Circuit Theory and Electronics Fundamentals

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April 5th, 2021

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1 Introduction

The objective of this laboratory assignment is to study an RC circuit containing a sinusoidal voltage source $v_s(t)$, a dependent voltage source V_d and a dependent current source I_b, as well as a capacitor C connected to 7 resistors. The circuit can be seen in Figure 1.

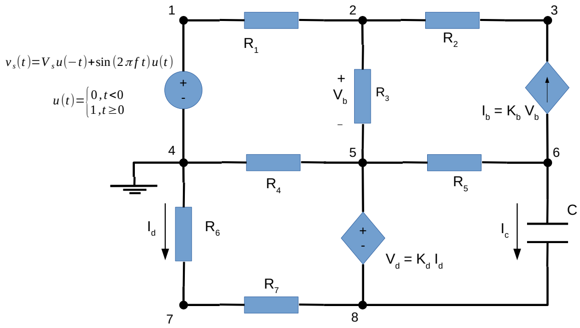

Figure 1: RC Circuit for Lab T2.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, mainly with Kirchoff's Current Law, initially for t<0, then the natural solution, forced and natural solution, and frequency response for t>0.

The circuit consists of 8 nodes. These were numbered from 1 to 8 as illustrated in Figure 1. Node 4 is the reference node, because it's connected to ground. The circuit contains 7 resistors, 1 capacitor, 1 independent source (V_s for voltage source) and 2 dependent sources (V_d for current controlled voltage source and I_b for voltage controlled current source).

2.1 RC circuit (t<0)

For t<0, vs(t)=Vs.

´ Apllying Kirchoff's Current Law (KCL) to nodes 2, 3, 4, 6 and 7 we get four equations:

$$(V_2 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0. \tag{1}$$

$$K_b(V_2 - V_5) + (V_2 - V_3)G_2 = 0. \tag{2}$$

$$V_4 = 0. \tag{3}$$

$$-I_c + (V_5 - V_6)G_5 - K_b(V_2 - V_5) = 0. \tag{4}$$

$$(V_4 - V_7)G_6 + (V_8 - V_7)G_7 = 0. \tag{5}$$

In addition, because voltage source V_s is connected between the reference node and a non reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

$$V_1 = V_s. \tag{6}$$

As the voltage source V_d is between two non reference nodes then it forms a supernode whose analysis is done as following:

$$(V_4 - V_5)G_4 + (V_2 - V_5)G_3 + (V_6 - V_5)G_5 + I_c + (V_7 - V_8)G_7 = 0. \tag{7}$$

$$(V_5 - V_8) = K_c(V_4 - V_7)G_6. \tag{8}$$

Recognizing a steady state in our analysis of this RC circuit, we can also see that:

$$I_c = 0. \tag{9}$$

The solution is obtained by solving Equations (6), (1), (2), (3), (7), (8), (4) and (5), which are shown in matrix form in the Appendix. The results are illustrated in Table 1.

Name	Value[A or V]
V_1	5.1057286642899999
V_2	4.8572771697009598
V_3	4.3546568389610742
V_4	0
V_5	4.8911888226937226
V_6	5.6686093715410824
V_7	-1.9301706667705107
V_8	-2.9000591228620491
$@I_R1$	-0.00023735363691793315
$@I_R2$	-0.00024812091856431809
$@I_R3$	-1.0767281646384539e-05
$@I_R4$	0.0011708849141782512
$@I_R5$	-0.00024812091856431766
$@I_R6$	0.00093353127726031847
$@I_R7$	0.00093353127726031847
$@I_b$	-0.00024812091856431793
$@I_c$	0
$@I_d$	0.00093353127726031847

Table 1: Nodal voltages and branch currents for t<0. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Determining R_eq

To determine the Natural Solution of V_6(t) for t>0, we must obtain the value of the equivalent resistance as seen from the capacitor. This is due to the time constant being R_eqC, thus explaining the need for this step.

