Pattern Recognition 2015
Support Vector Machines

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Support Vector Machines

Overview

1. Separable Case
2. Kernel Functions
3. Allowing Errors (Soft Margin)
4. SVM's in R.

Linear Classifier for two classes

Linear model

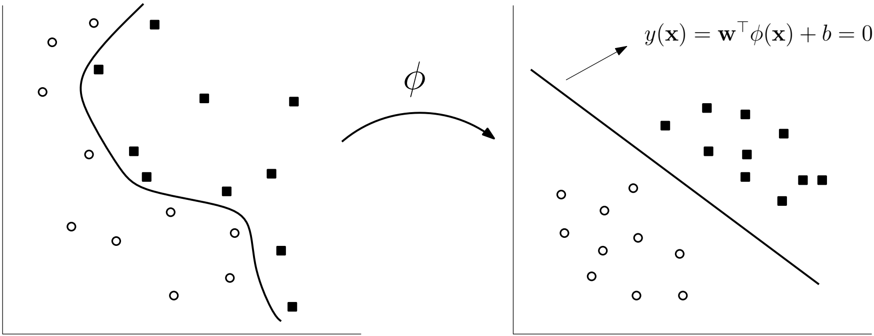
$$y(\mathbf{x}) = \mathbf{w}^\top \phi(\mathbf{x}) + b \tag{7.1}$$

with $t_n \in \{-1, +1\}$.

- Predict $t_0 = +1$ if $y(\mathbf{x}_0) \geq 0$ and $t_0 = -1$ otherwise.
- The decision boundary is given by $y(\mathbf{x}) = 0$.

This is a linear classifier in feature space $\phi(\mathbf{x})$.

Mapping



ϕ maps $\mathbf{x}$ into higher dimensional space where data is linearly separable.

Data linearly separable

Assume training data is linearly separable in feature space, so there is at least one choice of $\mathbf{w}, b$ such that:

1. $y(\mathbf{x}_n) > 0$ for $t_n = +1$;
2. $y(\mathbf{x}_n) < 0$ for $t_n = -1$;

that is, all training points are classified correctly.

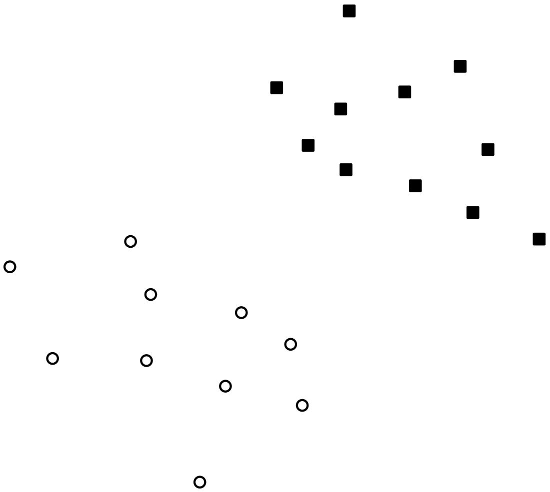
Putting 1. and 2. together:

$$t_n y(\mathbf{x}_n) > 0 \qquad n = 1, \ldots, N$$

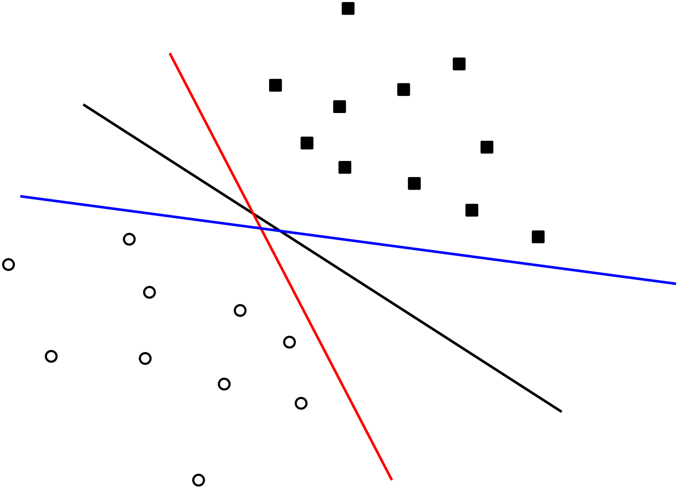
Maximum Margin

- There may be many solutions that separate the classes exactly.
- Which one gives smallest prediction error?
- SVM chooses line with maximal *margin*, where the margin is the distance between the line and the closest data point.
- In this way, it avoids "low confidence" classifications.

