Chapter 3

Models

3.1 Linear Expressions

This section addresses the following topics.

apparently to note only that one is in this section but I wasn't consistent

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

This section covers the following mathematical concepts.

- Solve *linear*, rational, quadratic, and exponential equations and formulas (skill)
- Read and interpret models (critical thinking)
- Use models including *linear*, quadratic, exponential/logarithmic, and trigonometric (skill)

Section 1.3 presented models in general. This section presents linear models. First, we look at some examples and learn how the pieces of a linear model work. Next, we learn to write linear models given a description of a problem. After that we practice solving for different parts of a linear equation. Section 3.3 will introduce a more in depth look at identifying linear models.

can cut into

3.1.1 Linear Models

Why is "linear" italicized?

This section presents examples of linear models and provides an explanation for the two parts of a linear model.

Deb may have said the same. However this is TILT

A linear model (equation) can be written in the following, equivalent forms.

- $y = mx + c = \dfrac{a}{b}x + c$
- $ay + bx + c = 0$

Please use y=mx+b. It matches other things and will help students connect this to what they already know.

The second form can be solved for y which will make it look like the first form.

Model of Temperature Change with Altitude. As a result of atmospheric physics, temperature decreases as the distance above the ground increases. For lower altitudes this can be modeled as

$$T_A = T_G - \left(\frac{3.5}{1000}\right) A.$$

- T_A is the expected temperature at the specified altitude.
- T_G is the temperature at ground level.

- A is the specified altitude in number of feet above ground level.
- $\frac{3.5^\circ}{1000 \text{ ft}}$ is the rate of temperature decrease.

All temperatures are in Fahrenheit.

I like this!

Before we can use this model we need to know the parameter T_G. A parameter is not a variable, rather it is a value (number) that we obtain from the circumstances and write into the model (equation) before we do any work.

In contrast the ratio $-\frac{3.5^\circ}{1000 \text{ ft}}$ is a constant (not a parameter), because it is a result of atmospheric physics that is not dependent on the location for this simplified model.

Temperature (T_A) and altitude (A) are variables which implies that the model shows a relationship between these two properties.

The model subtracts from the starting temperature results in a decrease of temperature from T_G. This implies that temperature decreases with altitude.

Every linear model (equation) has a rate. In this case $m = \frac{a}{b} = -\frac{3.5}{1000}$.

Every linear model has a shift, which may be zero. In this case $c = T_G$.

Example 3.1.1 If the temperature at ground level is 43° what is the temperature 1000 ft above ground level (AGL)? 2000 ft AGL, 3000 ft AGL, 3500 ft AGL?

Because fractions of a degree are not useful in making decisions like what to wear, we will round to units.

Solution. Note $T_G = 43^\circ$. We need to calculate T_A for $A = 1000, 2000, 3000, 3500$.

$$\begin{aligned}
T_{1000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(1000 \text{ ft}) \\
&= 39.5 \\
&\approx 40. \\
T_{2000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(2000 \text{ ft}) \\
&= 36. \\
T_{3000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(3000 \text{ ft}) \\
&= 32.5 \\
&\approx 33. \\
T_{3500} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(3500 \text{ ft}) \\
&= 30.75 \\
&\approx 31.
\end{aligned}$$

Notice that we now know that it will be below freezing just above 3000 ft. □

Model of Time to Altitude. A fixed wing aircraft flown optimally climbs from a starting altitude at a fixed climb rate.

$$A_t = A_G + C \cdot t.$$

there is only one

Okay, it is beginning to feel like a lot of abbreviations. Don't be too FAA Mark!

- A_t is the altitude after t minutes.
- A_G is the starting altitude (likely ground level) in feet mean sea level (MSL).
- C is the rate of climb in feet per minute.
- t is the time since the climb began in minutes.

Before we can use this model we need to know the parameters A_G and C. A parameter is not a variable, rather it is a value (number) that we obtain from the circumstances and write into the model (equation)

you already said this?

close enough? repetition teaches looking for balance before we do any work. A_G varies by airport, because they are at different altitudes. The rate C must be obtained for each plane and is often available in the aircraft's Pilot's Operating Handbook (POH).

Final altitude (A_t) and time (t) are the variables which implies that the model shows a relationship between time climbing and how high the plane is.

