Damping of a beam subjected to free and forced vibration.

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Nomenclature

c	damping coefficient
F	force
k	spring stiffness
m, M	mass
r	ω/ω_n.
R	Radius
x, X	amplitude

Greek Symbols

δ	logarithmic decrement
ζ	damping ratio
σ	standard deviation
τ_d	1 period of a vibration (s)
ω	frequency, angular velocity

Subscripts

c	critical
d	damped
n	natural frequency under free conditions
r	resonant

Introduction

A single degree of freedom (SDOF) system can often be used to approximate more complex structures if the resonances are well separated. If the system is undamped its preferred vibration frequency is the natural frequency.
Systems can be examined in both free and forced vibration situations.

Free vibration response

It is usually possible to obtain the damping of a SDOF system from the free vibration response. Shown in Figure 1 is a typical free vibrational response of a damped system

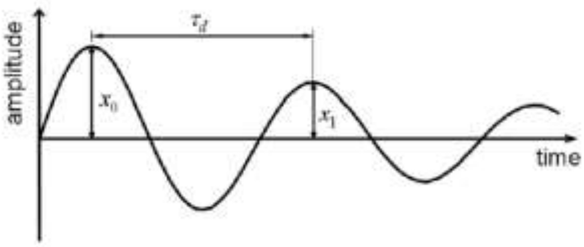

Figure 1: Free decay of vibration of a SDOF system

The damping is found by comparing the magnitude of successive oscillations following an initial impulse excitation. The logarithmic decrement is defined as,

$$\delta = log_e\left(\frac{x_i}{x_{i+1}}\right) \tag{1}$$

where x_i and x_{i+1} are any two consecutive amplitudes of the response separated one period (τ_d) as shown in Figure 1 . For harmonic motion δ can be obtained from displacement, velocity or acceleration measurements as the ratio between successive cycles is the same.

The damping coefficient, ζ, is related to the logarithmic decrement. When $\zeta < 0.05$, this can be approximated by

$$\zeta = \frac{\delta}{2\pi} \tag{2}$$

The natural frequency of the *undamped* system can be obtained using,

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}. \tag{3}$$

Forced vibration resulting from a rotating eccentric mass

Shown Figure 2 is a SDOF system with mass *m* supported by a spring of stiffness *k*. Between the mass and ground there is also a viscous damper with coefficient *c*. The system is excited by the action of a point mass *M* which rotates at a distance *R* about an axis with an angular velocity of ω rad/s.

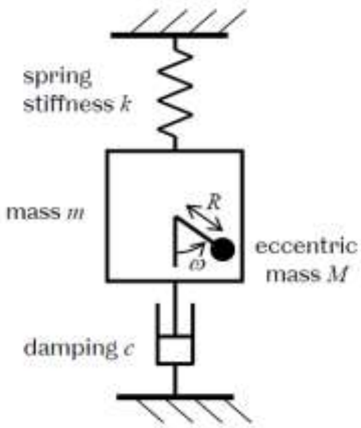

Figure 2: Forced system

The vertical force produced on mass *m* by the rotation of mass *M* at any instant during the vibration is

$$F = MR\omega^2 cos(\omega t). \tag{4}$$

The amplitude *X* of the steady state response is given by,

$$X = \frac{MR\omega^2}{k - \omega^2 m + j\omega c} \tag{5}$$

Equation 5 can be rewritten in terms of the undamped natural frequency ω_n, the damping ratio ζ and the critical damping c_c where: $\omega_n = \sqrt{k/m}$, $\zeta = c/c_c$, $c_c = 2\sqrt{km} = 2m\omega_n$.

$$X = \left(\frac{MR}{m}\right)\frac{r^2}{1 - r^2 + j2\zeta r} \tag{6}$$

where $r = \omega/\omega_n$. Damping affects the response near the natural frequency as illustrated in Figure 3.