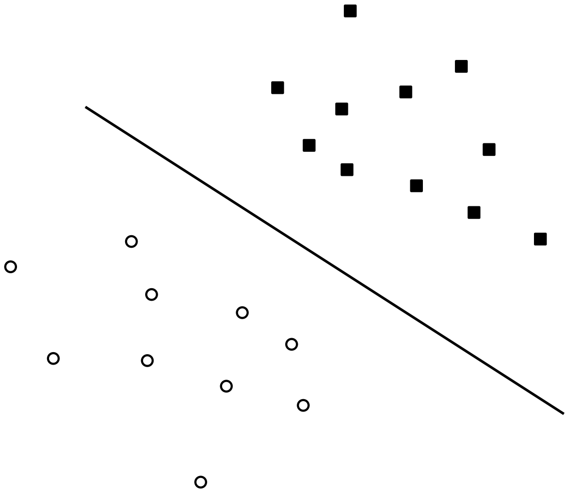
Two-class training data



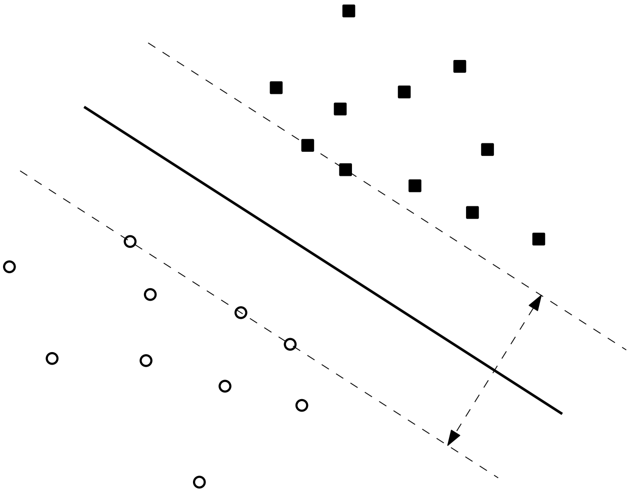
Many Linear Separators



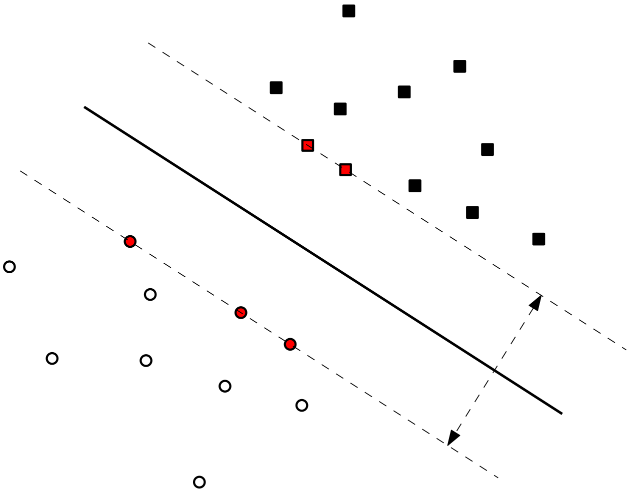
SVM Decision Boundary



Maximize Margin



Support Vectors



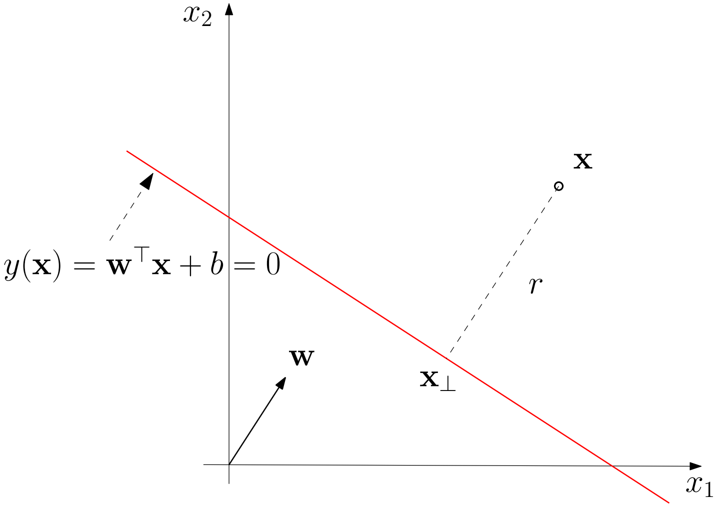
Weight vector is orthogonal to the decision boundary

Consider two points $\mathbf{x}_A$ and $\mathbf{x}_B$ both of which lie on the decision surface. Because $y(\mathbf{x}_A) = y(\mathbf{x}_B) = 0$, we have

$$(\mathbf{w}^\top \mathbf{x}_A + b) - (\mathbf{w}^\top \mathbf{x}_B + b) = \mathbf{w}^\top (\mathbf{x}_A - \mathbf{x}_B) = 0$$

and so the vector $\mathbf{w}$ is orthogonal to the decision surface.

