Mathematics in Trades and Life

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For Students

This book is intended for use as lessons in a course that emphasizes building the skills to read and use mathematics (such as in a technical manual), and to recognize mathematical concepts in things you see and read in life.

This book is not written as a how to manual for specific applications. That is, while this book provides examples of many specific applications from trades and life, it does not provide a step-by-step example to follow for every type of problem. Rather it provides initial examples, presents general concepts used in the example, and helps you practice recognizing what is important in an example and applying it to similar problems.

The topics include

1. Interpret data in various formats and analyze mathematical models
2. Read and use mathematical models in a technical document
3. Communicate results in mathematical notation and in language appropriate to the technical field

You will learn to work with the following mathematical concepts.

1. Precision and accuracy
   (a) Rounding (skill)
   (b) Significant Figures (skill)
   (c) Determining appropriate rounding from context (critical thinking)
2. Proportions
   (a) Setup and solve proportions (skill)
   (b) Calculate Percentages (skill)
   (c) Understand and interpret percentages (critical thinking)
   (d) Unit conversion (skill)
3. Rates
   (a) Identify rates as linear, quadratic, exponential, or other (critical thinking)
   (b) Identify data varrying directly or indirectly (critical thinking)
4. Solving
   (a) Solve linear, rational, quadratic, and exponential equations and formulas (skill)
   (b) Solve a system of linear equations (skill)
5. Models
   (a) Read and interpret models (critical thinking)

   (b) Use models including linear, quadratic, exponential/logarithmic, and trigonometric (skill)

6. Trigonometry
   (a) Analyze right triangles (skill)
   (b) Analyze non-right triangles (skill)
   (c) Identify properties of sine and cosine functions (skill)

For Faculty

This book is designed to be used for classes supporting trade programs in a variety of fields and also to satisfy baccalaureate general education requirements.

It is also written to be used in conjunction with active learning pedagogies rather than as a reference text. That is, it has been designed for students to read the examples, work initial problems to help them identify questions, and to then seek help while working on exercises. It can be easily incorporated into flipped classrooms and asynchronous learning. It intentionally does not provide an example to mimic for each question in the exercises. Rather each section provides an simple example, a more complex example, and some explanation about what is important. Then exercises are provided for them to test their ability to recognize and use the mathematics from that section.

The check points (exercises in the reading) are self-grading with feedback so the reader can determine what if any questions they need to ask. Videos, where included, are presentations of the introduction of the concept in that section. Homework by default is live, online problems that provide feedback. The scores on these cannot be saved in an LMS however. If you wish to use MyOpenMath, the problems, and even a shell, can be provided. If desired the PDF version can be used which does not have live homework.

The projects are an integral part of the general education goals. These are intended to be assigned after relevant material is covered. They require students to use topics from that chapter to perform calculations and then interpret the results of their work. The projects also provide a opportunity for students to express calculations using standard mathematical notation and to communicate mathematical results in clear language.

For context, here is a brief history. The first version of this text was written to transition to an OER for MATH A104 Technical Mathematics at the Univesity of Alaska Anchorage. Commercially available texts emphasized memorizing problem types with limited critical thinking. As such they could not be used to satisfy the general education requirement. There was also a desire to reduce costs including for textbooks and online homework systems. As a result this book was created with matching homework in MyOpenMath.

Served disciplines at UAA included auto/diesel, heavy equipment mechanics, welding and non-destructive testing, aviation mechanics, piloting, air traffic controll, and medical certification programs.

The general education outcome around which this was designed was: Quantitative courses develop abilities to reason mathematically and analyze quantitative and qualitative data to reach sound conclusions for success in undergraduate study and professional life. The indicators were

- Interprets info in mathematical form (equations, graphs, diagrams, tables, words)
- Represents and/or converts relevant quant info and explain its assumptions and limits
- Applies mathematical forms (equations, graphs, diagrams, tables, words) to quantitative problems to reach sound conclusions
- Communicates quantitative results appropriate to the problem or context

Contents

Chapter 1

Cross Cutting Topics

1.1 Units

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Communicate results in mathematical notation and in language appropriate to the technical field

and covers the mathematical concepts

- Unit conversion (skill)
- Setup and solve proportions (skill)

This book is designed to present mathematics in various contexts including a variety of trades. As a result numbers will frequently be connected to units such as length (feet, meters), time (seconds) or others. These units are part of the arithmetic, and so we must learn what they mean and how to perform this arithmetic. Sometimes the units even suggest to us how to setup the arithmetic.

In this section we will introduce the units so we can understand them when we read technical material (Item 2) and use them correctly to communicate results to others (Item 3). We will also learn to convert units (Item 2.d) which involves our first use of proportions (Item 2.a).

We measure many things such as distance, time, and weight. We describe these measurements in terms of units like mile, hour, and pounds. But have you ever stopped to think about how these units are defined?

The story of some of these units is lost in history. For example dividing the day into 24 units began with ancient Egyptians. They did not record, that we know of, the reason for choosing 24 units as opposed to 30 or any other number.

Other units, such as the metric (or SI) are much more modern. Initially many units were based on something physical. For example one calorie is the amount of heat it takes to raise the temperature of one gram of water 1° C. The meter was originally defined as one ten-millionth of the distance from the equator to the north pole. The problem with this type of measurement is that it is neither fixed (depends on where on the equator your begin) nor easy to measure.

Thus modern definitions were developed. The length of a meter was changed to mean the length of a bar of metal kept in special storage in France. The bar had been carefully constructed and was used to confirm other measurement devices were correctly calibrated. It was changed yet again to be based on wavelengths of radiation. These are uniform no matter where they are done, so they can be used by many people to construct simple measurement tools.

1.1.1 Types of Measurement

First we will look at the units (names of units) for different types of measurement. In a specific trade you will need to memorize the units you use most often. For this class you should ask your instructor which units must be memorized and which you may look up when working on problems.

Note that the U.S. Customary system (related to the British Imperial system) is non-uniform, so there are multiple names for some types of measurements. This is in contrast to the metric system (formally known as SI or international system) which has one name for each property and prefixes to indicate the size. Table 1.1.1 lists names of units for both systems. It is important to be able to recognize which unit (name) goes with which type of measurement (e.g., length, volume, …).

Table 1.1.1 Units of Measure

Measuring	US Customary	Metric
Length	inch (in) foot (ft) yard (yd) mile (mi) nautical mile (nm)	meter (m)
Volume	fluid ounce (oz) cup (c) pint (pt) quart (qt) gallon (g)	liter (L or ℓ)
Weight	ounce (oz) pound (lb)	gram (g)
Temperature	degrees Fahrenheit (F)	degrees Celsius (C)
Pressure	inches of mercury (inHg)	Pascal (Pa)
Time	second (s) minute (min) hour (hr)	

One name: two meanings Note that fluid ounces and weight ounces are not the same unit. 10 fluid ounces of milk does not weigh 10 ounces. You must determine which ounce is referenced by the context. This can be tricky in recipes which is a good reason to us SI units.

Note a gram is a unit of mass rather than than a unit of weight. Pound and ounce on the other hand are units of weight. Nevertheless gram is often used to describe weight because it is easy to switch between it and weight. Namely, mass can be obtained by dividing by the acceleration due to gravity (see a physics book for more information). The official unit for weight (a force) is a Newton, but we will not use that in this book.

1.1.2 U.S. Customary

Often we need to convert between units within the U.S. Customary system. This section provides the information needed for conversion and examples of performing them. It is an example of units suggesting how we setup the calculation.

Why would we need to convert units? This can occur because measurements were taken with different scales. For instance, we cannot add 3 inches to 2.2 feet without changing one to make the units match.

Why are there multiple units in the first place? There are different units for different scales (e.g., inches for small lengths and miles for long distances). This is a result of the U.S. Customary system being developed from the British Imperial system which was based on disparate measurements from multiple centuries ago (look up unit names in an etymological dictionary for fun). Converting between units therefore requires remembering special numbers for conversion. Most of these you likely know.

Table 1.1.2 Converting within U.S. Customary

Measuring	Unit 1	Unit 2
Length	1 nm	6076 ft
Length	1 mi	5280 ft
	1 yd	3 ft
	1 ft	12 in
Area	1 acre	43,560 ft^2
Volume	1 g	4 qts
	1 qt	2 pts
	1 pt	2 c
	1 c	8 oz
Weight	1 ton	2000 lbs
	1 lb	16 oz
Time	1 year	365 days
	1 day	24 hrs
	1 hr	60 mins
	1 min	60 secs

Of course a year is not always the same number of days. It is important to know whether in a given circumstance we can use the common approximation of 365 days without injury or loss.

Review each of these examples to see how to convert from one U.S. Customary unit to another.

Example 1.1.3 How many quarts is 2.3 gallons?

Solution. From Table 1.1.2 we know (or can look up) that each gallon is 4 quarts. This means we have $\frac{4 \text{ quarts}}{1 \text{ gallon}}$. This suggests that we can multiply 2.3 by this ratio, because the gallons will divide out.

$$2.3 \text{ gallons} \cdot \frac{4 \text{ quarts}}{\text{gallon}} = 9.2 \text{ quarts}$$

□

Example 1.1.4 How many cups is 1.7 gallons?

Solution. We do not have a number of cups per gallon. However, we can convert start by converting gallons to quarts. Then looking at the table again, we can convert quarts to pints, and finally we can convert pints to cups.

As with the previous example we can treat each conversion as a ratio of units. We setup the units so that multiplying will result in units dividing out.

$$1.7 \text{ gallons} \cdot \frac{4 \text{ quarts}}{\text{gallon}} \cdot \frac{2 \text{ pints}}{\text{quart}} \cdot \frac{2 \text{ cups}}{\text{pint}} = 27.2 \text{ quarts}$$

□

Example 1.1.5 How many days is 17 hours?

Solution. Here we are going from a small unit (hours) to a bigger one (days). This does not change our process. We can still multiply the amount by the unit conversion. Because we want to end up with days we use $\frac{1 \text{ day}}{24 \text{ hours}}$

$$17 \text{ hours} \cdot \frac{1 \text{ day}}{24 \text{ hours}} = 0.708\overline{3} \text{ days}$$

□

Standalone

Check that you can perform a unit conversion using this Checkpoint.

Checkpoint 1.1.6 Convert the measurement. You may find it useful to use this table[1].
96 cups = ____ gal

Solution.

- 6

This can be converted using the unit conversion ratios: 1 pint/2 cups, quart/2 pints, and 1 gallon/4 quarts. Multiply these so that all the units divide out except for gallons.

$$96 \text{ cups} \cdot \frac{1 \text{ pint}}{2 \text{ cups}} \cdot \frac{1 \text{ quart}}{2 \text{ pints}} \cdot \frac{1 \text{ gallon}}{4 \text{ quarts}} = \frac{96}{16}\text{gallons} = 6 \text{ gallons}.$$

1.1.3 Metric (SI)

Just as with U.S. Customary units we often need to convert between SI units. This section provides the information needed for conversion and examples of performing them. It is an example of units suggesting how we setup the calculation.

Rather than have different names for different scales, metric uses one name of the unit (e.g., liter) and then uses prefixes to indicate size. These can be converted easily, because each prefix is a power of ten (uniform).

You will need to memorize a few of the prefixes. As with units which ones depends on your work. Ask your instructor which prefixes you should memorize for this course.

[1] `mital.uaa.alaska.edu/section-units.html#table-customary-convert`

Table 1.1.7 Metric Prefixes

Multiple	Prefix
10^{12}	tera (T)
10^{9}	giga (G)
10^{6}	mega (M)
10^{3}	kilo (k)
10^{2}	hecto (h)
10	deka (da)
10^{-1}	deci (d)
10^{-2}	centi (c)
10^{-3}	milli (m)
10^{-6}	micro (μ)
10^{-9}	nano (n)
10^{-12}	pico (p)

Review each of these examples to see how to convert from one SI unit to another.

Example 1.1.8 How many centimeters is 3.8 meters?

Solution. From Table 1.1.7 we know one centimeter is 10^{-2} meters, that is, $\frac{1\text{ cm}}{10^{-2}\text{ m}}$. This suggests that we can multiply 3.8 by the ratio which will cause the meters units to divide out.

$$3.8\text{ m} \cdot \frac{1\text{ cm}}{10^{-2}\text{ m}} = 3.8 \cdot 100\text{ cm} = 380\text{ cm}.$$

Note, because this is a power of ten (10^2) the result is shifting the decimal place two positions. 3.8 meters becomes 380 centimeters.

Using this idea we can convert 0.76 meters to 76 centimeters by just shifting the decimal (no additional process necessary). □

Example 1.1.9 How many kilotons is 2.3 megatons?

Solution. We know one kiloton is 10^3 tons and one megaton is 10^6 tons. These are three powers apart $(6 - 3 = 3)$, which means we shift the decimal position three places. Because we are converting from a large unit to a smaller unit, we move the decimal place to the right (make the number bigger). 2.3 megatons is 2,300 kilotons. □

Example 1.1.10 How many centiliters is 13.6 milliliters?

Solution. We know one centiliter is 10^{-2} liters and one milliliter is 10^{-3} liters. This means we shift the decimal $-2 - (-3) = 1$ position. Because we are moving from a smaller unit to a larger unit, we move the decimal place to the left (make the number smaller). 13.6 milliliters is 1.36 centiliters. □

Checkpoint 1.1.11 Convert the units below. You may find it useful to use this table[2].

3050 mL = ______ L

Solution.

- 3.05

mL is $\dfrac{1}{1000} = 10^{-3}$ of a liter, and the conversion is from a smaller unit to a larger unit, so we move the decimal place 3 units to the left.

$3050\ \ \text{mL} \cdot \dfrac{1\ \ \text{L}}{1000\ \ \text{mL}} = 3.05\ \ \text{L}$

[2] `mital.uaa.alaska.edu/section-units.html#table-metric-sizes`

1.1.4 Converting between Systems

Commonly we end up with measurements in both U.S. Standard system and SI. We will need to convert all units to one system before using them together. This process is the same as converting one Standard unit to another (e.g., converting miles to feet). This section provides the information needed for conversion and examples of performing them. It is an example of units suggesting how we setup the calculation.

Table 1.1.12 U.S. Customary to SI

Measuring	Standard	SI
Length	1 nm	1.852 km
Length	1 mi	1.609344 km
	1 ft	0.3048 m
	1 in	2.54 cm
Volume	1 gal	3.785412 L
	1 oz	29.573532 mL
Weight	1 lb	0.453592 kg
	1 oz	28.349523 g

Table 1.1.13 SI to U.S. Customary

Measuring	SI	Standard
Length	1 km	0.621371 mi
	1 m	3.280840 ft
	1 cm	0.393701 in
Volume	1 L	0.264172 gal
	1 mL	0.033814 oz
Weight	1 kg	2.204623 lb
	1 g	0.035274 oz

Review each of these examples to see how to convert between U.S. Customary units and SI units.

Example 1.1.14 How many kilometers is 26.2 miles?

Solution. From Table 1.1.12 we know each mile is 1.609344 km; this means there is

$$\frac{1.609344 \text{ km}}{1 \text{ mi}}.$$

The ratio suggests that we can multiply 23.6 miles by the ratio, because the miles will divide out.

$$26.2 \text{ miles} \cdot \frac{1.609344 \text{ km}}{\text{mi}} \approx 42.2 \text{ km}$$

□

Example 1.1.15 How many inches is 15 centimeters?

Solution. From Table 1.1.13 we know each centimeter is 0.393701 inches; this means there is

$$\frac{0.393701 \text{ in}}{1 \text{ cm}}.$$

The ratio suggests that we can multiply 15 centimeters by the ratio, because the centimeters will divide out.

$$15 \text{ cm} \cdot \frac{0.393701 \text{ in}}{\text{cm}} \approx 5.9 \text{ in}$$

□

Example 1.1.16 How many inches is 1 meter?

Solution. From Table 1.1.13 we know each meter is 3.280840 feet. From Table 1.1.2 that each foot is 12 inches. We use the method of setting up a product of ratios so that the units divide out. We start with meters, so the first ratio must be feet per meters. We want to end with inches so the second ratio must be inches per feet.

$$1 \text{ m} \cdot \frac{3.280840 \text{ ft}}{1 \text{ m}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \approx 39 \text{ in}$$

□

Checkpoint 1.1.17 While purchasing gas on vacation in Canada, Kenneth wonders how many gallons he is purchasing. You may find it useful to use this table[3].

75 L = ____ gal

Solution.

- 19.8

According to the table there are 0.264172 gallons per liter. We can multiply the number of liters by the conversion ratio.

$$75 \text{ L} \cdot \frac{0.264172 \text{ gal}}{1 \text{ L}} = 19.8 \text{ gal}$$

1.1.5 Converting Compound Units

Some units, such as speeds, are compound. For example speed is distance per time. This section provides examples of converting compound units.

Example 1.1.18 How many meters per second is 15 miles per hour?

Solution. We start with $\frac{15 \text{ mi}}{1 \text{ hr}}$. We can convert miles to feet and feet to meters (multi-step conversion like Example 1.1.4). The conversion ratios suggest we can multiply the 15 mi/hr by the conversion ratios.

$$\frac{15 \text{ mi}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{0.3048 \text{ m}}{1 \text{ ft}} \approx \frac{24140 \text{ m}}{1 \text{ hr}}$$

We can use the same method (multiplying by conversion ratios to divide out units) to also conver hours to seconds.

$$\frac{24140 \text{ m}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 6.706 \frac{\text{m}}{\text{sec}}.$$

Note, we could perform that conversion in on step by multiplying all the conversion ratios at once. □

Example 1.1.19 How many pounds does a tablespoon of water weigh? Note that one gallon of water weights 8 lbs. Also a tablespoon is a half fluid ounce.

Solution. We need to conver the gallons into tablespoons. Because conversions are ratios, we multiply 8 lbs by the necessary conversions.

$$\frac{8 \text{ lbs}}{1 \text{ gal}} \cdot \frac{1 \text{ gal}}{4 \text{ qts}} \cdot \frac{1 \text{ qt}}{2 \text{ pints}} \cdot \frac{1 \text{ pint}}{2 \text{ cups}} \cdot \frac{1 \text{ cup}}{16 \text{ oz}} \cdot \frac{1 \text{ oz}}{2 \text{ tbs}} = \frac{1\text{lb}}{64 \text{ tbs}} = 0.015625 \frac{\text{lbs}}{\text{tbs}}.$$

□

Checkpoint 1.1.20 Convert 63 miles per hour to feet per second. You may wish to use this table[4].

63 miles per hour = _____ feet per second. Round your answer to the nearest tenth.

Solution.

- 92.4

To convert to feet per second we need to convert miles to feet and hours to seconds. There are 5280 ft per mile, 60 minutes per hour, and 60 seconds per minute. Note this means there are $\frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} =$

[3] `mital.uaa.alaska.edu/section-units.html#table-si-to-customary`

$\frac{3600 \ \ \text{sec}}{1 \ \ \text{hr}}$. The same relationship can be written $(1 \ \ \text{hr})(3600 \ \ \text{sec})$.

Thus there are $63\frac{\text{mi}}{\text{hr}} \cdot \frac{5280 \ \ \text{ft}}{1 \ \ \text{mi}} \cdot \frac{1 \ \ \text{hr}}{3600 \ \ \text{sec}} = 92.4\frac{\text{ft}}{\text{sec}}$

Another kind of compound unit is square units such as square feet or seconds squared. When converting these we must account for the square.

Example 1.1.21 Convert 2 acres to units of square miles.

Solution. First we note that an acre is $43,560 \text{ ft}^2$. From Table 1.1.2 we know that there are 5280 ft per mile. These conversion ratios suggest that we can multiply the 2 acres by the ratios to obtain the result in square miles.

$$\begin{aligned} 2\text{acres} \cdot \frac{43,560 \text{ ft}^2}{\text{acre}} \cdot \frac{\text{mi}}{5280 \text{ ft}} \cdot \frac{\text{mi}}{5280 \text{ ft}} &= \\ 2\text{acres} \cdot \frac{43,560 \text{ ft}^2}{\text{acre}} \cdot \left(\frac{\text{mi}}{5280 \text{ ft}}\right)^2 &= \\ 2\text{acres} \cdot \frac{43,560 \text{ ft}^2}{\text{acre}} \cdot \frac{\text{mi}^2}{5280^2 \text{ ft}^2} &= \frac{1}{320} \text{ mi}^2 \\ &= 0.003125 \end{aligned}$$

It is not necessary to write all of the steps above if you understand how the final conversion line is obtained. The steps are included here to show how the squares show up in the final conversion. □

Checkpoint 1.1.22 Convert 206 square inches to square feet. Round your answer to the nearest hundredth.

_____ ft^2

Solution.

- 1.4305555555556

Using that there are 12 inches per foot, we can convert square inches to square feet by multiplying by the convesion ratios.

$206 \ \ \text{in}^2 \cdot \frac{\text{ft}}{12 \ \ \text{in}} \cdot \frac{\text{ft}}{12 \ \ \text{in}} = 206 \ \ \text{in}^2 \cdot \frac{\text{ft}^2}{144 \ \ \text{in}^2} = 1.4305555555556 \ \ \text{ft}^2$

1.1.6 Exercises

1. Units. Select the correct units to complete the conversion below.

gallons ∗ ______________ = miles

Answer:

(a) 1/gallons

(b) miles/gallons

(c) miles

(d) gallons

(e) 1/miles

(f) gallons/miles

2. Units. Select the correct units to correctly complete the calculation below.

(people)∗(______________) = (dollars)

Answer:

(a) dollars/people

[4] mital.uaa.alaska.edu/section-units.html#table-customary-convert

(b) dollars

(c) 1/dollars

(d) people

(e) people/dollars

(f) 1/people

3. **Units.** Convert the measurement. You may find it useful to use this table[5].
96 cups = ____ gal

4. **Units.** Karen mixed 5 gallons of lemonade and poured it into six 3-quart jugs. How many cups of lemonade were left over after she filled the jugs?
This table[6] may be useful for this problem.
_ cups

5. **Units.** Convert $2\frac{1}{2}$ hours to minutes. Enter your answer as an integer or a reduced fraction.
$2\frac{1}{2}$ hours =______Preview Question 1 minutes

6. **Units.** Convert the measurement.
3 days = ________ sec

7. **Units.** A corn stalk grew 7 inches in the first month after it planted, since then it grow another 5 feet.
What is the total height of the corn in feet and inches? __________________ ft __________________ in
What is the to total height of the corn stalk in inches? ____________________Preview Question 1 Part 3 of 4
What is the total height of the corn stalk in feet? ____________________Preview Question 1 Part 4 of 4 Round your answer to 2 decimal places.

8. **Units.** Select the unit that best fits the scenario
The pitcher holds 8

(a) fluid ounce(s)

(b) cup(s)

(c) gallon(s)

of lemonade

9. **Units.** Add the following weights:
22 lb 12 oz + 27 lb 15 oz + 27 lb 9 oz

_______________ pounds ____________________ ounces

10. **Units.** You are in charge of drinks for a community barbecue. You need to supply at least 120 cups of beverage to provide enough for the projected number of people that will attend. So far, you have received the following donations:

- Enough mix to make 5 gallons of lemonade
- 7 bottles of fruit juice that each contain 64 fl. oz.

How many cups of beverage do you have?
____Preview Question 1 Part 1 of 2
Will you have enough for the barbecue?

[5] `mital.uaa.alaska.edu/section-units.html#table-customary-convert`
[6] `mital.uaa.alaska.edu/section-units.html#table-customary-convert`

(a) no

(b) yes

11. Units. Convert 63 miles per hour to feet per second. You may wish to use this table[7].
63 miles per hour = _____ feet per second. Round your answer to the nearest tenth.

12. Units. Convert 448 square inches to square feet.
Round your answer to the nearest hundredth.
_____ square feet

13. Units. Jean's bedroom is 12 feet by 13 feet. She has chosen a carpet which costs \$34.85 per square yard. This includes installation. Determine her cost to carpet her room.
\$ ____________________Preview Question 1 Part 1 of 2
How much would she have saved if she went with the carpet that costs \$31.30 per square yard instead?
\$ ____________________Preview Question 1 Part 2 of 2

14. Units. Wiyot is making a quilt and he has determined he needs 413 square inches of burgundy fabric and 1203 square inches of blue. How many square yards of each material will he need to purchase from the fabric store?
The store only sells fabric by the by the quarter yard.
The burgundy fabric: ____________________ square yards
The blue fabric: ____________________ square yards
How many total yards of fabric will she have to buy?
____________________ square yards

15. Units. A unit of measure sometimes used in surveying is the *link;* 1 link is about 8 inches. About how many links are there in 3 feet? Do not round your answer.
There are ___ links in 3 feet.

16. Units. 90 in. to yards, feet, and inches
__ yds __ ft __ in

17. Units. David has 14 yd. of material that he will cut into strips 13 in. wide to make mats. How many mats can David make?
___

18. Units. Part 1 of 2I saw a job opening for a Music Therapist that pays $\$40,000$ per year salary. Assume that a regular work week for this job is 40 hours, and that you will work 50 weeks in a year.What is the hourly pay for this job? We will answer the question by converting \$40,000 per year into dollars per hour.

If we begin with the fraction $\frac{\$40000}{\text{year}}$, we can multiply by two unit fractions to complete the conversion. What are these fractions? Choose the correct fractions in the calculation below:
$\frac{\$40000}{\text{year}} \times$

(a) 50 years/1 week

(b) 50 weeks/1 year

(c) 1 week / 50 years

(d) 1 year/50 weeks

$\times$

(a) 1 week/40 hours

(b) 40 hours/1 week

[7] mital.uaa.alaska.edu/section-units.html#table-customary-convert

(c) 40 weeks/1 hour

(d) 1 hour/40 weeks

Part 2 of 2Finally, what is the hourly pay for this job?
Hourly pay = ____________________ dollars/hour

19. Units. Part 1 of 2

Molly was driving at 94 feet per second on the freeway the other day. If the speed limit is 60 miles per hour, was she driving too fast? Answer the question by converting 94 feet per second into miles per hour.

