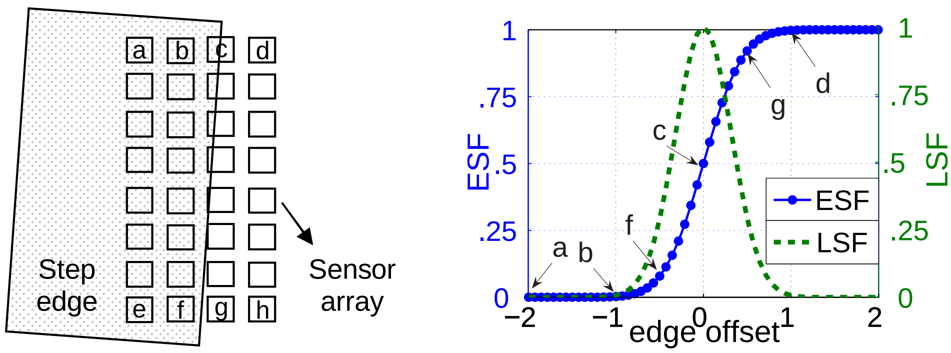
Appendix B

Model-based point spread function estimation

The Point Spread Function (PSF) is the space-invariant spatial response of an imaging system to a point light source. It often acts as a low-pass filter. Since a low-pass filter destroys small details, knowledge of the PSF reveals how sharp the details can be reproduced by the imaging system. In practice, a normalized magnitude of the Fourier transform of the PSF, i.e. the Modulation Transfer Function (MTF), is used for this purpose. The MTF measures the image contrast at a given spatial frequency. Although the system's MTF can be computed from a product of the transfer functions of all components along the imaging path, it is often measured directly from the output signal. A direct way to measure the signal contrast as a function of frequency is using a test image of sine gratings [87] or bar patterns [204] at increasing spatial frequencies. However, a simpler and therefore more popular method is to use a test image with a slanted edge, a.k.a. a tilted knife edge [167, 133]. This appendix extends the slanted edge method to elongated edges found in any images. The edges are automatically detected with subpixel localization. A Gaussian edge profile is then fitted to intensity samples around the edge in the direction perpendicular to the edge.

B.1 Slanted edge method

Measuring the PSF from a slanted step edge is advantageous over other test patterns. Tiny light dots [79], for example, have been used to measure the PSF. However, they often produce a poor signal-to-noise ratio that limits the accuracy of the measurements. An image of a step edge, on the other hand, offers enough redundancy on either side of the edge that noise is no longer a problem. Furthermore, the step edge can be captured in a slanted position to effectively increase the sampling rate of the Edge Spread Function (ESF) (figure B.1). This super-sampling also removes the aliasing artifacts caused by an under-Nyquist sampling rate of the sensor array [140]. To obtain a a one-dimensional Line Spread Function (1-D LSF), which is a projection of the 2-D PSF along the edge, the ESF is differentiated along the edge's normal direction.



(a) Slanted edge seen by a sensor array **(b)** Super-resolved edge profile

Figure B.1: A super-resolved edge spread function from a slanted step edge.

Due to the projection-slice theorem [39], the Fourier transform of an oriented LSF equals to a cross-section through the origin of the 2-D MTF in the direction parallel to the edge normal. The MTF can therefore be interpolated from multiple Fourier transforms of LSFs at different edge tilts [174].

However, multiple oriented edge measurements are not necessary if the system's PSF is isotropic. This is especially true for imaging systems that use circular microlenses and digital readout (circular microlenses form an isotropic photosensitive area per pixel, digital readout prevents signal degradation along a raster-scan pattern as occurs in analog video signals). As showed in figure B.2, the directional ESFs due to a square sensor alone already look similar in all directions. In reality, they resemble each other even more because many blur sources such as diffraction, out of focus, and cross-talk all contribute to the overall PSF. Due to a striking resemblance between the Airy disk and the Gaussian function [191] and the central limit theorem [49], the overall PSF can be modeled by an isotropic 2-D Gaussian function. The ESFs can therefore be modeled by an *erf* (*er*ror *f*unction) function. A real measurement of the directional ESFs of a Radiance HS infra-red camera in figure B.3 indeed confirms this model.

In this section, we assume an isotropic Gaussian PSF model and estimate its standard deviation σ_{psf} from a slanted edge. Two tasks are involved: sub-pixel edge localization and robust model fitting. Straight edges are automatically located from the gray-scale image by an adaptive Hough transform [190]. The Gaussian edge model is then fitted to the super-resolved ESF using a non-linear optimization method.

