Problem 1

Hand Written Work With Steps

Question 1

$X = [2\ 1\ 3\ 2\ 1\ 7\ 3\ 4\ 2\ 2\ 1\ 3\ 3\ 4\ 6\ 7\ 3\ 2\ 2\ 3\ 3\ 2]$

a) $Y_1 = \frac{2+1+3+2+1}{5} = 1.8$

$Y_2 = \frac{1+3+2+1+7}{5} = 2.8$

$Y_3 = \frac{3+2+1+7+3}{5} = 3.2$

$\vdots$

$Y = [1.8, 2.8, 3.2, 3.4, 3.4, 3.6, 2.4, 2.4, 2.2, 2.6, 3.4, 4.6, 4.6, 4.4, 4, 3.4, 2.6, 2.4]$

b) $Y_1 = \frac{(0\times2)+(1\times1)+(2\times3)+(1\times2)+(0\times1)}{0+1+2+1+0} = \frac{9}{4}$

$Y_2 = \frac{(0\times1)+(1\times3)+(2\times2)+(1\times1)+(0\times7)}{4} = 2$

$\vdots$

$y = [\frac{9}{4}, 2, 2.75, 4.5, 4.25, 3.75, 2.5, 1.75, 1.75, 2.5, 3.25, 4.25, 5.75, 5.75, 3.75, 2.25, 2.25, 2.75, 2.75]$

