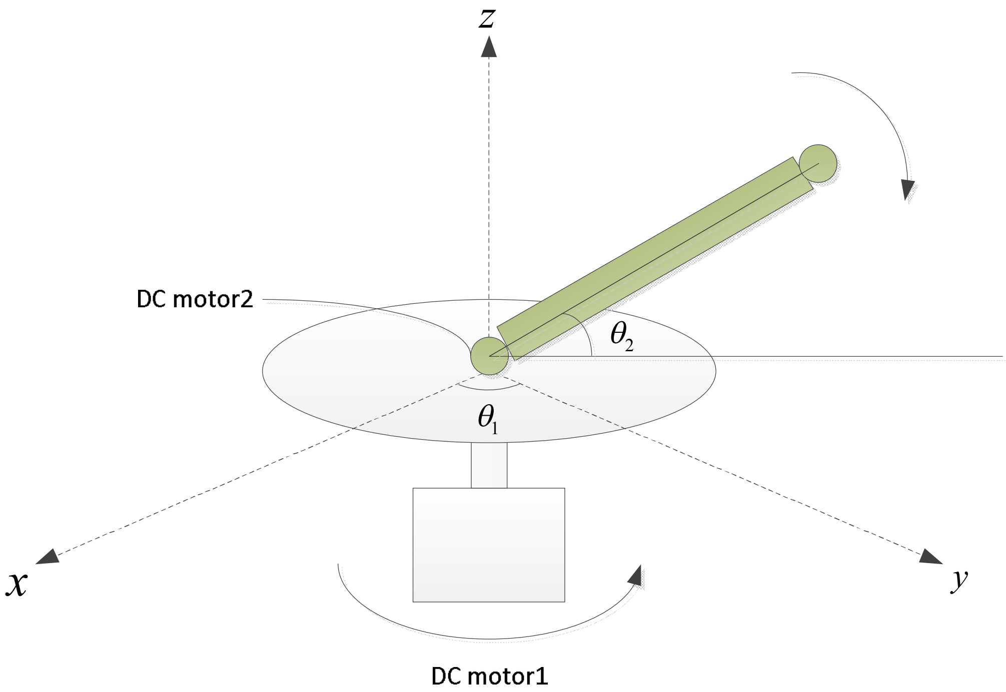
Two Joints Robot Arm Dynamics

Modelling part

Consider the following structure of a two degree freedom robot arm



Such a robotic system

- Widely used and simple form
- Called articulated robots

Step 1: Identify the inputs and outputs.

Input: u_1 applied to DC motor1;

u_2 applied to DC motor2;

Both variables are armature voltages applied to DC motor 1 and 2, respectively.

Output: θ_1 and θ_2

Notations

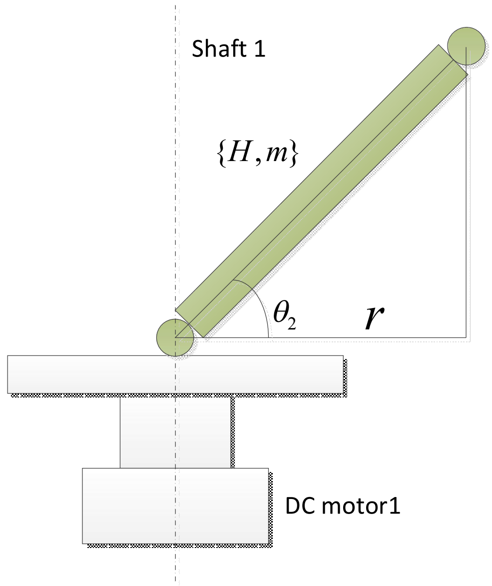
Shaft of DC motor1, it has a fixed part of the inertia J_0 that is obtained by taking the arm away.

In terms of the robot arm, it has length H and mass m, i.e. $\{H, m\}$.

The parameters of the DC motors are given by:

$$\text{DC motor1} \Rightarrow \{R_1, L_1, K_1^1, K_2^1\}$$

$$\text{DC motor2} \Rightarrow \{R_2, L_2, K_1^2, K_2^2\}$$



The equivalent inertia for the shaft driven by DC motor 1 is given by

$$J_0 + \frac{1}{2}mr^2 = J_0 + \frac{1}{2}mH^2\cos^2\theta_2 \tag{1}$$

Therefore the mechanical equation for DC motor1 (shaft1) is

$$\left[J_0 + \frac{1}{2}mH^2\cos^2\theta_2\right]\frac{d^2\theta_1}{dt^2} = K_2^1 i_1 \tag{2}$$

where i_1 is the armature current of DC motor1.

$$R_1 i_1 + L_1\frac{di_1}{dt} + K_1^1\frac{d\theta_1}{dt} = u_1 \tag{3}$$

Equation (3) is the electrical equation for the armature circuit of DC motor1.

Robot arm part modelling (DC motor2)

$$\begin{cases} J_2 \dfrac{d^2\theta_2}{dt^2} = K_2^2 i_2 - \dfrac{1}{2} mHg\cos\theta_2 \\ R_2 i_2 + L_2 \dfrac{di_2}{dt} + K_1^2 \dfrac{d\theta_2}{dt} = u_2 \end{cases} \tag{4}$$

Therefore the mechanical part of the model can be expressed as the following matrix form

$$M(q) = \begin{bmatrix} J_0 + \dfrac{1}{2} mH^2\cos^2\theta_2 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \dfrac{d^2\theta_1}{dt^2} \\ \dfrac{d^2\theta_2}{dt^2} \end{bmatrix} = \begin{bmatrix} K_2^1 & 0 \\ 0 & K_2^2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{2} Hmg\cos\theta_2 \end{bmatrix} \tag{5}$$

In the course work the left hand side of the above equation is extended to: $M(q,\dot{q})$ which is defined as inertia matrix for general multiple joint robot arm.

The above system can also be expressed as a state space form described before.