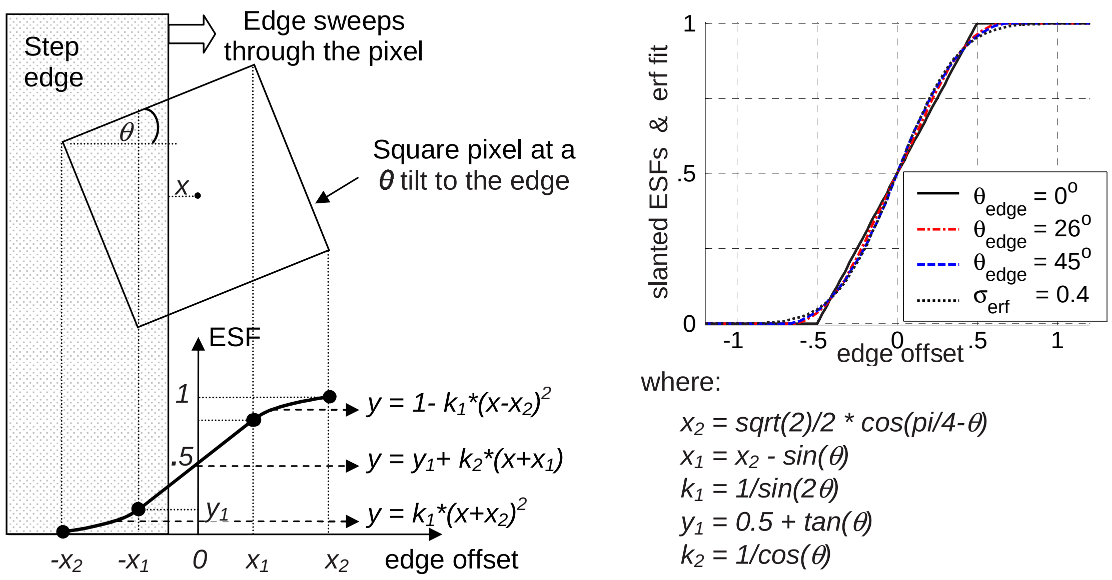
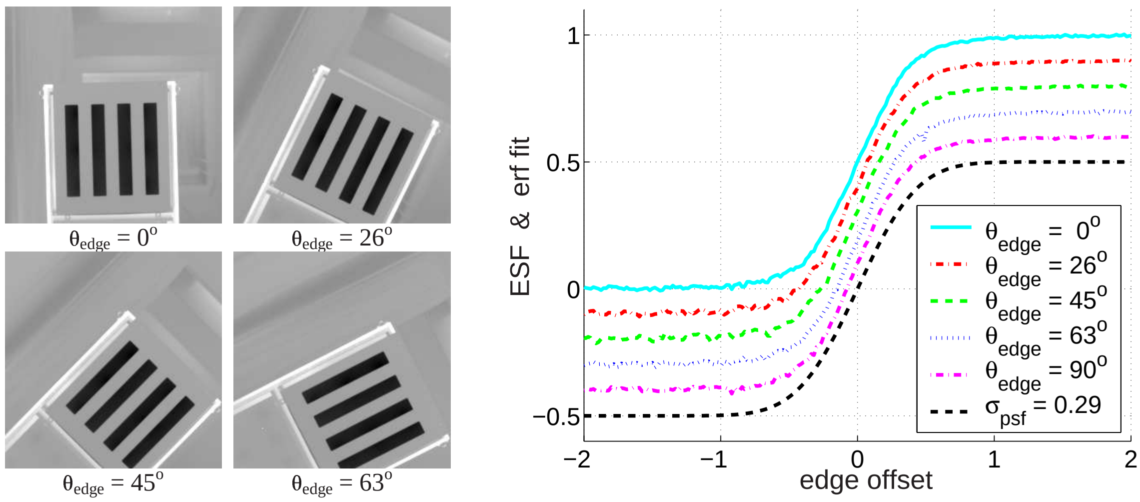


Figure B.2: Left: A tilted ESF due to a square sensor comprises of three sections: a convex quadratic section, a linear section and a concave quadratic section. Right: ESFs from different edge tilts are well approximated by an error function.