c) $y_1 = \text{median}(2,1,3,2,1) = 2$

$y_2 = \text{median}(1,3,2,1,7) = 2$

$y_3 = \text{median}(3,2,1,7,3) = 3$

ect

$y = [2\ 2\ 3\ 3\ 3\ 3\ 2\ 2\ 2\ 3\ 3\ 4\ 4\ 4\ 3\ 3\ 3\ 2]$

d) $y(t) = 0.3 \times X(t) + 0.7 \times y(t-1)$

$y[0] = 0$

$y[1] = 0.3^*(1) + 0.7^*(0) = 0.3$

$y[2] = 0.3^*(3) + 0.7^*(0.3) = 1.11$

to compute HPF use $Hpf = 1 - Lpf$

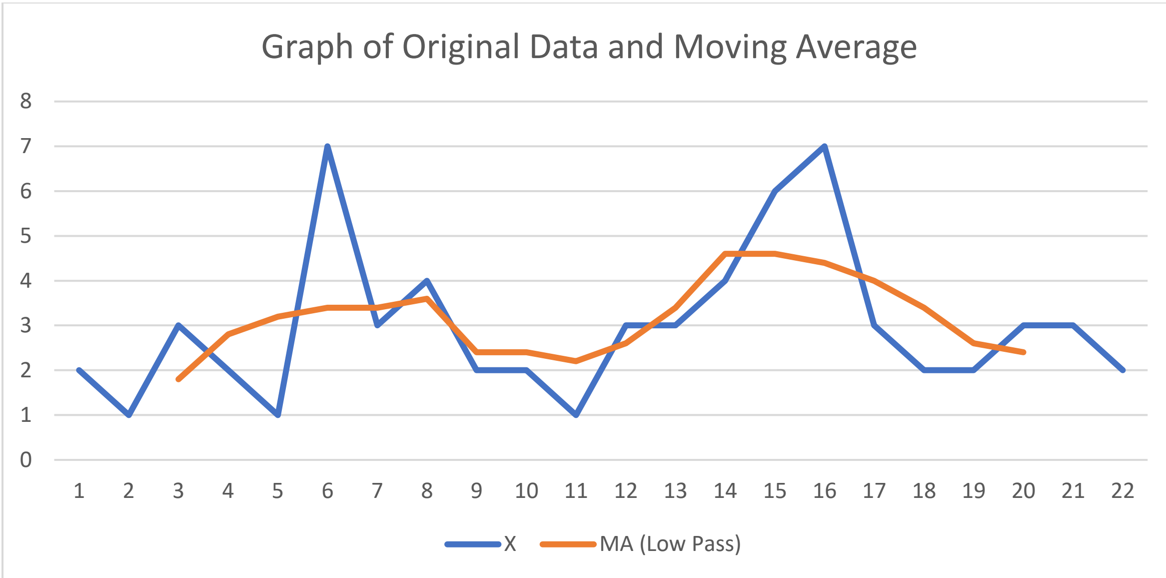
$\therefore X - Y_{LPF} = Y_{HPF}$

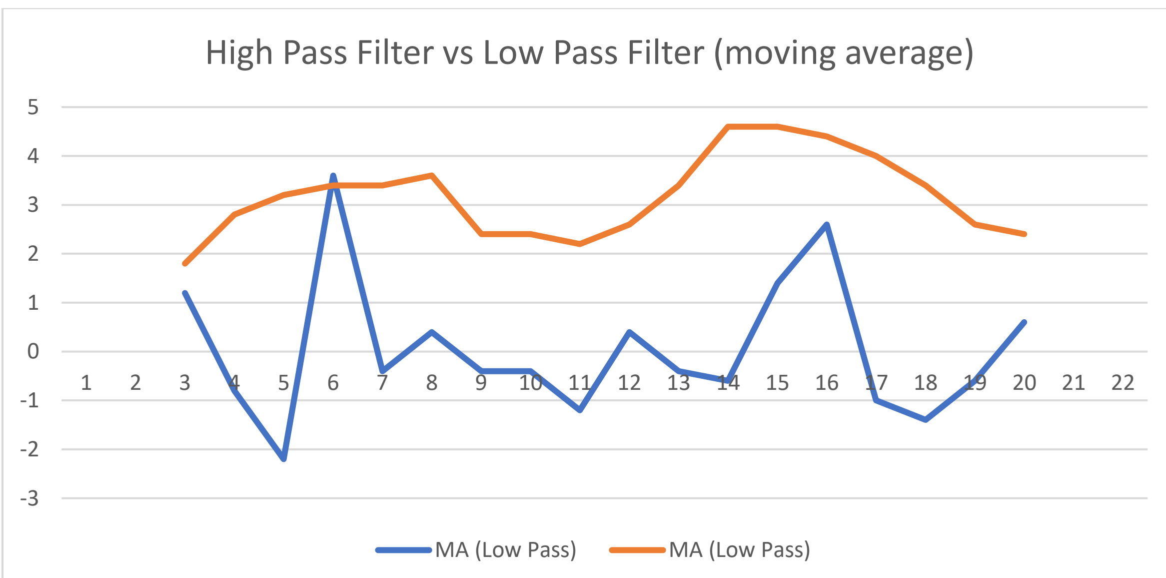
a) $Y_{HPF}[1] = 3 - 1.8 = 1.2$ $Y_{HPF}[2] = 2 - 2.8 = 0.2$