Distance of a point to a line



Distance to decision surface ($\phi(\mathbf{x}) = \mathbf{x}$)

We have

$$\mathbf{x} = \mathbf{x}_\perp + r\frac{\mathbf{w}}{\|\mathbf{w}\|}. \tag{4.6}$$

where $\frac{\mathbf{w}}{\|\mathbf{w}\|}$ is the unit vector in the direction of $\mathbf{w}$, $\mathbf{x}_\perp$ is the orthogonal projection of $\mathbf{x}$ onto the line $y(\mathbf{x}) = 0$, and r is the (signed) distance of $\mathbf{x}$ to the line. Multiply (4.6) left and right by $\mathbf{w}^\top$ and add b:

$$\underbrace{\mathbf{w}^\top\mathbf{x} + b}_{y(\mathbf{x})} = \underbrace{\mathbf{w}^\top\mathbf{x}_\perp + b}_{0} + r\frac{\mathbf{w}^\top\mathbf{w}}{\|\mathbf{w}\|}$$

So we get

$$r = y(\mathbf{x})\frac{\|\mathbf{w}\|}{\|\mathbf{w}\|^2} = \frac{y(\mathbf{x})}{\|\mathbf{w}\|} \tag{4.7}$$

Distance of a point to a line

The signed distance of $\mathbf{x}_n$ to the decision boundary is

$$r = \frac{y(\mathbf{x}_n)}{\|\mathbf{w}\|}$$

For lines that separate the data perfectly, we have $t_n y(\mathbf{x}_n) = |y(\mathbf{x}_n)|$, so that the distance is given by

$$\frac{t_n y(\mathbf{x}_n)}{\|\mathbf{w}\|} = \frac{t_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b)}{\|\mathbf{w}\|} \tag{7.2}$$

Maximum margin solution

Solve

$$\arg\max_{\mathbf{w},b} \left\{ \frac{1}{\|\mathbf{w}\|} \min_n [t_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b)] \right\}. \tag{7.3}$$

Since $\frac{1}{\|\mathbf{w}\|}$ does not depend on n, it can be moved outside of the minimization.

- Direct solution of this problem would be rather complex.
- A more convenient representation is possible.

Canonical Representation

The hyperplane (decision boundary) is defined by

$$\mathbf{w}^\top \phi(\mathbf{x}) + b = 0$$

Then also

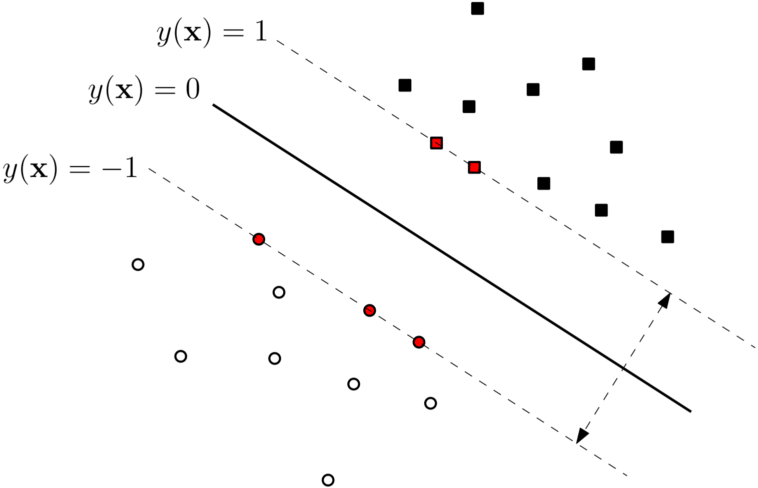
$$\kappa(\mathbf{w}^\top \phi(\mathbf{x}) + b) = \kappa\mathbf{w}^\top \phi(\mathbf{x}) + \kappa b = 0$$

so rescaling $\mathbf{w} \rightarrow \kappa\mathbf{w}$ and $b \rightarrow \kappa b$ gives just another representation of the same decision boundary. Choose scaling factor such that

$$t_i(\mathbf{w}^\top \phi(\mathbf{x}_i) + b) = 1 \tag{7.4}$$

for the points $\mathbf{x}_i$ closest to the decision boundary.

