Universidade Federal de Viçosa
Instituto de Ciências Exatas e Tecnológicas
Campus UFV - Florestal

Álgebra Linear A - Gabarito da Lista 2

Prof. Fernando Bastos

1. (i) $a=-2$ e $b=-9$ (ii) $a=\frac{4}{5}$ e $b=-2$ (iii) $a=-6$ e $b=8$
   (c) $|u|=5$, $\|v\|=\sqrt{29}$ e $\|w\|=\sqrt{a^2+b^2}$

2. (i) $a=7$, $b=-3$ e $c=-2$
   (ii) $a=9$, $b=-6$ e $c=-12$
   (iii) $a=5$, $b=0$ e $c=8$
   (c) $\|u\|=\sqrt{21}$, $\|v\|=\sqrt{29}$ e $\|w\|=\sqrt{a^2+b^2+c^2}$

3. $w=(-\frac{\sqrt{10}}{10},-\frac{3\sqrt{10}}{10})$ ou $w=(\frac{\sqrt{10}}{10},\frac{3\sqrt{10}}{10})$

4. Intensidade: 13 newtons. Direção: $\theta=\pi-arctg\left(\frac{5}{12}\right)$ Sentido: Noroeste

5. Vetor velocidade $v=(4,-3)$ e $\|v\|=5$ km/h

6. (a) U é subespaço de V
   (b) U não é subespaço de V
   (c) U não é subespaço de V
   (d) U não é subespaço de V
   (e) U é subespaço de V

7. (a) L.D. se $k=8$ e L.I. se $k\neq 8$
   (b) L.D. se $k\in\mathbf{R}$

8. $a-3b-5c=0$

9. Observe que $[u,v,w,p]=\{(x,y,z,t)\,;\; x+3y-7z-7t=0\}$
   (a) Sim (b) Não (c) Sim (d) Não

10. $[S]=\left\{\begin{bmatrix} x & y \\ z & t\end{bmatrix}\in M_{2\times 2}(\mathbf{R})\,;\; 7x-5y+18z+7t=0\right\}$

11. (a) (L.D.) (b) (L.D.) (c) (L.D.) (d) (L.I.) (e) (L.I.)

12. (a) (V) (b) (F) (c) (V) (d) (V) (e) (F)

13.

14. $\begin{cases} 2x+y+z=0 \\ 5x+y-t=0\end{cases}$

15. L.I.

16.

17. (b) $\begin{bmatrix} 0 & -2 \\ 0 & 1\end{bmatrix}\in W$ e $\begin{bmatrix} 0 & 2 \\ 3 & 1\end{bmatrix}\notin W$
    (c) Base: $\left\{\begin{bmatrix} 2 & 1 \\ 0 & 1\end{bmatrix},\begin{bmatrix} 0 & -2 \\ 0 & 1\end{bmatrix}\right\}$

18. (b) Base de $\mathbf{W}_1$:
$$\left\{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix},\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix},\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0\end{bmatrix},\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0\end{bmatrix},\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0\end{bmatrix},\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1\end{bmatrix}\right\}$$

19. (b) Base de $\mathbf{W}_2$: $\left\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \right\}$

20. Observe que para toda matriz $A \in M_{3\times 3}(\mathbf{R})$, temos:

$$\begin{aligned} A =& \underbrace{\text{Simétrica}} + \underbrace{\text{Anti-simétrica}} \\ =& \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right) \end{aligned}$$

21. $\dim D = 3$. Uma base para este subespaço é
$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

22. $\dim D = 6$. Uma base para este subespaço é
$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

23. (a) $\dim S = 2$ e $\mathcal{B}_S = \{(11, 0, -5, 1),\ (-2, 1, 0, 0)\}$
(b) $\dim S = 3$ e $\mathcal{B}_S = \{(-2, 1, 0, 0, 0),\ (5, 0, -2, 1, 0),\ (-7, 0, 2, 0, 1)\}$

24. (a) $\dim U = 2$, $\quad U = \{(x, y, 3y - 3x, y - 2x);\ x, y \in \mathbf{R}\}$
(b) $\dim W = 2$, $\quad W = \{(x, y, 2y - 2x, -y);\ x, y \in \mathbf{R}\}$
(c) $\dim(U \cap W) = 1$, $\quad U \cap W = \{(x, x, 0, -x);\ x \in \mathbf{R}\}$
(d) $\dim(U + W) = \quad \dim U + \dim W - \dim(U \cap W) = 3$

25. (b) $\mathcal{B}_{W_1} = \{(1, 0, -1, 0), (0, 1, 0, 0), (0, 0, 0, 1)\}$
$\mathcal{B}_{W_2} = \{(1, 0, 0, 0), (0, 1, 0, -1), (0, 0, 1, 0)\}$
$\mathcal{B}_{W_1 \cap W_2} = \{(1, 0, -1, 0), (0, 1, 0, -1)\}$
(c) $W_1 + W_2 = \mathbf{R}^4$, pois $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 + \dim W_1 \cap W_2 = 3 + 3 - 2 = 4$.

