Circuit Theory and Electronics Fundamentals

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March 22, 2021

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing one independent and one dependent voltage sources, V_a and V_c respectively, as well as one independent and one dependent current sources, I_d and I_b, connected to 7 resistors. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

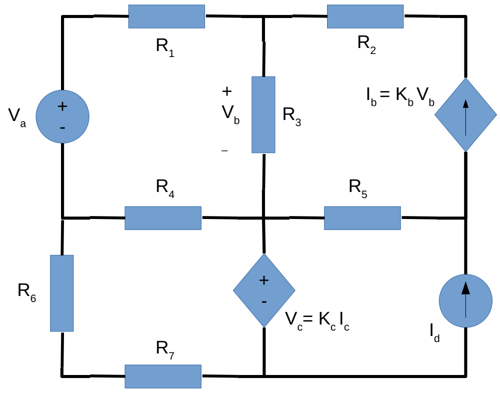

Figure 1: Circuit for Lab T1.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using the mesh and nodal methods.

2.1 Mesh Method

The circuit consists of 4 meshes, with 7 resistors, 2 independent sources (V_a for voltage source and I_d for current source) and 2 dependent sources (V_c for current controlled voltage source and I_b for voltage controlled current source).

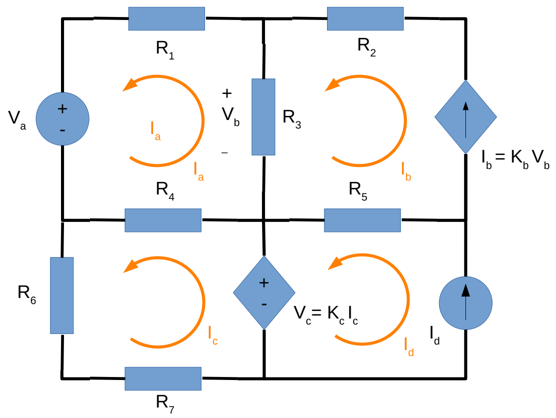

Figure 2: Mesh method analysis.

Apllying Kirchoff's Voltage Law (KVL) to the two meshes on the left (I_a is the current for the top one, I_c for the bottom one, I_b for the top right one, I_d for the bottom right, as ilustrated in Figure 2, we get two equations:

$$(R_4 + R_3 + R_1)I_a - R_3 I_b - R_4 I_c = 0. \tag{1}$$

$$-R_4 I_a + (R_6 + R_7 - K_c + R_4)I_c = 0. \tag{2}$$

The other two meshes have current sources, so we cannot apply KVL to them. Instead, we recognize the relation between the sources and the meshes' currents: I_d is equal to the current coming from the I_d source and the following equation:

$$I_b = K_b V_b. \tag{3}$$

Now, because

$$V_b = R_3(I_b - I_a). \tag{4}$$

we get:

$$I_b = K_b R_3(I_b - I_a). \tag{5}$$

The solution is obtained by solving Equations (1), (2) and (5) and is illustrated in Table 1.

Name	Value[A or V]
$@I_a$	-0.00023372996529640106
$@I_b$	-0.00024517951245893801
$@I_c$	0.00098327653344681271
I_d	0.00102526488404
V_b	-0.03496414637530082
V_c	7.931247031167528

Table 1: Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Nodal Method

The circuit consists of 8 nodes. These were numbered from 0 to 7 as illustrated in the image below. The node 0 was chosen as the reference node.

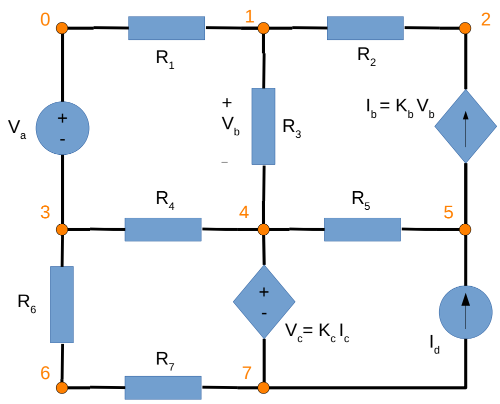

Figure 3: Node method analysis.

Apllying Kirchoff's Current Law (KCL) to nodes 1, 2, 5 and 6 we get four equations:

$$-V_1G_1 - (V_2 - V_1)G_2 + (V_4 - V_1)G_3 = 0. \tag{6}$$

$$K_b(V_1 - V_4) + (V_1 - V_2)G_2 = 0. \tag{7}$$

$$I_d + (V_4 - V_5)G_5 - K_b(V_1 - V_4) = 0. \tag{8}$$

$$(V_3 - V_6)G_6 + (V_7 - V_6)G_7 = 0. \tag{9}$$

In addition, because voltage source V_a is connected between the reference node and a non reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

$$V_3 = -V_a. \tag{10}$$

As the voltage source V_c is between two non reference nodes then it forms a supernode whose analysis is done as following:

$$(V_3 - V_4)G_4 + (V_1 - V_4)G_3 + (V_5 - V_4)G_5 - I_d + (V_6 - V_7)G_7 = 0. \tag{11}$$

$$(V_4 - V_7) = K_c(V_3 - V_6)G_6. \tag{12}$$

The solution is obtained by solving Equations (6), (7), (8), (9), (10), (11) and (12) and is illustrated in Table 2.

Name	**Value[A or V]**
V_0	0
V_1	-0.23790844246659815
V_2	-0.73947695823628645
V_3	-5.1277592116299999
V_4	-0.20294429609129594
V_5	3.6935601924823294
V_6	-7.1316871834296993
V_7	-8.1341913272588204
V_b	-0.034964146375302207
V_c	7.9312470311675245
$@I_b$	-0.00024517951245894094

Table 2: Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results for the circuit under analysis. The results are identical to the theoretical values for the node voltages and mesh currents (excluding aproximation differencies).

Name	Value [V]
@gib[i]	-2.45180e-04
@id[current]	1.025265e-03
@r1[i]	2.337300e-04
@r2[i]	2.451795e-04
@r3[i]	-1.14495e-05
@r4[i]	-1.21701e-03
@r5[i]	-1.27044e-03
@r6[i]	9.832765e-04
@r7[i]	9.832765e-04
v(1)	-2.37908e-01
v(2)	-7.39477e-01
v(3)	-5.12776e+00
v(4)	-2.02944e-01
v(5)	3.693560e+00
v(6)	-7.13169e+00
v(7)	-8.13419e+00
v(8)	-5.12776e+00
v(1,4)	-3.49641e-02
v(4,7)	7.931247e+00

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

We have successfully analysed theoretically the given circuit, through the nodal method and the mesh method, using the Octave maths tool. By comparing with the results obtained with the simulation done with the Ngspice tool, we could see they were identical, which allowed us to confirm the validity of the methods and their perfect precision in simple circuits like the one given.