(a) step edge image at different tilts **(b)** Identical ESFs across different orientations

Figure B.3: Super-resolved edge spread functions of a Radiance HS infra-red camera. The curves in (b) are separated vertically with equal offsets for visualization purposes.

B.1.1 Adaptive Hough transform for line detection

Hough transform [86] is a general technique for parametric shape detection from an image. Hough transform for line detection [38], for example, transforms a point $P(x_0, y_0)$ in the Cartesian coordinates to a sinusoidal curve $r = x_0\, cos\varphi + y_0\, sin\varphi$ in the polar coordinates. Each point on this curve corresponds to a line passing through P in the Cartesian space (figure B.4). To find straight line in the input, the $[\varphi, r]$ Hough space is quantized into a two-dimensional array of accumulating cells. Each time a sinusoidal curve pass through a cell, the cell is incremented by one. After the accumulation is done for all input samples, the cells with the highest counts will most likely correspond to straight lines in the input space.

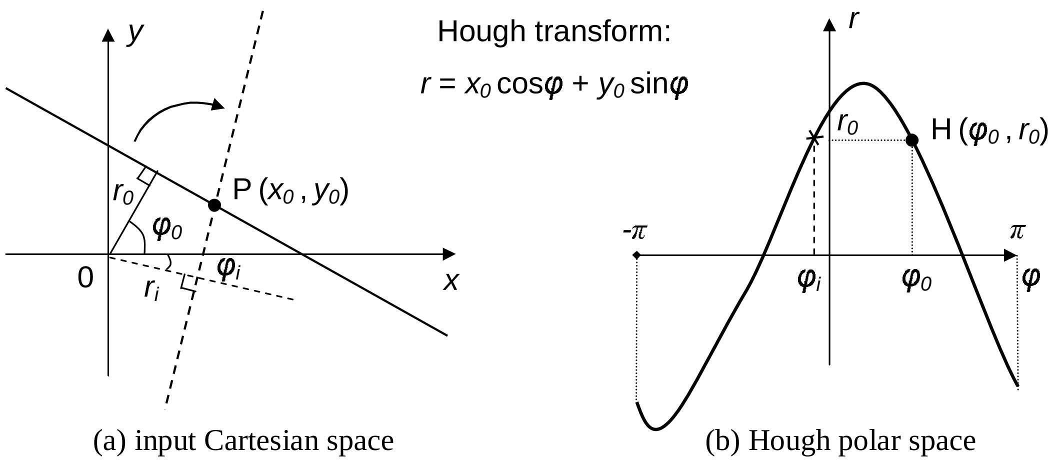
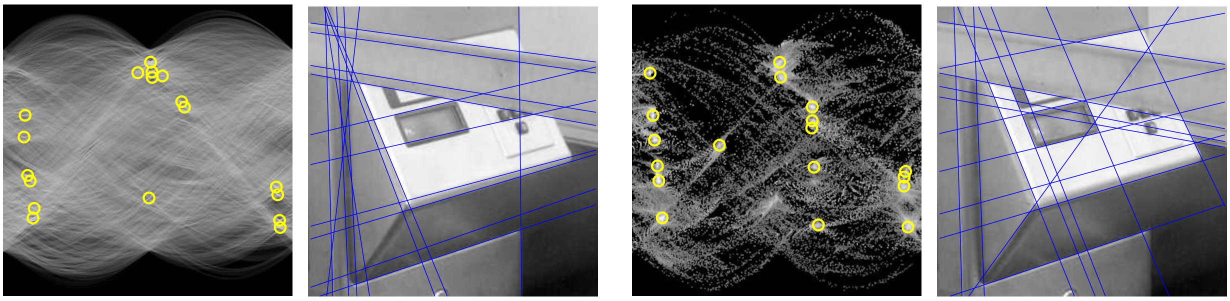


Figure B.4: Original Hough: point P transforms to a sinusoidal curve. Adaptive Hough: point P with orientation φ_0 transforms to a single point H on the curve.

