OASIS refinery optimization

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1 Mathematical Modeling

1.1 Sets and Indices

1. $v \in \text{Vessels} = \{v_1, v_2, ..., v_6\}$
2. $l \in \text{Location} = \{\text{PM}, \text{Sabah}, \text{Sarawak}, \text{Melaka}\}$
3. SourceLocation = {PM, Sabah, Sarawak}
4. $d \in \text{Days} = \{1, 2, ..., 30\}$
5. $c \in$ CrudeGrades = {Base, A, . . . , G}
6. $p \in$ Parcel - crude parcel which can be picked up by a vessel.
7. $s \in \text{Slots} = \{1, 2, \ldots, 60\}$ - slots on which crude blends are consumed in refinery. Each day is split into two slots, so day 1 has slots 1 and 2.
8. $b \in$ Blends = {(Base), (Base, A), . . . , (E, G)} - set of crude pairing/blending that can be fed to the refinery.

1.2 Parameters

1. PC_p - Crude grade in parcel p.
2. $\text{PD}_p \subset \text{Days}$ - Subset of days when parcel p is available.
3. $\text{PL}_p \in \text{SourceLocation}$ - Location at which the parcel p can be collected.
4. $\text{PV}_p \in [0, 700]$ - volume of parcel.
5. $\tau_{l_1,l_2} \in \{1, 2, 3, 4\}$ - days taken to travel from locations l_1 to l_2.
6. $\text{BR}_{c,b} \in [0, 1]$ - Blend ratio, The ratio in which crude grade c is mixed to create the blend b.
7. $\text{BC}_b \in \mathbb{R}_+$ - Blend capacity, the maximum amount of blend b which can processed on a day.
8. $\text{MR}_c \in \mathbb{R}_+$ - Margin Rate, the profit generated in USD by processing one barrel of crude grade c.
9. $\text{DR} \in \mathbb{R}_+$ - **D**emurrage **R**ate $\in \mathbb{R}_+$, the fees imposed on vessels per day.
10. $\text{OI}_c \in \mathbb{R}_+$ - **O**pening **I**nventory, the volume of crude c present in the inventory on the first day.
11. $\text{RC}_d \in \mathbb{R}_+$ -Refinery Capacity, maximum amount of crude that can be processed on day d by the plant.

1.3 Decision variables

1. $\text{AtLocation}(v, l, d) \in \{0, 1\}$ - denotes if a vessel v was at location l on the day d.
2. $\text{Discharge}(v, d) \in \{0, 1\}$ - If vessel v starts to discharge in *Melaka* on day d. Keep in mind that $\text{Discharge}(v, d) = 1$ means that discharging happens on the day d and $d + 1$.
3. $\text{Pickup}(v, p, d) \in \{0, 1\}$ - 1 indicates that the vessel v pick-ups parcel p on day d, otherwise 0.
4. $\text{Inventory}(c, d) \in \mathbb{R}_+$ is the amount of crude c in Kilo barrels present in the inventory on day d.
5. $\text{BlendFraction}(b, s) \in [0, 1]$ - The fraction of maximum capacity of crude blend b used on slot s.

1.4 Constraints

1.4.1 Vessel travel constraints

1. A vessel can only be at one location on a given day.

$$\sum_{l} \text{AtLocation}(v, l, d) \leq 1 \quad \forall v, d \tag{1}$$

2. A vessel cannot visit a location twice. This can be modeled by putting a constraint that for a given vessel and a location, a 1 turning to 0 from day d to $d+1$ can only happen once in the whole schedule. For this, we will introduce an auxiliary variable $\text{Departure}(v, l, d) \in \{0, 1\}$ to capture when a vessel leaves a location.

$$\text{Departure}(v, l, d) \geq \text{AtLocation}(v, l, d) - \text{AtLocation}(v, l, d+1) \tag{2}$$
$$\text{Departure}(v, l, d) \leq \text{AtLocation}(v, l, d) \tag{3}$$
$$\text{Departure}(v, l, d) \leq 1 - \text{AtLocation}(v, l, d+1) \tag{4}$$
$$\sum_{d} \text{Departure}(v, l, d) \leq 1 \quad \forall v, l \tag{5}$$