In this model everything is added which matches the increase of elevation over time (adding makes the altitude bigger).

Every linear model (equation) has a rate. In this case $m = \frac{a}{b} = \frac{C \text{ ft}}{1 \text{ min}}$.

Every linear model has a shift, which may be zero. In this case $c = A_G$.

Is it worth relating this back to y=mx+b and noting the shift is the y-coordinate of the y-intercept?

I hate this term

it does not have to be zero

clarify

Example 3.1.2 If a plane begins at 160 ft MSL and is climbing at 700 ft/min, how high will it be after 5 minutes? 10 minutes? 15 minutes?

These calculations are made as part of safety planning. The data is sufficiently accurate that rounding is not necessary. Rather we make conservative estimates of the parameters, so that there is always a safety buffer. In this case a conservative estimate for A_G is to round down: this will give us a lower altitude. If that lower altitude is safe, then one 5 feet higher will be safe as well. For the climb rate a conservative estimate is to round down as well. If we can reach an altitude at 700 ft/min, then we will reach it a little earlier at 720 ft/min.

I can see a student start to get overwhelmed by different rounding situations, and start thinking they need to understand a lot of about the situation in order to round. While I DO want students thinking about the situation, I personally feel this is too much information. I think a person reading this is going to be sidelined in thinking about rounding (and thinking "where did the "5 feet higher" bit come from?!"). The point (as I understand it) is about plugging in values into equations -- not about the rounding. In this particular case we don't even need to round, making this feel extra extraneous. (How is that for extraneous? "Extra extraneous" - haha!

Solution. Note

$$A_t = 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot t.$$

The expected altitudes are

$$\begin{aligned} A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 5 \text{ min} \\ &= 3660. \\ A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 10 \text{ min} \\ &= 7160. \\ A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 15 \text{ min} \\ &= 10660. \end{aligned}$$

maybe I move the rounding below?

This is vital in aviation

different rounding is major theme of MilaC. I don't expect them to memorize this, but I do expect they will look in context for value

Note if we need to climb above 4500 ft MSL we will achieve this in between 5 and 10 minutes (closer to 5). □

Model of Fuel Remaining Calculation. When operated at a fixed power setting a vehicle burns the same amount of gas per hour (or other time unit). This leads to the linear model

$$F_t = F_I - r \cdot t.$$

- F_t is the amount of fuel remaining after t minutes.
- F_I is the amount of fuel at the beginning.
- r is the rate (volume per time) at which fuel is being consumed.
- t is the time the vehicle has been operated.

Fuel amounts will be measured in units of volume like gallons or liters. Time will be measured in minutes or hours. The rate r is then in units such as gallons/hour or liters/min.

Before we can use this model we need to know the parameters F_I and r. These parameters are not variables (they remains the same the whole time the model is in use), rather they are values (numbers) that we obtain from the circumstances and write into the model (equation) before we do any work.

The initial fuel F_I is obtained by checking the fuel tanks or fuel gauges. The rate r is often is not shown during operation (fuel gauges show how much is remaining rather than how fast it is used). The rate can sometimes be obtained from vehicle documentation.

Final fuel (F_t) and time (t) are the variables which implies that the model shows a relationship between time flown and fuel available (left in the tanks).

Because fuel decreases the $r \cdot t$ term is subtracted decreasing the amount from F_I.

Every linear model (equation) has a rate. In this case $m = \frac{a}{b} = \frac{r \text{ gal}}{1 \text{ hr}}$ (or similar units).

Every linear model has a shift, which may be zero. In this case $c = F_I$.

Example 3.1.3 If a car begins with 20 gallons of fuel and burns 1.55 gallons per hour, how much fuel will it have after 1 hour, 2 hours, 3 hours, 36 minutes?

A gallon is a large amount so we will maintain one decimal place precision. For safety we should always assume a larger fuel burn, so we will round fuel remaining down.

Solution. Note

$$F_t = 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot t \text{ hr}.$$

Thus

$$\begin{aligned}
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 1 \text{ hr} \\
&= 18.45 \\
&\approx 18.4. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 2 \text{ hrs} \\
&= 16.9. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 3 \text{ hrs} \\
&= 15.35 \\
&\approx 15.3. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 0.6 \text{ hrs} \\
&= 19.07 \\
&\approx 19.0.
\end{aligned}$$

□

Use this Checkpoint to practice using a linear model.