b) $Y_{HPF}[1] = 3 - 2.25 = 0.75$ $Y_{HPF}[2] = 2 - 2 = 0$

Moving Average

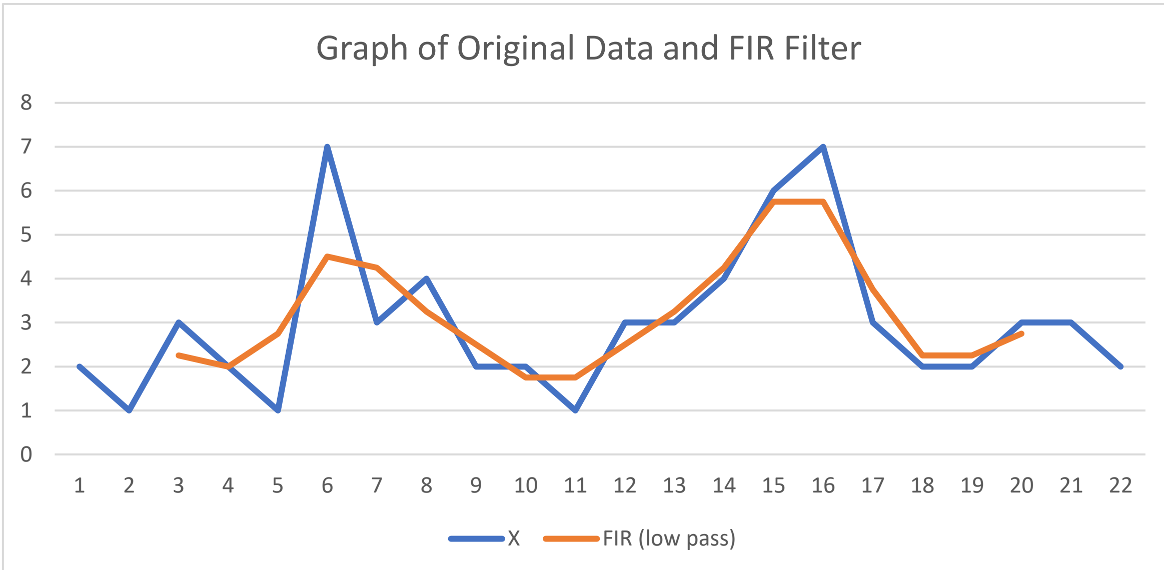
X	2	1	3	2	1	7	3	4	2	2	1	3	3	4	6	7	3	2	2	3	3	2
MA (Low Pass)			1.8	2.8	3.2	3.4	3.4	3.6	2.4	2.4	2.2	2.6	3.4	4.6	4.6	4.4	4	3.4	2.6	2.4		
MA (Low Pass)			1.2	-0.8	-2.2	3.6	-0.4	0.4	-0.4	-0.4	-1.2	0.4	-0.4	-0.6	1.4	2.6	-1	-1.4	-0.6	0.6		

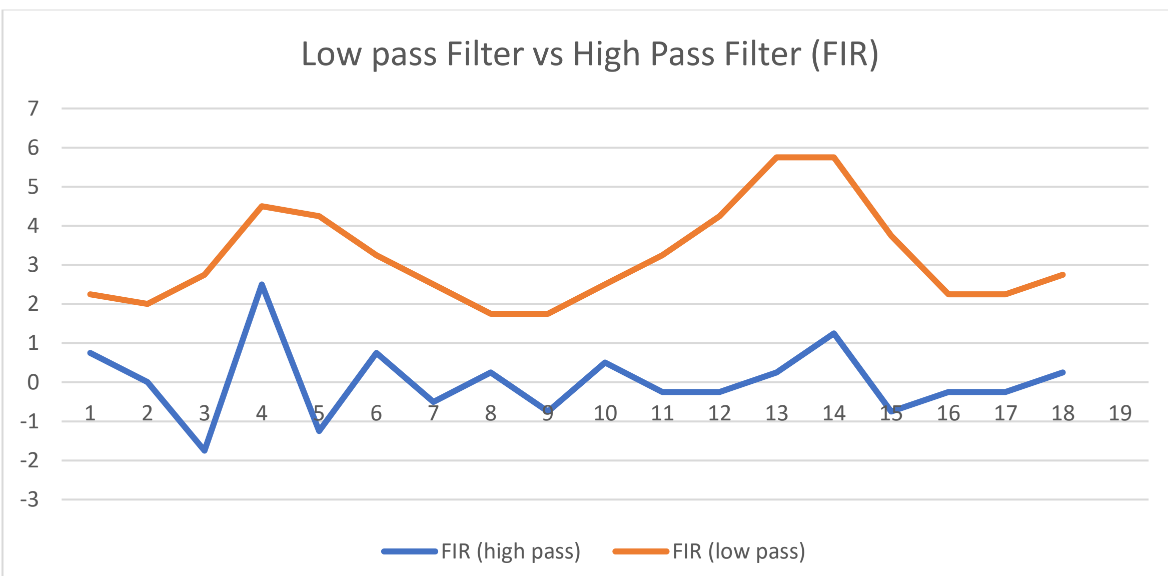

Graph of Original Data and Moving Average
8
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
X
MA (Low Pass)