Canonical Representation (square=1,circle=−1)



Canonical Representation

In this case we have

$$t_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b) \geq 1 \qquad n = 1, \ldots, N \tag{7.5}$$

Quadratic program

$$\arg\min_{\mathbf{w},b} \frac{1}{2}\|\mathbf{w}\|^2 \tag{7.6}$$

subject to the constraints (7.5).

This optimization problem has a unique global minimum.

Lagrangian Function

Introduce Lagrange multipliers $a_n \geq 0$ to get Lagrangian function

$$L(\mathbf{w}, b, \mathbf{a}) = \frac{1}{2}\|\mathbf{w}\|^2 - \sum_{n=1}^{N} a_n \{t_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b) - 1\} \qquad (7.7)$$

with

$$\frac{\partial L(\mathbf{w}, b, \mathbf{a})}{\partial \mathbf{w}} = \mathbf{w} - \sum_{n=1}^{N} a_n t_n \phi(\mathbf{x}_n)$$

Lagrangian Function

and for b:

$$\frac{\partial L(\mathbf{w}, b, \mathbf{a})}{\partial b} = -\sum_{n=1}^{N} a_n t_n$$

Equating the derivatives to zero yields the conditions:

$$\mathbf{w} = \sum_{n=1}^{N} a_n t_n \phi(\mathbf{x}_n) \tag{7.8}$$

and

$$\sum_{n=1}^{N} a_n t_n = 0 \tag{7.9}$$

Dual Representation

Eliminating $\mathbf{w}$ and b from $L(\mathbf{w}, b, \mathbf{a})$ gives the *dual representation*.

$$
\begin{aligned}
L(\mathbf{w}, b, \mathbf{a}) &= \frac{1}{2}\|\mathbf{w}\|^2 - \sum_{n=1}^{N} a_n \{t_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b) - 1\} \\
&= \frac{1}{2}\|\mathbf{w}\|^2 - \sum_{n=1}^{N} a_n t_n \mathbf{w}^\top \phi(\mathbf{x}_n) - b \sum_{n=1}^{N} a_n t_n + \sum_{n=1}^{N} a_n \\
&= \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m t_n t_m \phi(\mathbf{x}_n)^\top \phi(\mathbf{x}_m) \\
&- \sum_{n=1}^{N} \sum_{m=1}^{N} a_n t_n a_m t_m \phi(\mathbf{x}_n)^\top \phi(\mathbf{x}_m) + \sum_{n=1}^{N} a_n \\
&= \sum_{n=1}^{N} a_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} a_n t_n a_m t_m \phi(\mathbf{x}_n)^\top \phi(\mathbf{x}_m)
\end{aligned}
$$

Dual Representation

Maximize

$$\widetilde{L}(\mathbf{a}) = \sum_{n=1}^{N} a_n - \frac{1}{2} \sum_{n,m=1}^{N} a_n t_n a_m t_m \phi(\mathbf{x}_n)^\top \phi(\mathbf{x}_m) \tag{7.10}$$

with respect to **a** and subject to the constraints

$$a_n \geq 0, \qquad n = 1, \ldots, N \tag{7.11}$$

$$\sum_{n=1}^{N} a_n t_n = 0. \tag{7.12}$$

Kernel Function

- We map $\mathbf{x}$ to a high-dimensional space $\phi(\mathbf{x})$ in which data is linearly separable.
- Performing computations in this high-dimensional space may be very expensive.
- Use a kernel function k that computes a dot product in this space (without making the actual mapping):

$$k(\mathbf{x}, \mathbf{x}') = \phi(\mathbf{x})^\top \phi(\mathbf{x}')$$

Example: polynomial kernel

Suppose $\mathbf{x} \in \mathbb{R}^3$ and $\phi(\mathbf{x}) \in \mathbb{R}^{10}$ with

$$\phi(\mathbf{x}) = (1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_3, x_1^2, x_2^2, x_3^2, \sqrt{2}x_1x_2, \sqrt{2}x_1x_3, \sqrt{2}x_2x_3)$$

Then

$$\begin{aligned} \phi(\mathbf{x})^\top \phi(\mathbf{z}) &= 1 + 2x_1z_1 + 2x_2z_2 + 2x_3z_3 + x_1^2z_1^2 + x_2^2z_2^2 + x_3^2z_3^2 \\ &+ 2x_1x_2z_1z_2 + 2x_1x_3z_1z_3 + 2x_2x_3z_2z_3 \end{aligned}$$

But this can be written as

$$(1 + \mathbf{x}^\top \mathbf{z})^2 = (1 + x_1z_1 + x_2z_2 + x_3z_3)^2$$

which costs much less operations to compute.