To enable a fast detection of straight edges in an image, we apply an adaptive Hough transform [190] to the gradient magnitude image. Since the orientation of every pixel in an image can be computed from the gradient structure tensor (see chapter 3), a point $P(x_0, y_0)$ with orientation φ_0 in the input image corresponds to a *single* point $H(\varphi_0, r_0)$ in the Hough space [14]. This point-to-point instead of the point-to-curve correspondence significantly improves the efficiency of sample accumulation. To favor strong edges over weak ones, the transformed point H is accumulated to the output with a weight equal to the gradient magnitude at input point P. The weight is credited to the four nearest grid neighbors of H through a bilinear weighting scheme. Finally, the local maxima of the adaptive Hough transform is refined to a subpixel position through a parabola fit of the 3×3 neighborhood around a local peak. These improvements produce a very fast transform which facilitates an accurate localization of well-separated straight edges (figure B.5d).

B.1.2 Robust model fit to an edge profile

As previously shown in figure B.1, once the slanted edge is located, the edge profile can be interpolated from a scatterplot of pixel intensities versus distances from the edge. A robust fusion technique, such as the one in chapter 3, can then be used to obtain a super-sampled edge cross-section profile. If the edge model is known, however, it can be fitted directly to the scattered point set. The LSF can then be computed analytically. This subsection presents a robust curve fit to a scatter point set using Nelder-Mead optimization method [158].



(a) Hough transform [38] **(b)** 18 best lines from (a) **(c)** fast Hough transform **(d)** 18 best lines from (c)

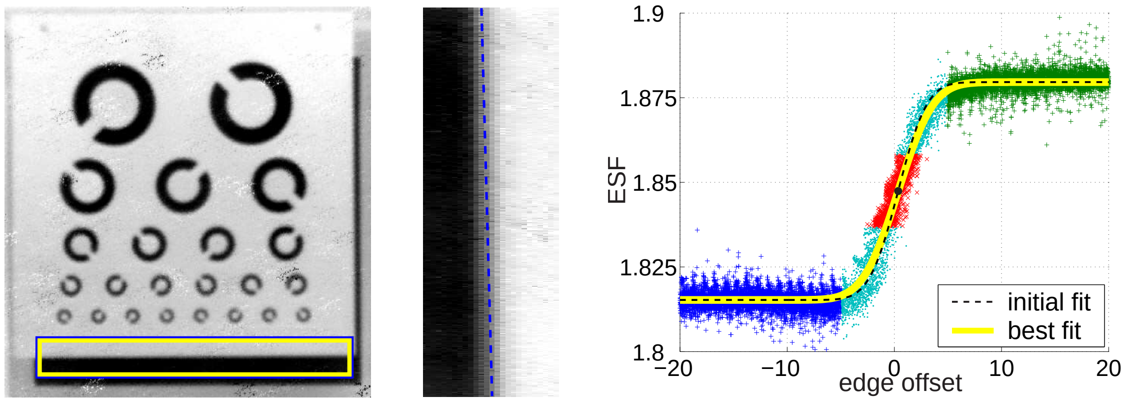
Figure B.5: A comparison of straight edge detection by the original and our adaptive Hough transforms: (a) Hough transform of the Canny edge map [28], and (c) adaptive transform of the gradient magnitude image. (b) and (d) plot 18 best edges detected by peak localization in (a) and (c).

Figure B.6 illustrates an algorithm of automatic Gaussian blur estimation from a straight step edge in an image. Figure B.6a shows part of an eight-times Super-Resolved (SR) *eyechart* image as a result of fusion from 100 Low-Resolution (LR) shifted frames. The Region Of Interest (ROI) at the bottom of figure B.6a is zoomed 6-times and the detected edge is plotted in figure B.6b. A scatterplot of the pixel intensities versus their distances from the edge is shown in figure B.6c. In this plot, the scattered points are partitioned into four different point sets based on their intensities. The middle and two outer point sets are used to estimate the initial parameters of the *erf* fit (floor and ceiling levels, inflection point and Gaussian spread). The mean squared distances of the inlier points to the current *erf* fit is minimized iteratively using the Nelder-Meat nonlinear optimization process[1]. Outlier points are excluded from the process if their residual from the fitted curve is larger than three standard deviation of all the residuals. With this robust regression scheme, the standard deviation of the PSF in figure B.6 finally converges to $\sigma_{psf} = 3.66$ HR pixels (or 0.46 pixel in the original LR sequence).

B.2 PSF from arbitrary edges under subpixel localization

In the previous section, the PSF is estimated from a cross-section profile of a straight edge. This requires the availability of straight edges in an image. Unfortunately, even this simple requirement is not always satisfiable. In that case, the ESF must be derived from curved edges instead. Although the edge needs not be straight, it should be long enough with no sharp bend. The contrast across the edge should also exceed the noise level for an accurate ESF estimation.