High Pass Filter vs Low Pass Filter (moving average)
5
4
3
2
1
0
-1
-2
-3
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
MA (Low Pass)
MA (Low Pass)

FIR

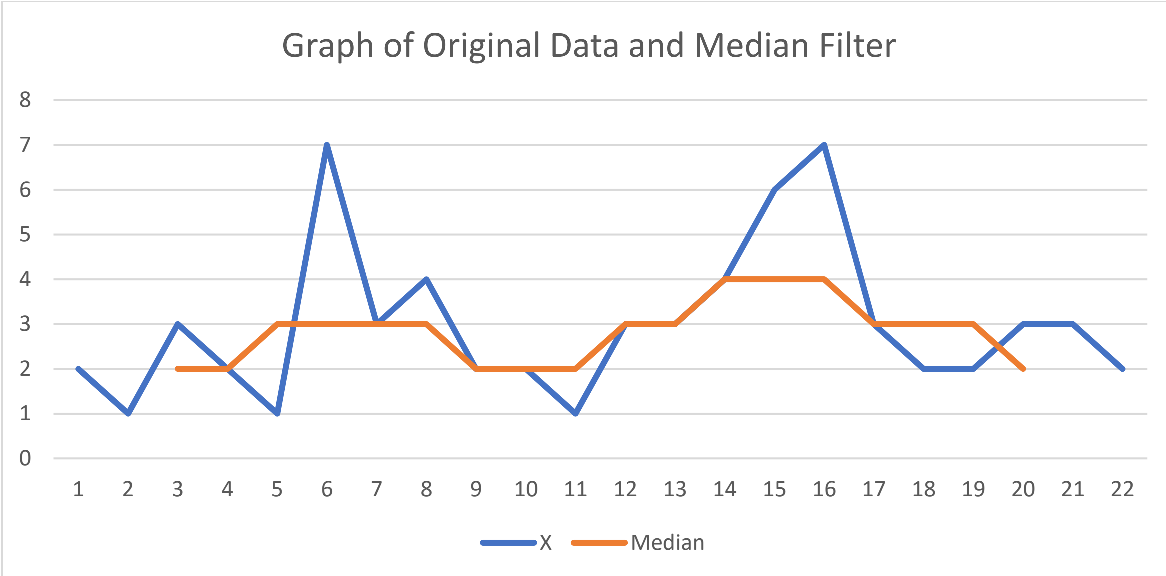
X	2	1	3	2	1	7	3	4	2	2	1	3	3	4	6	7	3	2	2	3	3	2
FIR (low pass)			2.25	2	2.75	4.5	4.25	3.25	2.5	1.75	1.75	2.5	3.25	4.25	5.75	5.75	3.75	2.25	2.25	2.75		
FIR (high pass)			0.75	0	-1.75	2.5	-1.25	0.75	-0.5	0.25	-0.75	0.5	-0.25	-0.25	0.25	1.25	-0.75	-0.25	-0.25	0.25		