Polynomial kernel: numeric example

Suppose $\mathbf{x} = (3, 2, 6)$ and $\mathbf{z} = (4, 1, 5)$.

Then

$$\begin{aligned} \phi(\mathbf{x}) &= (1, 3\sqrt{2}, 2\sqrt{2}, 6\sqrt{2}, 9, 4, 36, 6\sqrt{2}, 18\sqrt{2}, 12\sqrt{2}) \\ \phi(\mathbf{z}) &= (1, 4\sqrt{2}, 1\sqrt{2}, 5\sqrt{2}, 16, 1, 25, 4\sqrt{2}, 20\sqrt{2}, 5\sqrt{2}) \end{aligned}$$

Then

$$\phi(\mathbf{x})^\top \phi(\mathbf{z}) = 1 + 24 + 4 + 60 + 144 + 4 + 900 + 48 + 720 + 120 = 2025.$$

But

$$(1 + \mathbf{x}^\top \mathbf{z})^2 = (1 + (3)(4) + (2)(1) + (6)(5))^2 = 45^2 = 2025$$

is a more efficient way to compute this dot product.

Kernels

Linear kernel

$$k(\mathbf{x}, \mathbf{x}') = \mathbf{x}^\top \mathbf{x}'$$

Two popular non-linear kernels are the polynomial kernel

$$k(\mathbf{x}, \mathbf{x}') = (\mathbf{x}^\top \mathbf{x}' + c)^M$$

and Gaussian (or radial) kernel

$$k(\mathbf{x}, \mathbf{x}') = \exp(-\|\mathbf{x} - \mathbf{x}'\|^2 / 2\sigma^2), \tag{6.23}$$

or

$$k(\mathbf{x}, \mathbf{x}') = \exp(-\gamma \|\mathbf{x} - \mathbf{x}'\|^2),$$

where $\gamma = \frac{1}{2\sigma^2}$.

Dual Representation with kernels

Using $k(\mathbf{x}, \mathbf{x}') = \phi(\mathbf{x})^\top \phi(\mathbf{x}')$ we get dual representation:

Maximize

$$\widetilde{L}(\mathbf{a}) = \sum_{n=1}^{N} a_n - \frac{1}{2} \sum_{n,m=1}^{N} a_n t_n a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \tag{7.10}$$

with respect to **a** and subject to the constraints

$$a_n \geq 0, \qquad n = 1, \ldots, N \tag{7.11}$$

$$\sum_{n=1}^{N} a_n t_n = 0. \tag{7.12}$$

Is this dual "easier" than the original problem?

Prediction

Recall that

$$y(\mathbf{x}) = \mathbf{w}^\top \phi(\mathbf{x}) + b \tag{7.1}$$

Substituting

$$\mathbf{w} = \sum_{n=1}^{N} a_n t_n \phi(\mathbf{x}_n) \tag{7.8}$$

into (7.1), we get

$$y(\mathbf{x}) = b + \sum_{n=1}^{N} a_n t_n k(\mathbf{x}, \mathbf{x}_n) \tag{7.13}$$

Prediction: support vectors

KKT conditions:

$$a_n \geq 0 \tag{7.14}$$

$$t_n y(\mathbf{x}_n) - 1 \geq 0 \tag{7.15}$$

$$a_n\{t_n y(\mathbf{x}_n) - 1\} = 0 \tag{7.16}$$

From (7.16) it follows that for every data point, either

1. $a_n = 0$, or
2. $t_n y(\mathbf{x}_n) = 1$.

The former play no role in making predictions (see 7.13), and the latter are the *support vectors* that lie on the maximum margin hyper planes.

Only the support vectors play a role in predicting the class of new attribute vectors!

Prediction: computing b

Since for any support vector $\mathbf{x}_n$ we have $t_n y(\mathbf{x}_n) = 1$, we can use (7.13) to get

$$t_n \left(b + \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \right) = 1, \tag{7.17}$$

where $\mathcal{S}$ denotes the set of support vectors.

Hence we have

$$\begin{aligned} t_n b + t_n \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) &= 1 \\ t_n b &= 1 - t_n \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \\ b &= t_n - \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \qquad (7.17\text{a}) \end{aligned}$$

since $t_n \in \{-1, +1\}$ and so $1/t_n = t_n$.