Checkpoint 3.1.4 The expected temperature at a height above ground is given by

$T_A = T_G - \frac{3.5}{1000}A$

where T_A is the expected temperature in Fahrenheit

T_G is the temperature at ground level in Fahrenheit

A is the height above ground level in feet

If the temperature on the ground is 75°, what will it be at 2600 feet above ground level? ___

Answers should be rounded to the units place.

Solution.

- 66

The model for this situation is $T_A = 75 - \frac{3.5}{1000}A$.

Because we want to estimate the temperature at 2600 ft AGL, we calculate $T_A = 75 - \frac{3.5}{1000}2600 = 65.9 \approx 66$

3.1.2 Building Linear Models

The previous section presented linear models, and illustrated using ~~provided~~ models. This section presents problems that can be modeled as linear equations, and illustrates writing the model (equation) before using it.

A linear model has a starting point (shift, b) and rate (ratio, m). We need to identify these and then write the linear model

$$y = mx + b$$

Okay, now you have y=mx+b.
Why not before?

with these values. We should also label units and explain any parameters.

Example 3.1.5 Consider rope that costs \$0.93 per foot with a shipping charge of \$7.64. To produce a model for the cost of each purchase we will start by trying a couple specific orders.

Suppose we are purchasing 20 feet of this rope. The cost for the 20 feet will be $20 \text{ ft} \cdot \frac{\$0.93}{\text{ft}}$, because each foot is \$0.93. This is just like unit conversion: the units (\$/ft and ft) suggest multiplying.

Notice this multiplication is also the same as using a ratio (proportion). We could setup $\frac{\$0.93}{1\text{ft}} = \frac{C}{20 \text{ ft}}$ When we solve this we end up with the same multiplication $20 \text{ ft} \cdot \frac{\$0.93}{\text{ft}}$.

Next we must add the shipping charge. Thus the final cost is $20 \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = \$26.24$. Note there is no rounding because all numbers are exact (no measurements, so no signficant digits) and no fractions of a cent occurred.

Suppose we are purchasing 100 feet of this rope. The cost for the 100 feet will be

$$100 \text{ ft} \cdot \frac{\$0.93}{\text{ft}}.$$

Then we must add the shipping charge. Thus the final cost is $100 \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = \$100.64$.

Notice we could do this with any number of feet (unless the shipping charge increases for larger orders). So in general we can write the cost as

$$s \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = C.$$

Notice that this equation has a ratio (0.93/1), which is the cost per foot, but also has a shift (+7.64), which is the fixed shipping cost. Thus this is another linear equation. □

When cost is set per linear foot, or per square yard, or similar per unit pricing we often end up with a linear model.

Example 3.1.6 At lower altitudes the barometric pressure typically drops 1 inHg for every 1000 feet of elevation gained (the air is less dense higher up). To produce a model for pressure decrease we will start by calculating the pressure for a couple specific cases.

If the pressure on the ground is 29.76 inHg, what do we expect the pressure to be flying at 4500 ft above ground level?

The pressure drop is a ratio $\frac{1 \text{ inHg}}{1000 \text{ ft}}$. The units suggest we can multiply $4500 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}} = 4.5 \text{ inHg}$. This is the drop in pressure. To calculate the resulting pressure we need $29.76 \text{ inHg} - 4.5^\circ \text{ inHg} = 25.26 \text{ inHg}$. We retain 2 decimal places because that is the traditional amount for reporting by meteorologists. Written as one calculation this is $T = 29.76 - \left(4500 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right)$.

If the pressure on the ground is 30.02 inHg what do we expect the pressure to be flying at 6000 ft above ground level?

The pressure drop is a ratio $\frac{1 \text{ inHg}}{1000 \text{ ft}}$. The units suggest we can multiply $6000 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}} = 6 \text{ inHg}$. This is the drop in pressure. To calculate the resulting pressure we need $30.02 \text{ inHg} - 6 \text{ inHg} = 24.02 \text{ inHg}$. Written as one calculation this is $P = 30.02 - \left(6000 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right)$.

Notice we could do this same calculation for any altitude. So in general we can write

$$P_A = P_G - \left(A \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right).$$

P_A is the pressure at the specified altitude. P_G is the pressure at ground level. A is the altitude above