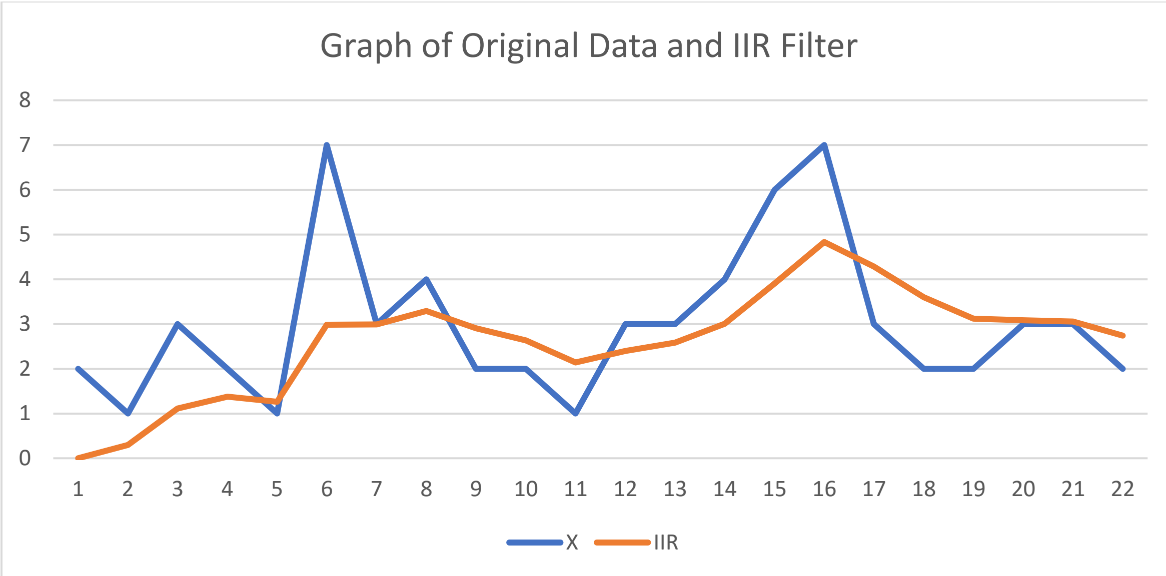
Median Filter

X	2	1	3	2	1	7	3	4	2	2	1	3	3	4	6	7	3	2	2	3	3	2
Median			2	2	3	3	3	3	2	2	2	3	3	4	4	4	3	3	3	2		



IIR

y[0]	0
y[1]	0.3
2	1.11
3	1.377
4	1.2639
5	2.98473
6	2.989311
7	3.292518
8	2.904762
9	2.633334
10	2.143334
11	2.400333
12	2.580233
13	3.006163
14	3.904314
15	4.83302
16	4.283114
17	3.59818
18	3.118726
	3.083108
	3.058176
	2.740723



Problem 2

Filter B:

Use [1,2,3,2,1]

Filter D:

Use $y(t) = 0.1 * x(t) + 0.4 * y(t-1)$

Problem 3

Question 3

$$x = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 1 & 0 \\ 2 & 3 & 9 \end{bmatrix} \quad \mu_{x_1} = \frac{1+1+2}{3} = \frac{4}{3}$$

$$\mu_{x_2} = \frac{4+1+3}{3} = \frac{8}{3}$$

$$\mu_{x_3} = \frac{5+0+9}{3} = \frac{14}{3}$$

$$\sigma_{x_1} = \sqrt{\tfrac{1}{2}\left(\left(1-\tfrac{4}{3}\right)^2+\left(1-\tfrac{4}{3}\right)^2+\left(2-\tfrac{4}{3}\right)^2\right)} = 0.5774$$

$$\sigma_{x_2} = \sqrt{\tfrac{1}{2}\left(\left(4-\tfrac{8}{3}\right)^2+\left(1-\tfrac{8}{3}\right)^2+\left(3-\tfrac{8}{3}\right)^2\right)} = 1.5275$$

$$\sigma_{x_3} = \sqrt{\tfrac{1}{2}\left(\left(5-\tfrac{14}{3}\right)^2+\left(0-\tfrac{14}{3}\right)^2+\left(9-\tfrac{14}{3}\right)^2\right)} = 4.5092$$

$$\bar{x} = \begin{bmatrix} -0.572 & 0.8707 & 0.0733 \\ -0.572 & -1.093 & -1.0350 \\ 1.161 & 0.2160 & 0.961 \end{bmatrix}$$

$$\bar{x}^T = \begin{bmatrix} -0.572 & -0.572 & 1.161 \\ 0.8707 & -1.093 & 0.2160 \\ 0.0733 & -1.0350 & 0.961 \end{bmatrix}$$

$$Cov(\bar{x}^T x) = \begin{bmatrix} -0.572 & -0.572 & 1.161 \\ 0.8707 & -1.093 & 0.2160 \\ 0.0733 & -1.035 & 0.961 \end{bmatrix} \begin{bmatrix} -0.572 & 0.8707 & 0.0733 \\ -0.572 & -1.093 & -1.0350 \\ 1.161 & 0.2160 & 0.961 \end{bmatrix}$$

$$= \begin{bmatrix} 2.002 & 0.3779 & 1.663 \\ 0.3779 & 1.994 & 1.4027 \\ 1.6658 & 1.4027 & 2.001 \end{bmatrix}$$