Prediction: computing b

A numerically more stable solution is obtained by averaging (7.17a) over all support vectors:

$$b = \frac{1}{N_{\mathcal{S}}} \sum_{n \in \mathcal{S}} \left(t_n - \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \right) \tag{7.18}$$

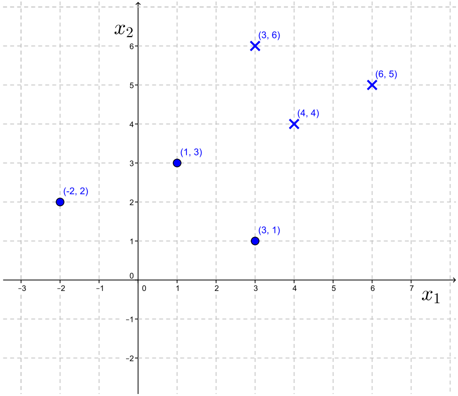
Prediction: Example

We receive the following output from the optimization software for fitting a support vector machine with linear kernel and perfect separation of the training data:

n	$x_{n,1}$	$x_{n,2}$	t_n	a_n
1	-2	2	-1	0
2	1	3	-1	1
3	3	1	-1	1
4	3	6	$+1$	0
5	4	4	$+1$	$\frac{9}{8}$
6	6	5	$+1$	0

Prediction: Example

The figure below is a plot of the same data set, where the dots represent points with class -1, and the crosses points with class $+1$.



Prediction: Example

(a) Compute the value of the SVM bias term b.
Data points with $a > 0$ are support vectors.
Let's take the point $x_1 = 4, x_2 = 4$ with class label $+1$:

$$b = t_m - \sum_{n=1}^{N} a_n t_n \mathbf{x}_m^\top \mathbf{x}_n = 1 + [4 \;\; 4] \begin{bmatrix} 1 \\ 3 \end{bmatrix} + [4 \;\; 4] \begin{bmatrix} 3 \\ 1 \end{bmatrix} - \tfrac{9}{8} [4 \;\; 4] \begin{bmatrix} 4 \\ 4 \end{bmatrix} = -3$$

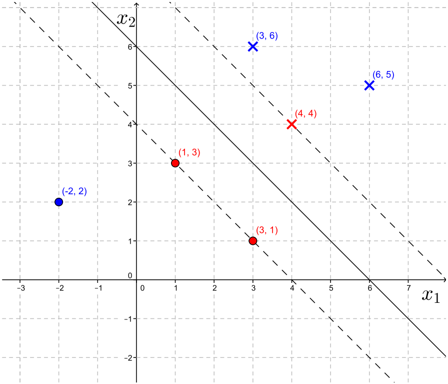
(b) Which class does the SVM predict for the data point $x_1 = 5, x_2 = 2$?

$$y(\mathbf{x}) = b + \sum_{n=1}^{N} a_n t_n \mathbf{x}^\top \mathbf{x}_n = -3 - [5 \;\; 2] \begin{bmatrix} 1 \\ 3 \end{bmatrix} - [5 \;\; 2] \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \tfrac{9}{8} [5 \;\; 2] \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \tfrac{1}{2}$$

Since the sign is positive, we predict class $+1$.

Prediction: Example

Decision boundary and support vectors.



Allowing Errors

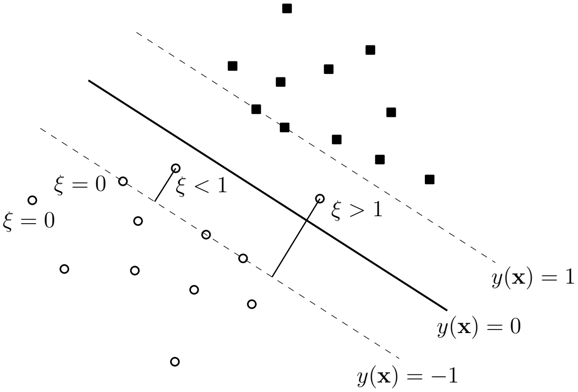
- So far we assumed that the training data points are linearly separable in feature space $\phi(\mathbf{x})$.
- Resulting SVM gives exact separation of training data in original input space $\mathbf{x}$, with non-linear decision boundary.
- Class distributions typically overlap, in which case exact separation of the training data leads to poor generalization (overfitting).

Allowing Errors

- Data points are allowed to be on the “wrong” side of the margin boundary, but with a penalty that increases with the distance from that boundary.
- For convenience we make this penalty a linear function of the distance to the margin boundary.
- Introduce slack variables $\xi_n \geq 0$ with one slack variable for each training data point.