3. After departure from a source location the vessel should reach destination location according to the vessel travel time.

$$\text{Departure}(v, l, d) \leq \sum_{l' \neq l} \text{AtLocation}(v, l', d + \tau_{l,l'}) \quad \forall v, l, d \tag{6}$$

4. Vessel can't reach the destination location before the travel time between the locations.

$$\text{AtLocation}(v, l_1, d_1) + \text{AtLocation}(v, l_2, d_2) \leq 1 \quad \forall v, l_1, l_2, d_1, d_2, \tag{7}$$

where $l_1 \neq l_2$ and $d_2 - d_1 \in [1, \ldots, \tau_{l_1,l_2} - 1]$.

1.4.2 Vessel loading constraints

1. Every vessel has to at least pick one parcel.

$$\sum_{p} \sum_{d \in \text{PD}_p} \text{Pickup}(v, p, d) \geq 1 \quad \forall v \tag{8}$$

2. A parcel can only be picked up by one vessel.

$$\sum_{v,d} \text{Pickup}(v, p, d) \leq 1 \quad \forall p \tag{9}$$

3. One vessel can only pick one parcel on a day.

$$\sum_{p} \text{Pickup}(v, p, d) \leq 1 \quad \forall v, d \tag{10}$$

4. A pickup can only happen on the day the parcel is available.

$$\sum_{d \notin \text{PD}_p} \sum_{v} \text{Pickup}(v, p, d) = 0 \quad \forall p \tag{11}$$

5. A vessel can only pickup a parcel if that vessel is in the location where parcel is present.

$$\text{Pickup}(v, p, d) \leq \text{AtLocation}(v, PL_p, d) \quad \forall v, p, d \tag{12}$$

6. If a vessel is visiting a location, it should at least pick one parcel from there. We introduce an auxiliary variable LocationVisited$(v, l) \in \{0, 1\}$ which is 1 when vessel v visited location l, 0 otherwise.

$$\sum_d \text{AtLocation(v,l,d)} \geq \text{LocationVisited}(v, l) \quad \forall v, l \in \text{SourceLocation} \tag{13}$$

$$\sum_d \text{AtLocation(v,l,d)} \leq M \times \text{LocationVisited}(v, l) \quad \forall v, l \in \text{SourceLocation} \tag{14}$$

$$\sum_{p:PL_p=l} \sum_d \text{Pickup}(v, p, d) \geq \text{LocationVisited}(v, l) \quad \forall v, l \in \text{SourceLocation} \tag{15}$$

7. A vessel can carry at max 3 different types of crude. For this we will need to introduce an auxiliary variable CrudeInVessel$(v, c) \in \{0, 1\}$ which is 1 if vessel v is carrying crude c, otherwise 0.

$$\sum_{p:\text{PC}_p=c} \sum_d \text{Pickup}(v, p, d) \geq \text{CrudeInVessel}(v, c) \quad \forall v, c \tag{16}$$

$$\sum_{p:\text{PC}_p=c} \sum_d \text{Pickup}(v, p, d) \leq \text{CrudeInVessel}(v, c) \times M \quad \forall v, c \tag{17}$$

$$\sum_c \text{CrudeInVessel}(v, c) \leq 3 \quad \forall v \tag{18}$$

where M is total number of crude grades (8).

8. The max crude volume which a vessel can carry depends on number of types of crude grades on that vessel. If the vessel is carrying 1 or 2 grades it can carry 700 Kb and if it is carrying 3 grades which is the upper limit, then that is 650 Kb. To acheive this, we introduce auxiliary variables: NumGrades12$(v) \in \{0, 1\}$ which is 1 if vessel v is carrying 1 or 2 grades and 0 otherwise. Similarly NumGrades3$(v) \in \{0, 1\}$ when number of crude grades is 3.

