Problem 1

QUESTION 1

a) False

→ we have $(\pi > 2) \wedge (24 \text{ is prime})$ which evaluates to

$(\text{true}) \wedge (\text{false})$

$\therefore$ False

b) True

→ we have $(2^{10} = 1024) \vee (\sqrt{3} \text{ is rational})$

$(\text{true}) \vee (\text{False})$

$\therefore$ True

c) True $(\text{all apples are red}) \Rightarrow (x = 5)$

→ False $\Rightarrow$ False

d) False $(\text{Some apples are red}) \Rightarrow (3 > 300)$

True $\Rightarrow$ False

e) True $(\text{earth is flat}) \Leftrightarrow (\pi = 2)$

False False

Problem 2

QUESTION 2

$\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$ $\neg(T \vee Q)$ $R \Rightarrow (\neg T \Rightarrow (\neg S \wedge P))$

$\neg P \vee Q \vee S \vee T$ $\boxed{R_0}$ $\neg T \wedge \neg Q$ $(\neg R \vee T \vee \neg S) \wedge (\neg R \vee T \vee P)$

$\neg T$ $\boxed{R_1}$

$\neg Q$ $\boxed{R_2}$

$\neg P \vee T \vee S$ $\boxed{R_0 R_2}$

$\neg R \vee T \vee \neg S$ $\boxed{R_3}$

R $\boxed{R_4}$

$\neg R \vee P \vee T$ $\boxed{R_5}$

$\neg P \vee T \vee \neg R$ $\boxed{R_0 R_2 R_3}$

$\neg R \vee T$ $\boxed{R_0 R_2 R_3 R_5}$

$\neg R$ $\boxed{R_0 R_2 R_3 R_5 R_1}$

when we add R_4 (R) we get a contradition leaving us withe tle empty set $\blacksquare$

Problem 3

QUESTION 3

R1: If I can get a loan, I can buy a car

R2: If I win the lottery, I can buy a car

a) $Loan \Rightarrow Car$

$Lottery \Rightarrow car$

b) $((Loan \Rightarrow Car) \vee (Lottery \Rightarrow car)) \Rightarrow ((Lottery \wedge Loan) \Rightarrow Car)$

c) $((Loan \Rightarrow car) \vee (Lottery \Rightarrow Car)) \Rightarrow ((Lottery \wedge Loan) \Rightarrow Car)$

$((\neg Loan \vee car) \vee (\neg Lottery \vee car)) \Rightarrow \neg(Lottery \wedge Loan) \vee Car)$

$\neg Loan \vee \neg Lottery \vee Car \Rightarrow \neg Lottery \vee \neg Loan \vee Car$

$\neg(\neg Loan \vee \neg Lottery \vee Car) \vee (\neg Lottery \vee \neg Loan \vee car)$

$Loan \wedge Lottery \wedge \neg Car \vee \neg Lottery \vee \neg Loan \vee Car$

Problem 4

QUESTION 4

a) $\exists x,y\ (Student(x) \wedge French(y) \wedge TakeInFall2020(x,y))$

b) $\forall x,y\ (Student(x) \wedge Greek(y) \wedge Take(x,y) \Rightarrow Pass(x,y))$

c) $\forall x, \exists y\ (Student(x) \wedge French(y) \wedge Fail(x,y) \Rightarrow TakeInFall2020(x,y)$

d) $\forall x, \exists y\ (Take(x,y) \wedge Greek(y) \Leftrightarrow Take(x,y) \wedge Pass(x,y))$

Problem 5

QUESTION 5

$\forall x\ [Apple(x) \Rightarrow Fruit(x)]$ is equiv. to $\neg Apple(x) \vee Fruit(y)$

$\forall x, z\ [Apple(x) \wedge SeedOf(z, x)]$

Show by contr. $(Fruit(Apple) \wedge SeedOf(Blackseed, Apple))$

$\neg(Fruit(Apple) \wedge SeedOf(Blackseed, Apple))$ $\boxed{R_0}$

$\neg Fruit(Apple) \vee \neg SeedOf(Blackseed, Apple)$

$Apple(x) \Rightarrow Fruit(x)$ $\boxed{R_1}$

$\neg Apple(x) \vee Fruit(x)$

$Apple(x) \wedge SeedOf(z, x)$ $\boxed{R_2}$

$SeedOf(z, x)$ $\boxed{R_3}$

$Apple(x)$ $\boxed{R_4}$

$\neg Apple(x) \vee \neg SeedOf(Blackseed, Apple)$ $x | Apple$ $\boxed{R_5}$

$\neg SeedOf(Blackseed, Apple)$ $\boxed{R_3 R_5}$

$\neg Fruit(Apple)$ $z | Blackseed$ $x | Apple$ $\boxed{R_6}$

$Fruit(x) = Apple$ $\boxed{R_7}$

$\therefore$ Contradiction as R_0 is not satisfiable $\boxed{R_6, R_7}$

Problem 6

QUESTION 6

$(a \vee b) \wedge (\neg b \vee c)$ using $A \rightarrow B \equiv \neg A \vee B$

$(\neg a \rightarrow b) \wedge (b \rightarrow c)$ using chain rule

$\neg a \rightarrow c$ using $A \rightarrow B \equiv \neg A \vee B$

$a \vee c$