Problem 4

Question 4

a) $X_1 = [1, 2, NaN, 3, 3, NaN, 0, 2, -2, -2, 1, 5, NaN, 9]$

$X_2 = [NaN, -1, 2, NaN, -2, -1, 3, 2, -2, NaN, -4, 1]$

Zero Padding

$X_1 = [1, 2, 0, 3, 3, 0, 0, 2, -2, -2, 1, 5, 0, 9]$

$X_2 = [0, -1, 2, 0, -2, -1, 3, 2, -2, 0, -4, 1]$

Sample & Hold

$X_1 = [1, 2, 2, 3, 3, 3, 0, 2, -2, -2, 1, 5, 5, 9]$

$X_2 = [0, -1, 2, 2, -2, -1, 3, 2, -2, -2, -4, 1]$

↑ assuming this is zero

linear interpolation

$X_1 = [1, 2, 2.5, 3, 3, 1.5, 0, 2, -2, -2, 1, 5, 7, 9]$

$X_2 = [0, -1, 2, 0, -2, -1, 3, 2, -2, -3, -4, 1]$

↑ assuming this is zero

```
X1 = [1 2 NaN 3 3 NaN 0 2 -2 -2 1 5 NaN 9];
X1_SH = [1 2 2 3 3 3 0 2 -2 -2 1 5 5 9];
X1_LI = [1 2 2.5 3 3 1.5 0 2 -2 -2 1 5 7 9];
X1_ZP = [1 2 0 3 3 0 0 2 -2 -2 1 5 0 9];

std_X1 = std(X1,'omitnan')
std_X1_SH = std(X1_SH,'omitnan')
std_X1_LI = std(X1_LI,'omitnan')
std_X1_ZP = std(X1_ZP,'omitnan')

X2 = [NaN -1 2 NaN -2 -1 3 2 -2 NaN -4 1];
X2_SH = [0 -1 2 2 -2 -1 3 2 -2 -2 -4 1];
X2_LI = [0 -1 2 0 -2 -1 3 2 -2 -3 -4 1];
X2_ZP = [0 -1 2 0 -2 -1 3 2 -2 0 -4 1];

std_X2 = std(X2,'omitnan')
std_X2_SH = std(X2_SH,'omitnan')
std_X2_LI = std(X2_LI,'omitnan')
std_X2_ZP = std(X2_ZP,'omitnan')
```

```
std_X1 =                std_X2 =

    3.1305                  2.3333

std_X1_SH =             std_X2_SH =

    2.8670                  2.1672

std_X1_LI =             std_X2_LI =

    3.0598                  2.1515

std_X1_ZP =             std_X2_ZP =

    2.8747                  1.9924
```

The blue boxes represent the standard deviations in the data with no data imputation.

The orange boxes represent the standard deviations with a sample and hold data imputation method.

The green boxes represent the standard deviations with a linear interpolation data imputation method.

The black boxes represent the standard deviations with a zero padding data imputation method.

As we can see when using the sample and hold method there is a decrease in standard deviation on both datasets. The same can be said for a linear interpolation data imputation method and a zero padding one.

However, for data set 1 a sample and hold method works the best, followed by zero padding and then lastly linear interpolation.

For data set 2 the zero padding method works the best followed by linear interpolation and then sample and hold.