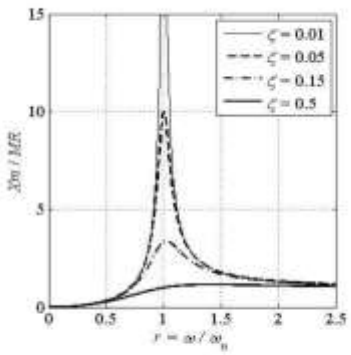

Figure 3: Frequency response of SDOF system excited by rotating unbalance

The response is maximised at resonance when,

$$\omega_r = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}} \tag{7}$$

where ω_r is the resonance frequency. Damping can be estimated from the frequency response using the Half Power method. This approach estimates the damping ratio ζ with

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_r} \tag{8}$$

where ω_1 and ω_2 are points on the response curve at either side of the resonance at which the amplitude is 0.707 times the peak response. Theoretical studies have shown that for ζ < 0.1, Equation 8 is accurate to 5%.

In this lab the system of interest was a horizontal steel beam supported by bearings at each end. The forced vibration was created by a motor rotating an unbalanced disk located on the centre of the beam. Damping was created using a viscous dashpot damper which can be attached to the beam. Experiments were performed under both free and forced vibration with and without damping.

The objectives were:

- To measure the free vibration response of the system following an impulse.
- To measure the steady-state frequency response of the system when excited by rotating unbalance.
- To compare estimates of natural frequency and damping obtained from free and forced responses.
- Determine the damper damping coefficient.

Experimental

The SDOF system used in the experiment consisted of a horizontal steel beam supported by bearings at each end as shown in Figure 4. The stiffness of the beam in the vertical direction was k = 2930 ± 200 N/m. The centre of the beam had a loading which consisted of the mass of the motor, disk assembly, sensors and cables. The vibrating mass for each rig was nominally 0.689 ± 0.026 kg. The beam centre was also the attachment point of a viscous dashpot damper.

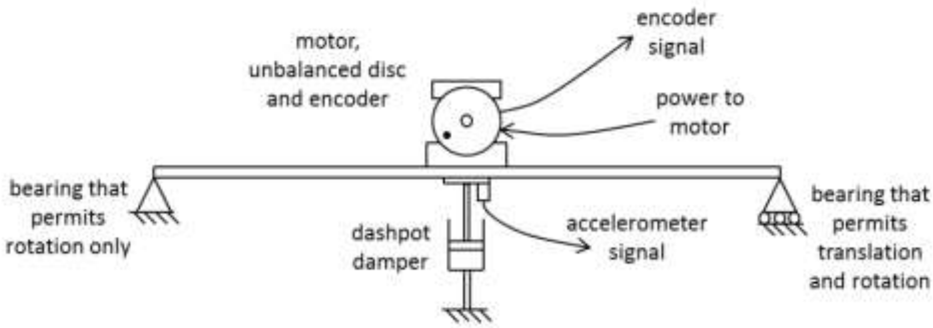

Figure 4: Beam apparatus used in the experiment

The force excitation to the system was provided by an eccentrically positioned mass on the disk which was rotated using a variable speed motor. The motor speed was measured using an encoder. A piezoelectric accelerometer which measured the acceleration at the midpoint of the beam was fitted to top of the motor assembly.

The beam vertical acceleration and motor rotational speed signals are acquired using a data acquisition card and were displayed using LabView™ software.

Measurements were performed under both free and forced vibration. Under free vibration the beam was set vibrating by pulling it and then releasing it. The DAC as activated and response captured. Using the displayed accelerometer response the log decrement δ and the period of oscillation, τ_d were recorded. Using this data the damping ratio ζ and undamped natural frequency ω_n were calculated. For forced vibration the motor speed was set and controlled using the computer software. The accelerometer response was displayed and maximum acceleration for each motor speed recorded. The maximum amplitude (resulting from resonance) was found and measurements taken around this to resolve the frequency response curve. The damping ratio was then calculated using the method described above (Equation 8). Experiments were repeated with the damper connected to the beam.