To answer this question, we will convert the numerator into miles and the denominator into hours. If we begin with the fraction $\frac{94 \text{ feet}}{1 \text{ second}}$, we can multiply by three unit fractions to complete the conversion. What are these fractions? Choose the correct fractions in the calculation below:

$\frac{94 \text{ feet}}{1 \text{ second}} \times$

(a) 60 sec/1 min

(b) 60 min/1 sec

(c) 1 sec/60 min

(d) 1 min/60 sec

$\times$

(a) 1 hr/60 min

(b) 60 hrs/1 min

(c) 60 min/1 hr

(d) 1 min/60 hrs

$\times$

(a) 1 ft/5280 mi

(b) 5280 ft/1 mi

(c) 1 mi/5280 ft

(d) 5280 mi/1 ft

Part 2 of 2The conversion calculation looks like:

$\frac{94 \text{ feet}}{1 \text{ second}} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{1 \text{ mi}}{5280 \text{ ft}}$

Was Molly driving too fast?
Answer:

(a) No

(b) Yes

...because she was driving ______ mi/hr (round using significant figures; all speeds are accurate to two sig figs) when the limit was 60 mi/hr

20. Units. Part 1 of 3Maisha wants to buy some specialty fabric to make her kindergardner Jose a halloween costume. The fabric store tells her that the fabric costs 7.66 $\frac{\text{dollars}}{\text{yard}^2}$. Note about this unit[8][Note: Typically, when a fabric store says ”yard”, they actually mean ”square yards”. In this problem, I have written it out in the correct format: yards^2]Maisha is sure she saw this advertised online for $0.47\frac{\text{cents}}{\text{inch}^2}$, and she wants to know if it is a better deal in the store or online. She will take a minute to convert the price from $\frac{\text{dollars}}{\text{yard}^2}$ to $\frac{\text{cents}}{\text{inch}^2}$ to see which is the better deal.

Maisha knows the following facts:

- 1 yard = 3 feet
- 1 ft = 12 inches
- 1 dollar = 100 cents

Maisha will perform the conversion by starting with the measured price as a fraction. Choose the correct unit fraction to multiply by in order to complete the conversion:

$\frac{7.66 \text{ dollars}}{\text{yard}^2}*$

(a) 1ft / 3yd

(b) 1yd / 3ft

(c) 3ft / 1yd

(d) 3yd / 1ft

*

(a) 3yd / 1ft

(b) 3ft / 1yd

(c) 1yd / 3ft

(d) 1ft / 3yd

*

(a) 12in / 1ft

(b) 1in / 12ft

(c) 12ft / 1in

(d) 1ft / 12in

*

(a) 1ft / 12in

(b) 12ft / 1in

(c) 1in / 12ft

(d) 12in / 1ft

*

(a) 1cent / 100dollars

(b) 100dollars / 1cent

(c) 100cents / 1dollar

(d) 1dollar / 100cents

Part 2 of 3The correct expression is:
$\frac{7.66 \text{ dollars}}{\text{yard}^2} * \frac{1 \text{ yd}}{3 \text{ ft}} * \frac{1 \text{ yd}}{3 \text{ ft}} * \frac{1 \text{ ft}}{12 \text{ in}} * \frac{1 \text{ ft}}{12 \text{ in}} * \frac{100 \text{ cents}}{1 \text{ dollar}}$
What is the final result, rounded to two decimal places?
Answer: ____________________ $\frac{\text{cents}}{\text{inch}^2}$
Part 3 of 3If Maisha wants to save the most money, should she shop at the store or online?
Answer:

(a) The Store

(b) Online

21. Units. Select the unit that best fits the scenario
The pancake recipe called for 300

(a) milliliter(s)

(b) liter(s)

(c) centimeter(s)

(d) kilogram(s)

(e) meter(s)

(f) kilometer(s)

of milk

22. Units. Select the unit that best fits the scenario
The coin weighs 5

(a) milligram(s)

(b) gram(s)

(c) kilogram(s)

(d) centimeter(s)

(e) meter(s)

(f) liter(s)

(g) milliliter(s)

23. Units. Select the unit that best fits the scenario
The hamster weighs 200

(a) milligram(s)

(b) gram(s)

(c) kilogram(s)

24. Units. How many millimeters are there in a meter? ____________________
How many liters are in a decaliter? ____________________
How many centigrams are in there in a gram? ____________________
Which prefix indicates a bigger quantity? kilo

[8] #

hecto
Which prefix indicates a bigger quantity? deca
deci
Which prefix indicates a bigger quantity? kilo
mega
Which prefix indicates a bigger quantity? milli
centi

25. Units. Convert the measurement
860 mL = ______ L

26. Units. Convert the measurement
3.1 g = ______ mg

27. Units. Convert the measurement
2 kg = ______ g

28. Units. Convert the measurement
2 m = ______ cm

29. Units. A bottle of Vitamin E contains 100 soft gels, each containing 10 mg of vitamin E. How many total grams of vitamin E are in this bottle?
There are __ grams of Vitamin E in the bottle.

30. Units. Convert the measurement
414 cm = ______ m

31. Units. Convert the measurement using the rules of SI prefixes.
1.5 terabyte = __________ gigabyte

32. Units. Convert 3.78 square meters to square centimeters.
_____ square centimeters

33. Units. A small rectangular panel measures 3.4 cm by 3.9 cm. What is its area in square millimeters?
_____ square millimeters

1.2 Accuracy and Precision

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Communicate results in mathematical notation and in language appropriate to the technical field

and covers the mathematical concepts

- Rounding (skill)
- Significant Figures (skill)
- Determining appropriate rounding from context (critical thinking)

While calculating devices will produce a lot of decimal places, these are not always meaningful nor useful. This section presents different purposes for rounding numbers, examples of using each one, and examples of interpreting numbers we consume in work and life.

First, we will consider what lots of decimal places do and do not mean which will lead to definitions, then we will present a method for reasonably tracking precision, then consider other motivations and matching methods for rounding, and later consider the importance of managing error.

1.2.1 Explanation

When working with measurements, we care about the reasonableness of the results. Suppose four people measure the length of a piece of wood and come up with 1.235 m, 1.236 m, 1.237 m, and 1.234 m. We might conclude that we are confident it is 1.23 m long but we are not certain about the millimeters position. This leads to the concepts of accuracy and precision.

Definition 1.2.1 Accuracy. The **accuracy** of a measurement is how close the measurement is to the actual value. ♢

Example 1.2.2 If the board referenced above is actually 1.2364 m long then all four measurements are accurate to the second decimal place. The second measurement (1.236 m) is accurate to the third decimal place. □

Example 1.2.3 Note $\frac{22}{7} \approx 3.142857$ is an approximation for π. Because π to six decimals places (not rounded) is 3.141592, this approximation is accurate to only 2 decimal places.

Note, π is not a measurement, rather it is defined theoretically. Thus we can produce an approximation that is as accurate as we have time and will to do. If curious, ask the nearest calculus instructor for details. □

Note, if we are measuring something, it is because it is not possible to know the actual value. In the example of measuring the board all we can do is use measuring tools and all such tools have a margin of error. The actual length of the board is a mystery. Because of this we cannot determine the exact value of many kinds of data nor determine how accurate our measurement is. Instead we will settle for repeatability. If we get the same result often enough, we can convince ourselves that it is accurate.

Definition 1.2.4 Error. The **error** of a measurement is the difference between the reported measurement and the actual value. ♢

Example 1.2.5 The number of people at an outdoor concert was 2453. If someone estimated that the number of people was 2500, then that estimate is accurate to 1000's place, but has an error of only $2500 - 2453 = 47$. □

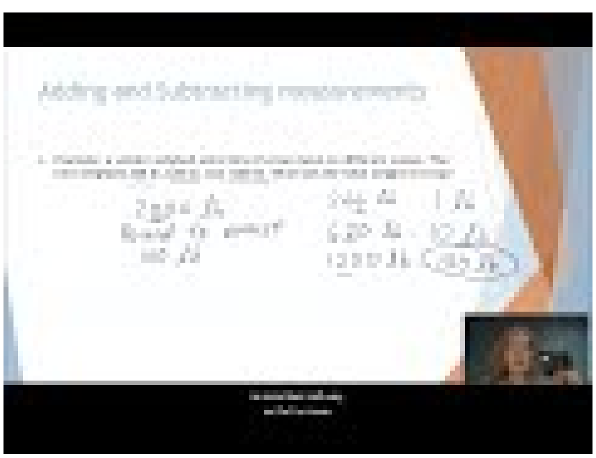
Definition 1.2.6 Precision. The **precision** of a measurement is the size of the smallest unit in it. ♢

Note we can have high precision with low accuracy. That is, just because we write a lot of decimal places does not mean that number is close to the actual value of the measurement.

Example 1.2.7 The answer to a homework question is 5.7632. If a response given is 5.7647 what are the precision and accuracy of the response?

Precision is effectively the number of decimal places. This is precise to 4 decimal places (the 10,000th position).

Because the response matches the actual value to the hundreds position, it is accurate to 2 decimal places. Because $5.7647 - 5.7632 = 0.0015$, the error is 0.0015. □

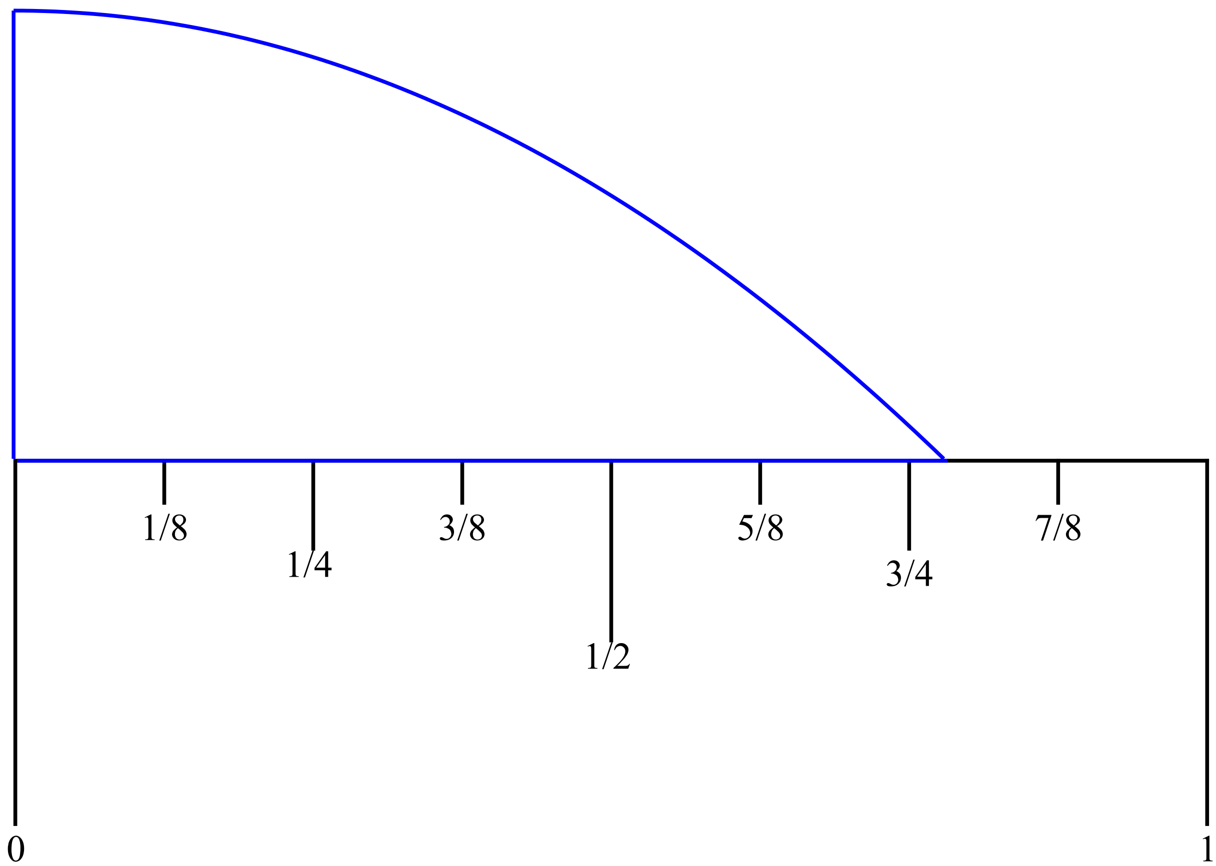


Standalone

Figure 1.2.8 Introduction to Precision and Accuracy

How do we end up with measurements parts of which are not accurate? Consider the following.

Example 1.2.9



When measuring the width of the blue, curved shape using the ruler (measurement in inches), it is clearly longer than 3/4'' and less than 7/8''. The right side of the shape appears to be less than half way between 3/4'' and 7/8''. Because it appears to be closer to 3/4'', we can state the width is 3/4''. Because the ruler does not have finer markings (e.g., 16ths or 32nds), we cannot be more precise.

We know this measurement is accurate to the nearest 1/8'', because the ruler has those marked and, in this case, we can be confident it is closer to the left side.

To estimate the error we note that the right edge is less than half way between the markings. Half way would be 13/16'' or 1/16'' farther. Thus we can state that the shape is 3/4'' wide with an error that is less than 1/16''. □

While other tools for measurement can be more precise, every tool has a limit to its precision that is similar to this example. We should always be aware of the limitations of measurements when we use them.

1.2.2 Significant Figures

It makes no sense to write numbers that are more precise than they are accurate. For example writing 3.142857 (from the approximation $\frac{22}{7}$) for π makes no sense, because it is only accurate to the hundreds position (3.14). It also makes no sense to perform arithmetic on digits that are not accurate. This section presents a reasonable way of tracking meaningful precision and rounding to maintain it. This will be used in most of the problems for the rest of the course.

When writing down measurements we need a way to indicate how precise the measurement is. **Significant digits**, also called **significant figures** or simply "sig figs", are a way to do this.

The rules for writing numbers with significant digits have two parts: non-zero digits, and zero digits.

1. All non-zero digits are signficant.

2. Zeros between non-zeros are significant.

3. Any zeros written to the right of the decimal point are significant.

4. If zeros between non-zero digits (on left) and the decimal point (on right) are supposed to be significant, a line is drawn over top of the last significant digit.

5. For numbers less than 1, zeros between the decimal point (on left) and non-zero digits (on right) are not significant.

We can summarize these rules as: write only digits that you mean, and if it is ambiguous clarify.

Example 1.2.10 Writing Significant Digits. Each of these numbers is written with five (5) significant digits.

- 10267
- 1.2400
- $7201\bar{0}$
- 2834100
- 0.0010527

□

Checkpoint 1.2.11 How many significant digits does 305 have? ___
How many significant digits does $2,6\bar{0}0$ have? ___
How many significant digits does 0.00138 have? ___

Solution.

- 3
- 3
- 3

We also need rules for arithmetic with significant digits. These are based on two principles

- A result of arithmetic cannot be more precise than the least precise measurement.
- The number of significant digits cannot increase.

For addition and subtraction the result (sum or difference) has the same precision as the least precise number added or subtracted. After adding or subtracting we round to the farthest left, last significant digit.

Example 1.2.12 Subtraction with Significant Digits. $11050 - 723 = 10330$. This is because the last significant digit of 11050 is the 10's position (with the 5 in it) whereas the last significant digit of 723 is the 1's position (with the 3 in it). We do not know the 1's position of 11050, so we cannot know the 1's position in the result. □

Example 1.2.13 Addition with Significant Digits. $311 + 8310 + 202200 \approx 210800$. This is because the farthest left, last significant digit is in the 100's position in 202200. The extra precision of the other two numbers is not useful. □

The significant digits addition/subtraction rule basically says that adding precise data to imprecise data does not increase the precision of the imprecise data. For those who are curious an explanation of why this rule works is in this video.

Standalone

Checkpoint 1.2.14 Calculate each of the following.

$951 + 46.33 + 540 =$ ____________________

$46.33 - 11 =$ ____________________

Solution.

- 1540
- 35

For multiplication and division the result (product or quotient) has the same number of significant digits as the least number of the input numbers.

Example 1.2.15 Division with Significant Digits. $11050/722 = 15.3$. This is because 722 has only 3 significant digits. □

Example 1.2.16 Multiplication with Significant Digits. $17 \times 14\bar{0} \times 3.178 = 7600$. This is because 17 has only two significant digits. □

The significant digit multiplication/division rule basically says that digits that were multiplied by imprecise data cannot be precise. For those who are curious an explanation of why this rule works is in this video.

Standalone

Checkpoint 1.2.17 Calculate the following. Round using significant digits.

$428 \times 20.41 =$ ____________________

$33.77 \div 81 =$ ____________________

Solution.

- 8,740
- 0.42

The rounding for significant digit rules is applied at the end of a calculation. That is if we have a mix of addition, subtraction, multiplication, and division then we do all of the operations, track the significant digits that should apply for each operation and apply the rounding at the end.

Example 1.2.18 Multi-Step Arithmetic with Significant Digits. Consider

$$11,728 + 39(17.9 + 1.23).$$

By order of operations we first calculate $17.9 + 1.23 = 19.13$. Note that the result 19.13 is significant only to the first decimal place. Second by order of operations we calculate $39 \times 19.13 = 746.07$. This is the product of a number with 2 significant digits and one with 3 significant digits so the result should have only 2 significant digits which would be the 10's place. The last calculation is $11,728+746.07 = 12,474.07$. 11,728 is significant to the one's place but the 746.07 is only significant to the 10's place. As a result the final result is rounded to the 10's place so

$$11,728 + 39(17.9 + 1.23) \approx 12,470.$$

□

Example 1.2.19 Calculate

$$21 \cdot 9 - \frac{1}{2} \cdot 9(21 - 5 - 4).$$

Note the 1/2 is an exact number here. The rest are measurements in centimeters.

$$\begin{aligned}
21 \cdot 9 - \frac{1}{2} \cdot 9(21 - 5 - 4) &= \\
21 \cdot 9 - \frac{1}{2} \cdot 9(12) &= && \text{all numbers are precise to the ones position} \\
189 - 54 &= && \text{only one significant digit remains in both numbers because 9 has only one} \\
105 &\approx && \text{189 was precise to only the hundreds position} \\
100.
\end{aligned}$$

□

Checkpoint 1.2.20 $10.9 \cdot 5.4 + 66.6 \cdot 8.4 =$ ____________________

Solution.

- 620

The two products multiply a number with 3 significant digits by a number with 2 significant digits. The result should have only 2 significant digits. For these products that will be the 100's and 10's positions.

The sum then is adding two numbers with the last significant digit in the 10's position, so the final result has its last significant digit in the 10's position.

$10.9 \cdot 5.4 + 66.6 \cdot 8.4 = 618.3 \approx 620$

1.2.3 Rounding

Significant digits uses rounding to remove non-useful precision. This section presents various motivations for rounding and types of rounding and motivations for each.

Table 1.2.21 Reasons for Rounding

Reality Constraints	For example we cannot buy partial packages or have fractional people
Remove Detail	For example when describing the population of a nation
Control Error	When used in significant digits

The reason for rounding determines how we do it. Consider the following reality constraints requiring rounding. For example if we need 21 eggs and eggs are sold in cartons of one dozen (12) eggs, we need

$21/12 = 1.75$ cartons. Since we cannot purchase part of a carton, we must round 1.75 up to 2, and purchase 2 cartons.

Note in this example reality requires us to round up to the nearest integer. We round to an integer because we cannot purchase fractional cartons of eggs. We had to round up, because rounding down would leave us with insufficient eggs (and we are hungry).

Suppose you have a bank account containing \$11410 that accrues 1.65% interest. The bank calculates the payment should be $\$11410 \cdot 0.0165 = \$188.265$. The bank will pay you \$188.26. They round to the nearest one hundredth because cents is a unit which can be paid. They round down, because they like paying less.

For removal of detail consider reporting the population of a country. We might report the population as over 9 million rather than 9,904,607. There are multiple motivations for this rounding. Note the population is likely changing multiple times per day, so more precision in the number does not equal more accuracy. Also, because of the scale (millions) the detail about how many ones, tens, hundred, and thousands looses meaning.

When reporting on salary ranges we might report a range between \$60,000 and \$80,000. That the range is actually \$61,233.57 and \$80,290.11 is unlikely to change a decision. The applicant will ask about the exact salary after deciding the position is a good fit. A common usage of removing detail is when we care about the scale of things rather than the count.

Rounding to control error is the use of significant digits.

Before considering context, we will practice rounding numbers. Note we can round to any digit. We can round up, down, or to the nearest number (what is meant by "round" if neither up nor down are specified). Context or instructions will specify which digit and which type of rounding.

Example 1.2.22 Rounding Up/Down.

(a) Round 72481 down to the nearest hundred.

Solution. 72400 is rounding down: we leave the 4 (hundred position) alone and "truncate" (turn to 0) all digits to the right. Note $72400 \leq 72481$.

(b) Round 72481 up to the nearest hundred.

Solution. 72500 is rounding up: we increase the 4 to a 5 and "truncate" (turn to 0) all digits to the right. Note $72500 \geq 72481$.

(c) Round 72481 the nearest hundred.

Solution. Because 72481 is closer to 72500 than it is to 72400, we round to 72500. We can recognize that we should round up because the tens position is $8 \geq 5$ which means rounding up results in a closer number. We could also recognize the need to round up by calculating $500-481 = 19$ and $481-400 = 81$ and noticing that $19 \leq 81$ (round up is closer).

□

Example 1.2.23 Rounding Up/Down.

(a) Round 72481 down to the nearest thousand.

Solution. 72000 is rounding down: we leave the 2 (thousands position) alone and "truncate" (turn to 0) all digits to the right. Note $72000 \leq 72481$.

(b) Round 72481 up to the nearest thousand.

Solution. 73000 is rounding up: we increase the 2 to a 3 and "truncate" (turn to 0) all digits to the right. Note $73000 \geq 72481$.

(c) Round 72481 to the nearest thousand.

Solution. Because 72481 is closer to 72000 than it is to 73000, we round to 72000. We can recognize that we should round up because the hundreds position is $4 < 5$ which makes it closer to go down. We could also recognize the need to round down by calculating $3000 - 2481 = 519$ and $2481 - 2000 = 481$ and noticing that $481 < 519$.

□

Example 1.2.24 Rounding to Different Precisions. Round 72321.83 to the specified precision.

- Thousands: 72000
- Ones:72322
- Tenths: 72321.8

□

Checkpoint 1.2.25 Round 735191 as indicated below.

To the 10's position: ____________________

To the 100's position: ____________________

Up in the 100's position: ____________________

Down in the 100's position: ____________________

Solution.

- 735190
- 735200
- 735200
- 735100

Next we need to consider when to use each type of rounding.

Example 1.2.26 Some floors are covered in carpet tiles. These are squares of carpet that are tiled to cover a floor. Suppose the carpet tiles are square with side length 20''. If a room is 50 feet by 38 feet, how many carpet square do we need?

First lets figure out how many tiles will go across the 50 feet. Note 50 feet is 50 ft $\times \frac{12 \text{ in}}{\text{ft}} = 600in$. This will require laying $\frac{600 \text{ in}}{20 \text{ in}} = 30$ tiles across.

Next, lets figure out how many tiles will go across the 38 feet. Note 38 feet is 38 ft $\times \frac{12 \text{ in}}{\text{ft}} = 456in$. This will require laying $\frac{456 \text{ in}}{20 \text{ in}} = 22.8$ tiles across. For each 0.8 of a tile we must cut a tile leaving only 0.2 of a tile left. This is too small to use elsewhere. Thus for each of these we will use a whole tile resulting in needing 23 tiles across (rounding up to have enough).

Finally we can count the number of tiles which is $30 \times 23 = 690$ tiles.

If you are wondering why we do not use four of the 0.2 parts of a tile to fill a space, it is because that would look bad. Also, with so many seams it is more likely to pull up. □

Example 1.2.27 Suppose you baked three (3) dozen cookies and are distributing them equally between 7 people. How many cookies does each person receive?

There are $3 \cdot 12 = 36$ cookies. Each person can have $36/7 \approx 5.1$ cookies. Because cutting cookies into pieces is typically a bad idea, we must round this down to 5 cookies per person.

Curious minds want to know what happens with the rest of the cookies. Notice there will be $7 \cdot 5 = 35$ cookies given away leaving just one cookie for the baker to enjoy. □

1.2.4 Greatest Possible Error

We have acknowledged that measurements will always have error. We have considered ways to round that are practical for the circumstances. Part of this depends on controlling the error. This section presents how to calculate the maximum error (worst case scenario). Typically we use this to ensure that error will not cause problems.

Because our rule for rounding is digits 0-4 round down and digits 5-9 round up, rounding will always have a greatest possible error of 5 in the position to the right of the one rounded. Consider the following.

Example 1.2.28 What is the greatest possible error if 130 was rounded to the nearest 10?

Solution. One possibility is that 130 was rounded down. Then the original number was one of 130, 131, 132, 133, or 134. 134 is the farthest away from 130 at $134 - 130 = 4$.

The other possibility is that 130 was rounded up. Then the original number was one of 125, 126, 127, 128, or 129. 125 is the farthest away at $130 - 125 = 5$.

Thus the greatest possible error was 5 from the case that 125 was rounded up.

Note in this solution we assumed the number rounded was an integer. However, if we allowed for 134.927 and 125.01 the result would be the same. the extra digits don't change the rounding. □

Example 1.2.29 What is the greatest possible error if 9.31 was rounded to the nearest hundredth?

Solution. The largest possible error is if 9.31 was rounded up from 9.305. Thus the greatest possible error is 5 one thousdandths. □

Example 1.2.30 What is the greatest possible error if 223 was rounded up to the nearest one?

Solution. 223 could have been rounded up from 222.1. But it could also have been rounded up from 222.01 or anything else. Thus the greatest possible error is less than 1 ($223 - 222 = 1$). □

Notice we have to know what type of rounding was used. In most measurements (i.e., significant digits) standard rounding will be used. For example think about measuring on a ruler: if the object isn't exactly on one of the lines, you will choose the closest one. The closest one requires rounding.

Checkpoint 1.2.31 What is the greatest possible error of 100000 if it was rounded to the nearest 1000? ___

Solution.

- 500

Checkpoint 1.2.32 What is the greatest possible error of $565,\bar{0}00$? ___

Solution.

- 50

1.2.5 Exercises

1. Significant Digits. How many significant figures does 52,800 s have?

____

2. Count Significant Digits. Tell how many significant digits there are in each measurement.

(a) 150.0 acres _____

(b) 20,305 ft _____

(c) 0.002 mm _____

(d) 423,000,000 ft _____

3. Count Significant Digits. How many significant digits does 3.990 590 0 have?

1
2
3
4
5
6
7
8

4. Count Significant Digits. Determine how many significant figures are in each measurement. If the measurement is exact, select "exact". Exact means there is no error in measurement.