$$\text{NumGrades12}(v) + \text{NumGrades3}(v) = 1 \quad \forall v \tag{19}$$

$$2 \cdot \text{NumGrades12}(v) + 3 \cdot \text{NumGrades3}(v) \geq \sum_c \text{CrudeInVessel}(v, c) \tag{20}$$

$$1 \cdot \text{NumGrades12}(v) + 3 \cdot \text{NumGrades3}(v) \leq \sum_c \text{CrudeInVessel}(v, c) \tag{21}$$

$$\sum_p \left[\text{PV}_p \times \sum_d \text{Pickup}(v, p, d)\right] \leq 700 \cdot \text{NumGrades12}(v) + 650 \cdot \text{NumGrades3}(v) \quad \forall v \tag{22}$$

1.4.3 Vessel discharge constraints

1. Every vessel should discharge and the process should start exactly on one day

$$\sum_d \text{Discharge}(v, d) = 1 \quad \forall v \tag{23}$$

2. Discharge only happens at Melaka and on that day and the next day vessel should be at Melaka.

$$2 \cdot \text{Discharge}(v, d) \leq \text{AtLocation}(v, \text{Melaka}, d) + \text{AtLocation}(v, \text{Melaka}, d + 1) \quad \forall v, d \tag{24}$$

3. No two vessels can discharge on the same day. Which means no two vessels can start their discharges on the same day or the days next to each other.

$$\sum_v \text{Discharge}(v, d) + \text{Discharge}(v, d + 1) \leq 1 \quad \forall d \tag{25}$$

4. Vessels stops after discharge. After the second day of discharge, vessel should not be present in any of the locations.

$$\text{AtLocation}(v, l, d') \leq 1 - \text{Discharge}(v, d) \quad \forall v, l, d, d' \tag{26}$$

where d' is all the days after $d + 1$, i.e. $d' > d + 1$.

5. Vessel cannot discharge if there is no enough ullage in the inventory. We introduce auxiliary variable $\text{VolumeDischarged}(v, c, d) \in \mathbb{R}_+$ the amount of grade c crude discharged by vessel v on day d. For ease of development we introduce another variable $\text{VolumeOnboard}(v, c) \in \mathbb{R}_+$, the amount of grade c crude present on vessel v.

$$\text{VolumeOnboard}(v, c) = \sum_{p:PC_p=c} \Big[\text{PV}_p \cdot \sum_{d'} \text{Pickup}(v, p, d')\Big] \quad \forall v, c \tag{27}$$

$$\text{VolumeDischarged}(v, c, d) \leq 700 \cdot \text{Discharged}(v, d) \quad \forall v, d \tag{28}$$

$$\text{VolumeDischarged}(v, c, d) \leq \text{VolumeOnboard}(v, c) \quad \forall v, d \tag{29}$$

$$\text{VolumeDischarged}(v, c, d) \geq \text{VolumeOnboard}(v, c) - 700 \cdot (1 - \text{Discharge}(v, d)) \tag{30}$$

These auxiliary variables are used for combine inventory update equation on the next section.

6. inventory updates moved to crude blending section below.

1.4.4 Crude blending constraints

1. Consume exactly one blend of crude per slot. To achieve this and to use in the next few constraints we introduce an auxiliary variable $\text{IsBlendConsumed}(b, s) \in \{0, 1\}$ which is 1 when blend b is consumed on slot s, otherwise 0.

$$\text{IsBlendConsumed}(b, s) \geq \text{BlendFraction}(b, s) \quad \forall b, s \tag{31}$$

$$\sum_b \text{IsBlendConsumed}(b, s) = 1 \quad \forall s \tag{32}$$

2. The sum of fractions of the crude blended for 2 slots in a day should be upper bounded by 1.

$$\sum_b \Big[\text{BlendFraction}(b, s) + \text{BlendFraction}(b, s+1)\Big] \leq 1 \quad \forall s : \text{s is odd} \tag{33}$$

3. We call a *transition* happened when the crude blend processed changes for slot s to s+1. The number of transitions has to be less than *MaxTransitions*. We introduce an auxiliary variable $\text{IsTransition}(b, s) \in \{0, 1\}$, which takes value 1 when we stop using blend b on slot s, 0 otherwise. In other words, when $\text{IsBlendConsumed}(b, s)$ goes from 1 to 0 on s to $s + 1$.