Results

Shown in Tables 1 to 4 are the measured results for vibration tests; undamped and damped also free and motored. The frequency response curve for both the motored tests is shown in Figure 5. Indicated on this figure are ω_1 and ω_2 the points on the response curve at either side of the resonance at which the amplitude was 0.707 times the peak response. These were used to determine the damping ratio using Equation 8.

The determined values of ζ, ω_n and c are given in Table 5 with associated errors and uncertainties also reported. The calculation of the errors are detailed in the Experimental Uncertainty.

Table 1. Measured data form the undamped free vibration tests

Test No.	**Peak No.**	**Time (ms)**	τ_d **(ms)**	ω_d **(Hz)**	**Acceleration (m/s²)**	δ	ζ	ω_n **(Hz)**
1	0	203			19.4			
	1	300	97.5	10.3	18.8	0.0313	0.00498	10.3
	2	398	97.5	10.3	18.6	0.0118	0.00187	10.3
	3	496	98.7	10.1	18.2	0.0214	0.00340	10.1
	4	594	97.5	10.3	17.7	0.0254	0.00404	10.3
	5	691	97.4	10.3	17.3	0.0269	0.00429	10.3
2	0	275			8.68			
	1	371	96.2	10.4	8.24	0.0518	0.00824	10.4
	2	470	98.7	10.1	8.10	0.0173	0.00275	10.1
	3	566	96.2	10.4	7.96	0.0177	0.00281	10.4
	4	666	100	10.0	7.68	0.0362	0.00576	10.0
	5	763	97.5	10.3	7.49	0.0249	0.00397	10.3
3	0	300			19.4			
	1	399	98.7	10.1	18.3	0.0568	0.00903	10.1
	2	498	98.8	10.1	18.0	0.0163	0.00259	10.1
	3	595	97.4	10.3	17.8	0.0140	0.00222	10.3
	4	692	97.5	10.3	17.2	0.0338	0.00538	10.3
	5	790	97.5	10.3	17.0	0.0129	0.00205	10.3

Table 2. Measured data form the undamped forced vibration tests

Speed (rpm)	Frequency (Hz)	Frequency (rad/s)	Acceleration (m/s^2)	Displacement (m)
560	9.3	58.6	2.2	0.000648
570	9.5	59.7	2.9	0.000811
580	9.7	60.7	3.9	0.001057
590	9.8	61.8	5.7	0.001496
600	10.0	62.8	9.7	0.002458
610	10.2	63.9	31.6	0.007754
620	10.3	64.9	30.7	0.007292
630	10.5	66.0	13.3	0.003053
640	10.7	67.0	8.5	0.001888
650	10.8	68.1	6.4	0.001379
660	11.0	69.1	5.1	0.001074
605	10.1	63.4	15.0	0.003739
615	10.3	64.4	56.7	0.013665
625	10.4	65.4	19.0	0.004438
612	10.2	64.1	52.5	0.012775
617	10.3	64.6	44.3	0.010611
622	10.4	65.1	25.3	0.005968
607	10.1	63.6	18.7	0.004631
614	10.2	64.3	62.0	0.015002
613	10.2	64.2	60.6	0.014694
611	10.2	64.0	43.9	0.010728
616	10.3	64.5	50.9	0.012240
618	10.3	64.7	39.4	0.009405