(a) $449.96

i. 1

ii. 2

iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(b) 2ft 2in

i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(c) 1100.0 m

i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(d) 1199.07 mL

i. 1
ii. 2
iii. 3
iv. 4
v. 5

vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(e) 1100 m

i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

5. Count Significant Digits. Determine the accuracy (i.e., the number of significant digits) of this number: 0.01001
123
4
5

6. Count Significant Digits. Determine the accuracy (i.e., the number of significant digits) of this number: $39\bar{0},000$
123
4
5

7. Significant Digits Arithmetic. Calculate the product below, and express the result with the correct number of significant figures.
$5.2 \times 6.801 =$ _____

8. Significant Digits Arithmetic. Calculate the quotient below, and express the result with the correct number of significant figures.
47 &;div; 6.563 = _____

9. Significant Digits Arithmetic. Calculate the sum below, and express the result with the correct number of significant figures.
$0.295 + 20.39 + 445 =$ _____

10. Significant Digits Rounding. Round off the approximate number as indicated.
19.78; 2 significant digits
____

11. Greatest Error. Determine the GPE (i.e., the greatest possible measurement error) of this number: 0.102

(a) ± 0.5

(b) ± 0.05

(c) ± 0.005

(d) ± 0.0005

(e) ± 0.00005

12. Greatest Error. Given the measurement 4.8 gal, find the following.
Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ gal

13. Greatest Error. Given the measurement 0.003 ft, find the following.
Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ ft

14. Greatest Error. Given the measurement 379 psi, find the following.
Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ psi

1.3 Working with Applications

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Solve *linear*, rational, quadratic, and exponential equations and formulas (skill)
- Read and interpret models (critical thinking)
- Use models including linear, quadratic, exponential/logarithmic, and trigonometric (skill)

In life when we figure out processes at work or in science we often express the result in mathematical notation. This includes equations, functions, and other options. These are collectively known as models. They allow us to communicate what we know and calculate results as needed. To succeed in many jobs and to fully enjoy life we need to be proficient at reading and using models.

This section begins by presenting models and illustrating calculating some results from them. It progresses to solving equations (models) as a review of algebra skills. Finally we present tips on how to identify and use models arising in applications. These topics are continued with specific models in later sections.

1.3.1 Calculating Results using Models

Fact 1.3.1 Ohm's Law. *Ohm's Law relates three properties of electricity: voltage, current, and resistance.* ***Voltage****, measured in volts (V), is analagous to the amount of pressure to move the electrons.* ***Current****, measured in amperes (amps), is how much electricity is moving.* ***Resistance*** *measured in Ohms (Ω), is, as it sounds, the resistance of a material to letting electricity flow.*

The relationship is

$$V = IR$$

where

- *V is voltage,*

- *I is current, and*
- *R is the resistance.*

Example 1.3.2 If we know that the current is 3 amps and the resistance is 8 ohms then we can calculate

$$V = 3 \cdot 8 = 24.$$

Thus in this system there must be 24 volts.

If the current is increased to 6 amps on the 8 ohm circuit, then

$$V = 6 \cdot 8 = 48.$$

To double the amps, we would need to double the voltage.

Similarly if we know that the current is 1.7 amps and the resistance is 6 ohms, then we can calculate

$$V = 1.7 \cdot 6 = 10.2$$

Thus in this system there must be 10.2 volts. □

Fact 1.3.3 Lift Equation. *Lift is the force that keeps aircraft in the air.The lift equation explains factors that control the strength of lift produced by an airfoil (think wing or propellor). The factors included are air density, surface area of the airfoil, the coefficient of lift, and velocity. Air density is the amount of air per volume; you may see this as highs and lows on a weather map. It is also related to pressurizing aircraft flying at high altitude. The coefficient of lift incorporates multiple factors that are part of the design of the airfoil and how it is in use during flight.*

The lift equation is

$$L = \frac{1}{2}\rho s C_L v^2$$

where

- *L is the lift in units of lbs or Newtons)*
- *ρ is air density in units of slugs per cubic feet or kilograms per cubic meter*
- *s surface area in units of square feet or square meters*
- *C_L is the coefficient of lift which is unitless*
- *v is velocity in units of feet per second or meters per second*

Example 1.3.4 If we know that air density is 0.002378 slugs per cubic feet, surface area is $125\,\mathrm{ft}^2$, $C_L = 1.5617$, and velocity is $84.4\,\frac{\mathrm{ft}}{\mathrm{s}}$, we can calculate the lift.

$$L = \frac{1}{2} \cdot 0.002378 \cdot 125 \cdot 1.5617 \cdot 84.4^2 \approx 1650.$$

Under these circumstances this airfoil can lift 1650 lbs.

If the air density is reduced to 0.001988 slugs per cubic feet then the lift is

$$L = \frac{1}{2} \cdot 0.001988 \cdot 125 \cdot 1.5617 \cdot 84.4^2 \approx 1380.$$

This represents the same aircraft flying 6000 feet higher (hence lower air density). Notice that without changing other factors (like velocity), it can lift (hold in the air) $1650 - 1380 = 270$ lbs less. □

Fact 1.3.5 Ideal Gas Law. *The ideal gas law is a relationship between the volume, pressure, temperature, and number of molecules of an ideal gas. The relationship is*

$$PV = nRT$$

where

- *P is the pressure in units of atmospheres (atm) or Pascals (Pa)*
- *V is the volume in units of cubic feet or cubic meters*
- *n is the number of moles (number of molecules, see a chemistry text for details)*
- *R is a constant specific to each gas (e.g., oxygen and nitrogen have different ones) in units that match the other values*
- *T is the temperature in degrees Rankine or Kelvin (these are shifted versions of Fahrenheit and Celsius).*

When the number of molecules remains fixed, such as in a closed container, this law can be used to produce the equation

$$\frac{P_1V_1}{T_1+273} = \frac{P_2V_2}{T_2+273}.$$

where

- *P_1, V_1, and T_1 are the initial pressure, volume, and temperature, and*
- *P_2, V_2, and T_2 are the pressure, volume, and temperature at another time.*

Note in both forms of the law the units can be other than those listed (especially different scale like centimeters rather than meters). However, they must always match including the constant R which is looked up in reference books.

For rounding note 273 is part of the definition of the Kelvin temperature scale, so it has infinite precision. It will not have any affect on signficant digits calculations.

Example 1.3.6 Suppose the initial conditions are $P_1 = 101.3$ Pa, $V_1 = 0.125$ m^3, and $T_1 = 10.2°$ C. Also $V_2 = 0.125$ m^3 and $T_2 = 50.7°$ C. We can calculate the new pressure.

$$\begin{aligned}
\frac{101.3 \cdot 0.125}{10.2+273} &= \frac{P_2 \cdot 0.125}{50.7+273}\\
0.0447122 &\approx 0.000386160P_2\\
\frac{0.0447122}{0.000386160} &\approx \frac{0.000386160P_2}{0.000386160}\\
115.8 &\approx P_2
\end{aligned}$$

This is a scientific calculation so the rounding is significant digits.

If instead $T_2 = -10.3°$ C, then we have the following.

$$\begin{aligned}
\frac{101.3 \cdot 0.125}{10.2+273} &= \frac{P_2 \cdot 0.125}{-10.3+273}\\
0.0447122 &\approx 0.000475828P_2\\
\frac{0.0447122}{0.000475828} &\approx \frac{0.000475828P_2}{0.000475828}\\
94.0 &\approx P_2
\end{aligned}$$

□

Checkpoint 1.3.7 Recall Ohm's Law states $V = IR$.

Calculate the voltage if the current is $I = 2.7$ and the resistance is $R = 8$. $V =$ ____

Solution.

- 21.6

$V = IR.$
$V = (2.7)(8).$
$V = 21.6.$

Checkpoint 1.3.8 Note $\dfrac{V_1P_1}{T_1+273}=\dfrac{V_2P_2}{T_2+273}$ with temperature in Celsius.

If $V_1=V_2=0.250$, $P_1=2322$, $T_1=11.0$, and $T_2=29.0$, what is P_2? ______

Solution.

- 2,470

$$\begin{aligned}&\frac{V_1P_1}{T_1+273}=\frac{V_2P_2}{T_2+273}.\\&\frac{0.250\cdot 2322}{11.0+273}=\frac{0.250\cdot P_2}{29.0+273}.\\&\frac{0.250\cdot 2322}{11.0+273}\cdot\frac{29.0+273}{0.250}=P_2.\\&2469.1690140845=P_2.\\&2470\approx P_2.\end{aligned}$$

1.3.2 Calculating Results Requiring Solving

The previous section illustrated calculating model results without solving. This section presents additional example requiring limited solving and finishes with solving before any values have been substituted.

It does not matter if the value we desire is by itself, we can solve using arithmetic.

Example 1.3.9 Recall the model for lift. Suppose we know the weight of the aircraft ($w = 2390$ lbs), the density of air ($\rho = 0.001869$ slugs/ft^3), wing surface area ($s = 165$ ft^2), and velocity ($v = 91.1$ ft/sec). Noting that lift must equal weight, what must the coefficient of lift be?

$$\begin{aligned}L&=\frac{1}{2}\rho s C_L v^2\\2390&=\frac{1}{2}(0.001869)(165)C_L(91.1)^2. && \text{Perform arithmetic.}\\2390&=1279.675938C_L.\\\frac{2390}{1279.675938}&=\frac{1279.675938C_L}{1279.675938} && \text{undo multiplication.}\\1.867660342&=C_L.\\1.87&\approx C_L.\end{aligned}$$

□

The desired value from the model may be in a denominator. We can solve for this using multiplication and division.

Example 1.3.10 Recall that under simplifying assumptions

$$\frac{P_1V_1}{T_1+273}=\frac{P_2V_2}{T_2+273}.$$

See Fact 1.3.5 for details. Suppose we know the initial conditions ($P_1 = 1.00$ atm, $V_1 = 1.35$ ft^3, $T_1 = 51.2°$ F) and also $P_2 = 1.00$ atm and $V_2 = 1.39$ ft^3. What must the new temperature (T_2) be?

$$\begin{aligned}\frac{1.001.35}{51.2+273}&=\frac{1.001.39}{T_2+273} && \text{Perform arithmetic..}\\0.004164096237&=\frac{1.39}{T_2+273}.\\0.004164096237(T_2+273)&=\frac{1.39}{T_2+273}(T_2+273). && \text{Multiply to move } T_2 \text{ out of denominator.}\\0.004164096237(T_2+273)&=1.39.\end{aligned}$$

$$\frac{0.004164096237(T_2+273)}{0.004164096237} = \frac{1.39}{0.004164096237}. \qquad \text{Divide to undo multiplication.}$$
$$T_2 + 273 = 333.8059259.$$
$$T_2 + 273 - 273 = 333.8059259 - 273. \qquad \text{Subtract to undo addition.}$$
$$T_2 = 60.8059259.$$
$$T_2 = 60.8.$$

Note we use significant digits for rounding because this is a science model. □

The previous examples solved for a variable in a model after substituting numbers for the other variables. The next examples illustrate solving first. Note this process is the same as solving after substituting (same algebra) though there may be more steps. We might wish to solve this way, so it is easier to use the model multiple times.

Example 1.3.11 Solve the equation $V = IR$ for R. Note, this model is explained in Fact 1.3.1.

$$V = IR.$$
$$\frac{V}{I} = \frac{IR}{I} \qquad \text{Divide to undo multiplication.}$$
$$\frac{V}{I} = R.$$

□

Example 1.3.12 Solve the lift equation $L = \frac{1}{2}\rho S C_L v^2$ for S.

$$L = \frac{1}{2}\rho S C_L v^2.$$
$$2L = 2\frac{1}{2}\rho S C_L v^2. \qquad \text{Multiply to undo division.}$$
$$2L = \rho S C_L v^2.$$
$$\frac{2L}{\rho} = \frac{\rho S C_L v^2}{\rho}.\text{Divide to undo multiplication.}$$
$$\frac{2L}{\rho} = S C_L v^2.$$
$$\frac{2L}{\rho C_L} = \frac{S C_L v^2}{C_L}.$$
$$\frac{2L}{\rho C_L} = S v^2.$$
$$\frac{2L}{\rho C_L v^2} = \frac{S v^2}{v^2}.$$
$$\frac{2L}{\rho C_L v^2} = S.$$

Notice that the steps are the same algebra as if there were numbers. Also we could divide by v^2 and we are not concerned with the square as part of solving for S. □

Checkpoint 1.3.13 Solve the lift equation $L = \frac{1}{2}\rho S C_L v^2$ for ρ.

1.3.3 Process Overview

Above we started with a model and were asked to do something with it. Normally we will start with a problem which does not identify a model to use. Very few problems you encounter in lifte come pre-labled with models. This section presents how to start with a problem and work to a solution by identifying the model first.

Our first task is to read the problem to understand it.

- Read the problem description a few times.
  - If you can paraphrase it, you understand it enough.
  - Drawing a picture and labeling parts may help.
- Identify what we are asked to do.
- Identify the information we are given. Note distinctions like measurements and rates.
- Identify any units. These often help us set up a model.
- Write everything! We do not model in our heads.

Next we write the mathematical model (equation or function).

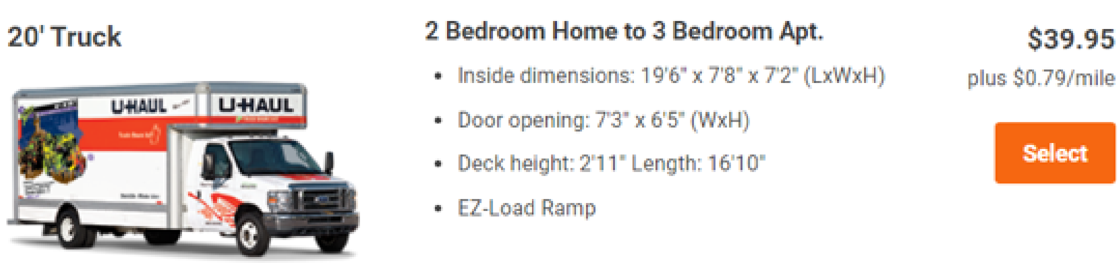
- Use the description to determine which application type (e.g., percent, proportion, linear model, etc.). Note units can suggest this (e.g., meters and meters squared indicate something was squared).
- Do not insert any numbers yet.
- Do not do any calculations yet.

Now we will have a model that matches our situation and possibly some numbers to insert.

- Insert numbers into the model. You may have to calculate some of these (e.g., you are given two points but not the slope you need).
- Solve for the desired value. Note it may help to do some calculations with the numbers first.
- State your answer and use units appropriately.

Finally we should check that our answer makes sense. We should not have negative prices (usually) or distances larger than the earth (when working with terrestrial problems).

Example 1.3.14 You moved across town and rented a 20 foot moving truck for the day. You want to make sure the bill you received is correct. If you paid \$81.03, for how many miles were you charged? Assume there were no extra fees.



Solution. We want to compare the bill we received to the price listed in the add. The question is about how many miles (not how much money).

We are given the price per mile (\$0.79 per mile). There is also a fixed cost for the rental (\$39.95). Adding the fixed cost and the milage cost will give us the total.

Our model is $C = \$39.95 + \$0.79m$ where C is the total cost and m is the number of miles.

We know the total cost, which will leaves m in the equation, the number of miles, which is what we want to calculate. We can use the solving technique in Section 3.1

$$\$81.03 = \$39.95 + \$0.79m$$

$$
\begin{aligned}
-\$39.95 + \$81.03 &= -\$39.95 + \$39.95 + \$0.79m \\
\$41.08 &= \$0.79m \\
\frac{\$41.08}{\$0.79} &= \frac{\$0.79}{\$0.79}m \\
52 &= m.
\end{aligned}
$$

The charge is for 52 miles. □

Standalone

Figure 1.3.15 Using math modeling for rental truck

1.3.4 Exercises

1. Minimum Grade. A student in a Pre-Calculus class has test scores of 68, 60, 69, and 78. The final exam is worth 3 test grades.

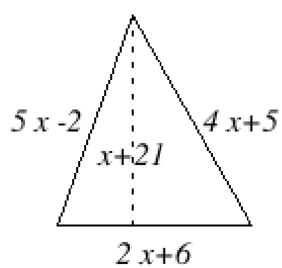
Write a linear equation that models this problem, where x is the grade on the final exam and y is the student's grade in the course. The whole grade is based on these tests and the final.

____________________Preview Question 1 Part 1 of 2

What grade is needed on the final to earn a B (average score of 80%)?

____

2. Triangle Properties.



Consider the triangle shown on the picture. Find the value of x, given that the perimeter of the triangle is 42 unit.

$x =$ _______ unit

Preview Question 1

3. Bike Rental. Amanda rented a bike from Ted's Bikes.

It costs $9 for the helmet plus $3.75 per hour.

If Amanda paid about $25.88, how many hours did she rent the bike?

a) Let h = the number of hours she rented the bike. Write the equation you would use to solve this problem.

____________________Preview Question 1 Part 1 of 2

b) Now solve your equation

Amanda rented the bike for ____________________ hours. (Round your answer to the nearest tenth of an hour.)

4. Pilot Training. Part 1 of 5

The cost of a private pilot course is \$1425. The flight portion costs \$461 more than the ground school portion. What is the cost of the flight portion alone?

a) Let x represent the cost of the ground school portion. Write a variable expression to represent the cost of the flight portion.

____________________Preview Question 1 Part 1 of 7

Part 2 of 5b) The total cost of the private pilot course can be represented by: Cost of ground school portion + Cost of flight portion = Total Cost

Fill in the boxes using the information from the problem and your expressions from part a:

Table 1.3.16

Cost of Ground School	+	Cost of flight portion
____________________Preview Question 1 Part 2 of 7	+	____________________Preview Question 1 Part 3 of 7

Part 3 of 5

c) Solve the equation $x + (x + 461) = 1425$ to answer the question.

$x =$ ____________________

Part 4 of 5

d) Since $x = 482$, this tells us:

(a) Flight school portion costs \$482

(b) Ground school portion costs \$482

Part 5 of 5

e) We used the expression $x+461$ to represent the cost of the flight portion. Knowing that $x = 482$, what is the cost of the flight portion alone?

Flight portion costs \$____________________

5. Thunder. In a thunderstorm, the formula:

$M = \frac{t}{5}$

gives the approximate distance, M, in miles, from a lightning strike if it takes t seconds to hear the thunder after seeing the lightning. If you are 3.7 miles away from the lightning flash, how long will it take the sound of the thunder to reach you.

Answer: It will take ______ seconds for the sound to reach you.

6. Speeding. In a Northwest Washington County, speeding fines are determined by the formula:

$F = 7(s - 60) + 40$

where F is the cost, in dollars, of the fine if a person is caught driving at a speed of s miles per hour.

If a fine comes to \$278, how fast was the person speeding?

Answer: The person's speed was ______ miles per hour.

7. Area of Triangle. The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where A is the area of the triangle, b is its base and h is its height.

Solve the formula, $A = \frac{1}{2}bh$, for b.

$b =$ ____________Preview Question 1 Part 1 of 2

Find the base of the triangle with height of 5 meters and area of 137 square meters. Write your answer as a decimal, rounded to the nearest hundredth, when necessary.

base = ______Preview Question 1 Part 2 of 2 meters

8. Unit cost. You and your classmates create 24 sculptures to sell at the PTA auction to raise money for your school. In order to pay for the materials you bought to make the sculptures, you need to sell each of the sculptures for \$4.75. When transporting the sculptures to the auction, one of your parents accidentally drops five of the sculptures in the street and they are run over by a Amtrak train. What is the new minimum price you need to set for the remaining sculptures in order to pay for the materials?

Keep in mind that you do not want to collect less money than you paid for the materials. This may affect how you round your answer.

\$____________________

9. Paychecks. Your weekly paycheck is 25 percent less than your coworker's. Your two paychecks total 775. Find the amount of each paycheck.

Your coworker's is : \$ __________ and yours is \$ __________ .

Given your answers to the nearest cent

10. Mixture. The radiator in a car is filled with a solution of 65 per cent antifreeze and 35 per cent water. The manufacturer of the antifreeze suggests that for summer driving, optimal cooling of the engine is obtained with only 50 per cent antifreeze.

If the capacity of the radiator is 3.7 liters, how much coolant (in liters) must be drained and replaced with pure water to reduce the antifreeze concentration to 50 per cent?

Round your answer to two significant figures.

____________________Preview Question 1 L

11. Area of Rectangle. A rectangular garden is 30 ft wide. If its area is 1800ft^2, what is the length of the garden?

Your answer is : ____________________Preview Question 1 ft

12. Shelving. A bookshelf containing 7 bookshelves is to be constructed. The floor-to-ceiling clearance is 9 ft 3.0 in. Each shelf is 1.0 in thick. An equal space is to be left between the shelves, the top shelf and the ceiling, and the bottom shelf and the floor. (There is no shelf on the ceiling or floor.)

What space should be between each shelf and the next? Round your answer to the nearest tenth of an inch.

____________________ in

13. Fax Cost. An online fax company, EFaxIt.com, has a customer plan where a subscriber pays a monthly subscription fee of \$18 dollars and can send/receive 120 fax pages at no additional cost. For each page sent or received past the 120 page limit, the customer must pay an overage fee of \$0.14 per page. The following expression gives the total cost, in dollars, to send p pages beyond the plan's monthly limit.

$C = 0.14p + 18$

If the monthly bill under this plan comes out to be \$26.54, what was the total number of pages that were sent or received?

Answer: The total number of pages sent/received was ____________________.

14. Population Decrease. The current population of a small city is 27000 people. Due to a loss of jobs, the population is decreasing by an average of 275 people per year. How many years (from now) will it take for the population to decrease to 22050 people?

A) Write an equation you can use to answer this question. Be sure all the numbers given above appear in your equation. Use x as your variable and use no commas in your equation.

The equation is ____________________Preview Question 1 Part 1 of 2

B) Solve your equation in part [A] for x.

Answer: $x =$ ____________________

15. Sales. After a 40% reduction, you purchase a new digital player for \$171. What was the price of the digital player before the reduction?

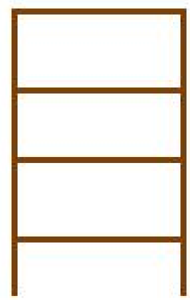
A) First write an equation you can use to answer this question. Use x as your variable and express any percents in decimal form in the equation.

The equation is ____________________Preview Question 1 Part 1 of 2

B) Solve your equation in part [A] to find the original price of the digital player.

Answer: The original price of the digital player was ____________________ dollars.

16. Bookcase. A bookcase is to have 4 shelves including the top as pictured below.



The width is to be 9 feet less than 2 times the height. Find the width and the height if the carpenter expects to use 24 feet of lumber to make it.

Width: _____ feet

Height: _____ feet

17. Knitting. It takes Rylla 18 hours to knit a scarf. She can only knit for 1.5 hours per day. How many days will it take her to knit the scarf?

Part 1: Let x be the number of days it will take her to knit the scarf. Choose the correct translation of this problem into an equation:

(a) 1.5 x=18

(b) x=(1.5)(18)

(c) 18-1.5=x

Part 2: Solve for x.

____________________Preview Question 1 Part 2 of 2

18. Rental Cost. A rental car company charges $30 plus 45 cents per each mile driven.

Part1. Which of the following could be used to model the total cost of the rental where m represents the miles driven.

(a) $C = 0.45m + 30$

(b) $C = 45m + 30$

(c) $C = 4.5m + 30$

Part 2. The total cost of driving 350 miles is;

$____________________Preview Question 1 Part 2 of 2

19. Mixture. You need a 55% alcohol solution. On hand, you have a 385 mL of a 65% alcohol mixture. How much pure water will you need to add to obtain the desired solution?

A) Write an equation using the information as it is given above that can be used to solve this problem. Use x as your variable to represent the amount of pure water you need to use. Equation: ______________________________Preview Question 1 Part 1 of 3

B) You will need

_____ mL of pure water

to obtain

_____ mL of the desired 55% solution.

20. Mixture. 14.5 liters of fuel containing 2.7% oil is available for a certain two-cycle engine. This fuel is to be used for another engine requiring a 5% oil mixture. How many liters of oil must be added?

Give your answer to 3 significant digits.

____

21. Wire Cutting. A wire 29 cm long is cut into two pieces. The longer piece is 7 cm longer than the shorter piece.

Find the length of the shorter piece of wire

____________________ cm

1.4 Project: Literal Formula

Project 1 Literal Formula. Most math books define the area of a circle as follows: $A = \pi r^2$, where A is the area of the circle and r is the radius of a circle. A text used in UAA's Automotive Diesel program defines the area of a circle as $A = 0.7854d^2$, where A is the area of the circle and d is the diameter of the circle.

The purpose of this project is to determine when each formula is most useful.

(a) What is the mathematical relationship between radius and diameter? Your answer can be a sentence or an equation.

(b) Show mathematically how to get from the formula $A = \pi r^2$ to the formula $A = 0.7854d^2$. This should take you multiple steps.

(c) Explain in words what you did in each step to change the first formula into the second. What assumptions did you have to make? Anyone reading this answer should be able to replicate the math by just reading your answer. That is, talk me through all the steps.

(d) Did you have any false starts or did you see how to change the formula right away? There is no wrong answer here; I just want you to think about your process.

(e) For this problem, you will need a tape measure or a ruler. If doing this on a device it must be a computer and ensure you are at 100% magnification. Your phone or a scaled version will distort the results. First *measure* the radius of the circle in Figure 1.4.1. Then *measure* the diameter of the circle below and record your answer. ***Do not calculate the diameter!*** This must be measured, not calculated. Try to be as precise as is reasonably possible. Include units.

Was it easier to measure the radius or the diameter?

(f) What is one reason why it might be more practical on a job to use the formula $A = 0.7854d^2$ instead of $A = \pi r^2$? If it helps, you may wish to ask yourself why the auto diesel students in particular use this less traditional formula.

(g) Determine how many significant figures are in each measurement. If the number is not a measurement or the measurement has no error then it is called 'exact'.

- π
- 0.7854

(h) Which of the two formulas is more accurate? Which is more precise? Give a reason to back up your answer.