To obtain this value, we "turn off" the independent sources in the circuit, and replace the capacitor with a voltage source V_x, whose value is V_6-V_8, V_6 and V_8 being the voltages obtained for t<0. Recognizing this continuity in the circuit, we use these as "inital conditions" to find the solution of R_eq.

First we replace Vs with a short circuit (Vs=0V) and replace equation (4) with the new equation for node 6:

$$V_6 - V_8 = V_x. \quad (10)$$

Then we replace equation (7) with a new supernode:

$$(V_3 - V_4)G_4 + (V_6 - V_7)G_7 + (V_2 - V_5)G_3 - I_b = 0. \quad (11)$$

R_eq will be the result of V_x/I_x, I_x=-I_c.

The solution is obtained by solving Equations (6), (1), (2), (3), (11), (8), (10) and (5), and is illustrated in Table 2.

Name	**Value[A or V or Ohm]**
V_1	0
V_2	-0
V_3	-0
V_4	0
V_5	-0
V_6	8.5686684944031306
V_7	-0
V_8	0
$@I_R1$	-0
$@I_R2$	0
$@I_R3$	0
$@I_R4$	-0
$@I_R5$	-0.0027347693611348952
$@I_R6$	0
$@I_R7$	0
$@I_b$	-0
$@I_c$	0
$@I_d$	0
$*R_eq$	0

Table 2: Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere and one preceded by * is of type *resistance* and is expressed in Ohm; other variables are of type *voltage* and expressed in Volt.

2.3 Determining V_6(t)

Now that we have R_eq, we can plot the natural solution as:

$$V_6(t) = Vx * exp(-t/R_eq * C). \quad (12)$$

This result is shown in figure 2:

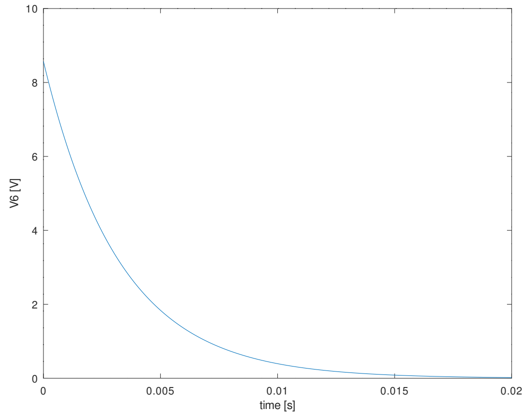

Figure 2: Natural Solution of $V_6(t)$ for t>=0.

2.4 Determining the Forced Solution for V_6(t)

To obtain the phasor voltage of V_6 (PV_6), we replace V_s with its complex amplitude 1, and replace the capacitor with its Impedance Z_c=1/(jwC).

First we obtain the equation for I_c:

$$I_c = (V_6 - V_8)/Z_c. \tag{13}$$

For the supernode, we use the following equation:

$$(V_1 - V_2)G_1 + (V_4 - V_5)G_4 + I_d = 0. \tag{14}$$

The solution is obtained by solving Equations (6), (1), (2), (3), (14), (8), (4) and (5), and is illustrated in Table 3.

Name	**Value[V]**
PV_1	1
PV_2	(0.95133868034806945,-1.759740368845301e-19)
PV_3	(0.85289625150235715,-7.0393369900758691e-18)
PV_4	0
PV_5	(0.9579805634606493,2.8709513998181016e-19)
PV_6	(-0.56397874882241394,-0.082062127109815114)
PV_7	(-0.37804019635244734,1.0133200843832287e-18)
PV_8	(-0.56800102660083862,1.1165776815218275e-18)

Table 3: Phasor voltages. Variables are of type *voltage* and expressed in Volt.