Table 3. Measured data form the damped free vibration tests

Test No.	Peak No.	Time (ms)	τ_d (ms)	ω_d (Hz)	Acceleration (m/s²)	δ	ζ	ω_n (Hz)
1	0	101			31.0			
	1	199	97.4	10.3	28.7	0.0748	0.0119	10.3
	2	298	98.8	10.1	26.5	0.0797	0.0127	10.1
	3	395	97.4	10.3	24.4	0.0840	0.0134	10.3
	4	492	97.5	10.3	22.6	0.0756	0.0120	10.3
	5	590	97.5	10.3	21.7	0.0426	0.0068	10.3
2	0	308			11.6			
	1	405	97.5	10.3	10.7	0.0819	0.0130	10.3
	2	503	97.4	10.3	9.7	0.0939	0.0150	10.3
	3	601	98.8	10.1	9.0	0.0755	0.0120	10.1
	4	699	97.4	10.3	8.9	0.0177	0.0028	10.3
	5	798	98.8	10.1	8.4	0.0546	0.0087	10.1
3	0	295			22.3			
	1	392	97.5	10.3	20.2	0.0996	0.0159	10.3
	2	491	98.7	10.1	19.1	0.0554	0.0088	10.1
	3	590	98.8	10.1	17.5	0.0910	0.0145	10.1
	4	687	97.4	10.3	16.6	0.0508	0.0081	10.3
	5	785	97.5	10.3	15.6	0.0636	0.0101	10.3

Table 4. Measured data form the damped forced vibration tests

Speed (rpm)	Frequency (Hz)	Frequency (rad/s)	Acceleration (m/s²)	Displacement (m)
560	9.3	58.6	2.3	0.00065
570	9.5	59.7	2.9	0.00081
580	9.7	60.7	4.0	0.00108
590	9.8	61.8	5.8	0.00153
600	10.0	62.8	10.1	0.00257
610	10.2	63.9	20.2	0.00494
620	10.3	64.9	20.3	0.00480
630	10.5	66.0	11.4	0.00261
640	10.7	67.0	7.7	0.00171
650	10.8	68.1	6.1	0.00131
660	11.0	69.1	4.9	0.00102
605	10.1	63.4	14.3	0.00357
615	10.3	64.4	24.2	0.00582
625	10.4	65.4	15.1	0.00352

612	10.2	64.1	22.6	0.00551
617	10.3	64.6	22.8	0.00546
622	10.4	65.1	17.7	0.00417
607	10.1	63.6	16.7	0.00412
614	10.2	64.3	24.1	0.00584

