

6. PRIMITIVAÇÃO (SOLUÇÕES)

6.1.

- | | | |
|---|---|---|
| a) $\frac{3}{2}x^2 + 2x;$ | b) $\frac{(x^3 - 3)^3}{9};$ | c) $\frac{x^5}{5} - \frac{2}{3}x^3 + x;$ |
| d) $\frac{5}{7}\sqrt[5]{x^7};$ | e) $\frac{e^{x^3}}{3};$ | f) $\frac{3}{16}\sqrt[3]{(1 + 2x^2)^4};$ |
| g) $-\frac{5}{12}\sqrt[5]{(2 - 3x)^4};$ | h) $\frac{5^x}{\ln 5};$ | i) $\frac{1}{2}\ln(1 + x^2);$ |
| j) $-\ln \cos x ;$ | k) $\frac{1}{3}\arctg(x^3);$ | l) $\frac{1}{2}\ln 2tgx + 1 ;$ |
| m) $\frac{\ln^2 x}{2};$ | n) $\frac{2}{1 - \operatorname{sen} x};$ | o) $\operatorname{sen}(\ln x);$ |
| p) $-\arctg(\cos x);$ | q) $-e^{\frac{1}{x}};$ | r) $\frac{1}{2}(tg x + x);$ |
| s) $\frac{1}{2}\operatorname{sen}(e^{2x});$ | t) $\frac{1}{\sqrt{5}}\arctg\left(\frac{x}{\sqrt{5}}\right);$ | u) $\frac{1}{3}\arctg^2\left(\frac{x}{3}\right);$ |
| v) $\arctg(e^x);$ | w) $\frac{1}{2}\operatorname{arc sen}(x^2);$ | x) $\ln\left tg\left(\frac{x}{2}\right)\right ;$ |
| y) $-x + tg x;$ | z) $\frac{1}{4}\arg \operatorname{senh}(x^4)$ | |

6.3.

- | | |
|--|--|
| a) $-x \cos x + \operatorname{sen} x, \text{ em } \mathbb{R};$ | b) $\frac{e^x}{2}(\cos x + \operatorname{sen} x), \text{ em } \mathbb{R};$ |
| c) $e^{x+2}(x - 1), \text{ em } \mathbb{R};$ | d) $e^x(x^2 - 2x + 2), \text{ em } \mathbb{R};$ |
| e) $x[\ln(2x) - 1] \text{ em } \mathbb{R}^+;$ | f) $\frac{\operatorname{sen} x \cos x + x}{2}, \text{ em } \mathbb{R};$ |
| g) $\frac{1}{2}\left[\operatorname{sen}(4x - 1) + \frac{1}{4}\cos(4x - 1)\right], \text{ em } \mathbb{R};$ | h) $x \arctg(2x) - \frac{1}{4}\ln(1 + 4x^2), \text{ em } \mathbb{R};$ |
| i) $x \operatorname{arc sen} x + \sqrt{1 - x^2}, \text{ em } [-1, 1];$ | j) $\frac{x^2}{2}(\ln x^2 - 1) \text{ em } \mathbb{R} \setminus \{0\};$ |

- $k) \quad \frac{1}{2} \left(x \operatorname{sen}^2 x + \frac{\cos x \operatorname{sen} x}{2} - \frac{x}{2} \right), \text{ em } \mathbb{R};$
 $l) \quad \frac{1}{4} \left(\frac{3}{2} x - \frac{3}{2} \operatorname{sen} x \cos x - \operatorname{sen}^3 x \cos x \right), \text{ em } \mathbb{R};$
- $m) \quad x (\ln^2 x - 2 \ln x + 2) \text{ em } \mathbb{R}^+;$
 $n) \quad \frac{2}{3} \sqrt{2+x^3} - \frac{4}{9} \sqrt{(2+x^3)^3}, \text{ em }]-\sqrt[3]{2}, +\infty[;$
- $o) \quad \frac{x}{2} [\operatorname{sen}(\ln x) - \cos(\ln x)], \text{ em } \mathbb{R}^+.$
 $p) \quad \frac{1}{2} (x^2 \operatorname{arctg} x - x + \operatorname{arctg} x), \text{ em } \mathbb{R};$
- $q) \quad x \arccos x - \sqrt{1-x^2}, \text{ em } [-1, 1];$

6.4.

- $a) \quad -2 \cos(\sqrt{x}) + c, \quad c \in \mathbb{R};$
 $b) \quad \frac{2}{9} \left(\frac{\sqrt{(1+3x)^5}}{5} - \frac{\sqrt{(1+3x)^3}}{3} \right) + c \quad c \in \mathbb{R};$
- $c) \quad \frac{\operatorname{arc} \operatorname{sen} x}{2} - \frac{x}{2} \sqrt{1-x^2} + c, \quad c \in \mathbb{R};$
 $d) \quad 2 \operatorname{arctg}(\sqrt{e^x - 1}) + c, \quad c \in \mathbb{R};$
- $e) \quad -\frac{1}{\sqrt{1+x^2}} + c, \quad c \in \mathbb{R};$
 $f) \quad \frac{\sqrt{3}}{3} \operatorname{arcsec} \frac{\sqrt{3}x}{3} + c, \quad c \in \mathbb{R};$
- $g) \quad \ln(2x) - \ln 2 \ln |\ln(4x)| + c, \quad c \in \mathbb{R};$
 $h) \quad \frac{2}{3} \sqrt{(1+\ln x)^3} - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R};$
- $i) \quad \frac{4}{3} \sqrt{(1+\sqrt{x})^3} + c, \quad c \in \mathbb{R}.$

6.5.