Definition of Slack Variables

We define $\xi_n = 0$ for data points that are on the inside of the correct margin boundary and $\xi_n = |t_n - y(\mathbf{x}_n)|$ for all other data points.



New Constraints

The exact classification constraints

$$t_n y(\mathbf{x}_n) \geq 1 \qquad n = 1, \ldots, N \tag{7.5}$$

are replaced by

$$t_n y(\mathbf{x}_n) \geq 1 - \xi_n \qquad n = 1, \ldots, N \tag{7.20}$$

with

$$\xi_n \geq 0 \qquad n = 1, \ldots, N$$

New objective function

Our goal is to maximize the margin while softly penalizing points that lie on the wrong side of the margin boundary. We therefore minimize

$$C \sum_{n=1}^{N} \xi_n + \frac{1}{2}\|\mathbf{w}\|^2 \tag{7.21}$$

where the parameter $C > 0$ controls the trade-off between the slack variable penalty and the margin. Alternative view (divide by C and put $\lambda = \frac{1}{2C}$:

$$\sum_{n=1}^{N} \xi_n + \lambda \sum w_i^2$$

First term represents lack-of-fit (hinge loss) and second term takes care of regularization.

Optimization Problem

The Lagrangian is given by

$$L(\mathbf{w}, b, \mathbf{a}) = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{n=1}^{N}\xi_n - \sum_{n=1}^{N} a_n\{t_n y(\mathbf{x}_n) - 1 + \xi_n\} - \sum_{n=1}^{N}\mu_n\xi_n \tag{7.22}$$

where $a_n \geq 0$ and $\mu_n \geq 0$ are Lagrange multipliers.

The KKT conditions are given by:

$$a_n \geq 0 \tag{7.23}$$

$$t_n y(\mathbf{x}_n) - 1 + \xi_n \geq 0 \tag{7.24}$$

$$a_n(t_n y(\mathbf{x}_n) - 1 + \xi_n) = 0 \tag{7.25}$$

$$\mu_n \geq 0 \tag{7.26}$$

$$\xi_n \geq 0 \tag{7.27}$$

$$\mu_n\xi_n = 0 \tag{7.28}$$

Dual

Take derivative with respect to $\mathbf{w}$, b and ξ_n and equate to zero:

$$\frac{\partial L}{\partial \mathbf{w}} = 0 \Rightarrow \mathbf{w} = \sum_{n=1}^{N} a_n t_n \phi(\mathbf{x}_n) \tag{7.29}$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{n=1}^{N} a_n t_n = 0 \tag{7.30}$$

$$\frac{\partial L}{\partial \xi_n} = 0 \Rightarrow a_n = C - \mu_n \tag{7.31}$$

Dual

Using these to eliminate **w**, b and ξ_n from the Lagrangian, we obtain the dual Lagrangian: Maximize

$$\widetilde{L}(\mathbf{a}) = \sum_{n=1}^{N} a_n - \frac{1}{2} \sum_{n,m=1}^{N} a_n t_n a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \tag{7.32}$$

with respect to **a** and subject to the constraints

$$0 \leq a_n \leq C, \qquad n = 1, \ldots, N \tag{7.33}$$

$$\sum_{n=1}^{N} a_n t_n = 0. \tag{7.34}$$

Note: we have $a_n \leq C$ since $\mu_n \geq 0$ (7.26) and $a_n = C - \mu_n$ (7.31).

Prediction

Recall that

$$y(\mathbf{x}) = \mathbf{w}^\top \phi(\mathbf{x}) + b \tag{7.1}$$

Substituting

$$\mathbf{w} = \sum_{n=1}^{N} a_n t_n \phi(\mathbf{x}_n) \tag{7.8}$$

into (7.1), we get

$$y(\mathbf{x}) = \sum_{n=1}^{N} a_n t_n k(\mathbf{x}, \mathbf{x}_n) + b \tag{7.13}$$

with $k(\mathbf{x}, \mathbf{x}_n) = \phi(\mathbf{x})^\top \phi(\mathbf{x}_n)$.

Interpretation of Solution

We distinguish two cases:

- Points with $a_n = 0$ do not play a role in making predictions.
- Points with $a_n > 0$ are called support vectors. It follows from KKT condition

$$a_n(t_n y(\mathbf{x}_n) - 1 + \xi_n) = 0 \tag{7.25}$$

  that for these points

$$t_n y_n = 1 - \xi_n$$

  Again we have two cases:

  - If $a_n < C$ then $\mu_n > 0$, because $a_n = C - \mu_n$. Since $\mu_n \xi_n = 0$ (7.28), it follows that $\xi_n = 0$ and hence such points lie on the margin.
  - Points with $a_n = C$ can be on the margin or inside the margin and can either be correctly classified if $\xi_n \leq 1$ or misclassified if $\xi_n > 1$.