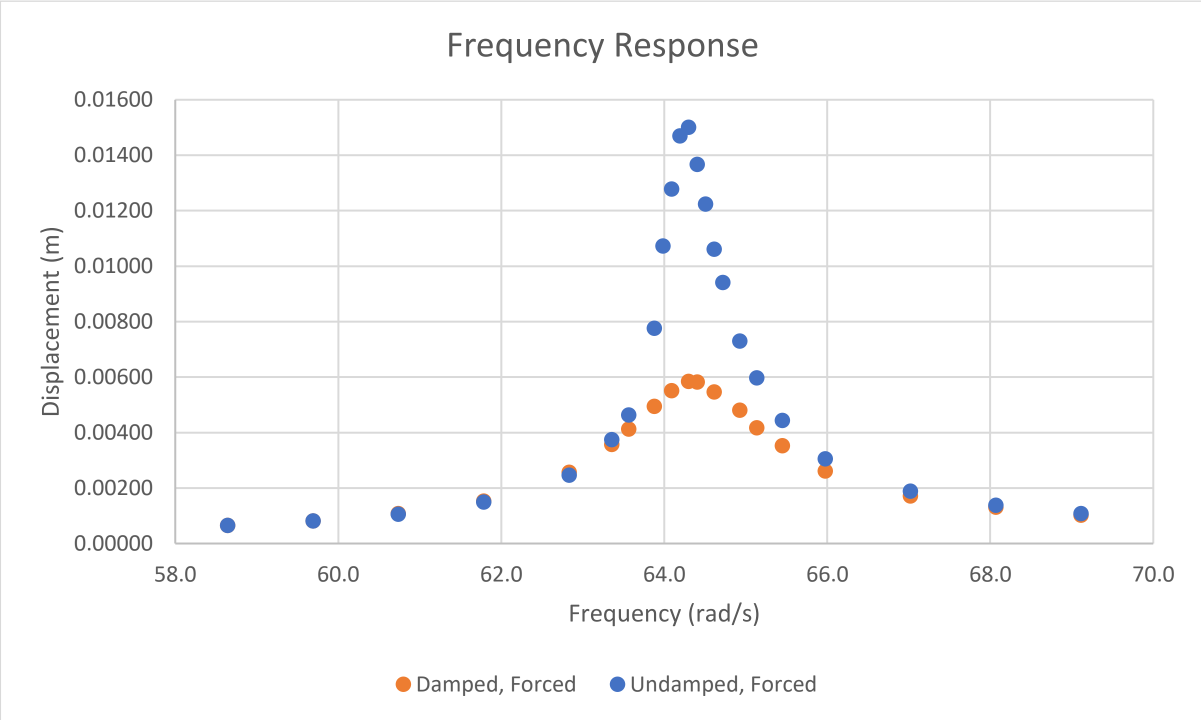

Figure 5. Measured displacement plotted against motor frequency for the motored vibration tests.

Table 5. Determined values of ζ, ω_n and c with associated errors and uncertainties.

Free vibration	ζ	*error*	ω_d (Hz)	ω_n (Hz)	*Error in* ω_d
Undamped	0.0042	±0.0044	10.23	10.23	±0.21
Damped	0.0110	±0.0070	10.22	10.22	±0.136
Forced vibration			ω_r (Hz)		*Error in* ω_r
Undamped	0.0136	±0.002	11.0		±0.3
Damped	0.0233	±0.04	11.0		±0.3
Damper damping coefficient					
	Ns/m				
c	0.377	±0.198	Based on Free Results		

Experimental Uncertainty

Free Vibration Tests Source of Error

Some damping caused by the bearings at the end due to the clamping arrangements, therefore, inducing further damping in the tests. Furthermore, systematic error in reading the peaks accurately and the difference in the force exerted for the vibrations is also a source of error. The uncertainty in the damping ratio is nearly as large as the damping ratio, due to the use of a conservative definition of uncertainty (uncertainty = $\pm 2\sigma$).

Description of 2 Sigma Results

The damping ratio was the average of the damping ratio of the 5 peaks over 3 experiments, to provide accurate results and dismiss the effects of any anomalies. The error is assumed to be 2 times the standard deviation as 95% of the results lie in this range, for a normally distributed result.

Method Used for the Forced Tests

The damping coefficient was calculated from the frequency response using the Half Power Method mentioned in equation 8. ω_1 and ω_2 points for both damped and undamped shown in Figure 6.

The uncertainty in the forced tests were calculated by first assuming that the uncertainty I the displacement measured was 10 percent of the RMS value of the peak displacement. Therefore, RMS +10% and RMS -10% lines were drawn on the graph for each damped and undamped graphs, and the uncertainty calculated by subtracting Δx_1 from Δx_2, as shown by Figure 6.

Compare the Magnitude of the Different Errors

The error associated with the damped forced vibrations is the largest as the peak is not very dense and flattens off very quickly, as shown by the orange points in Figure 6. Therefore, the difference between the frequency at peak displacement and the frequencies at the RMS value is much larger, hence inducing a larger uncertainty.

The free vibration tests percentage uncertainty is quite large due to damping associated with the bearings on each end of the beam.

Equation of Error Propagation for Damping Damper Coefficient

The camper damping coefficient was calculated using the following equation:

- $C_{da} = \zeta 2 m \omega_n$

The error associated with the damper damping ratio was calculated using the following equation:

- $\Delta C_{da} = C_{da}\sqrt{\left(\frac{\Delta\zeta_{da}}{\zeta_{da}}\right)^2 + \left(\frac{\Delta m}{m}\right)^2 + (\frac{\Delta\omega_n}{\omega_n})^2}$, where $\omega_n = \sqrt{\frac{m}{k}}$

Justifying Choice of Data for Damper Damping Coefficient

The free vibration results were used to calculate the damper damping coefficient as the impact of the error within the motor's vibration is not present.