2.5 Determining the total solution

Now we finally have the solution for V_6(t) (t>0), with the following expression:

$$V_6(t) = Vx * exp(-t/R_eq * C) + PV_6 * sin(wt). \tag{15}$$

Because in subsection 2.1 we calculated the values for t<0, we can plot the final solution, as shown in figure 3:

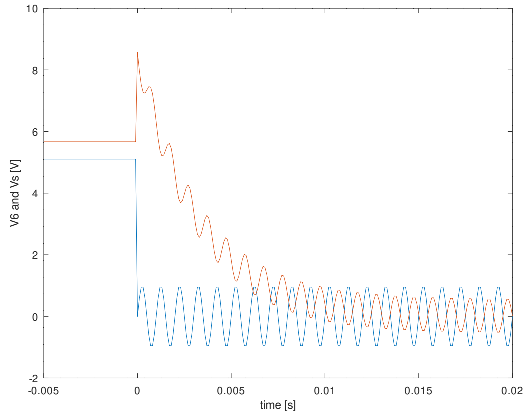

Figure 3: Final solution of $V_6(t)$ (orange) and $V_s(t)$ (blue).

2.6 Frequency Response

To analize the frequency response, we used the same system of equations as found in subsection (2.4), calculating PV_6,PV_s and PV_6-PV_8 (PV_c) for different frequencies. For each result of these complex vectors, the absolute value and the angle was saved.

The absolute values are shown in figure 4, in dB, representing the magnitude response, with the frequencies in a logarithmic scale.

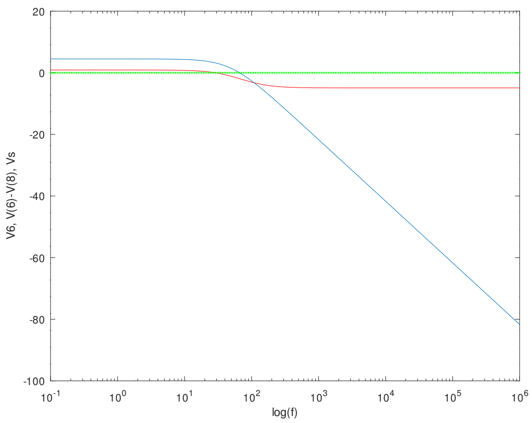

Figure 4: Magnitude response of $V_6(t)$ (red), $V_c(t)$ (blue) and $V_s(t)$ (green)

The magnitude of $V_s(t)$ doesn't change with the frequency of the signal and therefore it always´1 (0 in dB). However, the magnitude of V_6 changes with the frequency, firstly reducing and then staying constant. The magnitude of $V_c(t)$, which is a low pass filter, changes accordingly to what is expected of an RC circuit, with the increase of the frequency reducing the module of impedance and magnitude (Z_c=1/jwC).

Because the phase of the Voltage Signal V_s is 0, the phase of each voltage, for each frequency, is just the angles saved, in degrees. The plot for these is also shown in figure 5, with the frequencies also in a logarithmic scale.

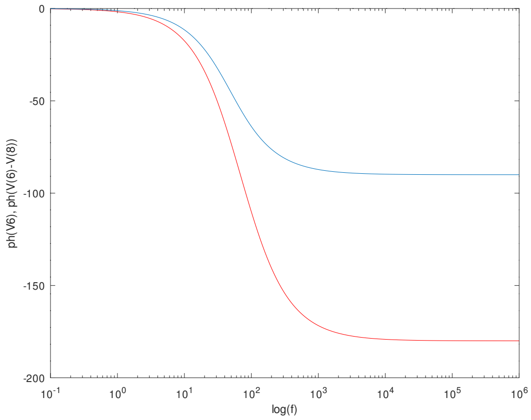

Figure 5: Phase response of $V_6(t)$ (red) and $V_c(t)$ (blue).

3 Simulation Analysis

3.1 Operating Point Analysis for t<0

Table 4 shows the simulated operating point results for the RC circuit for t<0.

Name	Value [V or A]
@c1[i]	0.000000e+00
@gib[i]	-2.48120e-04
@r1[i]	2.373536e-04
@r2[i]	2.481204e-04
@r3[i]	-1.07668e-05
@r4[i]	-1.17088e-03
@r5[i]	-2.48120e-04
@r6[i]	9.335311e-04
@r7[i]	9.335311e-04
v(1)	5.105729e+00
v(2)	4.857278e+00
v(3)	4.354658e+00
v(5)	4.891188e+00
v(6)	5.668607e+00
v(7)	-1.93017e+00
v(8)	-2.90006e+00
v(9)	0.000000e+00

Table 4: Operating point for RC circuit for t<0. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

The relative error between the theoretical and simulation values is of the order of magnitude of 10 to the power of -5 %. The errors are insignificant due to the simplicity of the circuit.