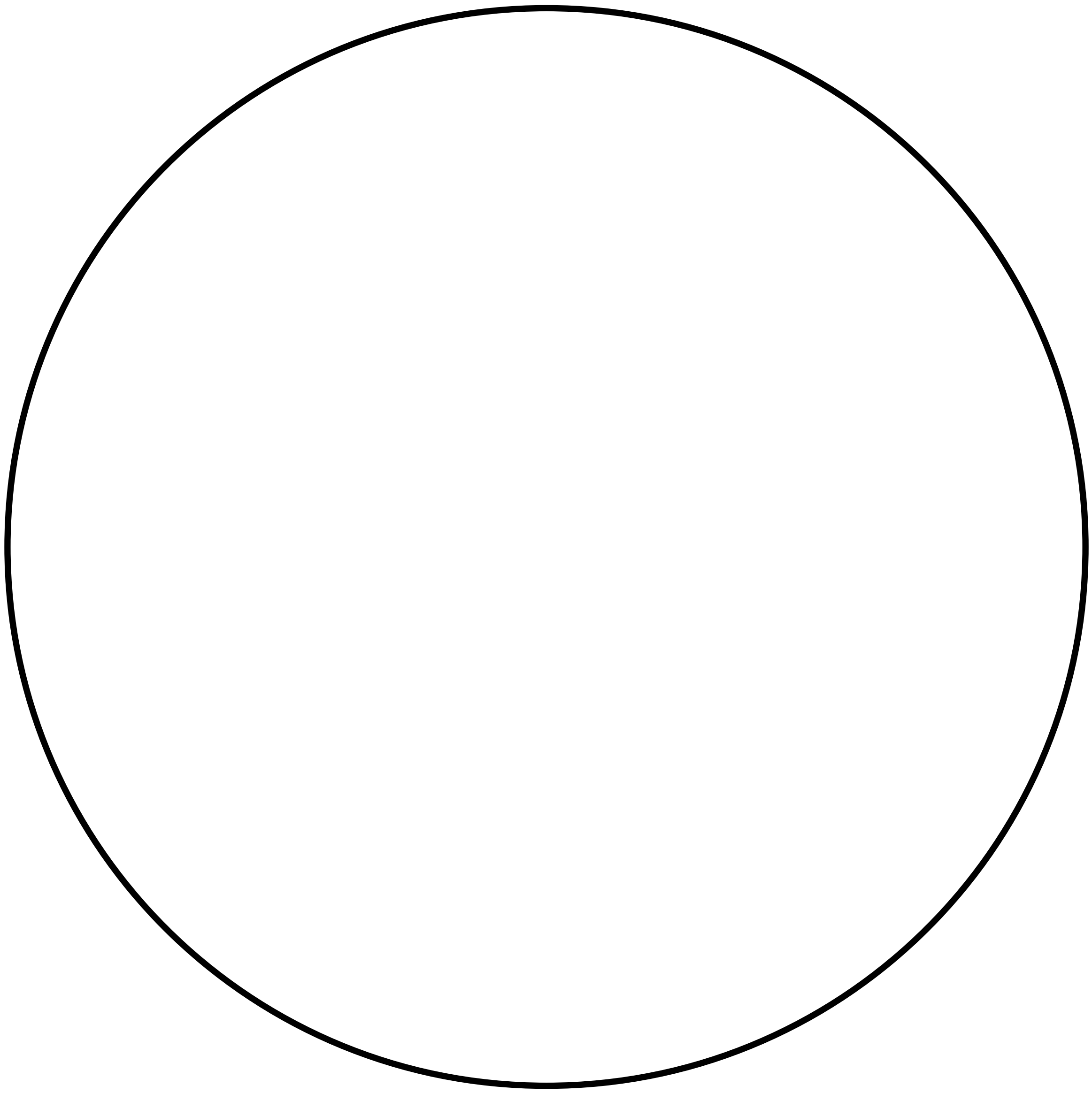


Figure 1.4.1 Circle

Chapter 2

Ratios

2.1 Percents

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Communicate results in mathematical notation and in language appropriate to the technical field

and covers the mathematical concepts

- Calculate Percentages (skill)
- Understand and interpret percentages (critical thinking)

Percentages are an often convenient way to express the relative size of two quantities such as the number of people who like lemon meringue pie to the number of those who like pie.

We will learn how to calculate a percent (Subsection 2.1.1, Item 2.b), how to convert between percents and the numbers (also Subsection 2.1.1, Item 2.b), how to describe growth in terms of percents (Subsection 2.1.2, Item 2.c), and how to recognize what a percent does and cannot tell us (Subsection 2.1.3, Item 2.c).

2.1.1 Calculating Percents

Definition 2.1.1 Percent. A **percent** is a ratio of part of something to the whole of that thing that is written as parts per hundred. ◊

Example 2.1.2 Calculate a Percent. In a class there are 34 students. Of them 21 are female. In this case female students is part of the whole (all students). Thus the percent is calculated as

$$\frac{\text{part}}{\text{whole}} = \frac{21}{34} \approx 0.6176.$$

This number says there are 61 hundreths (remembering our numbering system), so the percent is written as 61.76%.

Rounding to two (2) decimal places was chosen to illustrate how we convert a ratio in decimal form to a percent. If we were reporting this information we would most likely round to 62%. This would convey the same meaning because the difference between 61.76% and 62% for 34 people is less than one person. □

Standalone

Generally, we calculate a percent by

$$100 \times \frac{\text{part}}{\text{whole}}.$$

Example 2.1.3 In the class there are 34 students. Of them 13 are male. The percent is calculated as

$$100 \times \frac{13}{34} = 100 \times 0.3824 = 38.24\%.$$

□

Now that we have presented two examples of calculating a percent from counts, use the check point below to test that you can setup and calculate one yourself.

Checkpoint 2.1.4 In another class there are 76 students and 44 are female. What percent of the students are female? ______

Solution.

- 58

A percent is the ratio of part (44 female) to whole (76 total). So this is $\frac{44}{76} \approx 0.5789$. This is 58 hundreths, so it is 58%

Note in the first pair of examples we had a whole class of 34 students with 21 female and 13 male. Of course $21+13 = 34$, that is the two parts add up to the whole. Because of this $61.76\% + 38.24\% = 100\%$ as well.

Sometimes we are given the size of the whole and a percent, and we are interested in calculating how many are in the part.

Example 2.1.5 In a class of 22 students, 18% are Alaska Native. How many students are Alaska Native?

Solution 1. We use the same setup as before, but we don't know the part yet.

$$\begin{aligned} 100 \cdot \frac{P}{22} &= 18\% \\ \frac{P}{22} &= \frac{18}{100} \\ P &= 22 \cdot \frac{18}{100} \\ &\approx 3.96. \end{aligned}$$

Notice that 3.96 does not make sense as a result when counting people, so we expect that the correct result is 4. We can confirm this by checking that

$$\frac{\text{part}}{\text{whole}} =$$

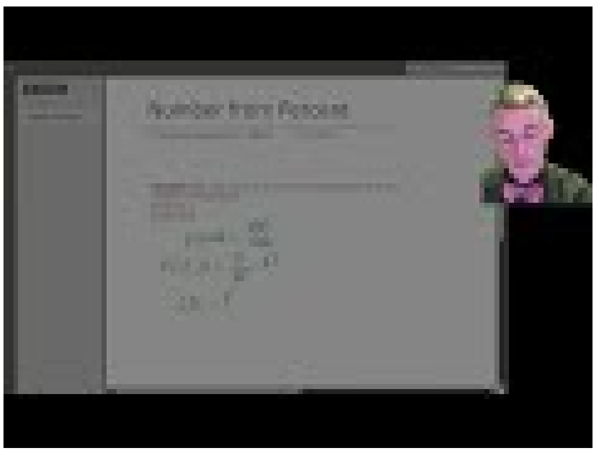
$$\begin{aligned}\frac{4}{22} &= 0.18\overline{18}\\ &\approx 18.18\%\end{aligned}$$

This suggests that the original 18% was rounded. Likely it was rounded to the ones position out of convenience.

Solution 2. We know that a percent is a number out of 100, so we can skip a step from the previous example.

$$\begin{aligned}\frac{P}{22} &= 0.18\\ P &= 22 \cdot 0.18\\ P &= 3.96\end{aligned}$$

□



Standalone

Checkpoint 2.1.6 There are 45 students in a class. Below are percents for each racial group tracked. Calculate the number of students in each group.

Table 2.1.7

Group	Percent	Number
Alaska Native	6.25%	___
Asian	12.5%	___
Black	6.25%	___
White	71.88%	___
Other	2.22%	___

Solution.

- 3
- 6
- 3
- 32
- 1

Sometimes we know the size of a part and what percent it is of the whole. From this information we can calculate the size of the whole.

Example 2.1.8 In a class 2 Alaska Native students make up 6.25% of the class. How many students are in the class?

Solution. Again we use the same setup, but we don't yet know the whole.

$$\begin{aligned}
\frac{\text{part}}{\text{whole}} &= \text{percent}.\\
\frac{2}{W} &= 0.0625.\\
2 &= 0.0625 \cdot W.\\
\frac{2}{0.0625} &= W.\\
32 &= W.
\end{aligned}$$

□

Example 2.1.9 How to Use an Example: Percents. Consider the following question.

If the first chapter of a certain book is 18 pages long and makes up 2% of the book, how many pages does the entire book have?

Because we see "2%" we recognize this as a percent problem. Without more information we can begin writing our steps. In Example 2.1.8 the first step is writing the definition of percent.

$$\frac{\text{part}}{\text{whole}} = \text{percent}$$

In the example 0.0625 is written on the right (in place of percent). In this problem we know the percent is 2, and we know that in a calculation we write the percent as a decimal. Thus our next step is

$$\frac{\text{part}}{\text{whole}} = \frac{2}{100}$$

In the next step in the example the entries for part and whole are entered. In this problem the 18 pages is stated as one chapter and is contrasted to the "entire" book. Thus the 18 is the part. As with the example, the whole is not known so we leave it as a variable.

$$\frac{18}{\text{whole}} = \frac{2}{100}$$

Finally in the example they solve for the variable. Note the steps of solving may vary depending on what we know, so rather than follow the rest of the example, we apply our algebra skills. For convenience we will write W instead of "whole".

$$\begin{aligned}
\frac{18}{W} &= \frac{2}{100}\\
\frac{18}{W}W &= \frac{2}{100}W\\
18 &= \frac{2}{100}W\\
\frac{100}{2} \cdot 18 &= \frac{100}{2} \cdot \frac{2}{100}W\\
900 &= W
\end{aligned}$$

Thus we know the entire book has 900 pages. □

[Standalone]

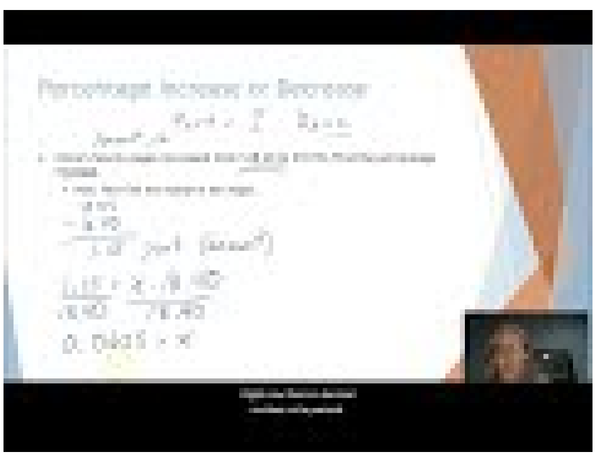
In this next check point the terminology is different but something is still part of a whole and the amount can be calculated using the same approach as above.

Checkpoint 2.1.10 Find the number of millilitres of alcohol needed to prepare 140 mL of solution that is 7% alcohol. ______

Solution.

- 9.8

We need to know what 7% of 140 mL is. Percent = part/whole, and we know the percent and whole.
$\frac{7}{100} = \frac{A}{140}$
$140 \cdot \frac{7}{100} = A$
$9.8 = A$
This video covers the topics above.



[Standalone]

2.1.2 Percent Increase/Decrease

A common use of percents is to indicate how much something has increased (or decreased) from one time to the next.

Example 2.1.11 In spring there were 22 students in a class. In the following fall there were 34 students in the same class. This was an increase of 34-22=12 students. We can calculate what percent the increase of 12 is with respect to the original (spring) class size of 22.

$$100 \times \frac{12}{22} = 55\%$$

□

We say that the class size had a **percent increase** of 55%. Note this says the **increase** was 55% of the previous **whole**.

We can think of this in another way.

Example 2.1.12 In spring there were 22 students in a class. In the following fall there were 34 students in the same class.

We calculate the percentage the fall class size is with respect to the spring class size.

$$100 \times \frac{34}{22} = 155\%$$

Because the fall class size (in the role of "part") is greater than the spring class size (in the role of whole), the percent ends up being greater than 100%. For percent increase we should always expect a percent greater than 100%.

Because this is 55% greater than 100%, the percent increase was 55% over the previous semester. □

Checkpoint 2.1.13 What is the percent increase if enrollment in a class was 53 in spring and 87 in the following fall? _____

Round to the nearest percent (ones).

Solution.

- 64

Because 87 is bigger than 53, we know this is a percent increase.

The increase is $87 - 53 = 34$. Thus the percent increase is

$100 \cdot \dfrac{34}{53} = 64$

Example 2.1.14 What is the percent increase or decrease if enrollment in a class was 78 in fall and 38 in the following spring?

Solution 1. Because 38 is less than 78 this is a decrease. Similar to the percent increase we can calculate the decrease first and then calculate the percent. $78 - 38 = 40$. Thus the percent decrease is

$$100 \cdot \frac{40}{78} = 51\%.$$

Solution 2. As with the percent increase we can start by simply computing what percent the fall enrollment is with respect to the prior spring enrollment. The ratio is $100 \cdot \frac{38}{78} \approx 0.49$. Because the new enrollment *is* 49% of the previous enrollment the decrease is 100%-49%=51%. □

Checkpoint 2.1.15 Suppose enrollment in a class was 50 in the fall and 51 in the following spring.

It was a ____________________ percent

1. decrease

2. increase

Solution.

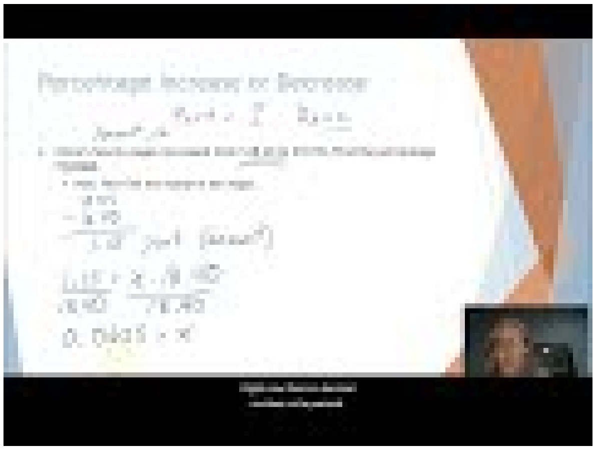
- 2

- B: increase

Because 51 is bigger than 50 it is a percent increase.

The percent increase is $100\left(\dfrac{51-50}{50}\right) = 100 \cdot \dfrac{1}{50} \approx 2\%$.

This video covers percent increase topics.



Standalone

2.1.3 Limitations

We use percents because they can make the difference in scale between two quantities clear to us. However presenting a percent by itself can be deceptive.

Example 2.1.16 Which of the following do you suppose represents a greater reduction in students?

Percent reduction	Total
18%	495
1.85%	54
60%	5

Solution.

Percent reduction	Total	Number reduced
18%	495	90
1.85%	54	1
60%	5	3

The 18% of 495 represents the largest number of students. The 60% is a higher percent, but because the total is so small it represents very few students. A percent is more useful if we also know the total number.

Did you calculate 89 for 18% of 495? Compare the following to see why both are reasonable responses. 89 is what percent of 495? 90 is what percent of 495? □

While percent is defined a parts per one hundred, there are times when percents, sensibly, add to more than one hundred.

Example 2.1.17 Table 2.1.18 contains data from the 2020 U.S. Census. It contains the percent of the state population who checked the box for that race. Note the total is 149.4%. The reason is that a person can select more than one race. As a result a large number of people are counted more than once. Naturally the total is greater than 100 as a result.

When interpreting percents and data in general we should ask about the assumptions are before we draw conclusions. □

Table 2.1.18 Declared Race in Alaska

Race	Percent
Alaska Native/Native American	21.9%
Native Hawaiian/Pacific Islander	2.5%
Asian	8.4%
Black	40.8%
White	70.4%
Other	5.4%

2.1.4 Exercises

Exercise Group. Questions about the definition, terminology, and notation.

1. **Decimal to Percentage.** The decimal 0.62 is equivalent to what percent?
   ____ %
   (Do not enter the % sign)
2. **Fraction to Percentage.** The fraction $\frac{3}{5}$ is equivalent to what percent? ____________________ %
   (Do not enter the % sign)
3. **Percent of Whole.** 124 is what percent of 80.
   ____%

Exercise Group. Use percentages in various settings.

4. **Application of Percent.** There are 11000 students attending the community college. Find the percent of students that attend classes in the evening if there are 5852 evening students.
   ____________________Preview Question 1 %
   Round to units. Do not type the %
5. **Percent of Whole.** There are $24,000$ students attending a private college. There are $7,200$ evening students. What percent of all students are the evening students?
   Evening students are _% of all the college's students.
6. **Use Percent to Calculate Total.** If the first chapter of a certain book is 36 pages long and makes up 4% of the book, how many pages does the entire book have?
   ____________________ pages
7. **Calculate Original from Percent.** This week Chase got a promotion at work that came with a 2 % pay increase. If now his monthly salary is $ 2397 , how much was he making before the raise?
   Enter your answer as a decimal. If needed, round to the nearest penny. Do not use commas.
   $ ________
8. **Calculate Whole from Part and Percent.** Santos is running a race. He has completed 10 km, which happens to be 50% of the total race. How long is the race? ____________________Preview Question 1 km
9. **Percent Decrease.** Alex's car insurance bill this year has decreased by 8%. He will be paying $62.32 less this year than last. How much did he pay last year?
   ____________________Preview Question 1
10. **Percents as Fractions.** Out of the last 42 days, it rained 17 days. What percent of these days did it rain? ____________________ %
    Round to 2 decimal places.
11. **Percents as Fractions.** In 70.83 % of the last 24 days it rained. How many days did it not rain?
    ____________________ days
    Round to the nearest whole.
12. **Percents as Fractions.** In 30 % of the last 40 days it rained. How many days did it not rain?
    ____________________ days
    Round to the nearest whole.
13. **Percents as Fractions.** An e-book regularly costs $21.99. How much does it cost if it's on sale for 63% of the regular cost?
    ____________________ dollars
    Round to the nearest cent.
14. **Percents as Fractions.** Laura invested $6200 in stocks which were later sold for $7100. What percent of the initial investment were they sold for?
    ____________________ %

Round to 2 decimal places.

15. Percents as Fractions. A city has a population 550,000 in a state with population 5,000,000. What percent of the state live in this city?

____________________ %

Round to 2 decimal places.

16. Repeated Percents. Joseph loves donuts so much that after returning from Costco with some donuts, he eats 45% of the donuts he just bought. The next day he eats 45% of the remaining donuts and continues to eat another 45% each day. What percent of the donuts will Joseph have left after eight days? (Round your answer to the nearest whole percent.)

____________________ %

17. Percent Increase or Decrease. Identify as an increase or decrease. Then find the percent of increase or decrease. If necessary, round to the nearest percent.

Original: 100
New: 90
_____%

(a) Increasing

(b) Decreasing

Exercise Group. Add problems asking students if the total should be 100% or greater.

Add problems asking students which percent and which amount are larger.

18. Dummy entry. Identify as an increase or decrease. Then find the percent of increase or decrease. If necessary, round to the nearest percent.

Original: 100
New: 90
_____%

(a) Increasing

(b) Decreasing

2.2 Mixtures

This section addresses the topics

- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Calculate Percentages (skill)
- Understand and interpret percentages (critical thinking)

This section continues the topic of percents through a set of applications.

There are many situations where we desire to mix two or more substances together in precise ratio. These include mixing medicines in diluents (like water) and mixing ingredients in a recipe. This section presents how to calculate the ratio of substances after mixing multiple solutions, and the reverse problem how to calculate the amount of each solution to mix for a desired ratio of substances.

2.2.1 Calculate the Result

In some circumstances we know the concentrations of substances in multiple solutions and how much of each has been combined. From this we can calculate the concentration of substances in the resulting solution.

2.2.1.1 Mix Multiple Solutions

Example 2.2.1 Suppose we have a container with a solution that is 22% sugar and the rest water and another container with a solution that is 16% sugar and the rest water. If we combine 150 g of the first solution and 250 g of the second solution, what is the percent of sugar in the resulting solution?

Solution. To calculate a percent we need the amount of the part and the total amount. We can calculate the total directly: $T = 150\text{ g} + 250\text{ g} = 400\text{ g}$. Note we know we can add these because both represented total amounts and they have the same units (g).

To calculate the part we need to know how much (rather than what percent) sugar was obtained from the two solutions. We can calculate that from the given percents and amounts.

$$\begin{aligned} 150\text{ g} \cdot 0.22 &= 33\text{ g}. \\ 250\text{ g} \cdot 0.16 &= 40\text{ g}. \\ P &= 33\text{ g} + 40\text{ g} \\ &= 73\text{ g}. \end{aligned}$$

Thus the percent of sugar in the mixture is $P/T = \frac{73\text{ g}}{400\text{ g}} = 18.25\%$. □

Example 2.2.2 Suppose we have 11.3 lbs of 4140 steel which is a type of steel containing 40% carbon and 9.2 lbs of 4150 steel which contains 50% carbon. If we melt and mix these two metals, what is the resulting percent carbon?

Solution. To calculate a percent we need the amount of the part and the total amount. We can calculate the total directly: $T = 11.3\text{ lbs} + 9.2\text{ lbs} = 20.5\text{ lbs}$. Note we know we can add these because both represented total amounts and they have the same units (g).

To calculate the part we need to know how much (rather than what percent) carbon was obtained from the two metals. We can calculate that from the given percents and amounts.

$$\begin{aligned} 11.3\text{ lbs} \cdot 0.40 &\approx 4.5\text{ lbs}. \\ 9.2\text{ lbs} \cdot 0.50 &= 4.6\text{ lbs}. \\ P &= 4.5\text{ lbs} + 4.6\text{ lbs} \\ &= 9.1\text{ lbs}. \end{aligned}$$

Thus the percent of carbon in the resulting metal is $P/T = \frac{9.1\text{ lbs}}{20.5\text{ lbs}} \approx 44\%$.

Note $\approx$ is used where rounding was needed for significant digits. □

Checkpoint 2.2.3 Suppose 2.0 oz of a solution that is 70.% alcohol is mixed with 5.0 oz of a solution that is 99% alcohol. The resulting solution is what percent alcohol? _____

Solution.

- 91

2.2.1.2 Dilute a Solution

This section presents a slight variation of the mixture calculation problem. In this case we are adding only diluent resulting in diluting the solution.

The next problem is producing a isopropyl alcohol with a lower concentration of alcohol than the original solution. We begin with 16.0 oz of a 91.0% isopropyl alcohol solution. The other ingredient is water.

Example 2.2.4 Dilute Alcohol. Suppose we begin with 16.0 oz of a 91.0% isopropyl alcohol and add 4.00 oz of water to this mixture. What will the percent alcohol of the resulting solution be?

Solution. The percent alcohol is the amount of alcohol (part) divided by the total volume (whole). Only the original solution has alcohol so based on the meaning of percent the volume of alchol is $0.910(16.0\text{ oz}) \approx 14.6\text{ oz}$. We are adding 4.00 oz total to the solution, thus the total volume is $16.0\text{ oz} + 4.00\text{ oz} = 20.0\text{ oz}$ oz.

The final percent alcohol is

$$\frac{14.6 \text{ oz}}{20.0 \text{ oz}} = 0.730$$

or 73.0%. □

If we added more water would the percent alcohol be greater or less? Note that if we are using all of the alcohol solution, the amount of water we add determines the percent alcohol.

Checkpoint 2.2.5 Suppose 6.5 cups of lemonade is 18% lemon. If 7.5 oz of sparkling water is added, what is the percent lemon in the resulting drink? _____%

Solution.

- 8.0

2.2.2 Producing a Desired Solution

In the previous section we calculated the result of mixing two solutions. In this section the goal is to figure out how much diluent to add to achieve a specific concentration. That is, the previous section calculated in this section we solve.

Example 2.2.6 Calcluate Dilution. If we start with 16.0 oz of 91.0% alcohol solution, how much water do we add to get (at least) 25.0 oz of a 55.0% alcohol solution?

How much solution total does this produce? Note it will not necessarily be 25.0 oz.

Solution. This is a percent problem with the total alcohol unchanged and adding only some amount w of water. We need w to be such that $\frac{A}{16.0+w} = 0.550$ where A is the amount of alcohol. Notice we do not use the 25.0 oz at this time. We will address that at the end.

The amount of alcohol is $(0.910)16.0 \text{ oz} \approx 14.6 \text{ oz})$. Thus we setup

$$\begin{aligned}
\frac{(0.910)16.0 \text{ oz}}{16.0 \text{ oz} + w \text{ oz}} &= 0.550\\
(0.910)16.0 \text{ oz} &= 0.550(16.0 \text{ oz} + w \text{ oz})\\
(0.910)16.0 \text{ oz} &= (0.550)16.0 \text{ oz} + (0.550)w \text{ oz}\\
14.6 \text{ oz} &= 8.80 \text{ oz} + (0.550)w \text{ oz}\\
5.8 \text{ oz} &= (0.550)w \text{ oz}\\
\frac{5.8 \text{ oz}}{0.550} &= w \text{ oz}\\
11 \text{ oz} &\approx w \text{ oz}
\end{aligned}$$

Note this means we end up with $16 \text{ oz} + 11 \text{ oz} = 27$ oz of new solution. If we had added less water to get exactly 25 oz we would have had a more concentrated solution. We have at least the 25 oz we need and it is the correct concentration, so this we have achieved our goal.

Also note that the percent alcohol is $\frac{0.910(16.0 \text{ oz})}{16.0 \text{ oz}+11 \text{ oz}} = 0.54$ This is not exactly 55% because of the rounding in the calculations. □

Checkpoint 2.2.7 If we have 13.0 oz of a 91.0% alcohol solution left in a bottle and we want 17.0 oz of a 63.0% alcohol solution, how much water should we add? _____

Solution.

- 5.8

Standalone

2.2.3 Exercises

1. Mixture. You need a 55% alcohol solution. On hand, you have a 385 mL of a 65% alcohol mixture. How much pure water will you need to add to obtain the desired solution?

A) Write an equation using the information as it is given above that can be used to solve this problem. Use x as your variable to represent the amount of pure water you need to use. Equation: ______________________________Preview Question 1 Part 1 of 3

B) You will need
_____ mL of pure water
to obtain
_____ mL of the desired 55% solution.

2. Mixture. 14.5 liters of fuel containing 2.7% oil is available for a certain two-cycle engine. This fuel is to be used for another engine requiring a 5% oil mixture. How many liters of oil must be added?

Give your answer to 3 significant digits.

____

3. Medical Proportion. Quinidine gluconate is a liquid mixture, part medicine and part water, which is administered intravenously. There are 110.0 mg of quinidine gluconate in each cubic centimeter (cc) of the liquid mixture. Dr. Bernal orders 275 mg of quinidine gluconate to be administered daily to a patient with malaria.

How much of the solution would have to be administered to achieve the recommended daily dosage?
____________________ cc

4. Medical Proportion. Albuterol is a medicine used for treating asthma. It comes in an inhaler that contains 17 mg of albuterol mixed with a liquid. One actuation (inhalation) from the mouthpieces delivers a 90 μg dose of albuterol. (Reminder: 1 mg = 1000 μg .)

a.) Dr. Olson orders 2 inhalations 3 times per day. How many micrograms of albuterol does the patient inhale per day?