Problem 5

Question 5

x = [1 3 4 5 0 1 11 -1 3 -2 5 6]

a) x = [1 3 4 5 0 1 11 -1 3 -2 5 6 ∅]

↑padding

x_max = [4,5,11,11,5,6]

b) x = [1 3 4 5 0 1 11 -1 3 -2 5 6 ∅ ∅]

↑padding

x_min = [1,0,-1,-2,-2,∅]

c) x = [1 3 4 5 0 1 11 -1 3 -2 5 6 ∅ ∅]

↑padding

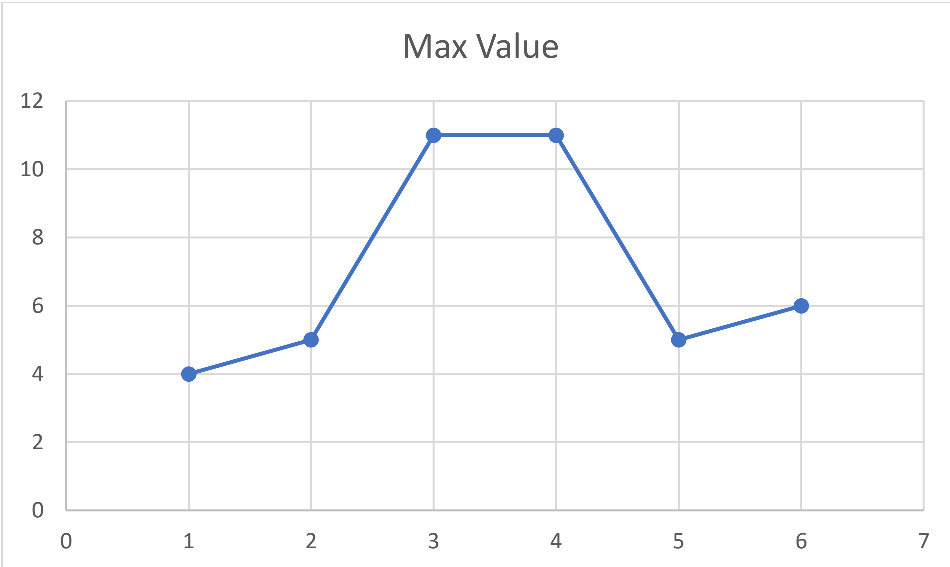
x_median = [3,0,3,∅]

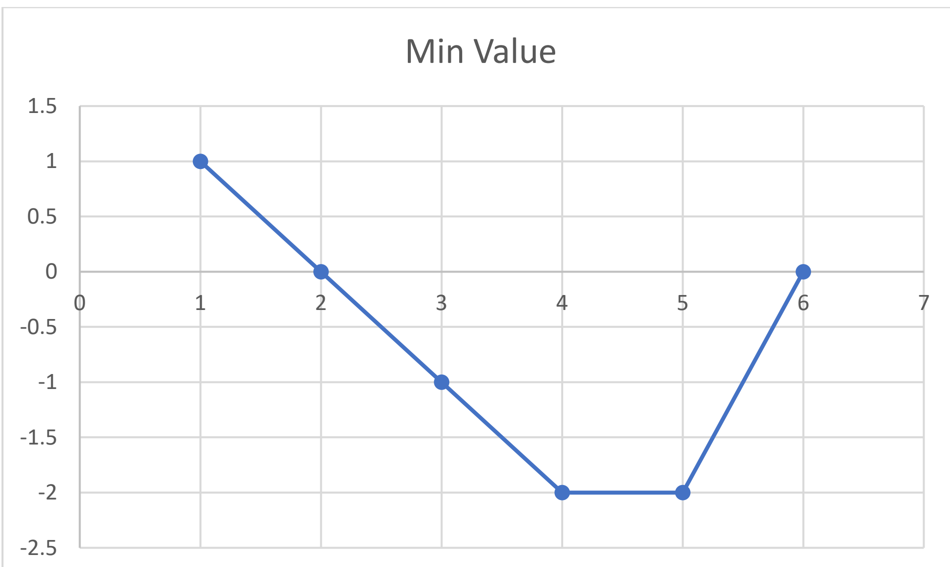
x_mean = [2.6,3.2,3.2,1.8]