- $a) \quad f(x) = x^4 + \frac{x^3}{3} - 3x^2 + x + 1;$
- $b) \quad f(x) = \frac{1}{2} \ln(1+x^2) + 2;$
- $c) \quad f(x) = \operatorname{arc} \operatorname{tg} x - \frac{1}{2} \ln(1+x^2) + 2x - 1.$

6.6. $3m/s^2.$

6.7.a) $8m/s.$

b) $18m.$

6.8.

- $a) \quad \ln|x+1| + c, \quad c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-1\};$
- $b) \quad \frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + c, \quad c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-1\};$

- c) $x + \frac{1}{2} \ln |x - 1| - \frac{1}{2} \ln |x + 1| + c, c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-1, 1\};$
- d) $2 \ln |x - 1| - \ln |x| - \ln |x + 1| + c, c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-1, 0, 1\};$
- e) $2 \ln \left| \frac{x+2}{x+1} \right| - \frac{4}{x+2} + c, c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-2, -1\};$
- f) $\frac{1}{2} \ln |x - 1| - \frac{1}{4} \ln (x^2 + 1) - \frac{1}{2} \operatorname{arc\,tg} x + c, c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{1\};$
- g) $\frac{x^2}{2} - 2x + \frac{16}{3} \ln |x + 2| + \frac{1}{6} \ln |x - 1| - \frac{1}{2} \ln |x + 1| + c, c \in \mathbb{R}, \text{ em } \mathbb{R} \setminus \{-2, -1, 1\};$
- h) $\frac{1}{2} \ln (x^2 + 2x + 3) - \frac{1}{\sqrt{2}} \operatorname{arc\,tg} \left(\frac{x+1}{\sqrt{2}} \right) + c, c \in \mathbb{R}, \text{ em } \mathbb{R};$
- i) $2 \operatorname{arc\,tg} x + \frac{2\sqrt{3}}{3} \operatorname{arc\,tg} \left(\frac{2}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \right) + c, c \in \mathbb{R}, \text{ em } \mathbb{R}.$

6.9.

- a) $\ln |3\sqrt[3]{x} + 1| + c, c \in \mathbb{R};$
- b) $-\frac{1}{2} \frac{1}{\sqrt{x^4 - 1}} + c, c \in \mathbb{R};$
- c) $-\frac{\sqrt{4 - x^2}}{x} - \operatorname{arc\,sen} \left(\frac{x}{2} \right) + c, c \in \mathbb{R};$
- d) $-\ln |1 + \cos x| + c, c \in \mathbb{R};$
- e) $\ln |\operatorname{sen} x + \cos x| + c, c \in \mathbb{R};$
- f) $\frac{\sqrt{3}}{3} \ln \left| \frac{3e^x - \sqrt{3}}{3e^x + \sqrt{3}} \right| + c, c \in \mathbb{R};$
- g) $\ln \left| \frac{\ln x - 1}{\ln x} \right| - \frac{1}{\ln x - 1} + c, c \in \mathbb{R};$
- h) $\ln \left| \frac{e^x - 1}{e^x} \right| + c, c \in \mathbb{R};$
- i) $\frac{6}{7} \sqrt[6]{(x-1)^7} + \frac{6}{5} \sqrt[6]{(x-1)^5} + 2\sqrt{x-1} + 6\sqrt[6]{x-1} + 3 \ln \left| \frac{\sqrt[6]{x-1} - 1}{\sqrt[6]{x-1} + 1} \right| + c;$

$$k) \quad \frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + c, \quad c \in \mathbb{R};$$

$$l) \quad \ln \left| \frac{tg\left(\frac{x}{2}\right) - 1}{tg\left(\frac{x}{2}\right)} \right| + c, \quad c \in \mathbb{R};$$

$$m) \quad 2\sqrt{x} - 4\sqrt[4]{x} + 4 \ln |\sqrt[4]{x} + 1| + c, \quad c \in \mathbb{R};$$

$$n) \quad \frac{6}{7}\sqrt[6]{x^7} - \frac{6}{5}\sqrt[6]{x^5} - \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} - 6\sqrt[6]{x} + 6\operatorname{arctg}(\sqrt[6]{x}) - 3 \ln |\sqrt[3]{x} + 1| + c, \quad c \in \mathbb{R};$$

$$o) \quad \frac{\operatorname{arctg}^5 x}{5} + c, \quad c \in \mathbb{R};$$

$$p) \quad -\frac{2}{e^x} + \frac{1}{2} \ln(e^{2x} + 3) + \frac{\sqrt{3}}{3} \operatorname{arctg}\left(\frac{e^x}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R};$$

$$q) \quad \frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} + c, \quad c \in \mathbb{R};$$

$$r) \quad 2\sqrt{x} - x - \ln |1 + 2\sqrt{x}| + c, \quad c \in \mathbb{R};$$

$$s) \quad x + c, \quad c \in \mathbb{R};$$

$$t) \quad \ln |\ln x| + c, \quad c \in \mathbb{R};$$

$$u) \quad \ln |x| \ln(\ln x) + c, \quad c \in \mathbb{R}.$$

6.10.

$$a) \quad f(x) = x^3 + \ln x + 1, \quad em \quad (0, +\infty);$$

$$b) \quad f(x) = \frac{x^4}{4} + x \ln x - x - e^3 x + e^3 + \frac{11}{4}, \quad em \quad (0, +\infty).$$

$$\mathbf{6.11.} \quad g(x) = \ln \left(1 + \frac{1}{e^x} \right) + \frac{\pi}{2};$$