____________________ μg

b.) How many actuations are contained in one inhaler?
____________________ actuations

c.) Erica is going away for 6 months and wants to take enough albuterol to last for that time. Her physician has prescribed 2 inhalations 3 times per day. How many inhalers will Erica need to take with her for the 6 period? Assume 30-day months.

Hint: she can't bring a fraction of an inhaler, and she does not want to run out of medicine while away.

____

5. Medical Ratio. Amoxicillin is a common antibiotic prescribed for children. It is a liquid suspension composed of part amoxicillin and part water.

In one formulation there are 175 mg of amoxicillin in 6 cubic centimeters (cc's) of the liquid suspension. Dr. Scarlotti prescribes 350 mg per day for a 2-yr old child with an ear infection.

How much of the amoxicillin liquid suspension would the child's parent need to administer in order to achieve the recommended daily dosage?

____

6. Medical Proportion. Diphenhydramine HCL is an antihistamine available in liquid form, part medication and part water. One formulation contains 17 mg of medication in 4 mL of liquid. An allergist orders 34-mg doses for a high school student. How many milliliters should be in each dose?

____________________ mL

7. Percent Concentration. How many mL of sodium hydroxide are required to prepare 900 mL of a 8.5% solution? Assume the sodium hydroxide dissolves in the solution and does not contribute to the overall volume.

____________________ mL

8. Dilution Ratio. You are asked to make a 1/11 dilution using 1 mL of serum. How much diluent do you need to use?

____________________ mL

9. Dilution Ratio. A clinical lab technician determines that a minimum of 65 mL of working reagent is needed for a procedure. To prepare a $\frac{1}{10}$ dilution ratio of the reagent from a stock solution, one should measure 65 mL of the reagent and ____________________ mL of the diluent.

10. Dilution Ratio. A patient's glucose result is suspected to be outside the range of the analyzer, so the techs decide to dilute the sample before running it. 45 microliters of serum is added to 180 microliters of diluent and the diluted sample is analyzed. The analyzer reads that the glucose value of the diluted sample is 50 $\frac{mg}{dL}$.

What was the ratio the sample was diluted to?

____________________Preview Question 1 Part 1 of 2

What is the glucose value of the original sample?

____________________ $\frac{mg}{dL}$

11. Dilution Ratio. A thyroid peroxidase antibody test was performed on a 45 year old man. The dilution sequence was 20 μL serum added to 140 μL of diluent in tube 1. Then 70 μL from tube 1 was added to 560 μL of diluent in tube 2. Finally 55 μL from tube 2 was added to 220 μL of diluent in tube 3.

All dilution ratios should be given as fractions.

a.) What is the dilution ratio in tube 1?

____________________Preview Question 1 Part 1 of 4

b.) What is the dilution ratio in tube 2?

____________________Preview Question 1 Part 2 of 4

c.) What is the dilution ratio in tube 3?

____________________Preview Question 1 Part 3 of 4

d.) What is the overall (serial) dilution ratio?

____________________Preview Question 1 Part 4 of 4

2.3 Ratios

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Setup and solve proportions (skill)

A ratio expresses a fixed relationship between two quantities. This section illustrates using ratios to calculate amounts subject to a ratio and presents how to recognize what a ratio does and cannot tell us. Note, percents are simply ratios scaled to parts per 100, that is, what we know about percents is applicable here.

2.3.1 Example Ratios

Example 2.3.1 Simple syrup consists of one cup of sugar and one cup of water which is heated until the sugar is disolved. There are multiple ratios that express this combination. $\frac{\text{1 cup sugar}}{\text{1 cup water}}$, $\frac{\text{7.05 oz}}{\text{8 oz}}$ ratio of sugar to water by weight, $\frac{\text{7.05 oz}}{\text{15.05 oz}}$ ratio of sugar to simple syrup.

Each of these ratios indicates that there is a fixed amount of sugar relative to the water or resulting syrup. That is, is if we made simple syrup with 2 cups of sugar we would need 2 cups of water because

$$\frac{\text{2 cups sugar}}{\text{2 cups water}} = \frac{\text{1 cup sugar}}{\text{1 cup water}}$$

by reducing fractions. □

Example 2.3.2 In a neighborhood there are 7 dogs and 12 cats. To express the relative number dogs and cats we can write the ratio

$$\frac{\text{7 dogs}}{\text{12 cats}}$$

Note that $\frac{7}{12} \approx 0.58 < 1$. Because it is less than one the ratio tells us that there are fewer dogs than cats in this neighborhood.

We could also write $\frac{\text{12 cats}}{\text{7 dogs}}$ to express the exact same relationship. Note that $\frac{12}{7} \approx 1.7 > 1$. Because it is greater than one the ratio tells us that there are more cats than dogs in the neighborhood (same result as above). □

Example 2.3.3 Rates are expressed as ratios. The following rates are all written as ratios.

$$\frac{\text{35 miles}}{\text{1 hour}}$$

$$\frac{\text{8 gallons}}{\text{1 minute}}$$

Notice that these (and most rates) are expressed with the denominator being one. This makes it easier to calculate as will be seen below. Frequently we will skip writing the one in the denominator, e.g., $\frac{\text{35 miles}}{\text{hour}}$.

Rates are not required to have a denominator of one and sometimes a different denominator is easier for calculation. For example

$$\frac{\text{3 nm}}{\text{2min}} = \frac{\text{1.5 nm}}{\text{1 min}}$$

□

2.3.2 Using Ratios

Just like percents (which are ratios written in decimal form) if we know a ratio and one of the amounts we can calculate the other amount. The method is the same as with percents, namely multiplying the correct number or solving an equation.

Example 2.3.4 Ratio: Airspeed. The Diamond DA-20 cruises at the rate (speed) of

$$\frac{\text{110 nautical miles}}{\text{1 hour}}.$$

Cruise is a portion of flight in which the speed is typically constant. How far can the plane cruise in 2.5 hours?

The units suggest we can multiply these.

$$\frac{110 \text{ nm}}{1 \text{ hr}} \cdot 2.5 \text{ hr} = 275 \text{ nm}.$$

This works because we are multiplying by hours and dividing by hours which leaves us with nautical miles as desired. □

Example 2.3.5 Ratio: Airspeed by Table. There is another approach to the same question.

The Diamond DA-20 cruises at the rate (speed) of

$$\frac{110 \text{ nautical miles}}{1 \text{ hour}}.$$

Cruise is a portion of flight in which the speed is typically constant. How far can the plane cruise in 2.5 hours?

Because this ratio remains the same during cruise we can break the problem down into pieces. Based on the ratio (speed) in the first hour the plane will cruise 110 nm. In the second hour it will cruise another 110 nm. In the last half hour it will fly half of the distance which is $110/2 = 55$ nm. Thus in 2.5 hours it will cruise $110 + 110 + 55 = 275$ nm.

Some manuals provide tables to make this method convenient. See Table 2.3.6 for a table of this sort. How far can the DA-20 cruise in 2.7 hours? We need to write 2.7 as the sum of numbers in the table. One way is $2.7 = 2 + 0.5 + 0.2$. From the table we know it will cruise 220 nm in 2 hours, 55 nm in an additional 0.5 hours, and 22 nm in the final 0.2 hours. Thus it will cruise $220 + 55 + 22 = 297$ nm.

While this method makes sense, it takes more time than simply multiplying. □

Table 2.3.6 Airspeed and Distance

Time	Distance
0.1 hour	11 nm
0.2 hour	22 nm
0.3 hour	33 nm
0.4 hour	44 nm
0.5 hour	55 nm
1 hour	110 nm
2 hour	220 nm
3 hour	330 nm

Example 2.3.7 Water is flowing out of a hose at a rate of 11 gallons per minute. How many gallons have come out after 2.7 minutes?

Solution. We can set this up like the calculation in Example 2.3.4. In that example multiplied the ratio (nm/hr) by the number of hours which gave us nm. Here we multiply the ratio (gal/min) by the number of minutes which will give us gallons.

$$\frac{11 \text{ gal}}{1 \text{ min}} \cdot 2.7 \text{ min} = 29.7 \text{ gal} \approx 30 \text{ gal}.$$

Here we use significant digits because these are measurements. □

Example 2.3.8 Based on data from the FDA the average amount of mercury found in fresh or frozen salmon is 0.022 ppm (parts per million). This means there are 0.022 mg of mercury in one liter of salmon. If a meal portion of salmon is 0.0020 liters how much mercury is consumed?

Solution. We can use the ratio $\frac{0.022 \text{ mg}}{1\ \ell}$. We apply this ratio to the given volume of 0.0020 liters.

$$\frac{0.022 \text{ mg}}{1\ \ell} \cdot 0.0020\ \ell = 0.000044 \text{ mg}$$

□

Checkpoint 2.3.9 Calculate how far a plane flying at 90 nm/hour would travel in 1.2 hours. ______________

Solution.

- 108

The ratio (nm/hour) suggests that we can multiply by hours to determine distance (nautical miles or nm).

$\frac{90 \;\; \text{nm}}{1 \;\; \text{hr}} \cdot 1.2 \;\; \text{hr} = 108 \;\; \text{nm}$

We could also change the ratio to be in terms of 1.2 hours.

$\frac{90 \;\; \text{nm}}{1 \;\; \text{hr}} \cdot \frac{1.2}{1.2} = \frac{108 \;\; \text{nm}}{1.2 \;\; \text{hr}}.$

The method for ratios that have a number other than one in the denominator is the same.

Example 2.3.10 A saline solution intended for nasal rinsing has a ratio of 2.5 g of salt (sodium chloride) per 240 mL of pure water. How much salt is needed to make a half liter of saline solution?

Solution. We can apply the given ratio (2.5 g/240 mL) to the given amount (0.5 L). First it will be convenient to convert a half liter to milliliters. This is also a ratio problem. See Table 1.1.7 for the conversion ratio.

$$\frac{1000 \text{ mL}}{1 \text{ L}} \cdot 0.5 \text{ L} = 500 \text{ mL}.$$

Next we can multiply the saline solution ratio by the volume.

$$\frac{2.5 \text{ g}}{240\text{mL}} \cdot 500 \text{ mL} = 5.2 \text{ g}.$$

Note we use two significant digits because the measurements 2.5 and 240 both have two significant digits. The 500 is part of a definition so it has infinite significant digits. □

Example 2.3.11 At 90 nm/hr a plane travels 3 nm/2 min. How far will it travel in 6 minutes?

Solution. We can multiply the given ratio (nm/hr) by the given amount (min) to calculate distance (nm).

$$\frac{3 \text{ nm}}{2 \text{ min}} \cdot 6 \text{ min} = 9 \text{ nm}.$$

□

Checkpoint 2.3.12 One formulation of amoxicillin, a drug used to treat infections in infants, contains 125 mg of amoxicillin per 5.00 mL of liquid. How much amoxicillin is in 13.0 mL of liquid? ______________

Solution.

- 325

2.3.3 Understanding Ratios

Like percents ratios tell us a relationship between two quantities but do not tell us how much. For example if a cookie recipe calls for 2 cups of milk for every 3 eggs, we do not know how many eggs are needed for a dozen cookies. We would also need to know either how many cups of milk per dozen or how many eggs per dozen.

Ratios may be based on rounded numbers. For example the unit convesion $\frac{8 \text{ kilometers}}{5 \text{ miles}}$ is convenient for quick calculations. However, $8/5 = 1.6 \approx 1.609344$ which is a more accurate conversion rate. If we are trying to convert "Tempo 30" (a speed limit of 30 kph) to mph this ratio is fine. If we are sending a satellite to another planet, we will need a more accurate conversion.

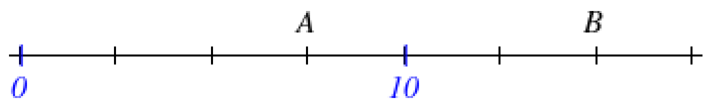
Rounding may occur to make the ratio easier to comprehend. For example according to an article by the U.S. Census Bureau[1] 1 in 6 people in the U.S. was aged 65 and over. Because this ratio uses small numbers it is easy to understand. It is much easier to read and use than a more precise estimate of $\frac{57822315}{333287562}$.

[1] www.census.gov/library/stories/2023/05/2020-census-united-states-older-population-grew.html

2.3.4 Exercises

1. Enter the ratio as a fraction in lowest terms
3 ft to 72 in.
____________________Preview Question 1

2. Identify the decimals labeled with the letters A and B on the scale below.



Letter A represents the number ____
Letter B represents the number ____

3. Write the ratio as a ratio of whole numbers in lowest terms
\$1.50 to \$1.00
____________________Preview Question 1

4. Consider the rectangle with width 14 ft and length 16 ft, write a ratio of the length to the width.
____________________Preview Question 1

5. If you spend 4.5 hours a week studying for English and 2 hours studying for math what is the ratio of time spent studying in math to studying for English?
____________________Preview Question 1

6. An employee pays \$475 towards health insurance, while the employer pays \$425. What is the ratio of the employers contribution to the employees contribution?
____________________Preview Question 1

7. Enter the ratio as a fraction in lowest terms (no decimals).
1.4 m to 0.12 m
____________________Preview Question 1

8. At a recent Pac-12 sporting event, there were 22,000 Buffaloes fans and 10,600 Bruins fans. Write each ratio as as a reduced fraction.
A) The ratio of Buffaloes fans to Bruins fans. ______Preview Question 1 Part 1 of 2
B) The ratio of Buffaloes fans to total fans. ______Preview Question 1 Part 2 of 2

9. In a recent survey, 20 percent of people claimed to like math. Write this ratio as a reduced fraction.
______Preview Question 1

10. Nadia bought 17 fruit in a week. The table shows the how many of each fruit Nadia bought.

Table 2.3.13

fruit	amount:
oranges	4
apples	7
bananas	6

What is the ratio of oranges to total fruit Nadia has in a week?
__ : __

11.

The labeling on this package of Granola shows that 12.34 ounces = 350 grams.Use that conversion factor to determine the weight in grams of a 42 ounce box of Granola.

Round your answer to the nearest whole gram. 42 ounces = _____ grams

12.

In the United States you can commonly purchase cans of Coca-Cola with a volume of 7.5 fluid ounces (fl oz), which is equivalent to 222 milliliters (ml).In Japan they sell 160 ml cans. How many fluid ounces is that?

*Round your answer to 2 decimal places.*160 ml =___fl oz

13. A few winters ago, it was very cold in northern New Hampshire, and Vicente thought about leaving the kitchen faucet running overnight (so the water pipes wouldn't freeze). Vicente's roommate Noa was a little concerned that they would be wasting a lot of water, so they performed an experiment.

Noa turned the kitchen faucet on so it was dripping water at a constant rate. Then she held up a 1/2 teaspoon under the faucet, and it filled in 7 seconds. So, the water was "flowing" at a rate of 0.5 teaspoons per 7 seconds.

Question: They were going to sleep and planned to get up 9 hours later to turn off the faucet. How many *gallons of water* would have gone down the drain in that time?

[Answer this question by converting 9 hours into gallons. Give your answer as a decimal number, rounded correctly to the nearest thousandth of a gallon.]

Some Useful Unit Conversions

- 1 fluid ounce = 2 tablespoons
- 128 fluid ounces = 1 gallon
- 1 hour = 60 minutes
- 7 seconds = 0.5 teaspoons
- 1 tablespoon = 3 teaspoons
- 60 seconds = 1 minute

In all, approximately ________ gallons of water will flow down the drain in 9 hours.

14. A 40 oz bottle of dish soap sells for $2.14. A 43 oz bottle of dish soap sells for $3.59.
(round all answers to four decimal places)
The unit price of the 40 oz bottle is $____ per oz
The unit price of the 43 oz bottle is $____ per oz
Which of the two is a better deal?

(a) The 40 oz bottle for $2.14

(b) The 43 oz bottle for $3.59

15. You can purchase a 15 fl oz bottle of window cleaner for $2.73 or a 18 fl oz bottle for $3.3. Which bottle of window cleaner is the better deal? What is the unit price of this bottle?

(a) 18 fl oz bottle for 18.33 cents per fl oz

(b) 18 fl oz bottle for 5.45 cents per fl oz

(c) 15 fl oz bottle for 18.2 cents per fl oz

(d) 15 fl oz bottle for 5.49 cents per fl oz

(e) None of these

16. Add some about interpreting ratios here.

2.4 Proportions

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Setup and solve proportions (skill)

When a ratio remains fixed, we can use it to solve for quantities. These are called proportions. This section presents a variety of problems in which this is useful and~~, indirectly,~~ reviews the algebra needed to solve them.

OK, maybe do keep "fixed"

Fixed ratios make sense in examples like conversion of units. For example 1 gallon is always 4 quarts. In contrast rates often change: your average speed may be 25 mph, but you must have driven slower and faster during that drive. For the ratios that do not change we can write equations and solve for properties.

good

?

2.4.1 Proportion Examples

Proportion problems start with a fixed ratio. Because it is fixed we can write ratio equals ratio. This gives us something to solve. There are multiple ways to solve these, each of which is demonstrated below.

an equation

Standalone

The first example shows a straight forward proportion with the simplest solving method. This is like solving a percent problem.

Example 2.4.1 Cheesecake Groceries: Double. For a particular cheesecake recipe there is $15\bar{0}$ g of eggs and $150\bar{0}$ g of cream cheese. We will determine how many grams of eggs we need if we double the recipe. This means everything will be in ratio of 2/1.

The proportion is based on original recipe and doubled recipe. That is every ingredient will be doubled: we must perform the calculation for eggs and cream cheese separately. We want the ratio of number of eggs in the doubled recipe to the number of eggs in the original recipe to be 2/1. Thus

$$\frac{2}{1} = \frac{E \text{ g}}{15\bar{0} \text{ g}}.$$

Because the quantity to solve is in the numerator we can simply multiply to isolate that quantity (variable).

$$\begin{aligned} \frac{2}{1} &= \frac{E \text{ g}}{15\bar{0} \text{ g}}. \\ \frac{2}{1} \cdot 150 \text{ g} &= \frac{E \text{ g}}{15\bar{0} \text{ g}} \cdot 150 \text{ g. Eliminating the denominator.} \\ \frac{2}{1} \cdot 15\bar{0} \text{ g} &= E \\ 30\bar{0} \text{ g} &= E \text{ of eggs.} \end{aligned}$$

Note because we are using measurements in grams (mass/weight) we used significant figures for rounding. In commercial recipes (and quality home cooking), weights are used because items like eggs are not uniform in mass. If we always use 3 eggs it might be more or less than we need ~~messing up the food~~. Also note the 2 and 1 have infinite significant figures (they are exact numbers rather than approximated measurements). ☐

This example may seem overly simple because doubling is easy, but the arithmetic is the same for any scaling.

Example 2.4.2 Guido needs 6 dozen cookies. A recipe makes 4 dozen. If that recipe calls for $30\bar{0}$ g of flour, how much flour does he need for the 6 dozen cookies?

First, we determine the ratio for scaling. We want 6 dozen and the recipe makes 4 dozen so our ratio is $6/4 = 3/2$. Thus to determine the amount of flour needed we setup the proportion

$$\begin{aligned} \frac{3}{2} &= \frac{F\text{ g}}{30\bar{0}\text{ g}} \\ \frac{3}{2}\cdot 30\bar{0}\text{ g} &= \frac{F\text{ g}}{30\bar{0}\text{ g}} 30\bar{0}\text{ g} \\ 45\bar{0}\text{ g} &= F \end{aligned}$$

□

This example shows methods for handling proportions when the desired quantity (variable) ends up in the denominator.

Example 2.4.3 Cheesecake Groceries: Unknown. For a particular cheesecake recipe there is $15\bar{0}$ g of eggs and $150\bar{0}$ g of cream cheese. If we have $35\bar{0}$ g of egg how much cream cheese do we need? We know that the egg to cream cheese ratio must be $150/1500$. We notice $150/1500 = 1/10$. This means we need to solve

$$\frac{1}{10} = \frac{35\bar{0}\text{ g}}{C\text{ g}}.$$

Because a ratio expresses the relationship between two quantities, it is not important which is numerator or denominator. Thus it is equally valid to write

$$\begin{aligned} \frac{10}{1} &= \frac{C\text{ g}}{35\bar{0}\text{ g}}. \\ \frac{10}{1}\cdot 350\text{ g} &= \frac{C\text{ g}}{350\text{ g}}\cdot 350\text{ g}. \\ 35\bar{0}0\text{ g} &= C\text{ of cream cheese.} \end{aligned}$$

□

This example shows an alternate way to solve for the desired quantity when it is in the denominator.

Example 2.4.4 Proportion: Solving. The Diamond DA-20 an airplane, cruises at the rate of

$$\frac{110\text{nautical miles}}{1\text{ hour}}.$$

How long will it take to travel 236 nm?

Because cruise speed is a fixed ratio we can write

$$\begin{aligned} \frac{110\text{ nm}}{\text{hour}} &= \frac{236\text{ nm}}{t\text{ hours}}. \\ t\text{ hours}\cdot\frac{110\text{ nm}}{1\text{ hour}} &= t\text{ hours}\cdot\frac{236\text{ nm}}{t\text{ hours}}. \\ t\cdot 110\text{ nm} &= 236\text{ nm}. && \text{Clearing the denominators.} \\ \frac{t\cdot 110\text{ nm}}{110\text{ nm}} &= \frac{236\text{ nm}}{110\text{ nm}}. \\ t &= \frac{236}{110}. \\ t &\approx 2.1\text{ hours.} \end{aligned}$$

Do we say anything about the rounding?

□

Checkpoint 2.4.5 If the Diamond DA-20 climbs at the rate of $\frac{450\text{ feet}}{1.0\text{ minute}}$ how long will it take it to climb $3,500$ ft? ____________

Solution.

- 7.8

Because the climb rate is treated as a constant we setup the proportion

$$\frac{450 \ \text{ feet}}{1.0 \ \text{ minute}} = \frac{3,500 \ \text{ feet}}{t \ \text{ minutes}}$$
$$\frac{450 \ \text{ feet}}{1.0 \ \text{ minute}} \cdot (t \ \text{ minutes}) = \frac{3,500 \ \text{ feet}}{t \ \text{ minutes}} \cdot (t \ \text{ minutes})$$
$$(450 \ \text{ feet})(t) = 3,500 \ \text{ feet}$$
$$\frac{(450 \ \text{ feet})(t)}{450 \ \text{ feet}} = \frac{3,500 \ \text{ feet}}{450 \ \text{ feet}}$$
$$t = 7.8 \text{ minutes}$$

2.4.2 Multiple Proportions

When we experience math in the wild, problems do not come labeled with solving methods. We must recognize the math and apply our knowledge appropriately. The next example illustrates identifying ratios (proportions) more than once when answering a question.

Example 2.4.6 Suppose a Diamond DA-20 climbs at the rate of $\frac{550 \text{ feet}}{1.0 \text{ minute}}$ and $\frac{72.8 \text{ nm}}{\text{hour}}$ across the ground during this climb. How far forward does the plane fly during a climb of 3500 feet?

Because we are told how far the plane climbs, we must use the rate of climb ratio first. As before we can calculate how long it will take to climb.

$$\frac{3500 \text{ ft}}{550 \text{ ft/min}} = 6.4 \text{ min.}$$

This is a jump from before.
Maybe that's okay?

Now that we know how long the plane flies during this climb we can use that with the ground speed ratio to calculate how far foward it flies. However, first we must convert the speed to feet per minute or the time to hours.

$$\frac{72.8 \text{ nm}}{\text{hr}} \cdot \frac{6076 \text{ ft}}{\text{nm}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \approx \frac{7370 \text{ ft}}{\text{min}}$$

Finally we can use this rate and the time of climb to calculate the desired distance.

$$\frac{7370 \text{ ft}}{\text{min}} = \frac{s \text{ ft}}{6.4 \text{ min}}$$
$$\frac{7370 \text{ ft}}{\text{min}} \cdot (6.4 \text{ min}) = \frac{s \text{ ft}}{6.4 \text{ min}} \cdot (6.4 \text{ min})$$
$$47000 \approx s \text{ ft}$$

If desired we can convert this to nautical miles which is

$$(47000 \text{ ft}) \cdot \frac{1\text{nm}}{6076 \text{ ft}} = 7.8 \text{ nm}$$

□

Example 2.4.7 A recipe for hush puppies calls for 150 g of flour for 340 g of buttermilk. If we have 465 g of flour and 918 g of buttermilk, how much of the flour and buttermilk can we use? Which one constrains us (limits size of our batch)? Note quality kitchen scales are accurate to a single gram.

The ingredients must remain in the ratio $\frac{340 \text{ g buttermilk}}{150 \text{ g flour}} = \frac{34 \text{ g buttermilk}}{15 \text{ g flour}}$. We can select either ingredient and see how much the ratio tells us we need of the other ingredient.

Suppose we use all 465 g of flour. Then we can setup the proportion

$$\frac{34 \text{ g buttermilk}}{15 \text{ g flour}} = \frac{B \text{ g}}{465 \text{ g flour}}$$
$$\frac{34 \text{ g buttermilk}}{15 \text{ g flour}} \cdot (465 \text{ g flour}) = \frac{B \text{ g}}{465 \text{ g flour}} \cdot (465 \text{ g flour})$$
$$1054 \text{ g} = B.$$

Notice that this is more buttermilk than we have. That means the buttermilk is the limiting ingredient. We will be able to use all of the buttermilk, but only some of the flour. To determine how much we setup the proportion but this time solve for flour.

We will use all 918 g of buttermilk. Then we can setup the proportion

$$\frac{15 \text{ g flour}}{34 \text{ g buttermilk}} = \frac{F \text{ g}}{918 \text{ g buttermilk}}$$
$$\frac{15 \text{ g flour}}{34 \text{ g buttermilk}} \cdot (918 \text{ g buttermilk}) = \frac{F \text{ g}}{918 \text{ g buttermilk}} \cdot (918 \text{ g buttermilk})$$
$$405 \text{ g} = F.$$

Thus we can use all 918 g of buttermilk and 405 of the 465 g of flour. Note we have rounded everything to one gram because that is as accurate as we can measure with our scale. If a single recipe uses 340 g of buttermilk, then we will be making

$$\frac{918 \text{ g}}{340 \text{ g}} = 2.7$$

times as much. □

Checkpoint 2.4.8 A cheesecake recipe calls for 150 g of eggs and 750 g of cream cheese. If you currently have 250 g of eggs and 1058 g of cream cheese, determine which ingredient is the limiting one (will use all of it) and the amount of each you will use.

Limiting ingredient:

I like these limiting questions.

1. eggs
2. cream cheese

Eggs: ______ g Cream cheese: ______ g

Solution.

- B: cream cheese
- 212
- 1058

To determine which ingredient limits us, we compare the ratios $\frac{250}{1058} = 0.23629489603025$ to the recipe's ratio of $\frac{1}{5} = 0.2$. Because the ratio is greater than 0.2, we know that the cream cheese is the limiting ingredient.