Even though the percentage uncertainty of the damping ratio of the free vibration test was high, it was still lower than the error associated with the forced vibrations; therefore, the free vibration test results were used to calculate the damper damping coefficient.

Discussion

Frequency

The 5 frequencies were very similar, but not exactly equal; the damped frequency was slightly lower than the natural frequency due to the induced damping.

Damping Ratio

The largest damping ratio is observed by the damped forced vibration due to the opposing external force acting by the motor.

Reflecting on the Experiment & Sources of Error/Improvements

The undamped forced vibrations had the highest displacement as there was no vicious damping and a large vibration was being generated by the motor.

Comparatively, the undamped forced vibrations did not have as much displacement, which however induced a larger uncertainty due to a lower dense peak, as shown by Figure 6 below.

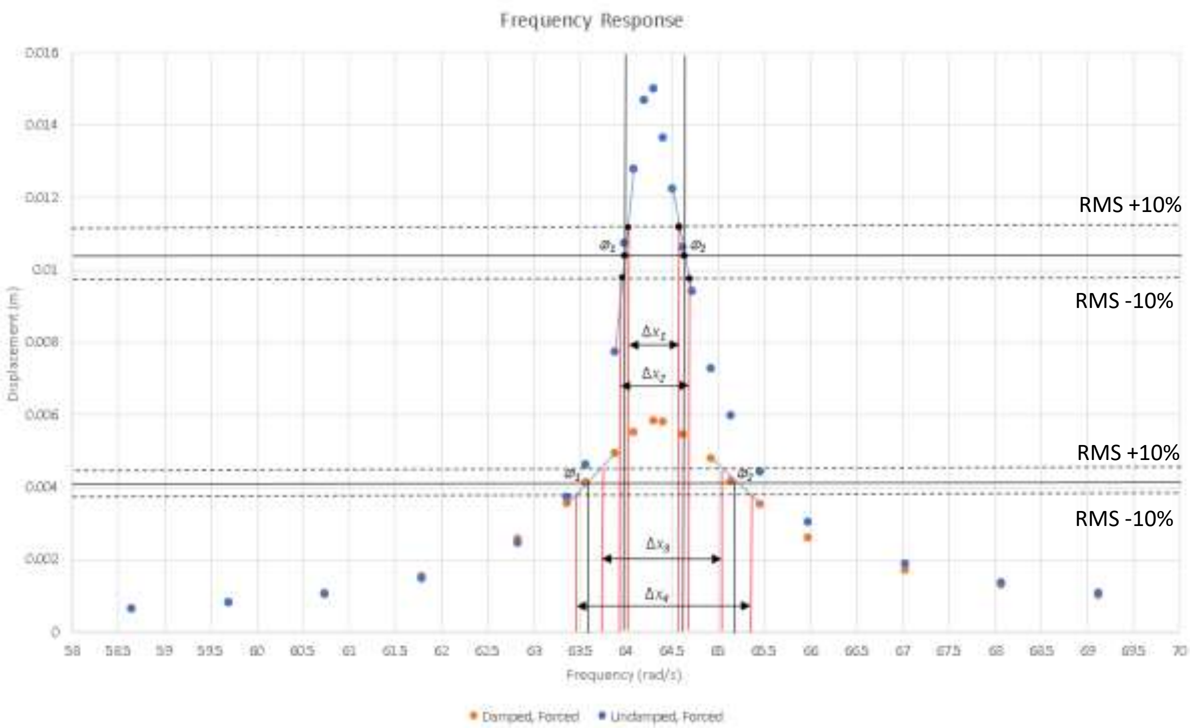

Figure 6 – Errors Associated with Forced Vibrations

The experiment could be improved by experimentally calculating the uncertainty in the displacement, instead of just assuming it to be ten percent of the RMS value of the peak displacement. Finally, further error could be reduced if the internal damping, caused by the bearings at each end, is calculated as it impacts the tests results by inducing a small degree of damping.