$$\text{IsTransition}(b, s) \geq \text{IsBlendConsumed}(b, s) - \text{IsBlendConsumed}(b, s + 1) \quad \forall b, s \tag{34}$$

$$\text{IsTransition}(b, s) \leq \text{IsBlendConsumed}(b, s) \quad \forall b, s \tag{35}$$

$$\text{IsTransition}(b, s) \leq 1 - \text{IsBlendConsumed}(b, s + 1) \quad \forall b, s \tag{36}$$

$$\sum_{b,s} \text{IsTransition}(b, s) \leq MaxTransitions \tag{37}$$

4. The plant has daily capacity limits. This generally is at 100 Kbd for all the day. But in Scenario-4 for few days this goes to 80 Kbd.

$$\sum_b \text{BC}_b \cdot \Big[\text{BlendFraction}(b, 2d - 1) + \text{BlendFraction}(b, 2d)\Big] \leq \text{RC}_d \quad \forall d \tag{38}$$

1.4.5 Inventory constraints

1. The amount of crude in inventory on the zeroth day is as per the parameter shared.

$$\text{Inventory}(c, 0) = \text{OI}_c \quad \forall c \tag{39}$$

2. Inventory updates. A day d's inventory is $d - 1$'s inventory with new crude added through vessel discharges and crude subtracted when consumed in refinery.

$$\begin{aligned} \text{Inventory}(c, d) = \text{Inventory}(c, d - 1) + \sum_v \text{VolumeDischarged}(v, c, d - 4) \\ - \sum_b \text{BC}_b \cdot \text{BR}_{c,b} \cdot \Big[\text{BlendFraction}(b, 2d - 1) + \text{BlendFraction}(b, 2d)\Big] \end{aligned} \tag{40}$$

$$\sum_c \text{Inventory}(c, d) \leq 1180 \quad \forall d \tag{41}$$

1.5 Objective function

1.5.1 Maximizing net profit

1. Demurrage from vessels staying at the source locations. If the vessel is not picking up any crude while at source location then that day gets counted for demurrage.

$$\text{DemurrageAtSource} = \sum_{l \neq \text{Melaka}} \sum_{v,d} \text{AtLocation}(v,l,d) - \sum_{v,p,d} \text{Pickup}(v,p,d) \tag{42}$$

2. Demurrage for staying in Melaka for more than 2 days (vessel takes 2 days to discharge).

$$\text{DemurrageAtMelaka} = \sum_{v} \left[\left(\sum_{d} \text{AtLocation}(v, \text{Melaka}, d) \right) - 2 \right] \tag{43}$$

3. Total profit generated by processing crude.

$$\text{TotalProfit} = \sum_{c,b,s} \text{MR}_c \cdot \text{BR}_{c,b} \cdot \text{BC}_b \cdot \text{BlendFraction}(b,s) \tag{44}$$

4. Net profitt = Total Crude Margin - demurrage cost - additional freight cost. We are not dealing with additional freight cost in this mathematical model. We will just subtract a 650,000 USD from net profit when using 6 vessels.

   Among two schedules which produces the same net profit or throughput rate we would prefer one which has least number of blend transitions. We are assigning an small penalty of 10 USD for each transition to capture this objective.

$$\begin{aligned} \text{NetProfit} = \text{TotalProfit} - \text{DR} \cdot \Big[& \text{DemurrageAtMelaka} + \text{DemurrageAtSource} \Big] \\ & - 10 \times \sum_{s} \sum_{b} \text{IsTransition}(b,s) \end{aligned} \tag{45}$$

1.5.2 Maximizing throughput

1. The total throughput objective with a small penalty on transition counts.

$$\text{Throughput} = \sum_{b,s} \text{BC}_b \cdot \text{BlendFraction}(b,s) - 10 \times \sum_{b,s} \text{IsTransition}(b,s) \tag{46}$$