Now we setup the proportion $\frac{1}{5} = \frac{E}{1058}$

Solving gives us 212 g of eggs

2.4.3 Similar polygons

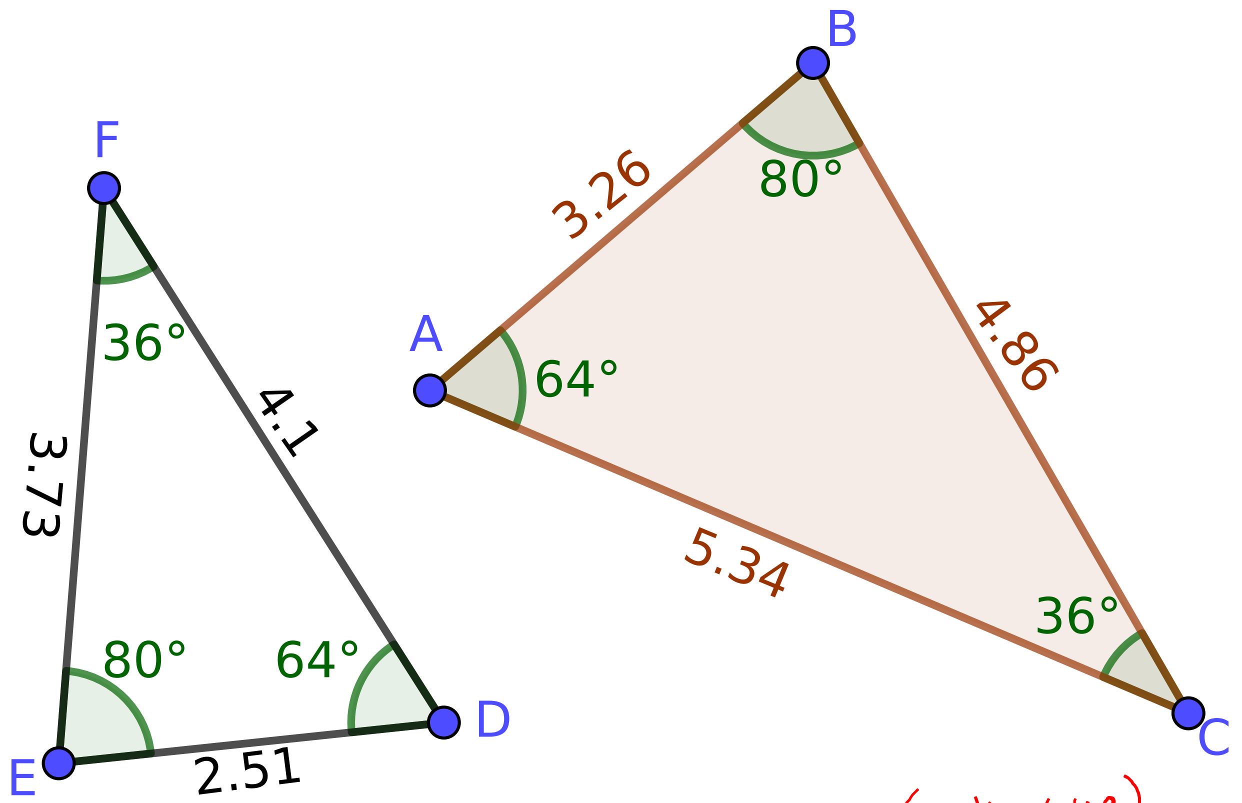
This section presents a geometric fact which is expressed as proportions. This geometry can be used to solve for distances or lengths in some circumstances. First we define and illustrate the geometric fact.

2.4.3.1 Explaining Similar Triangles

Definition 2.4.9 Similar Triangles. Two triangles are **similar** if and only if corresponding angles are the same. ◊

When triangles are similiar their corresponding side lengths are proportional. Corresponding sides are the sides from both triangles that are across from an angle of the same measure. This is illustrated in the following example.

Example 2.4.10



(both 64°)

The triangles $\triangle ABC$ and $\triangle DEF$ are similar. Notice that the angles at A and D are the same measure as are the angles at B and E (80°) and the angles at C and F. (36°).

Note that BC is the side opposite the angle at A and EF is the side opposite the angle at D. Because A and D are the same measure, the sides opposite them are the corresponding pairs.

Similarly CA is the side opposite the angle at B and FD is the side opposite the angle at E. Because B and E are the same measure, the sides opposite them are the corresponding pairs.

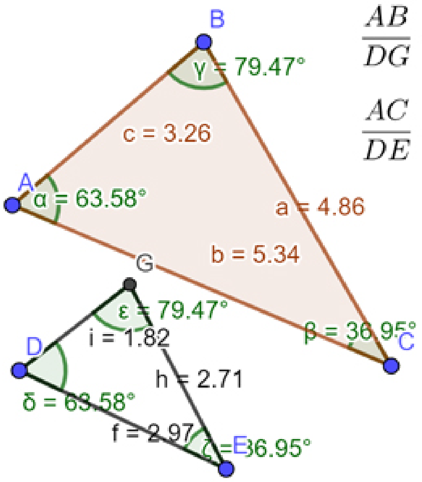
What is the third pair of corresponding sides?

As a result the following ratios of sides are the same

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}.$$

You can confirm this by dividing the lengths $(\frac{3.26}{2.51} = \frac{4.86}{3.73} = \frac{5.34}{4.1})$. □

From a single example we might think this was just a special case. To convince yourself use the following interactive example.



$$\frac{AB}{DG} = \frac{3.26}{1.82} = 1.8$$

$$\frac{AC}{DE} = \frac{5.34}{2.97} = 1.8$$

Standalone
Embed

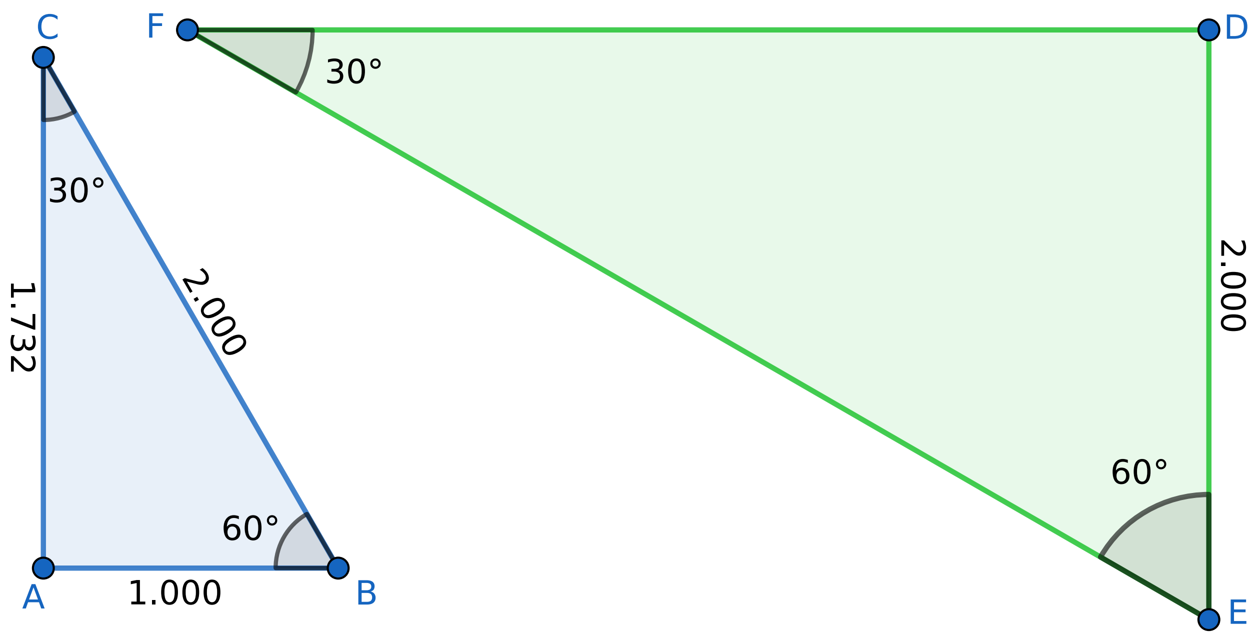
In instructions: You have "Note that every ratio of corresponding sides would be the same." Maybe change to "Every ratio of corresponding..."

Figure 2.4.11 Similar Triangles

2.4.3.2 Calculating Using Similar Triangles

We can use the proportionaliy of similar triangle sides lengths to calculate the lengths using the same technique as Example 2.4.4.

Example 2.4.12



Suppose triangle ABC has angles $90°, 60°, 30°$ with the lengths of the sides opposite them $2.000, 1.732, 1.000$. If triangle DEF also has angles $90°, 60°, 30°$ it is similar. Suppose the length of the side DE opposite the $30°$ angle at point F is 2.000.

First, we identify the corresponding sides. $\overline{AB}$ and $\overline{DE}$ are opposite $30°$ angles so they are corresponding. $\overline{CA}$ and $\overline{FD}$ are opposite $60°$ angles so they are corresponding. Finally, $\overline{BC}$ and $\overline{EF}$ are opposite $90°$ angles so they are corresponding. This means

$$\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{CA}}{\overline{FD}} = \frac{\overline{BC}}{\overline{EF}}.$$

Because we know the ratio $\frac{\overline{AB}}{\overline{DE}}$ we can use the proportion to solve for the other two side lengths on triangle DEF. We invert the ratios for easier solving.

$$\begin{aligned}
\frac{\overline{DE}}{\overline{AB}} &= \frac{\overline{DF}}{\overline{CA}} \\
\frac{2.000}{1.000} &= \frac{\overline{DF}}{1.732}. \\
\frac{2.000}{1.000} \cdot 1.732 &= \frac{\overline{DF}}{1.732} \cdot 1.732. && \text{Clearing the denominators.} \\
3.464 &= \overline{DF}.
\end{aligned}$$

We can calculate the length of the third side in the same way.

$$\begin{aligned}
\frac{\overline{DE}}{\overline{AB}} &= \frac{\overline{EF}}{\overline{BC}}. \\
\frac{2.000}{1.000} &= \frac{\overline{EF}}{2.000}. \\
\frac{2.000}{1.000} \cdot 2.000 &= \frac{\overline{EF}}{2.000} \cdot 2.000. \\
4.000 &= \overline{EF}.
\end{aligned}$$

□

Checkpoint 2.4.13 Suppose triangle A has angles 40°, 80°, and 60° and the lengths of the sides opposite are 10, 15.32, and 13.47 respectively. If triangle B has the same angle measures and the side opposite the angle of measure 40° is length 44, what are the other two side lengths?

Length opposite angle of measure 80°: ______

Length opposite angle of measure 60°: ______

Solution.

- 67.408
- 59.268

Because these are similar triangles we can setup the proportions below.

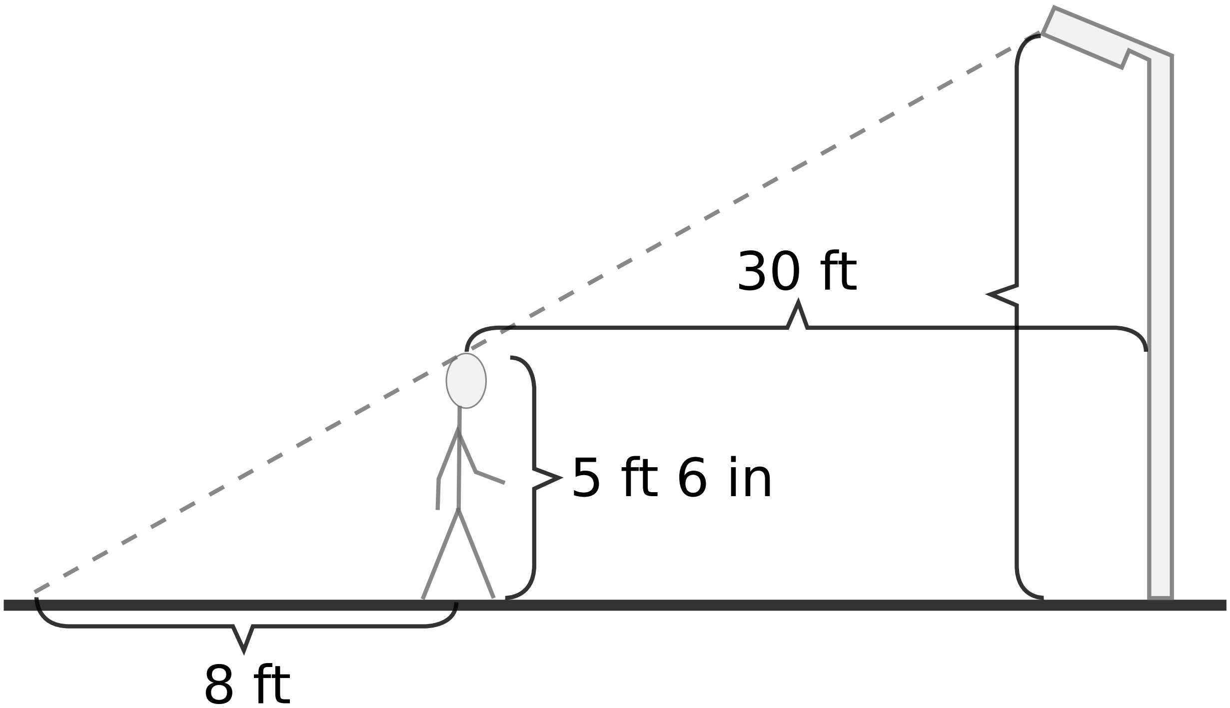
$\frac{44}{10} = \frac{S_{80}}{15.32}$, so $S_{80} = \left(\frac{44}{10}\right)15.32 = 67.408$

$\frac{44}{10} = \frac{S_{60}}{13.47}$, so $S_{60} = \left(\frac{44}{10}\right)13.47 = 59.268$

2.4.3.3 Similarity in Applications

Similar triangles can be found in a variety of circumstances. This example uses ~~recognizes~~ similar triangles in a context which has been used many time in history for indirect measurement.

Example 2.4.14



A person is standing 30 ft from a light pole. The shadow cast by the light is 8 ft long. If the person is 5 ft 6 in tall, how high is the point on the light that is casting the shadow?

In this image we have two (right) triangles that will be useful. The smaller one has legs of length 8 ft and 5 ft 6 in. The third side (hypotenuse) is the dashed gray line, but we will not need it. The other triangle has a leg that is the entire bottom (length 8 ft plus 30 ft). The other leg is the height of the light. The angles of the two triangles are the same. Because they are both right triangles (we are supposing the light post is straight up and the person is standing straight up), the angles at the persons feet and the base of the light are the same. They share the angle on the left (between dashed line and ground). The third angle must match because the first two do.

Because these have the same angles they are similar and we can use the proportionality of ratios of corresponding side lengths. Before we do we will convert the height of the person to decimal. 5 ft 6 in is 5.5 ft.

$$\begin{aligned} \frac{5.5\text{ ft}}{8\text{ ft}} &= \frac{h}{38\text{ ft}}. \\ \frac{5.5\text{ ft}}{8\text{ ft}} \cdot (38\text{ ft}) &= \frac{h}{38\text{ ft}} \cdot (38\text{ ft}). \\ 26.125\text{ ft} &= h. \end{aligned}$$

If these measurements were taken with a tape measure we can reasonably suppose they are accurate to the nearest inch. We can convert the 0.125 ft into inches.

$$0.125\text{ ft} \cdot \frac{12\text{ in}}{1\text{ ft}} = 1.5\text{ in}.$$

Thus the height to that point on the light is 26 ft and 2 in.

Note we use this type of measurement because it is simpler. We can measure across the ground much more easily than we can climb the pole and measure its height. It was not important that the person chose to be 30 ft from the pole. If they chose to be 20 ft, then the shadow would also be shorter (proportional). If we are too close it will be hard to accurately measure the short shadow (ever try to measure a shadow?). □

2.4.3.4 Similarity Beyond Triangles

Shapes other than triangles can be similar. For example there are similar rectangles and similar pentagons. To be similar they must have the same number of sides, corresponding angles must be the same, and corresponding sides must be in the same ratio. Note that just having the same angles is insufficient: any two rectangles have all the same angles (right angles) but not every pair is similar.

Similar Polygons. One way to define similar polygons that avoids the trap illustrated by rectangles is to require every triangle made from vertices to have the same angles. Take two, non-similar rectangles and find the triangles that don't match.

One place where similar shapes (beyond triangles) is used is scale drawing and scale models. If you ever built a model of a car or a plane or some such there was most likely a scale given. For example they may be 1/32 scale. This means that one inch on the model is 32 inches on the actual object.

2.4.4 Exercises

Example here too?

Exercise Group. Solve each of these proportions.

1. Find the unknown number in the proportion
$$\frac{x}{6} = \frac{8}{16}$$
____

2. Solve for the variable in $\dfrac{x}{7.4} = \dfrac{3.44}{8.6}$

$x =$________

3. Find the unknown number in the proportion
$$\frac{9}{12} = \frac{6}{x}$$
____

Exercise Group. Identify a proportion in each application. Set it up, and solve for the requested value(s).

4. Cellular phone service that charges per-minute will charge $60 for 250 minutes. How much would 988 minutes cost?

Round your answer to the nearest cent.

$____

5. Ben goes to the grocery store at a rate of 5 times a week. How many times would he be expected to go to the grocery store in 13 weeks? Use x as the variable.

Table 2.4.15

Translate to a proportion:	________________	Preview Question 1 Part 1 of 2
$x =$___ times in 13 weeks		

6. A carpet store charges $364.00 to install 56 square yards of carpet. Assuming they charge the same rate per square yard regardless of the amount of carpet installed, how much would they charge to install 100 square yards of carpet? Use the variable x in setting up the proportion.

What is the unit price for installation per square yard of carpet?

Table 2.4.16

Translate to a proportion:	________________	Preview Question 1 Par
They would charge $___ to install 100 square yards of carpet.		

$___ per square yard

7. Gwen's Gravel Company supplied a homeowner with 20 cubic yards of gravel for his driveway at a cost of $1,610.00. Assuming they charge the same rate per cubic yard regardless of the amount of gravel supplied, what would they charge for 38 cubic yards of gravel?

Table 2.4.17

Translate to a proportion: ____________________ Preview Question 1 Part 1 of 2
$_____ for 38 cubic yards of gravel

8. If a 35-acre alfalfa field produces 280 tons of hay, how many acres would be needed for a field to produce 424 tons of hay?

Table 2.4.18

Translate to a proportion: ____________________ Preview Question 1 I
A field would need to be _____ acres to produce 424 tons of hay.

9. A recipe for lemon tea cookies calls for $1\frac{1}{4}$ cups of flour for every $\frac{3}{4}$ cup of sugar. How many cups of sugar are needed if $2\frac{2}{3}$ cups of flour are used?

For $2\frac{2}{3}$ cups of flour you need __________Preview Question 1 cups of sugar.

10. A label reads: "2.5 mL of solution for injection contains 1,000 mg of streptomycin sulfate." How many millilitres are needed to give 900 mg of streptomycin?

____________________Preview Question 1

11. A floor plan has a 64 : 1 scale. On the drawing, one of the rooms measures $3\frac{3}{4}$" by $2\frac{5}{8}$". Show answers to the nearest .01 The actual dimensions would be: ___________________Preview Question 1 Part 1 of 3 feet by: ____________________Preview Question 1 Part 2 of 3 feet. The area of the room would be ____________________Preview Question 1 Part 3 of 3 square feet.

12. While planning a hiking trip, you examine a map of the trail you are going on hike. The scale on the map shows that 2 inches represents 3 miles.

If the trail measures 16 inches on the map, how long is the trail?

____________________ miles

Exercise Group. Use the property of side ratios for similar triangles to find the values requested.

13. The side lengths of ΔABC are: $AB = 3$ $BC = 9$ $AC = 10$

The side lengths of ΔRST are: $RS = 9$ $ST = 27$ $RT = 5$

Simplify the given corresponding side ratios:

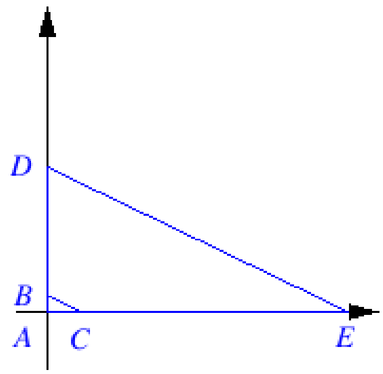
$\frac{RS}{AB} =$ _____ $\frac{ST}{BC} =$ _____ $\frac{RT}{AC} =$ _____

Is $\Delta ABC \sim \Delta RST$?

(a) Yes

(b) No

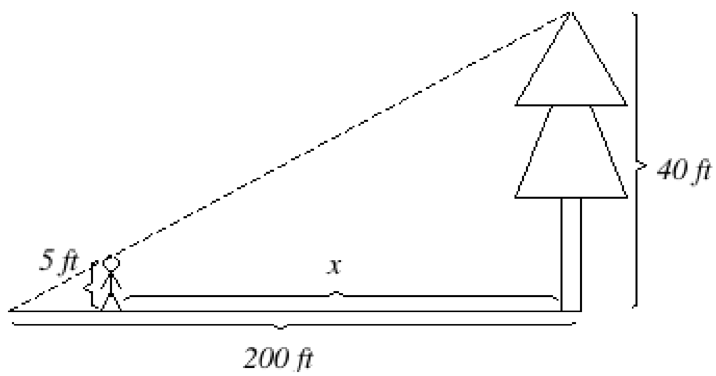
14.



Find the coordinates of point E so that $\Delta ABC \sim \Delta ADE$
$A = (0,0), B = (0,1), C = (4,0), D = (0,9)$
$E =$ (__ ,__)

Exercise Group. Identify similar triangles in each application, then use the property of side ratios to find the requested value(s).

15. Suppose you are standing such that a 40-foot tree is directly between you and the sun. If you are 5 feet tall and the tree casts a 200-foot shadow, how far away from the tree can you stand and still be completely in the shadow of the tree?

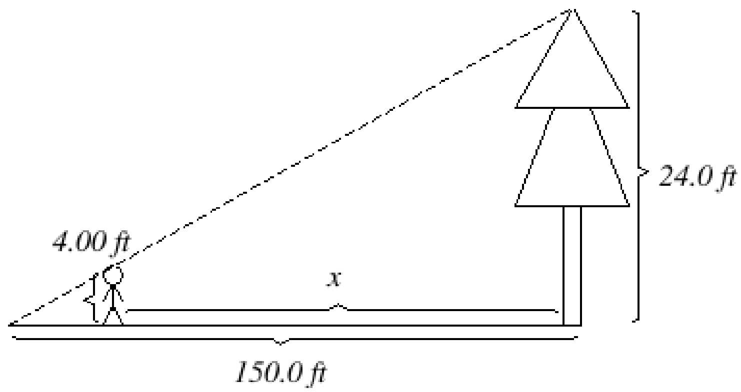


The distance between you and the tree is ________________Preview Question 1 ft (If needed, round to 1 decimal place.)

16. A stick 1.0 meter long casts a shadow 1.3 meters long. A building casts a shadow 16.0 meters long. How tall is the building? Use the rules of working with significant figures to round.
__________________Preview Question 1 meters

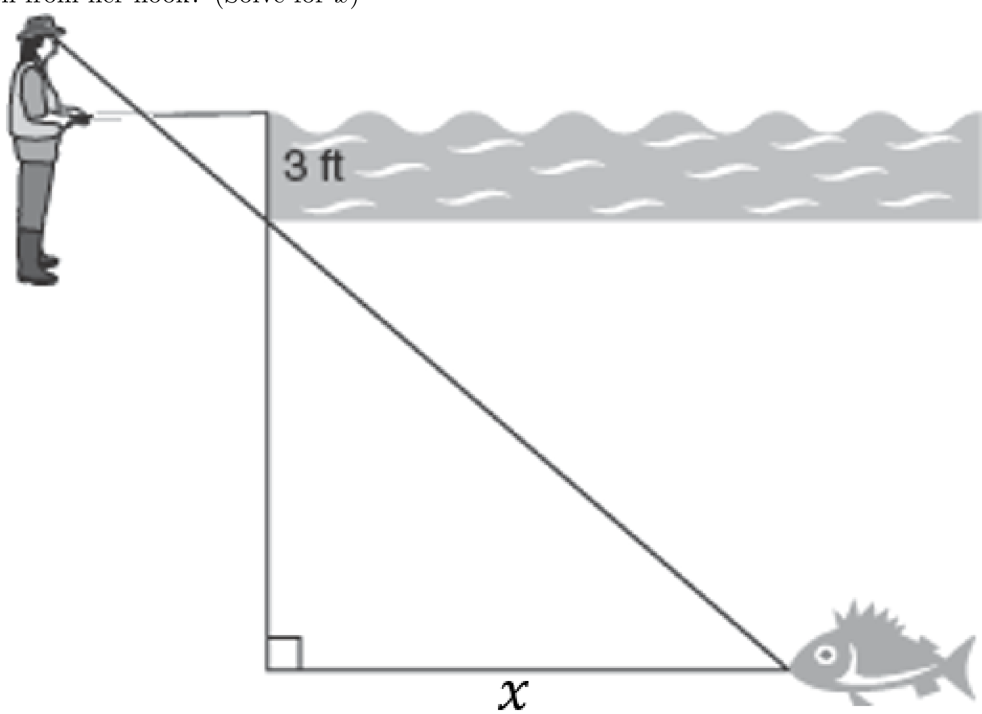
17. Suppose you are standing such that a 24.0-foot tree is directly between you and the sun. If you are 4.00 feet tall and the tree casts a 150.0-foot shadow, how far away from the tree can you stand

and still be completely in the shadow of the tree?



The distance between you and the tree is _____ ft (Use the rules of working with significant figures to round.)

18. Victoria holds a fishing pole with fishing line extended according to the picture below. How far is the fish from her hook? (Solve for x)



$4ft19.5ftx =$ _____

(a) °

(b) ft

(c) in

(d) cm

(e) m

2.5 Medical Ratios

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Setup and solve proportions (skill)

This section uses medical applications, primarily determine medicine dosages, to illustrate the use of ratios including change of units and proportions. In each example look for how the ratio is recognized and how the information provided is used to setup a calculation.

A common application of ratios in medicine is creating drugs of a desired strength. For example some drugs need to be administered based on the body weight of the patient. This requires the medical personnel to mix the drug they have on hand to the needed strength.

We will work the following three types of medical problems.

- Measure drug concentration
- Dilute a drug to a lower concentration
- Determine how much drug to use

2.5.1 Terminology

This section defines terminology used in medicine and sciences about solution concentrations that we need for the ratio examples in this section.

The active ingredient in a drug is often added to a inactive ingredient (often liquid) to administer it. This liquid is known as a **diluent**. The diluent might be water, saline solution, or other substances.