3.2 Determining initial conditions

To be able to plot the voltages for t>0, we need to obtain the initial conditions of the capacitor nodes.

Table 5 shows the simulated operating point results for the same circuit with its independent source turned off and with the capacitor replaced with a voltage source V_x, whose value is V_6-V_8, V_6 and V_8 being the nodal voltages obtained in the previous section.

Name	**Value [V or A]**
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.734769e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(5)	0.000000e+00
v(6)	-8.56867e+00
v(7)	0.000000e+00
v(8)	0.000000e+00
v(9)	0.000000e+00

Table 5: Operating point results for the circuit described in this section. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

The relative error between the theoretical and simulation values is of the order of magnitude of 10 to the power of -5 %. The errors are insignificant due to the simplicity of the circuit.

3.3 Natural Solution

Figure 6 shows the simulated transient analysis results for natural solution of $V_6(t)$. The initial solutions of V_6 and V_8 were provided by the octave script to simplify the project, since the relative error is irrelevant.

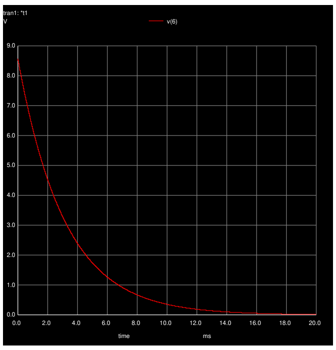

Figure 6: Natural Solution of $V_6(t)$ for t>=0.

Compared to the theoretical analysis results, one notices that the plots of figure 2 and figure 6 are very similar and any differences can be explained by aproximation errors.

3.4 Total solution

Figure 7 shows the simulated transient analysis results for $V_6(t)$.

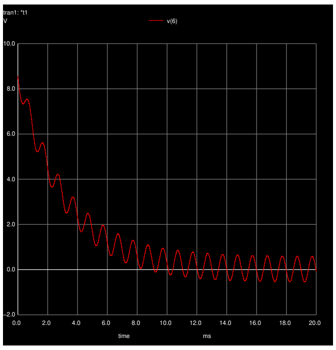

Figure 7: Total solution of $V_6(t)$.

Figure 8 shows the simulated transient analysis results for $V_s(t)$.

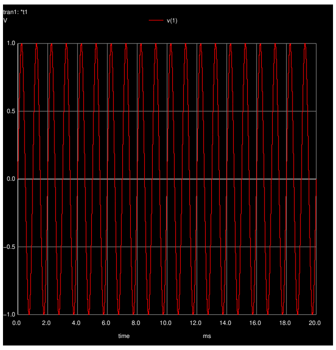

Figure 8: Total solution of $V_s(t)$.

Compared to the theoretical analysis results, one notices that, for t>0, the plots of figure 3 and figures 7 and 8 are very similar and any differences can be explained by aproximation errors.

3.5 Frequency Response

Figure 9 shows the magnitude response for $V_6(t)$ and $V_s(t)$.

Compared to the theoretical analysis results of figure 4, we can see that the simulated results in figure 9 are very similar and any differences can be explained by aproximation errors.

