Solutions

Name:

IE 364 Midterm Examination March 30, 2013

(Duration: 100 minutes. Books and notes allowed. No exchange of materials)

1. Parts arrive at a two-stage machine according to a Poisson process at the rate of 20/hr. The first stage (punching) is served by two parallel stations with the process time having a distribution of Normal(5 min, 4 min²). Second stage (bending) has a single station and takes 2 min, constant. There are waiting lines in front of both stages.

a) Draw a time diagram of events and mark each event as B (bound) or C (conditional)

1	35
2	15
3	35
Q+CW	15
	100

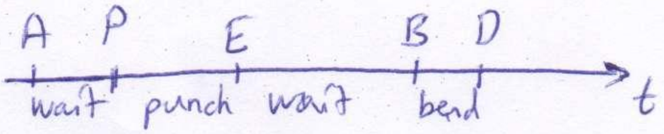
A Arrival —— Bound

P begin Punching —— conditional

E End punching —— B "

B begin Bending —— conditional

D Departure —— B "



b) Determine the standard deviation (in minutes) for the following random variables:

inter-arrival time $1/\lambda = 3$ min $\lambda = 20/\text{hr} \rightarrow 1/\lambda = 3$ min

punching time $\sigma = 2$ min

bending time 0

c) Determine the utilization of each stage. Which queue will be more crowded on the average?

Punching $\rho_1 = \frac{\lambda}{c\mu} = \frac{5}{2 \times 3} = 0.83$ ← more crowded

Bending $\rho_2 = \frac{2}{3} = 0.67$

d) When clock = 418 minutes, FEL (future events list) contains E420, A422, E425
Assuming both queues are empty, how many parts are there in the system at that time?

$\left.\begin{array}{l}\text{both punchers are busy} \\ \text{bender is idle} \\ \text{queues are empty}\end{array}\right\} L = 2$

e) Show a possible FEL after executing the first two events

420: bending starts
D is scheduled A422, D422, E425

422: customer arrives
punching starts
A + E scheduled D422, E425, E426, A428

2. Repair time X of a certain machine has the cdf given below (X is measured in hours)

$F(x) = 0$ if $x \leq 0$

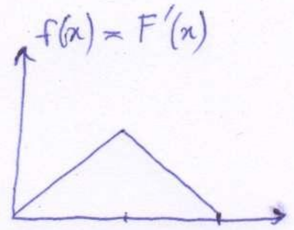
$F(x) = x^2/2$ if $0 < x \leq 1$

$F(x) = 2x-1-x^2/2$ if $1 < x \leq 2$

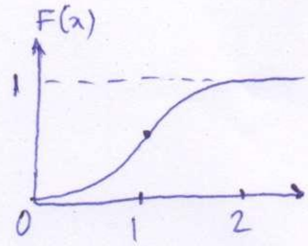
$F(x) = 1$ if $2 < x$

a) Determine the probability density function (pdf) $f(x)$

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$



b) Plot $f(x)$ and $F(x)$



c) Determine $E(X)$ (Hint: $f(x)$ is symmetrical)

$$E(X) = \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx = \frac{1}{3} + 3 - \frac{7}{3} = 1$$

d) Determine the probability of X exceeding 0.5 hr

$$P(X > 0.5) = 1 - P(X \leq 0.5) = 1 - F(0.5) = 1 - \frac{1}{8} = \frac{7}{8}$$

3. Customers arrive in front of a server according to a Poisson process with $\lambda = 12$ per hour. The service time S has a mean of 4 minutes.

a) Determine service rate μ and utilization ρ

$\mu = 15/hr = 0.25/min$ $\qquad$ $\rho = \frac{\lambda}{\mu} = 0.8$

b) Estimate L and w assuming service time S is constant

c.v. = 0

$$L_Q = \frac{\rho^2}{2(1-\rho)} = \frac{0.8 \times 0.8}{2 \times 0.2} = 1.6$$

$L = \rho + L_Q = 2.4$ $\qquad$ $w = \frac{L}{\lambda} = 0.2\ hr = 12\ min$

c) Estimate L and w assuming S has an unknown distribution with $\sigma = 1.2$ min.

c.v. = 0.3 $\longrightarrow$ $\frac{1+cv^2}{2} = 0.55$

$L_Q = 1.74$

$L = 2.54$ $\qquad$ $w = 0.212\ hr = 12.7\ min$

d) Determine the probability of at least two arrivals within 10 minutes

$N \sim$ Poisson (2)

$$P(N \geq 2) = 1 - P(0) - P(1) = 1 - e^{-2} - 2e^{-2}$$

$$= 1 - F(1) = 1 - 0.406 = 0.594$$