The substance (active ingredient for medicine) which may be a powder or another liquid to which the diluent is added is called the **solute**. The solute is disolved in the diluent. For example salt (solute) is disolved in water (diluent) to make saline solution.

Even if the drug can be administered directly (e.g., is already liquid) we sometimes need to dilute the **stock solution** (undiluted drug) for ease of use.

In some problems the drug mixture will be divided into parts. These parts are sometimes called **aliquots**. For example when testing substances (like blood samples) we may divide the sample drawn into multiple aliquots, one for each test to be run.

The most important concept is measuring how concentrated a solution is. This enables providing sufficient and safe amounts of drugs. There are three common ways concentration is written. These three are examples of how ratios can present the relationship between quantities in different ways. Being able to change between the different presentations of concentration will demonstrate your ability to understand and use ratios accurately.

Definition 2.5.1 Dilution Ratio. The **Dilution Ratio** is the ratio of solute (drug) to diluent.

If the solute is a liquid, then this is in units of volume per volume (e.g., mL per mL). For example a dilution ratio of 1:4 means 1 mL of drug to 4 mL of diluent giving 5 mL of solution.

This expression of concentration is unlikely to be used for dry solutes. ◊

Definition 2.5.2 Dilution Factor. The **Dilution Factor** is the ratio of solute (drug) to the resulting solution.

If the solute is liquid, then this is in units of volume per volume. For example a dilution factor of 5 means 1 unit of the drug in every 5 units of solution implying 4 units of diluent (1/4 dilution ratio).

If the solute is solid (e.g, powder) then this is in units of mass per volume. For example, 5 g of drug in a total of 100 mL of solution. Note we do not care how much diluent was added (hence we cannot calculate dilution ratio). This dilution factor can be achieved by putting in the dry ingredient, then adding part of the diluent to disolve the dry ingredient, then pouring in enough additional diluent to reach the desired volume. ◊

Definition 2.5.3 Percent Concentration. The **Percent Concentration** is the ratio of mass of solute (drug) to 100 mL of diluent.

If the solute is liquid, then this is in units of volume per volume. For example if the are 2 mL of drug per 100 mL of solution, then the percenet concentration is 2/100 or 2%.

If the solute is solid (e.g, powder) then this is in units of mass per volume. For example, if there are 2 mg of drug per 100 mL of solution, then the percent concentration is 2/100 or 2%. Note this is neither percent by volume nor percent by mass as would be used in science. ◊

These examples illustrate the meaning of these terms.

Example 2.5.4 A solution is produced from 3 mL of concentrated chloroform and 37 mL of water.

(a) What is the dilution ratio?

Solution. The dilution ratio is the ratio of the solute to the diluent. We are given both. The dilution ratio is 3/37.

(b) What is the dilution factor?

Solution. The dilution factor is the ratio of the total to the substance. The total is substance plus diluent. Here that is $3 + 37 = 4\bar{0}$. The ratio then is $\frac{4\bar{0}}{3} \approx 13$.

(c) Calculate the percent concentration.

Solution. The percent concentration is the ratio of the solute to the total solution by volume written as a percent. We are given the volume of the solute (3 mL) and have calculated the total volume in the previous task (40 mL). Thus the percent concentration is $\frac{3}{40} = 0.075$ which is 7.5%.

□

Example 2.5.5 Saline solution consists of the solute salt (sodium chloride) dissolved in the diluent pure water. The saline solution most commonly used in medical applications is 9 g of the solute salt, which is a solid, dissolved in enough water to make 1 liter of solution.

Because the solution is produced by adding enough water (amount not specified) it is easiest to express the concentration as a dilution factor. For medicine these are most commonly expressed in terms of milliliters, so we will convert units (metric terminology). The dilution factor is

$$\frac{9\text{ g}}{1\text{ L}} \cdot \frac{1\text{ L}}{1000\text{mL}} = \frac{9\text{ g}}{1000\text{mL}}.$$

We can also calculate the percent concentration. To do this we need to express volume in milliliters (percent concentration is ratio of solute to 100 mL). We can use unit conversion from above. The percent concentration is

$$\begin{aligned} \frac{9\text{ g}}{1000\text{ mL}} &= \\ \frac{9\text{ g}}{1000\text{ mL}} \cdot \frac{1/10}{1/10} &= \\ \frac{0.9\text{ g}}{100\text{ mL}} &= 0.009 \end{aligned}$$

or 0.9%.

Note how comparing the information provided to the definition showed us we needed to perform a unit conversion. The definitions also stated which number is numerator and denominator and what form to use for a final expression (e.g., percent or fraction). □

Example 2.5.6 Clorox® Disinfecting Bleach contains 7.0% sodium hypochlorite which is a liquid. This means the percent concentration is 7.0%. From this information we can calculate the dilution ratio and dilution factor.

One commercially available size of bleach contains 11 oz. To calculate the dilution ratio and dilution factor we need to know the amount of solute (sodium hypochlorite) in the 11 oz. Because it has a percent concentration of 7.0% there is $11 \cdot 0.070 = 0.77$ oz of sodium hypochlorite.

This is all we need for the dilution factor which is $\frac{0.77}{11}$. This would be easier to read if we reduce it $\frac{0.77}{11} = 0.7$ or $\frac{0.7}{1}$ expressed in ounces of bleach to ounces of water.

For the dilution ratio we need the amount of water added. Because the total solution is 11 oz and we know 0.77 oz is bleach, the water is $11 - 0.77 = 10.23$ oz. Thus the dilution ratio is $\frac{0.77}{10.23}$.

This ratio is hard to interpret, so we should reduce the fraction. We can perform this reduction multiple ways.

$$\begin{aligned}
\frac{0.77}{10.23} &= \frac{0.77}{10.23} \cdot \frac{1/0.77}{1/0.77} \\
&= \frac{1}{10.23/0.77} \\
&\approx \frac{1}{13.29}.
\end{aligned}$$

Another options is

$$\begin{aligned}
\frac{0.77}{10.23} &= \frac{1}{R}. \\
\frac{0.77}{10.23} \cdot 10.23 \cdot R &= \frac{1}{R} \cdot 10.23 \cdot R. \\
0.77R &= 10.23. \\
R &= \frac{10.23}{0.77}. \\
R &\approx 13.29.
\end{aligned}$$

The ratio $\frac{1}{13.29}$ means there is one ounce of bleach for every 13.29 ounces of water. □

Note a pure substance has dilution factor 1/1 (the total volume of the solution is just the volume of the solute). The percent concentration for a pure substance is 100%.

Standalone

Use these Checkpoints to test your ability to calculate these ratios.

2.5.2 Dilution

This section shows how to use knowledge of proportions to perform calculations required in medicine. Dilution ratios or factors tell us a desired ratio, and we know the initial ratio. This pair allows us to setup a proportion.

This first example shows how to produce a solution with a desired dilution factor.

Example 2.5.7 How much diluent do we need to add to produce a solution containing 3.0 mL of concentrated chloroform that will have a dilution factor of $5\bar{0}$?

The dilution factor is the ratio of the total to the substance. We want that to equal $5\bar{0}$, so we can write the proportion

$$\frac{\text{total}}{\text{solute}} = \frac{5\bar{0}}{1}.$$

We are not given the total volume, but the total is the volume of the solute plus the volume of the diluent. We do know the volume of solute (3.0 mL), and the volume of diluent is what we want to calculate. We can call the volume of diluent D. The volume of the solution is $3.0 + D$ where D is the volume of diluent to add.

Because we are starting with 3.0 mL of concentrated chloroform (no dilution) our proportion is

$$\begin{aligned} \frac{3.0 + D}{3.0} &= \frac{5\bar{0}}{1.0}. \\ 3.0 \cdot \frac{3.0 + D}{3.0} &= 3.0 \cdot \frac{5\bar{0}}{1.0}. \\ 3.0 + D &= 150. \\ -3.0 + 3.0 + D &= -3.0 + 150. \\ D &= 147. \end{aligned}$$

So we need 147 mL of diluent. Notice once we had the proportion set up we needed only use algebra. □

This example shows us how to apply a dilution ratio (dilute our solution). We can calculate the resulting dilution factor afterward.

Example 2.5.8 A doctor orders 120 mL of 50% solution of Ensure every two hours. How much Ensure (liquid) and water is needed?

50% is a percent concentration. This means the Ensure should be 50% of the total volume (120 mL). $120 \cdot 0.50 = 60$ mL of Ensure. This leaves $120 - 60 = 60$ mL of water (diluent).

The dilution ratio is 1/1, because there is the same volume of solute (Ensure) and diluent (water). The dilution factor is 1/2, because we have 60 mL of Ensure in 120 mL of solution ($60/120 = 1/2$). □

Working on dilutions is a proportion problem. This next example presents a scenario where we work a dilution problem backwards. Notice that the setup is still a proportion and the solving is still just the algebra steps needed.

Example 2.5.9 One usage of dilution is to reduce the concentration so that instruments can accurately measure it. Consider trying to measure an acid without dissolving the tools used to measure it.

A sample of a suspected high blood glucose value was obtained. According to the manufacturer of the instrument used to read blood glucose values, the highest glucose result which can be obtained on this particular instrument is 500 mg/dL. When the sample was run, the machine gave an error message (concentration too high).

The serum was then diluted to 1/10 and retested. The machine gave a result of 70 mg/dL. What was the initial concentration?

Note that the ratio is milligrams to decilitres (weight to volume). In these types of problems the amount of substance is so small that it does not affect the volume.

Solution. Before we jump into an equation, let's try an experiment. That's right, in math we do not have to know what to do when we start. We will try something and learn from it, maybe revising our approach after the first try.

This is a dilution problem which means we can setup the proportion

$$\frac{\text{solute mg}}{\text{diluent dL}} = \frac{70 \text{ mg}}{\text{dL}}.$$

We are trying to find the amount of blood sugar in the sample, so the solute portion is unknown. We also do not have the size of sample taken. We will experiment to see how this affects the problem.

Suppose we take 1 dL of the original serum. Because the blood sample is so small, we can calculate as if all the volume is the diluent. That is we started with 1 dL and added more to dilute. To dilute to a ratio of 1/10 we need to add $10-1=9$ dL of diluent. No blood glucose was added thus the concentration is changed only by the diluent. Thus the concentration proportion is now

$$\begin{aligned}
\frac{C+0 \text{ mg}}{1+9 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} \\
\frac{C \text{ mg}}{10 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} && \text{clearing the denominators} \\
(C \text{ mg})(\text{dL}) &= (70 \text{ mg})(10 \text{ dL}) \\
\frac{(C \text{ mg})(\text{dL})}{\text{dL}} &= \frac{(70 \text{ mg})(10 \text{ dL})}{\text{dL}} \\
C &= 700 \text{ mg}
\end{aligned}$$

Did this result depend on our selecting 1 dL of the original serum? If we are uncertain we can try the problem again and select 2 dL of the original serum. To figure out the total amount of which 2 is 1/10, we can treat this like Example 2.3.7

$$\begin{aligned}
\frac{1}{10} &= \frac{1}{10}\frac{2}{2} \\
&= \frac{2}{20}.
\end{aligned}$$

This means we need $20-2=18$ dL of diluent to have the desired dilution ratio. Also we will have twice as much of the blood glucose.

$$\begin{aligned}
\frac{2C+0 \text{ mg}}{2+18 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} \\
\frac{2C \text{ mg}}{20 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} && \text{clearing the denominators} \\
(2C \text{ mg})(\text{dL}) &= (70 \text{ mg})(20 \text{ dL}) \\
\frac{(2C \text{ mg})(\text{dL})}{2\text{dL}} &= \frac{(70 \text{ mg})(20 \text{ dL})}{2\text{dL}} \\
C &= 700 \text{ mg}
\end{aligned}$$

Notice the result is the same. This makes sense, because we are setting up a proportion, and ratios do not depend on the amount.

We can be confident that the original serum sample had a blood glucose level of 700 mg/dL. □

Sometimes we dilute more than one time. Here we experiment to determine what the effect of **serial dilution** is upon the resulting dilution factor.

Example 2.5.10 Suppose you have a solution consisting of 10 mL of acyl chloride and 90 mL of water. If this is diluted to a dilution ratio of 1/2 and then diluted again to a dilution ratio of 1/3, what is the final dilution ratio?

Solution. We can do the calculations one at a time. First we calculate the original concentration.

$$\begin{aligned}
\frac{10 \text{ mL}}{10+90 \text{ mL}} &= \frac{10}{100} \\
&= \frac{1}{10}.
\end{aligned}$$

To dilute to a ratio of 1/2 we can calculate the amount of diluent to add as a proportion problem like in

Example 2.4.1.

$$\begin{aligned} \frac{1}{2} &= \frac{100 \text{ mL}}{T \text{ mL}} \\ 1 \cdot (T \text{ mL}) &= 2 \cdot (100 \text{ mL}) \\ T &= 200 \text{ mL}. \end{aligned}$$

The total will be 200 mL so we need to add 200 mL − 100 mL = 100 mL of additional diluent. Note at this point the concentration is

$$\frac{10 \text{ mL acyl chloride}}{200 \text{ mL diluent}} = \frac{1}{20}.$$

To dilute again to a ratio of 1/3 we can calculate the amount of diluent to add

$$\begin{aligned} \frac{1}{3} &= \frac{200 \text{ mL}}{T \text{ mL}} \\ 1 \cdot (T \text{ mL}) &= 3 \cdot (200 \text{ mL}) \\ T &= 600 \text{ mL}. \end{aligned}$$

The total will be 600 mL so we need to add 600 mL − 200 mL = 400 mL of additional diluent. Note at this point the concentration is

$$\frac{10 \text{ mL acyl chloride}}{600 \text{ mL diluent}} = \frac{1}{60}.$$

Now we can determine what the resulting dilution ratio after diluting twice (1/2 and then 1/3).

$$\begin{aligned} \frac{1}{10} \cdot F &= \frac{1}{60} && \text{clear the denominator on the left} \\ 10 \cdot \frac{1}{10} \cdot F &= 10 \cdot \frac{1}{60}. \\ F &= \frac{1}{6}. \end{aligned}$$

Notice that $\frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$. This relationship is always true for serial dilution. □

2.5.3 Dosage

If we know the concentration of a drug, we can determine how much is needed for a given dose. These are proportion problems that require change of units.

In medicine some substances are measured in **International Unit** or IU. For each substance this is defined by the effect of that amount of the drug.

Example 2.5.11 One IU of insulin is 0.0347 mg. A common concentration of insulin is U-100 which is 100 IU/mL. This is produced by combining 100 units of insulin in one mL of diluent.

If a person needs 2 units of insulin, how many mL of solution will that be?

Solution 1. This can be solved as a proportion because are asked for an amount that matches a ratio. The ratio (concentration) is 100 IU/mL. Because we are solving for a number of mL, we will write the ratio as

$$\begin{aligned} \frac{\text{mL}}{100 \text{ IU}} &= \frac{v \text{ mL}}{2 \text{ IU}} \\ \frac{\text{mL}}{100 \text{ IU}} \cdot (2 \text{ IU}) &= \frac{v \text{ mL}}{2 \text{ IU}} \cdot (2 \text{ IU}) && \text{multiply to isolate the variable} \\ \frac{2 \text{ mL}}{100} &= v \text{ mL} \\ \frac{1}{50} \text{ mL} &= v. \\ 0.02 \text{ mL} &= v. \end{aligned}$$

Solution 2. Because we have a ratio of desired amount to provided amount we can also solve this problem as a percent.

The ratio is desired amount/provided amount. In this case $\frac{2\text{ IU}}{100\text{ IU}} = \frac{1}{50} = 0.02$ which is 2%. We therefore want 2% of the 1 mL (from 100 IU/1 mL) or 0.02 mL. □

Example 2.5.12 A label reads "2.5 mL of solution for injection contains 1000 mg of streptomycin sulfate." How many milliliters are needed to contain 800 mg of streptomycin?

Solution 1. Because a ratio is given (1000 mg/2.5 mL) and we want to scale this (to 800 mg), this can be setup as a proportion.

Because we want to solve for volume (mL) we setup the proportion as follows.

$$\frac{2.5\text{ mL}}{1000\text{ mg}} = \frac{v\text{ mL}}{800\text{ mg}}.$$
$$\frac{2.5\text{ mL}}{1000\text{ mg}} \cdot (800\text{ mg}) = \frac{v\text{ mL}}{800\text{ mg}} \cdot (800\text{ mg}).$$
$$2\text{ mL} = v.$$

Solution 2. Because we have a ratio of desired amount to provided amount we can also solve this problem as a percent.

The ratio is desired amount/provided amount. In this case $\frac{800\text{ IU}}{1000\text{ IU}} = \frac{4}{5} = 0.8$ which is 80%. We therefore want 80% of the 2.5 mL does or 2 mL. □

Standalone

Checkpoint 2.5.13 A physician ordered Omnicef (cefdinir) 500 mg. Omnicef (cefdinir) has a concentration of 125 mg per 5 milliliters.

What volume should be administered? ____ mL

Solution.

- 20

The unit suggest we can multiply. $500\ \text{ mg} \cdot \dfrac{5\ \text{ mL}}{125\ \text{ mg}} = 20\text{ mL}$

By dividing we determine how many 125 mg units we need to deliver the total 500 mg.

A physician may prescribe a medicine and specify a total amount and a speed at which it should be delivered. For IV's this is called **drop factor** and is specified as a number of drops per minute. Medical personnel must calculate how long to operate the IV so that the total amount of drug prescribed is delivered in the specified time.

Example 2.5.14 Give 1500 mL of saline solution IV with a drop factor of 10 drops per mL at a rate of 50 drops per minute to an adult patient. Determine how long in hours the IV should be administered.

Solution. The rate is specified in drops and the amount is specified in mL which means we need to convert units. This will be done like Example 1.1.18.

$$\frac{50 \text{ drops}}{\text{minute}} \cdot \frac{\text{mL}}{10 \text{ drops}} = \frac{5 \text{ mL}}{\text{minute}}$$

Now that we know the rate in mL, we can setup a proportion so that time calculated per total grams of medication matches the specified rate. Notice how we invert the rate to make the algebra easier.

$$\begin{aligned} \frac{T \text{ min}}{1500 \text{ mL}} &= \frac{1 \text{ min}}{5 \text{ mL}}. \\ \frac{T \text{ min}}{1500 \text{ mL}} \cdot (1500 \text{ mL}) &= \frac{1 \text{ min}}{5 \text{ mL}} \cdot (1500 \text{ mL}). \\ T &= 300 \text{ min}. \end{aligned}$$

The final step is to convert minutes to hours. This is another unit conversion problem using a conversion from Table 1.1.2. The units suggest we can multiply the 300 minutes by the conversion ratio.

$$300 \text{ minutes} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = 5 \text{ hours}$$

□

Standalone

Example 2.5.15 Amoxicillin is an antibiotic obtainable in a liquid suspension form, part medication and part water, and is frequently used to treat infections in infants. One formulation of the drug contains 125 mg of amoxicillin per 5 mL of liquid. A pediatrician orders 150 mg per day for a 4-month-old child with an ear infection. How much of the amoxicillin suspension would the parent need to administer to the infant in order to achieve the recommended daily dose?

Solution. Here we need to scale the amount (from 125 mg to 150 mg). This is a proportion problem, that is, the ratio of medicine to volume is the same so we can setup an equation based on the drug concentration.

$$\begin{aligned} \frac{125 \text{ mg}}{5 \text{ mL}} &= \frac{150 \text{ mg}}{A \text{ mL}} && \text{clear the denominators} \\ (125 \text{ mg})(A \text{ mL}) &= (5 \text{ mL})(150 \text{ mg}) && \text{divide to isolate the variable} \\ \frac{(125 \text{ mg})(A \text{ mL})}{125 \text{ mg}} &= \frac{(5 \text{ mL})(150 \text{ mg})}{(125 \text{ mg})} \\ A \text{ mL} &= \frac{(5 \text{ mL})(150 \text{ mg})}{(125 \text{ mg})} \\ A &= 6 \text{ mL}. \end{aligned}$$

□

Checkpoint 2.5.16 A 5% dextrose solution (D5W) contains 5 g of pure dextrose per 100 mL of solution. A doctor orders 500 mL of D5W IV for a patient. How much dextrose does the patient receive from that IV?

Solution. Once again we need to scale the amount (from 100 mL to 500 mL). This is also a proportion problem, that is, the ratio of medicine to volume is the same so we can setup an equation based on the drug concentration.

$$\begin{aligned}
\frac{5\text{ g}}{100\text{ mL}} &= \frac{D\text{ g}}{500\text{ mL}} && \text{clear the denominators}\\
(5\text{ g})(500\text{ mL}) &= (D\text{ g})(100\text{ mL}) && \text{divide to isolate the variable}\\
\frac{(5\text{ g})(500\text{ mL})}{(100\text{ mL})} &= \frac{(D\text{ g})(100\text{ mL})}{(100\text{ mL})}\\
25\text{ g} &= D
\end{aligned}$$

Example 2.5.17 A sample of chloroform water has a dilution factor of 40. If 2 mL of chloroform are needed how many milliliters total are needed?

Solution. A dilution factor of 40 indicates that 1 mL of chloroform is in 40 mL total of solution. We can setup a proportion to answer this.

$$\begin{aligned}
\frac{1\text{ mL}}{40\text{ mL}} &= \frac{2\text{ mL}}{T\text{ mL}} && \text{clear the denominators}\\
(1\text{ mL})(T\text{ mL}) &= (2\text{ mL})(40\text{ mL})\\
\frac{(1\text{ mL})(T\text{ mL})}{1\text{ mL}} &= \frac{(2\text{ mL})(40\text{ mL})}{1\text{ mL}}\\
T &= 80\text{ mL.}
\end{aligned}$$

□

2.5.4 Exercises

1. **Medical Ratio.** A 1 litre (1,000 mL) IV bag of dextrose solution contains 70 g of dextrose. Find the ratio of grams per millilitre of dextrose. (Enter your answer in fraction form.) ________________Preview Question 1

2. **Medical Ratio.** Find the flow rate (in drops/min) for the given IV (assume a drop factor of 15 drops/mL).

   1100 mL in 5.0 h

   ____________________Preview Question 1 drops/min

3. **Medical Ratio.** Find the length of time (in h) the IV should be administered (assume a drop factor of 11 drops/mL).

   1,000 mL at a rate of 40 drops/min

   ____________________Preview Question 1 h

4. **Medical Proportion.** A label reads: "2.5 mL of solution for injection contains 1,000 mg of streptomycin sulfate." How many millilitres are needed to give 600 mg of streptomycin?

   ____________________Preview Question 1

5. **Medicine to Solution.** Quinidine gluconate is a liquid mixture, part medicine and part water, which is administered intravenously. There are 110.0 mg of quinidine gluconate in each cubic centimeter (cc) of the liquid mixture. Dr. Bernal orders 275 mg of quinidine gluconate to be administered daily to a patient with malaria.

   How much of the solution would have to be administered to achieve the recommended daily dosage?

   ____________________ cc

6. **Medical Ratio with Rounding.** Albuterol is a medicine used for treating asthma. It comes in an inhaler that contains 17 mg of albuterol mixed with a liquid. One actuation (inhalation) from the mouthpieces delivers a 90 μg dose of albuterol. (Reminder: 1 mg = 1000 μg .)

a.) Dr. Olson orders 2 inhalations 3 times per day. How many micrograms of albuterol does the patient inhale per day?

____________________ μg

b.) How many actuations are contained in one inhaler?

____________________ actuations

c.) Erica is going away for 6 months and wants to take enough albuterol to last for that time. Her physician has prescribed 2 inhalations 3 times per day. How many inhalers will Erica need to take with her for the 6 period? Assume 30-day months.

Hint: she can't bring a fraction of an inhaler, and she does not want to run out of medicine while away.

____

7. Concentration. Amoxicillin is a common antibiotic prescribed for children. It is a liquid suspension composed of part amoxicillin and part water.

In one formulation there are 175 mg of amoxicillin in 6 cubic centimeters (cc's) of the liquid suspension. Dr. Scarlotti prescribes 350 mg per day for a 2-yr old child with an ear infection.

How much of the amoxicillin liquid suspension would the child's parent need to administer in order to achieve the recommended daily dosage?

____

8. Concentration. Diphenhydramine HCL is an antihistamine available in liquid form, part medication and part water. One formulation contains 17 mg of medication in 4 mL of liquid. An allergist orders 34-mg doses for a high school student. How many milliliters should be in each dose?

____________________ mL

9. Concentration. How many mL of sodium hydroxide are required to prepare 900 mL of a 8.5% solution? Assume the sodium hydroxide dissolves in the solution and does not contribute to the overall volume.

____________________ mL

10. Dilution Ratio. You are asked to make a 1/11 dilution using 1 mL of serum. How much diluent do you need to use?

____________________ mL

11. Dilution Ratio. A clinical lab technician determines that a minimum of 65 mL of working reagent is needed for a procedure. To prepare a $\frac{1}{10}$ dilution ratio of the reagent from a stock solution, one should measure 65 mL of the reagent and ____________________ mL of the diluent.

12. Dilution Ratio. A patient's glucose result is suspected to be outside the range of the analyzer, so the techs decide to dilute the sample before running it. 45 microliters of serum is added to 180 microliters of diluent and the diluted sample is analyzed. The analyzer reads that the glucose value of the diluted sample is 50 $\frac{mg}{dL}$.

What was the ratio the sample was diluted to?

____________________Preview Question 1 Part 1 of 2

What is the glucose value of the original sample?

____________________ $\frac{mg}{dL}$

13. Serial Dilution. A thyroid peroxidase antibody test was performed on a 45 year old man. The dilution sequence was 20 μL serum added to 140 μL of diluent in tube 1. Then 70 μL from tube 1 was added to 560 μL of diluent in tube 2. Finally 55 μL from tube 2 was added to 220 μL of diluent in tube 3.

All dilution ratios should be given as fractions.

a.) What is the dilution ratio in tube 1?

____________________Preview Question 1 Part 1 of 4

b.) What is the dilution ratio in tube 2?

____________________Preview Question 1 Part 2 of 4

c.) What is the dilution ratio in tube 3?

____________________Preview Question 1 Part 3 of 4

d.) What is the overall (serial) dilution ratio?

____________________Preview Question 1 Part 4 of 4

2.6 Project: False Position

Project 2 Method of False Position. In this project, we are going to learn about an ancient algebraic technique that is built around correcting guesses. We may gain greater appreciation for the value of *wrong* guesses and what we can gain from them.

(a) Solve the following equation any way you would like.

$$x\left(1+\frac{1}{3}+\frac{1}{4}\right)=14.$$

Check your answer using technology.

(b) Notice that 12 is the least common multiple of 3 and 4: the denominators. Distribute 12 in the following expression.

$$12\left(1+\frac{1}{3}+\frac{1}{4}\right).$$

Is this bigger, equal to, or smaller than 14?