Computing the intercept

To compute the value of b, we use the fact that those support vectors with $0 < a_n < C$ have $\xi_n = 0$ so that $t_n y(\mathbf{x}_n) = 1$, so like before we have

$$b = t_n - \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \tag{7.17a}$$

Again a numerically more stable solution is obtained by averaging (7.17a) over all data points having $0 < a_n < C$:

$$b = \frac{1}{N_{\mathcal{M}}} \sum_{n \in \mathcal{M}} \left(t_n - \sum_{m \in \mathcal{S}} a_m t_m k(\mathbf{x}_n, \mathbf{x}_m) \right) \tag{7.37}$$

Model Selection

- As usual we are confronted with the problem of selecting the appropriate model complexity.
- The relevant parameters are C and any parameters of the chosen kernel function.

SVM in R

LIBSVM is available in package `e1071` in `R`.
It can also perform regression and non-binary classification.

Non-binary classification is performed as follows:

- Train $K(K-1)/2$ binary SVM's on all possible pairs of classes.
- To classify a new point, let it be classified by every binary SVM, and pick the class with the highest number of votes.
- This is done automatically by function `svm` in `e1071`.

How to in R: analysis of optdigits data

```
# SVM with radial kernel and gamma=1/62 and cost=1 (default settings)

> optdigits.svm <- svm(optdigits.train[, -c(1, 40,65)],optdigits.train[,65])

# make predictions on test set

> svm.pred <- predict(optdigits.svm,optdigits.test[, -c(1, 40,65)])

> table(optdigits.test[,65],svm.pred)
   svm.pred
      0   1   2   3   4   5   6   7   8   9
  0 177   0   0   0   1   0   0   0   0   0
  1   0 179   0   0   2   0   0   0   1   0
  2   0   7 167   0   3   0   0   0   0   0
  3   0   0   3 172   2   2   0   1   2   1
  4   0   1   0   0 179   0   0   0   1   0
  5   0   0   0   0   0 181   0   0   0   1
  6   1   0   0   0   1   0 179   0   0   0
  7   0   0   0   0   0   0   0 172   0   7
  8   0   6   0   0   3   0   0   0 159   6
  9   0   0   0   2   0   1   1   1   1 174

# accuracy on test sample is somewhat better than regularized multinomial logit
# which had "only" 95% accuracy

> sum(diag(table(optdigits.test[,65],svm.pred)))/nrow(optdigits.test)
[1] 0.967724
```

How to in R: : analysis of optdigits data

```
# tune cost parameter with cross-validation
> optdigits.svm.tune <- tune.svm(optdigits.train[, -c(1, 40,65)],
                                 optdigits.train[,65],cost=1:10)
Warning messages:
1: In svm.default(list(V2 = c(0L, 0L, 6L, 0L, 3L, 1L, 0L, 0L, 0L, 0L,  :
  Variable(s) V57 constant. Cannot scale data.
# V57 is almost always zero, let's remove it
> optdigits.svm.tune <- tune.svm(optdigits.train[, -c(1, 40,57,65)],
                                 optdigits.train[,65],cost=1:10)
# show performance for each value of the cost parameter
> optdigits.svm.tune$performances
   cost      error  dispersion
1     1 0.01490643 0.005227509
2     2 0.01202958 0.005934869
3     3 0.01229136 0.005376797
4     4 0.01176917 0.005119188
5     5 0.01150739 0.004803942
6     6 0.01150739 0.004803942
7     7 0.01176849 0.005115891
8     8 0.01150739 0.004803942
9     9 0.01176849 0.005115891
10   10 0.01176849 0.005115891
```

How to in R: : analysis of optdigits data

```
# cost=5 is the smallest among the parameter values that minimize
# cross-validation error
# We fit the SVM with this parameter value on the whole training set:

> optdigits.svm.tuned <- svm(optdigits.train[, -c(1, 40,57,65)],
                             optdigits.train[,65],cost=5)

# Generate predictions on the test set

> svm.tuned.pred <- predict(optdigits.svm.tuned,
                            optdigits.test[, -c(1, 40,57,65)])

# Compute accuracy on test set

> sum(diag(table(optdigits.test[,65],svm.tuned.pred)))/nrow(optdigits.test)
[1] 0.9727323
```