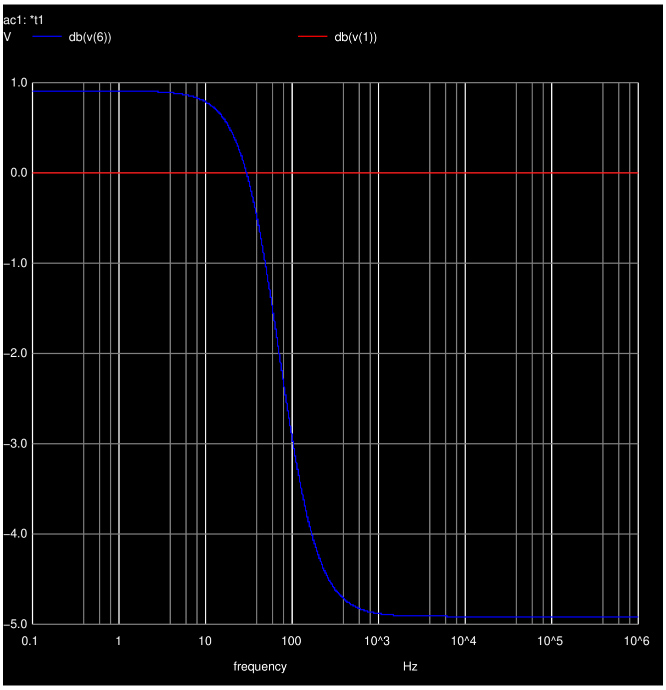

Figure 9: Magnitude Response of $V_6(t)$ (blue) and $V_s(t)$ (red).

The magnitude of $V_s(t)$ doesn't change with the frequency of the signal and therefore it always´1 (0 in dB). However, the magnitude of V_6 changes with the frequency, firstly reducing and then staying constant.

Figure 9 shows the phase response for $V_6(t)$ and $V_s(t)$.

Compared to the theoretical analysis results of figure 5, one notices that the simulated results in figure 10 are very similar and any differences can be explained by aproximation errors.

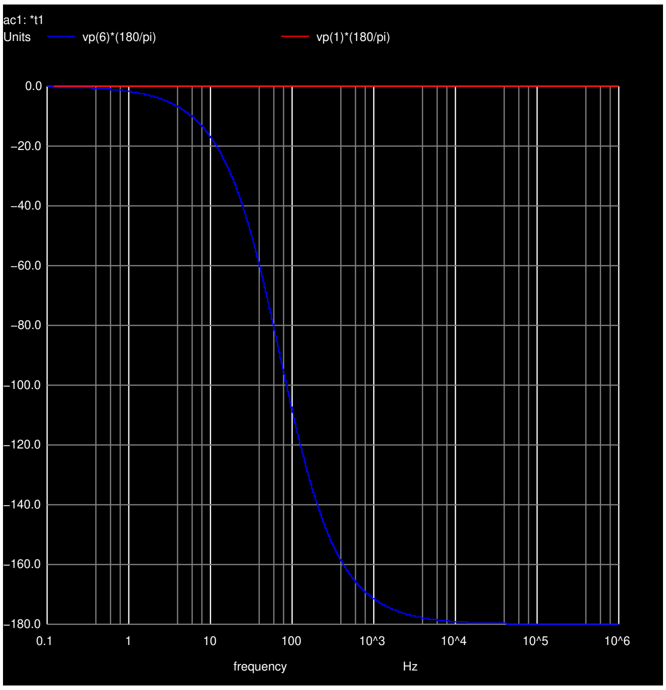

Figure 10: Phase Response of $V_6(t)$ (blue) and $V_s(t)$ (red).

4 Conclusion

We have successfully analysed theoretically the given circuit using the Octave maths tool to calculate the transient and frequency response. By comparing with the results obtained with the simulation done with the Ngspice tool, we could see they were mostly identical with relative errors of the ordem of magnitude 10 to the power of -5, which allowed us to confirm the validity of the methods and their precision in simple circuits like the one given.

5 Appendix

$$\begin{pmatrix} -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & 0 & -Kb & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -Kb & 0 & 0 & G5+Kb & -G5 & 0 & 0 \\ 0 & 0 & 0 & G6 & 0 & 0 & -G6-G7 & G7 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G3 & 0 & G4 & -G4-G3-G5 & G5 & G7 & -G7 \\ 0 & 0 & 0 & -KcG6 & 1 & 0 & KcG6 & -1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vs \\ 0 \\ 0 \end{pmatrix} \tag{18}$$