Bayesian X-ray Computed Tomography Using Material Class Knowledge

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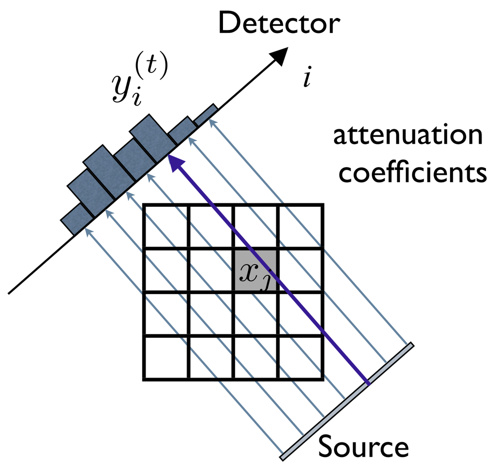
Computed Tomography (CT)

CT reconstructs tomographic images from their projections



Detectors are positioned on the opposite side of the x-ray source, and they rotate around the object

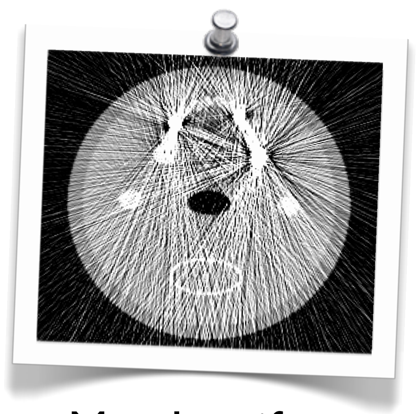
x_j: attenuation coefficient at location j
y_i: detected number of photons at ith detector
$\mathbf{y}^{(t)} = \{y_1^{(t)}, ..., y_I^{(t)}\}$: tth projection data
$\mathcal{D} = \{\mathbf{y}^{(1)}, \cdots, \mathbf{y}^{(T)}\}$: observed data set

Our aim is to reconstruct the x-ray attenuation coefficients x form projection data D

Demands for CT

1. Reduction of X-ray exposure
   X-ray exposure should be minimized in order to avoid an overdose of radiation but limitation of X-ray exposure makes the observed data noisy

2. Reduction of metal artifact
   Presence of high density objects such as metal prostheses and dental fillings cause streak or star artifacts



Metal artifact

Bayesian Approach to CT

Bayesian approach can regularize unwanted solutions by incorporating suitable prior knowledge.
Also, the estimation is robust to the probabilistic fluctuations by considering the uncertainty regarding the unknown random variables.

Estimate $\boldsymbol{x}^* = \int \boldsymbol{x}\, p(\boldsymbol{x}|D)\, d\boldsymbol{x}$
where,

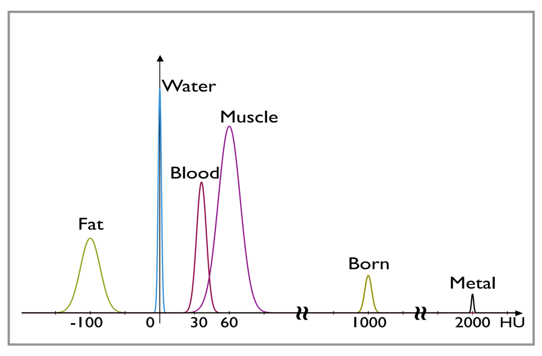
$$p(\mathbf{x}|\mathcal{D}) = \frac{p(\mathcal{D}|\mathbf{x})\ p(\mathbf{x})}{\int p(\mathcal{D}|\mathbf{x})\ p(\mathbf{x})d\mathbf{x}}$$

$p(D|x)$: likelihood function
$p(x)$: prior distribution over x-ray attenuation coefficients x

The design of likelihood and prior is crucial

Hierarchical Prior

A priori knowledge
i) X-ray attenuation depend on materials
ii) Human body is composed only a limited number of materials
iii) Materials have spacial continuity



Distribution over the material classes (schematic illustration)

To incorporate a priori knowledge, we introduce a hierarchical prior model

$$p(\mathbf{x}) = \sum_{\mathbf{z}} p(\mathbf{x}|\mathbf{z})p(\mathbf{z})$$

K-dimensional hidden variables z_j represent material classes.
z_{jk} is 1 when the pixel j belongs to the kth material class and otherwise 0

When the material class is given, the x-ray attenuation x obey a Gaussian

$$p(\mathbf{x}|\mathbf{z}) = \prod_{j=1}^{J}\prod_{k=1}^{K} \mathcal{N}(x_j|\nu_k, r_k^2)^{z_{jk}}$$

Boltzmann distribution is used for the prior distribution over materials classes

$$p(\mathbf{z}) = \frac{1}{Z_J}\exp\left\{\sum_k \left(J_k^{\mathrm{self}}\sum_j z_{jk} + J^{\mathrm{inter}}\sum_j\sum_{s\in\eta(j)} z_{jk}z_{sk}\right)\right\}$$

relative proportion of each material

represent spatial continuity

($J^{\mathrm{self}}, J^{\mathrm{inter}}$: constant)
($\eta(j)$: neighboring pixels of j)