26. (a) $\mathcal{B}_W = \{(1, 0, 0, -1), (0, 1, 0, -1), (0, 0, 1, 1)\}$ e $\dim W = 3$
(b) $u = (2, -3, 2, 2) \notin W$
(c) $x + y - z + t = 0$

27. $\dim[S] = 6$, $\dim[R] = 3$, $\dim([S] \cap [R]) = 3$ e $\dim([S] + [R]) = 6$

28. (a) $[I]^{\gamma}_{\mathcal{C}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ $\quad$ (b) $[u]_{\gamma} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

29. (b) (i) $[v]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\quad$ (ii) $[w]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

30. $\mathcal{B}_s = \left\{x^3 - 3x - 2,\ x^2 - 2x - 3\right\}$ e $\dim S = 2$.

31.
$\dim U = 3$, $\quad \mathcal{B}_U = \{(1, 0, 0, 0), (0, 1, 0, -1), (0, 0, 1, -1)\}$
$\dim W = 2$, $\quad \mathcal{B}_W = \{(1, -1, 0, 0), (0, 0, 2, 1)\}$
$\dim(U \cap W) = 1$, $\quad \mathcal{B}_{U\cap W} = \{(3, -3, 2, 1)\}$
$\dim(U + W) = 4$, $\quad \mathcal{B}_{U+W} = \{(1, 0, 0, 0), (0, 1, 0, -1), (0, 0, 1, -1), (1, -1, 0, 0)\}$

32. (a) $\left\{ \begin{array}{l} -3x-4y+z+w=0 \\ 4x+2y+t=0 \end{array} \right.$ e $\left\{ \begin{array}{l} 9x+2y+z+w=0 \\ -4x-2y+t=0 \end{array} \right.$

(b) $\mathcal{B}_{\mathbf{U}\cap\mathbf{W}} = \{(1,-2,-5,0,0)\,,\ (0,0,-1,0,1)\}$ e $\dim(\mathbf{U}\cap\mathbf{W}) = 2$

(c) $\dim(\mathbf{U}+\mathbf{W}) = 4$

33. $\alpha = \left\{ \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right) \right\}.$

34. (a) $[v]_\alpha = \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$ (b) $[v]_\beta = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

35. $[I]_{\beta_1}^{\beta_2} = \begin{bmatrix} \frac{4}{11} & -\frac{3}{11} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$ e $[(5,-8)]_{\beta_1} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

36. (a) (i) $[I]_\beta^{\beta_1} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $[I]_{\beta_1}^{\beta} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (iii) $[I]_{\beta_2}^{\beta} = \begin{bmatrix} \frac{\sqrt{3}}{6} & \frac{1}{2} \\ \frac{\sqrt{3}}{6} & -\frac{1}{2} \end{bmatrix}$ (iv) $[I]_{\beta_3}^{\beta} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

(b) (i) $[v]_\beta = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ (ii) $[v]_{\beta_1} = \begin{bmatrix} -\frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$ (iii) $[v]_{\beta_2} = \begin{bmatrix} -\frac{2-\sqrt{3}}{2} \\ \frac{2+\sqrt{3}}{2} \end{bmatrix}$ (iv) $[v]_{\beta_3} = \begin{bmatrix} \frac{3}{2} \\ -1 \end{bmatrix}$

(c) (i) $[v]_\beta = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$ (ii) $[v]_{\beta_2} = \begin{bmatrix} \frac{6-2\sqrt{3}}{3} \\ \frac{-6-2\sqrt{3}}{3} \end{bmatrix}$ (iii) $[v]_{\beta_3} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

37. (a) (i) $[I]_{\beta_1}^{\beta_2} = \begin{bmatrix} -1 & 1 \\ 0 & \frac{1}{2} \end{bmatrix}$ (ii) $[I]_{\beta_2}^{\beta_3} = \begin{bmatrix} 0 & -1 \\ -1 & -1 \end{bmatrix}$ (iii) $[I]_{\beta_1}^{\beta_3} = \begin{bmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

(iv) $[I]_{\beta_1}^{\beta_2}\,[I]_{\beta_2}^{\beta_3} = \begin{bmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $[I]_{\beta_1}^{\beta_3} = [I]_{\beta_1}^{\beta_2}\,[I]_{\beta_2}^{\beta_3}$