(c) We multiplied by a convenient number, which is not quite right. Because it is multiplication we can scale (multiply) our not quite right guess to make it right. Consider

$$y\cdot 12\left(1+\frac{1}{3}+\frac{1}{4}\right)=14.$$

Replace 12 times the sum with your result from the previous step.

Solve the resulting equation for y.

(d) Note that y is the correction to our guess of 12. Calculate $y \cdot 12$.

This will match your original solution. If not, check your calculations.

(e) This method is called *false position* because it guesses a convenient number which is typically false then corrects it. One of the original motivations for this method was the lack of a useful notation for fractions (it dates to the Sumerians and ancient Egyptians).

Many people today use similar methods when dealing with fractions. What is a reason people might distribute a convenient number before doing the solving?

2.7 Project: Arclength Estimation

Project 3 Estimating Arc Lengths. In aviation it is sometimes useful to estimate a distance between points as the length of a circular arc. This results from navigation methods (search for VOR and DME arc if curious). To estimate on the fly they use what is known as the 60:1:1 approximation. It means that 60 miles from a point a one degree arc is approximately one mile in length. Note in aviation the distances would be in nautical miles (nm), but the ratio does not change if we use statute miles (the usual type).

Here we will practice using the method to approximate then check why it works.

(a) *Using the Ratio.*

- **(i)** View Example 2.7.1 to Example 2.7.3.
- **(ii)** What is the arclength of 2 degrees at a distance of 30 miles?
- **(iii)** What is the arclength of 5 degrees at a distance of 30 miles?

(iv) What is the arclength of 10 degrees at a distance of 20 miles?

(b) *Explaining the Ratio.*

(i) Calculate the perimeter of a circle with radius 60 miles using the formula $P = 2\pi r$ where P is the perimeter and r is the radius.

(ii) Calculate the perimeter of a semi-circle (half circle) with radius 60 miles.

(iii) Calculate the perimeter of a quarter of a circle with radius 60 miles.

(iv) Calculate the perimeter of 1/360 of a circle with radius 60 miles.

(v) Note that the previous task is the 60:1:1 ratio (1 degree is 1/360th of a circle). Does your result match (i.e., is the result approximately 1 mile)?

Example 2.7.1 Calculate the arclength of 3 degrees at 60 miles.

Solution. If each degree is one mile then 3 degrees is $A = 3 \cdot 1 = 3$ miles. □

Example 2.7.2 Calculate the arclength of 1 degree at 30 miles.

Solution. At 30 miles we are only half way $(30/60 = 1/2)$, so the length is $A = \frac{1}{2} \cdot 1 = \frac{1}{2}$ miles. □

Example 2.7.3 Calculate the arclength of 4 degree at 18 miles.

Solution. The radius is $(18/60 = 3/10)$ of the usual. Thus each degree is $\frac{3}{10}$ of a mile. This arc is $4°$ so the length is $A = \frac{3}{10} \cdot 4 = 1.2$ miles. □

Chapter 3

Models

3.1 Linear Expressions

This section addresses the topics

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document

and covers the mathematical concepts

- Solve *linear*, rational, quadratic, and exponential equations and formulas (skill)
- Read and interpret models (critical thinking)
- Use models including *linear*, quadratic, exponential/logarithmic, and trigonometric (skill)

Having looked at models in general this section presents linear models. First, we look at some examples and learn how the pieces of a linear model work. Next, we learn to write linear models given a description of a problem. After that we practice solving for different parts of a linear equation. Section 3.3 will introduce a more in depth look at identifying linear models.

3.1.1 Linear Models

This section presents examples of linear models and provides an explanation for the two parts of a linear model.

A linear model (equation) can be written in the following, equivalent forms.

- $y = \frac{a}{b}x + c$
- $by - ax - bc = 0$

Model of Temperature Change with Altitude. As a result of atmospheric physics temperature decreases as the distance above the ground increases. For lower altitudes this can be modeled as

$$T_A = T_G - \left(\frac{3.5}{1000}\right) A.$$

- T_A is the expected temperature at the specified altitude.
- T_G is the temperature at ground level.
- A is the specified altitude in number of feet above ground level.

- $\frac{3.5^\circ}{1000 \text{ ft}}$ is the rate of temperature decrease.

All temperatures are in Fahrenheit.

Note T_G is a parameter, that is, this model depends on the temperature at ground level which of course differs by location and time. We need to obtain that before we can use the model. This is in contrast to the rate 3.5/1000 which is fixed (a result of atmospheric physics). The model states that temperature decreases with altitude, which is why altitude (A) is the variable.

To model the temperature decreasing the rate is negative: subtracting from the starting temperature results in a decrease. All linear models have a rate.

The decrease begins from T_G. This ground level temperature is a starting point from which change occurs. All linear models have a starting point (or shift).

Example 3.1.1 If the temperature at ground level is 43° what is the temperature 1000 ft above ground level (AGL)? 2000 ft AGL, 3000 ft AGL, 3500 ft AGL?

Because fractions of a degree are not useful in making decisions like what to wear, we will round to units.

Solution. Note $T_G = 43°$. We need to calculate T_A for $A = 1000, 2000, 3000, 3500$.

$$\begin{aligned}
T_{1000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(1000 \text{ ft}) \\
&= 39.5 \\
&\approx 40. \\
T_{2000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(2000 \text{ ft}) \\
&= 36. \\
T_{3000} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(3000 \text{ ft}) \\
&= 32.5 \\
&\approx 33. \\
T_{3500} &= 43^\circ - \frac{3.5^\circ}{1000 \text{ ft}}(3500 \text{ ft}) \\
&= 30.75 \\
&\approx 31.
\end{aligned}$$

Notice that we now know that it will be below freezing just above 3000 ft. □

Model of Time to Altitude. A fixed wing aircraft flown optimally climbs from a starting altitude at a fixed climb rate.

$$A_t = A_G + C \cdot t.$$

- A_t is the altitude after t minutes.
- A_G is the starting altitude (likely ground level) in feet mean sea level (MSL).
- C is the rate of climb in feet per minute.
- t is the time since the climb began in minutes.

Note A_G is a parameter, that is, this model depends on the elevation from which the plane begins to climb. This varies by airport (if the climb is started from the ground) or by the situation (if the plane needs to climb from some elevation at which it is flying). The rate C is also a parameter which must be obtained for each plane and is often available in the aircraft's Pilot's Operating Handbook (POH). The model states that altitude increases with time, which is why time (t) is the variable.

In this model everything is added which matches the increase of elevation over time (adding makes the altitude bigger).

Note the same plane will reach a given altitude after different amounts of time depending on how high it was to start with. A_G shifts the model up or down to represent this.

Example 3.1.2 If a plane begins at 160 ft MSL and is climbing at 700 ft/min, how high will it be after 5 minutes? 10 minutes? 15 minutes?

These calculations are made as part of safety planning. The data is sufficiently accurate that rounding is not necessary. Rather we make conservative estimates of the parameters, so that there is always a safety buffer. In this case a conservative estimate for A_G is to round down: this will give us a lower altitude. If that lower altitude is safe, then one 5 feet higher will be safe as well. For the climb rate a conservative estimate is to round down as well. If we can reach an altitude at 700 ft/min, then we will reach it a little earlier at 720 ft/min.

Solution. Note

$$A_t = 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot t.$$

The expected altitudes are

$$\begin{aligned}
A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 5 \text{ min} \\
&= 3660. \\
A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 10 \text{ min} \\
&= 7160. \\
A_t &= 160 \text{ ft} + \frac{700 \text{ ft}}{\text{min}} \cdot 15 \text{ min} \\
&= 10660.
\end{aligned}$$

Note if we need to climb above 4500 ft MSL we will achieve this in between 5 and 10 minutes (closer to 5). □

Model of Fuel Remaining Calculation. When operated at a fixed power setting a vehicle burns the same amount of gas per hour (or other time unit). This leads to the linear model

$$F_t = F_I - r \cdot t.$$

- F_t is the amount of fuel remaining after t minutes.
- F_I is the amount of fuel at the beginning.
- r is the rate (volume per time) at which fuel is being consumed.
- t is the time the vehicle has been operated.

Fuel amounts will be measured in units of volume like gallons or liters. Time will be measured in minutes or hours. The rate r is then in units such as gallons/hour or liters/min.

Note F_I is a parameter, that is, this model depends on the amount of fuel with which we began. The rate r is also a parameter which must be obtained for each vehicle. Often this is not shown during operation (fuel gauges show how much rather than how fast). The model states that the amount of fuel decreases with time, which is why time (t) is the variable.

Because fuel decreases the $r \cdot t$ term is subtracted decreasing the amount from F_I.

Note the same plane will reach a given remaining fuel amount (like empty) at different times depending on how much we have at the beginning. F_I shifts the model up or down to represent this.

Example 3.1.3 If a car begins with 20 gallons of fuel and burns 1.55 gallons per hour, how much fuel will it have after 1 hour, 2 hours, 3 hours, 36 minutes?

A gallon is a large amount so we will maintain one decimal place precision. For safety we should always assume a larger fuel burn, so we will round fuel remaining down.

Solution. Note

$$F_t = 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot t \text{ hr}.$$

Thus

$$\begin{aligned}
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 1 \text{ hr} \\
&= 18.45 \\
&\approx 18.4. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 2 \text{ hrs} \\
&= 16.9. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 3 \text{ hrs} \\
&= 15.35 \\
&\approx 15.3. \\
F_t &= 20 \text{ gal} - \frac{1.55 \text{ gal}}{\text{hr}} \cdot 0.6 \text{ hrs} \\
&= 19.07 \\
&\approx 19.0.
\end{aligned}$$

□

Use this Checkpoint to practice using a linear model.

Checkpoint 3.1.4 The expected temperature at a height above ground is given by

$T_A = T_G - \frac{3.5}{1000}A$

where T_A is the expected temperature in Fahrenheit

T_G is the temperature at ground level in Fahrenheit

A is the height above ground level in feet

If the temperature on the ground is 36°, what will it be at 6200 feet above ground level? ___

Answers should be rounded to the units place.

Solution.

- 14

The model for this situation is $T_A = 36 - \frac{3.5}{1000}A$.

Because we want to estimate the temperature at 6200 ft AGL, we calculate $T_A = 36 - \frac{3.5}{1000}6200 = 14.3 \approx 14$

3.1.2 Building Linear Models

The previous section presented linear models, and illustrated using provided models. This section presents problems that can be modeled as linear equations, and illustrates writing the model (equation) before using it.

A linear model has a starting point (shift, b) and rate (ratio, m). We need to identify these and then write the linear model

$$y = mx + b$$

with these values. We should also label units and explain any parameters.

Example 3.1.5 Consider rope that costs \$0.93 per foot with a shipping charge of \$7.64. To produce a model for the cost of each purchase we will start by trying a couple specific orders.

Suppose we are purchasing 20 feet of this rope. The cost for the 20 feet will be 20 ft $\cdot \frac{\$0.93}{\text{ft}}$, because each foot is \$0.93. This is just like unit conversion: the units (\$/ft and ft) suggest multiplying.

Notice this multiplication is also the same as using a ratio (proportion). We could setup $\frac{\$0.93}{1\text{ft}} = \frac{C}{20 \text{ ft}}$ When we solve this we end up with the same multiplication $20 \text{ ft} \cdot \frac{\$0.93}{\text{ft}}$.

Next we must add the shipping charge. Thus the final cost is $20 \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = \$26.24$. Note there is no rounding because all numbers are exact (no measurements, so no signficant digits) and no fractions of a cent occurred.

Suppose we are purchasing 100 feet of this rope. The cost for the 100 feet will be

$$100 \text{ ft} \cdot \frac{\$0.93}{\text{ft}}.$$

Then we must add the shipping charge. Thus the final cost is $100 \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = \$100.64$.

Notice we could do this with any number of feet (unless the shipping charge increases for larger orders). So in general we can write the cost as

$$s \text{ ft} \cdot \frac{\$0.93}{\text{ft}} + \$7.64 = C.$$

Notice that this equation has a ratio (0.93/1), which is the cost per foot, but also has a shift (+7.64), which is the fixed shipping cost. Thus this is another linear equation. □

When cost is set per linear foot, or per square yard, or similar per unit pricing we often end up with a linear model.

Example 3.1.6 At lower altitudes the barometric pressure typically drops 1 inHg for every 1000 feet of elevation gained (the air is less dense higher up). To produce a model for pressure decrease we will start by calculating the pressure for a couple specific cases.

If the pressure on the ground is 29.76 inHg, what do we expect the pressure to be flying at 4500 ft above ground level?

The pressure drop is a ratio $\frac{1 \text{ inHg}}{1000 \text{ ft}}$. The units suggest we can multiply $4500 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}} = 4.5$ inHg. This is the drop in pressure. To calculate the resulting pressure we need $29.76 \text{ inHg} - 4.5^\circ \text{ inHg} = 25.26$ inHg. We retain 2 decimal places because that is the traditional amount for reporting by meteorologists. Written as one calculation this is $T = 29.76 - \left(4500 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right)$.

If the pressure on the ground is 30.02 inHg what do we expect the pressure to be flying at 6000 ft above ground level?

The pressure drop is a ratio $\frac{1 \text{ inHg}}{1000 \text{ ft}}$. The units suggest we can multiply $6000 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}} = 6$ inHg. This is the drop in pressure. To calculate the resulting pressure we need $30.02 \text{ inHg} - 6 \text{ inHg} = 24.02$ inHg. Written as one calculation this is $P = 30.02 - \left(6000 \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right)$.

Notice we could do this same calculation for any altitude. So in general we can write

$$P_A = P_G - \left(A \text{ ft} \cdot \frac{1 \text{ inHg}}{1000 \text{ ft}}\right).$$

P_A is the pressure at the specified altitude. P_G is the pressure at ground level. A is the altitude above ground level. This is a linear equation with a ratio of (-1/1000) which is the drop in pressure with altitude, and a shift of P_G, which is the pressure on the ground. □

Example 3.1.7 We will find a model (equation) that converts temperature in Fahrenheit to temperature in Celsius. Note that every 9 degrees F is only 5 degrees C, so to convert we must scale the degrees. Also they use different values for the starting point (which is the freezing point of water). Fahrenheit starts at 32° and Celsius starts at 0°. Notice we have a ratio and a shift, so this already looks like a linear model.

To begin with we will convert 52° F to Celsius. We will round to units, because this is just an example (no one will be injured in the demonstration of this model).

The first step is to determine how many degrees above freezing. Because 32° F is the freezing temperature of water, 52° F is $52^\circ - 32^\circ = 20^\circ$ above freezing.

The next step is to scale the degrees. The conversion ratio is $\frac{9^\circ \text{ F}}{5^\circ \text{ C}}$. Converting the 20° above freezing is now a unit conversion. The units suggest we can multiply $20^\circ\text{ F} \cdot \frac{5^\circ\text{ C}}{9^\circ\text{ F}} \approx 11^\circ\text{ C}$. Notice we flipped the conversion ratio so the Fahrenheit degrees would divide to one. The result is 11° C above the freezing point of water in Celsius.

Finally we can add the degrees Celsius to the starting point (freezing temperature of water). Because that is 0°, we have $0^\circ + 11^\circ = 11^\circ$ C.

If we write all of that as one step we obtain

$$(52^\circ\text{ F} - 32^\circ\text{ F})\frac{5^\circ\text{ C}}{9^\circ\text{ F}} + 0^\circ\text{ C} = 11^\circ\text{ C}.$$

Notice we could do this with any temperature. So if we call the temperature to convert T we have

$$C = (T - 32)\frac{5}{9} + 0.$$

This may not look like a starting point plus a ratio scaled. If we expand the expression we obtain

$$C = (T - 32)\frac{5}{9} + 0 = \frac{5}{9}T - \frac{160}{9}.$$

So this is a linear model. We prefer the first form of the equation because the numbers have meaning (e.g., 32° is the freezing point of water as opposed to 160/9 which has no useful meaning. □

The temperature conversion example illustrates an idea about models. We describe a linear model as having a ratio and a (that is one) shift. However, in the temperature conversion example there is a shift a ratio and another shift, or two shifts. We showed these can be combined as one shift. That is always true. However, sometimes the version with multiple shifts is easier to understand. This will be true when we look at graphs of quadratics and exponentials (other forms of models).

3.1.3 Solving Linear Equations

The first section demonstrated using linear models to calculate values. However, sometimes we know the result and want to know the input. This requires solving the linear equation. This section reviews solving linear equation starting with non-contextualized examples and then using some of the models presented above.

Before reading farther solve the equation $5x - 7 = 12$. What steps did you use? Why do they work? Example 3.1.8 is an example of solving another linear equation.

Example 3.1.8 Solve $-8x - 3 = 5$.

Solution.

$$\begin{aligned}
-8x - 3 &= 5.\\
-8x - 3 + 3 &= 5 + 3.\\
-8x &= 8.\\
\frac{-8x}{-8} &= \frac{8}{-8}.\\
x &= -1.
\end{aligned}$$

Note we added three because it eliminates the -3 (undoes subtraction of 3). We divided by negative eight because it eliminates the -8 (undoes the multiplication by -8). □

Checkpoint 3.1.9 What is the solution to $13 = 3x + 4$? ___Preview Question 1

Solution.

- 3

$13 = 3x + 4$. We undo the $+4$ by subtracting from both sides.
$13 - 4 = 3x + 4 - 4$.

$9 = 3x$. We undo the multiplication by 3 by dividing by 3 on both sides.
$\frac{9}{3} = \frac{3x}{3}$.
$3 = x$.

Some linear equations need one more technique. What would you need to solve $17 - 4y = 14 - y$? Below is an example of solving a similar linear equation.

Example 3.1.10 Solve $17 - 4y = 5 + 2y$.

Solution.

$$\begin{aligned} 17 - 4y &= 5 + 2y. \\ -5 + 17 - 4y &= -5 + 5 + 2y. && \text{Undo } +5 \text{ by subtracting.} \\ 12 - 4y &= 2y. \\ 12 - 4y + 4y &= 2y + 4y. && \text{Bring terms with variable to the same side.} \\ 12 &= (2+4)y. && \text{Factoring leaves only one variable.} \\ 12 &= 6y. \\ \frac{12}{6} &= \frac{6y}{6}. && \text{Undo multiplication with division.} \\ 2 &= y. \end{aligned}$$

Notice we had to combine like terms (factor and add). □

Checkpoint 3.1.11 What is the solution to $6x + 3 = 3x + 12$? ___Preview Question 1

Solution.

- 3

$6x + 3 = 3x + 12$.
$6x + 3 - 12 = 3x + 12 - 12$.
$6x - 9 = 3x$.
$6x - 9 - 6x = 3x - 6x$.
$-9 = -3x$.
$-\frac{9}{-3} = \frac{-3x}{-3}$.
$3 = x$.

Another linear equation is $\frac{x}{3} + \frac{x}{4} = \frac{7}{12}$. How would you solve it?

We can solve this the same as in Example 3.1.10 but there is another technique as well whic is shown below.

Example 3.1.12 Solve $\frac{x}{5} + \frac{2x}{7} = \frac{34}{35}$

Solution.

$$\begin{aligned} \frac{x}{5} + \frac{2x}{7} &= \frac{34}{35}. \\ 5 \cdot \left(\frac{x}{5} + \frac{2x}{7}\right) &= 5 \cdot \frac{34}{35}. \\ \frac{5x}{5} + \frac{10x}{7} &= 5 \cdot \frac{34}{35}. \\ x + \frac{10x}{7} &= \frac{34}{7}. \\ 7 \cdot \left(x + \frac{10x}{7}\right) &= 7 \cdot \frac{34}{7}. \\ 7x + \frac{7 \cdot 10x}{7} &= 7 \cdot \frac{34}{7}. \\ 7x + 10x &= 34. \end{aligned}$$

$$\begin{aligned}(7+10)x &= 34.\\ 17x &= 34.\\ \frac{17x}{17} &= \frac{34}{17}.\\ x &= 2.\end{aligned}$$

This is referred to as clearing denominators. We are once again eliminating division by multiplying. Always remember to distribute. Note, we could multiply once if we figured out the correct number (it would be 35 in this case), but there are no prizes for doing this fast, so you can do this either way. □

Now that we have practiced solving linear equations, we can use this skill with the models.

Example 3.1.13 Given the temperature model in Model of Temperature Change with Altitude and supposing the temperature at ground level is 65, determine at what altitude we expect the temperature to be freezing.

In this case the model is $T_A = 65° - \frac{3.5°}{1000 \text{ ft}}A$. We know $T_A = 32°$ and we want to calculate A, the altitude in feet.

$$\begin{aligned}32° &= 65° - \frac{3.5°}{1000 \text{ ft}}A\\ -65° + 32° &= -65° + 65° - \frac{3.5°}{1000 \text{ ft}}A\\ -33° &= -\frac{3.5°}{1000 \text{ ft}}A\\ -\frac{1000 \text{ ft}}{3.5°} \cdot -33° &= -\frac{1000 \text{ ft}}{3.5°} \cdot -\frac{3.5°}{1000 \text{ ft}}A\\ 9428.571428 \text{ ft} &= A.\\ 9430 \text{ ft} &\approx A.\end{aligned}$$

We round to the tens position because that is the precision shown by many altimeters. □

Example 3.1.14 Given the time to altitude model in Model of Time to Altitude and supposing that we are climbing from 80 ft MSL to 5000 ft MSL with a climb rate of 700 ft/min, how long will it take to complete the climb?

In this case the model is $A_t = 80 \text{ ft} + \frac{700 \text{ ft}}{\text{min}}t$. We know $A_t = 5000$ ft, and we want to know the time t.

$$\begin{aligned}5000 \text{ ft} &= 80 \text{ ft} + \frac{700 \text{ ft}}{\text{min}}t.\\ -80 \text{ ft} + 5000 \text{ ft} &= -80 \text{ ft} + 80 \text{ ft} + \frac{700 \text{ ft}}{\text{min}}t.\\ 4920 &= \frac{700 \text{ ft}}{\text{min}}t.\\ \frac{\text{min}}{700 \text{ ft}} \cdot 4920 &= \frac{\text{min}}{700 \text{ ft}} \cdot \frac{700 \text{ ft}}{\text{min}}t.\\ 7.028571429 &= t.\\ 8 &= t.\end{aligned}$$

We round up as a safety margin: it is better to assume we need 8 minutes and be higher than to hope we can recognize 0.03 of a minute (not quite 2 seconds). □

Use this Checkpoint to try solving a linear model.

Checkpoint 3.1.15 How long can you fly if you start with 43 gallons of fuel, burn 14 gallons per hour, and want land with one hour worth of fuel remaining? ____

Round down to the nearest tenth for a conservative estimate.

Solution.

- 2

The model is $F_t = 43 - 14t$. We want one hour of fuel remaining which is 14 gallons, so we solve the following.

$14 = 43 - 14t.$
$14 - 43 = 43 - 43 - 14t.$
$-29 = -14t.$
$-\frac{29}{-14} = \frac{-14t}{-14}.$
$2.0714285714286 = t.$
$2 \approx t.$

3.1.4 Identifying Linear Expressions

All of the equations in this section are linear. What can we use to identify linear expressions or linear equations? Table 3.1.16 shows examples of linear expressions and non-linear expressions.

Table 3.1.16 Linear and Non-linear

Linear	Non-linear
$5x + 3$	$5x^2 - x + 3$
$y = 11 - \frac{7}{13}x$	$y = \frac{17}{x}$
$7x - 9y = 8$	$3 - 2xy = 12$

Some equations that may not appear to be linear can be solved using the same methods.

Example 3.1.17 Solve $\frac{11}{x} + 2 = \frac{18}{x} - 5$.

Solution.

$$\begin{aligned}\frac{11}{x} + 2 &= \frac{18}{x} - 5.\\ x \cdot \left(\frac{11}{x} + 2\right) &= x \cdot \left(\frac{18}{x} - 5\right).\\ 11 + 2x &= 18 - 5x.\\ -11 + 11 + 2x &= -11 + 18 - 5x.\\ 2x &= 7 - 5x.\\ 2x + 5x &= 7 - 5x + 5x.\\ 7x &= 7.\\ \frac{7x}{7} &= \frac{7}{7}.\\ x &= 1.\end{aligned}$$

□

In Section 3.3 we will learn to identify linear models from data.

3.1.5 Exercises

1. Solve. Solve the equation below.
$5(x - 9) - 7 = 31x - 156$
Answer: $x =$ ____________________

2. Solve. Solve $5(x - 4) + 4 = -6(x - 2)$ for x algebraically. If your answer is a fraction, write it in reduced, fractional form. Do NOT convert the answer to a decimal.
$x =$____________________Preview Question 1

3. **Solve.** Solve the equation for the given variable:
$\frac{-8x+3}{2} = -6$
If your answer is a fraction, write it in fraction from and reduce it completely. Do NOT convert to decimals.
$x =$ ________Preview Question 1

4. **Solve.** Solve the equation $\frac{1}{4}y - 3 = \frac{1}{10}y$.
$y =$ ____________________Preview Question 1

5. **Solve.** Solve the equation for the given variable. If your answer is a fraction, write it in reduced, fractional form. Do NOT convert the answer to a decimal.
$\frac{y}{3} + \frac{y}{5} = \frac{7}{8}$
Answer: $y =$ ____________________Preview Question 1

6. **Solve.** In certain deep parts of oceans, the pressure of sea water, P, in pounds per square foot, at a depth of d feet below the surface, is given by the following equation:
$P = 14 + \frac{4d}{13}$
If a scientific team uses special equipment to measures the pressure under water and finds it to be 146 pounds per square foot, at what depth is the team making their measurements?
Answer: The team is measuring at ______ feet below the surface.

7. **Solve.** Solve for k in the equation: $-\frac{8}{9}k - \frac{3}{4} = 7 - \frac{2}{3}k$.
Round your answer to three decimal places. *Note: round only on the last step!*
$k =$ __________

8. **Solve.** Solve the following formula for x
$y = 9mx + 4b$
$x =$ ____________________Preview Question 1
Enter your answer as an expression.
But be careful...to enter an expression like $\frac{a+b}{3+m}$ you need to type *(a+b)/(3+m)*. You need parentheses for both the numerator and denominator.

9. **Solve.** Solve the following formula for m
$c = amt$
$m =$ ____________________Preview Question 1
Enter your answer as an expression.
But be careful...to enter an expression like $\frac{a+b}{3+m}$ you need to type *(a+b)/(3+m)*. You need parentheses for both the numerator and denominator.

10. **Solve.** Solve the formula $d = rt$ for t.
$t =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

11. **Solve.** Solve the formula $V = \pi r^2 h$ for h. HINT: type π as pi.
$h =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

12. **Solve.** Solve the formula $A = \frac{1}{2}bh$ for b.
$b =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!