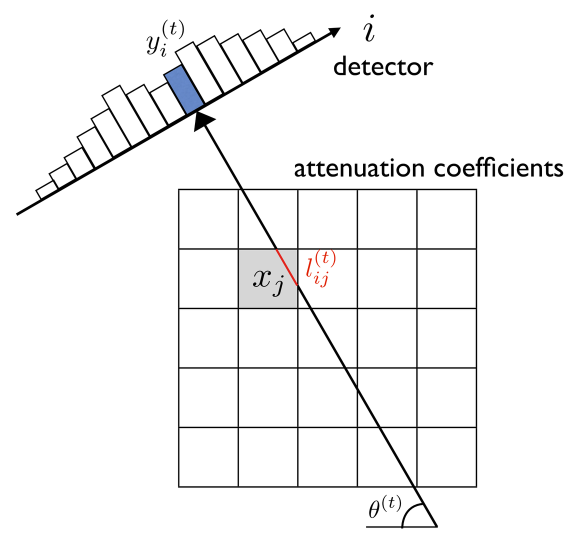
Likelihood function

The quantum nature of x-ray photons fluctuations y, making the probability the independent Poisson distribution

$$p(\mathcal{D}|\mathbf{x}) = \prod_{t=1}^{T}\prod_{i=1}^{I} \frac{\hat{y}_i^{(t)y_i^{(t)}} e^{-\hat{y}_i^{(t)}}}{y_i^{(t)}!}$$

Mean number of detected photons:

$$\hat{y}_i^{(t)} = b_i^{(t)}\exp\{-\sum_{j=1}^{J} l_{ij}^{(t)} x_j\}$$



y: detected number of photons
b: detected photons in the absence of any absorbers
$l^{(t)}_{ij}$: effective contribution of attenuation coefficient x_j to projection line i when projected from the direction $\theta^{(t)}$

Variational Bayes Approximation

Since direct calculation of the posterior distribution is difficult, we employ the variational Bayes method in order to derive a practical algorithm

Trial distribution is determined so as to minimize the KL divergence

$$D_{\mathrm{KL}}(q||p) \equiv \left\langle \ln\frac{q(\mathbf{x},\mathbf{z})}{p(\mathbf{x},\mathbf{z}|\mathcal{D})}\right\rangle_{q(\mathbf{x},\mathbf{z})}$$

under the following assumptions on the trial distribution

$$q(\mathbf{x},\mathbf{z}) = \prod_j q(x_j)q(\mathbf{z}_j) \qquad q(x_j) = \mathcal{N}(x_j|\mu_j, \sigma_j^2)$$

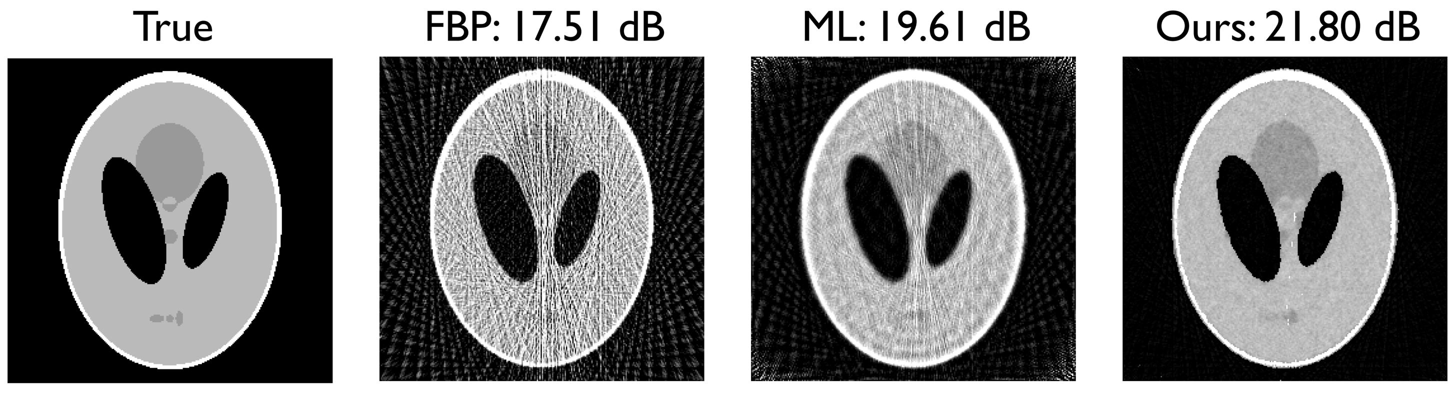
Trial distribution $q(x)$ and $q(z)$ is optimized alternately

Experiments

Phantom Data without Metal

The phantom that do not include metal is used
Number of projections is severely restricted (36 projection over 180°)
367 detectors are used and image size of 256 × 256 pixels are reconstructed

True | FBP: 17.51 dB | ML: 19.61 dB | Ours: 21.80 dB



FBP: filtered back projection, ML: maximum likelihood

Phantom Data with Metal Inserted

The metal inserted phantom is used
Number of projections is increased (1800 projection over 180°)
367 detectors are used and image size of 256 × 256 pixels are reconstructed

True | FBP: 23.77 dB | PCLIS: 30.35 dB | ML: 27.38 dB | Ours: 33.39 dB

PCLIS: projection completion method based on a linear interpolation in the sinogram

The proposed method shows better performance even under the limited number of projections and significantly reduce metal artifacts compared to the existing method
Good smoothing within each region is obtained by our method

Performance is evaluated by PSNR:
$\mathrm{PSNR} = 10\log_{10}(\mathrm{MAX}_I^2 / \mathrm{MSE})$
MAXi: max pixel value, MSE: mean squared error

Conclusion

The hierarchical model allows us to represent a realistic physical process
Our proposed method shows better performance compared to the existing algorithm