[1]using function `fminsearch` in the Matlab optimization toolbox



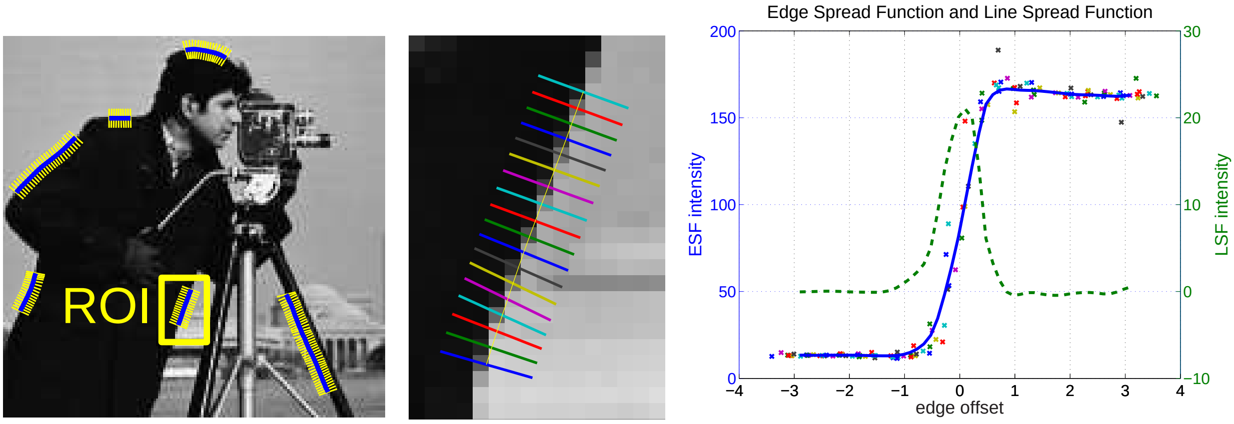
(a) input with detected edge **(b)** ROI **(c)** Erf fit to the edge profile ($\sigma_{psf} = 3.66$)

Figure B.6: Automatic PSF estimation from an image: Region of Interest (ROI) around a sharpest edge is extracted and an *erf* function is fitted to the edge profile.

An ESF can be estimated from an elongated edge segment that is sufficiently isolated from other edges or textured regions (at least twice the edge spread away from other disturbances). These proper edges can be extracted from a Canny edge map [28] using a series of image processing operations. First, the skeletonized Canny edge map is AND'ed with the skeleton of its binary closing image to remove edges that are too close to each other. Pixels with high curvature [188] and pixels at T-junctions are removed from the edge map. Next, short edge segments are removed, leaving only segments longer than a certain threshold. The remaining edge candidates are refined to a subpixel accuracy [92] and the ESF is estimated from the scatterplot of pixel intensities versus edge offsets as done in the previous section. Finally, the sharpest ESF is selected to produce the system PSF.

An experiment on the generalized edge-based PSF estimation is illustrated in figure B.7a, where good edge segments are shown as dark thick lines on top of bright normal bundles. The sharpest edge segment is shown in a rectangular bounding box, whose zoomed-in version can be seen in figure B.7b. The normals at equal intervals are drawn to facilitate the computation of distances from nearby pixels to the edge. Each pixel in the vicinity of the edge is projected geometrically to the nearest normal; the distance from the projection point to the normal-edge crossing is taken as the edge offset of the current pixel.

The computed ESF and LSF of the 256×256 Cameraman image is shown in figure B.7c. Thanks to the robust component of the estimation, the continuous ESF curve is not deflected by background outliers. The full-width half maximum of the dashed LSF curve is less than one pixel, indicating that the image is under-sampled. The under-sampling produces a sharp look of the image, which is especially noticeable around the outline of the cameraman against the background.



(a) good edge candidates **(b)** best edge **(c)** ESF (continuous) and LSF (dashed)

Figure B.7: Line spread estimation computed from arbitrary edges in an image.

Summary

We have presented a method for point spread function estimation from step edges. If straight edges are present in an image, they are accurately localized by a fast adaptive Hough transform. Otherwise, sharp and elongated edges can also be used to construct an edge spread function. A line spread function is then derived from the ESF using either a first-order robust normalized convolution or a robust model fit. The two-dimensional PSF is finally built from multiple oriented LSFs using the Fourier slice theorem.