Ciencia dos DADOS - Sabrina e Pedro

$$\hat{y}_i = \beta_0 + \beta_i x_i \qquad y_i = \beta_0 + \beta_i X_i + \varepsilon_i$$

$$\varepsilon_i = y_i - \hat{y}_i \longrightarrow \sum_{i=1}^{n} \left[\varepsilon_i^2\right] = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \sum_{i=1}^{n} (y_i - \beta_0 - \beta_i x_i)^2$$

$\times(-1)$ Regra da cadeia em adendo no final

$$\begin{cases} \dfrac{\partial E}{\partial \beta_0} = 0 \\ \dfrac{\partial E}{\partial \beta_1} = 0 \end{cases}$$

$\partial \beta_0$

$$-2 \cdot \left[\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i)\right]$$

distribuir a somatória

$$\frac{\partial E}{\partial \beta_0} = -2 \left[\sum_{i=1}^{n} y_i - \sum^{n} \beta_0 - \beta_1 \sum_{i=1}^{n} X_i\right] \left(\text{divide por } n\right)$$

$$\frac{\partial E}{\partial \beta_0} = -2 \left[\underbrace{\frac{\sum y_i}{n}}_{\bar{y}} - \frac{\sum^{n} \beta_0}{n} - \beta_1 \cdot \underbrace{\frac{\sum x_i}{n}}_{\bar{X}}\right]$$

$$\frac{\partial E}{\partial \beta_0} = -2(\bar{y} - \beta_0 - \beta_1 \bar{x}) = 0$$

$$-\cancel{2}\bar{y} + \cancel{2}\beta_i \bar{x} = -\cancel{2}\beta_0$$

$$-\bar{y} + \beta_i \bar{x} = -\beta_0 \longrightarrow \beta_0 = -\beta_1 \bar{X} + \bar{y}$$

(Parte 1)

☆

$$\frac{\partial E}{\partial \beta_1} = (-2)\sum_{i=1}^{n} x_i(y_i - \beta_0 - \beta_1 x_i) = 0$$

$$= (-2)\left(\sum x_i y_i - \beta_0 \sum x_i - \beta_1 \sum x_i^2\right) = 0$$

Resolver o Sistema: Substitui o β_0 por *

$$\sum_{i=1}^{n} x_i y_i - \beta_1 \sum_{i=1}^{n} x_i^2 + (\beta_1 \bar{x} - \bar{y}) \cdot \sum_{i=1}^{n} x_i = 0$$

$$\beta_1 = \frac{\bar{y}\sum x_i - x_i y_i \sum}{-\sum_{i=1}^{n} x_i^2 + \bar{x} \cdot \sum x_i} = \frac{\overbrace{\sum x_i y_i - n\bar{y}\bar{x}}^{\sum (x_i - \bar{x})(y_i - \bar{y})}}{\underbrace{\sum x_i^2 - n\bar{x}^2}_{\sum (x_i - \bar{x})^2}}$$

Portanto: $\beta_0 = \bar{y} - \beta_1 \bar{x}$ e

$$\beta_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{COV(xy)}{VAR(x)}$$

$$E(\beta_0,\beta_1) = (\underbrace{y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1}_{w})^2$$

$$E(\beta_0, \beta_1) = w^2$$

$$\frac{\partial E(\beta_0,\beta_1)}{\partial \beta_0} = \frac{\partial (w)^2}{\partial w} \frac{\partial w}{\partial \beta_0}$$

$$\boxed{\frac{\partial E}{\partial \beta_0} (y_1 - \beta_0 - \beta_1 x_1) = -1}$$

$$f(x^2)' = 2x \qquad 2 \cdot -1 = -2$$