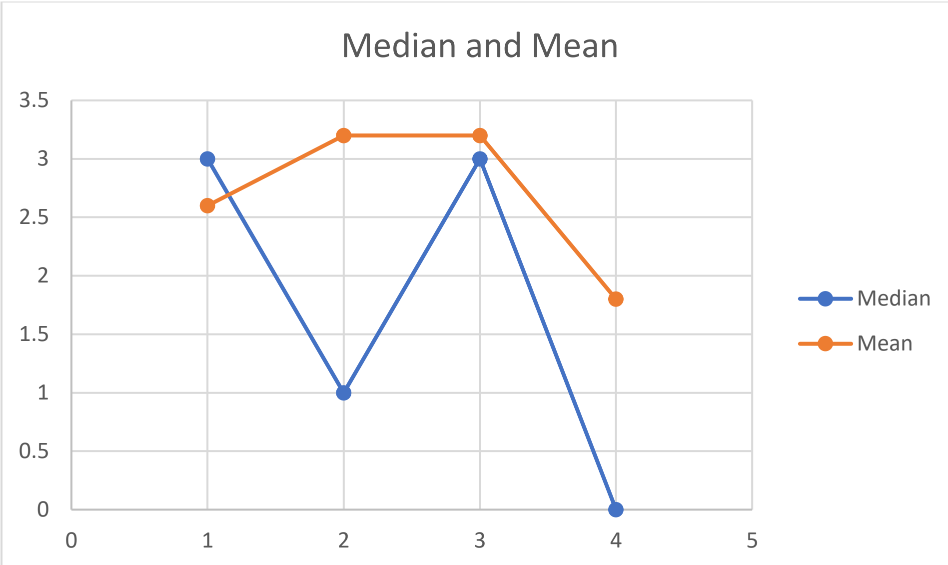
d) x = [1 3 4 5 0 1 11 -1 3 -2 5 6]

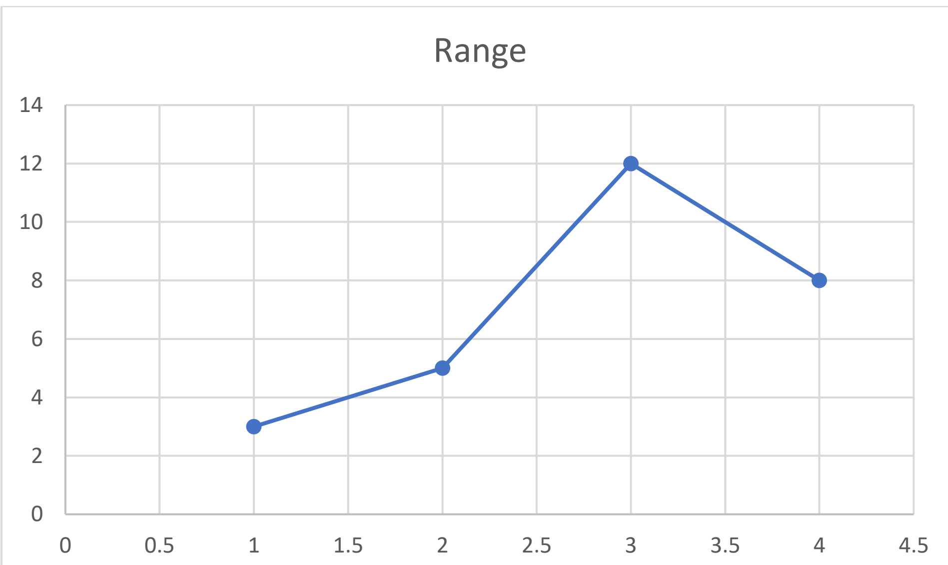
x_range = [3,5,12,8]

Window	1	2	3	4	5	6
Max Value	4	5	11	11	5	6
Min Value	1	0	-1	-2	-2	0
Median	3	0	3	0		
Mean	2.6	3.2	3.2	1.8		
Range	3	5	12	8		


Max Value
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7


Min Value
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7


Median and Mean
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
Median
Mean


Range
14
12
10
8
6
4
2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5