Mathematics in Trades and Life

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For Students

This book is designed for use in learning specific mathematical skills and topics useful in a variety of trades. The skills include

- Interpret data in various formats and analyze mathematical models
- Read and use mathematical models in a technical document
- Communicate results in mathematical notation and in language appropriate to the technical field

You will learn to work with the following mathematical concepts.

- Precision and accuracy
  - Rounding
  - Significant Figures
- Rates
  - Identify rates as linear, quadratic, exponential, or other
  - Direct and indirect variation
- Proportions
  - Setup and solve
  - Percentage
  - Unit conversion
  - Mixtures
  - Similarity of shapes
- Solving
  - Solve linear, rational, quadratic, and exponential equations and formulas
  - Solve a system of linear equations
- Linear Relationships
  - Linear models
  - Determine slope
  - Interpret and extrapolate from data
- Other Relationships
  - Quadratic models
  - Exponential models

  - Logarithmic models
- Trigonometry
  - Analyze right triangles
  - Analyze non-right triangles
  - Sine and cosine functions
  - Trigonometric models

This book is not written as a how to manual for specific applications. That is, this book does not provide a step-by-step example to follow for every type of problem. Rather it provides an initial example, presents general concepts used in the example, and helps you practice recognizing the same concept and applying it to other problems.

This book is intended for use in a course that emphasizes building the skills to read and use mathematics (such as in a manual), and to recognize mathematical concepts in things you see and read.

For Faculty

This book is designed to be used for classes supporting trade programs in a variety of fields and also to satisfy baccalaureate general education requirements. The first version was written to transition to an OER for MATH A104 Technical Mathematics at the Univesity of Alaska Anchorage. Served disciplines included auto and heavy equipment mechanics, welding and non-destructive testing, aviation mechanics, pilots and air traffic controller, and medical certification programs.

Because of a desire to enable students in these programs to extend their studies into a baccalaureate, the course was designed to fufill the general education requirements meaning the students would not need to take another mathematics or quantitative literacy course. The general education outcome was: Quantitative courses develop abilities to reason mathematically and analyze quantitative and qualitative data to reach sound conclusions for success in undergraduate study and professional life. The indicators were

- Interprets info in mathematical form (equations, graphs, diagrams, tables, words)
- Represents and/or converts relevant quant info and explain its assumptions and limits
- Applies mathematical forms (equations, graphs, diagrams, tables, words) to quantitative problems to reach sound conclusions
- Communicates quantitative results appropriate to the problem or context

The projects are an integral part of the general education goals. These are intended to be assigned after relevant material is covered. They require students to recognize the material on their own and apply it. The projects are designed to give students opportunity to express calculations using standard mathematical notation and to communicate mathematical results in clear language.

The book is written to be used in conjunction with active learning pedagogies. Each section introduces concepts providing a couple examples and then provides a self-grading exercise with feedback for the reader to test comprehension of the concept. Videos where included are presentations of the introduction of the concept. Homework by default is live, online problems that provide feedback. The scores on these cannot be saved. Because this is implemented through MyOpenMath, that tool can be easily paired with the book. It is possible to produce a version of the text where the homework is not live.

Contents

Chapter 1

Cross Cutting Topics

1.1 Units

We measure many things such as distance, time, and weight. We describe these measurements in terms of units like mile, hour, and pounds. But have you ever stopped to think about how these units are defined?

The story of some of these units is lost in history. For example dividing the day into 24 units began with ancient Egyptians. They did not record, that we know of, the reason for choosing 24 units as opposed to 30 or any other number.

Other units, such as the metric (or SI) are much more modern. Initially many units were based on something physical. For example one calorie is the amount of heat it takes to raise the temperature of one gram of water 1° C. The meter was originally defined as one ten-millionth of the distance from the equator to the north pole. The problem with this type of measurement is that it is neither fixed (depends on where on the equator your begin) nor easy to measure.

Thus modern definitions were developed. The length of a meter was changed to mean the length of a bar of metal kept in special storage in France. The bar had been carefully constructed and was used to confirm other measurement devices were correctly calibrated. It was change yet again to be based on wavelengths of radiation. These are uniform no matter where they are done, so they can be used by many people to construct simple measurement tools.

1.1.1 Types of Measurement

First we will look at the units (names of units) for different types of measurement. Note that the U.S. Customary system (related to the British Imperial system) is non-uniform, so there are multiple names for some types. For the metric (formally known as SI or international system). Table 1.1.1 lists names of units.

Table 1.1.1 Units of Measure

Measuring	US Customary	Metric
Length	inch (in) foot (ft) yard (yd) mile (mi)	meter (m)
Volume	fluid ounce (oz) cup (c) pint (pt) quart (qt) gallon (g)	liter (L or ℓ)
Weight	ounce (oz) pound (lb)	gram (g)
Temperature	degrees Fahrenheit (F)	degrees Celsius (C)
Pressure	inches of mercury (inHg)	Pascal (Pa)
Time	second (s) minute (min) hour (hr)	

Note that fluid ounces and weight ounces are not the same unit. 10 fluid ounces of milk does not weight 10 ounces. You must determine which ounce is referenced by the context. This can be tricky in recipes which is a good reason to us SI units.

Note a gram is a unit of mass rather than weight. Mass times the acceleration due to gravity is weight. However, pound and ounce are units of weight. The mass can be obtained by dividing by the acceleration due to gravity. However gram is often used to describe weight because it is easy to switch between it and weight. The unit official unit for weight (a force) is a Newton.

1.1.2 U.S. Customary

Because the British Imperial system from which the U.S. Customary system was developed was based on disparate measurements from many years ago. As a result there are different units for different scales (e.g., inches for small lengths and miles for long distances). Converting between units therefor requires remembering special numbers for conversion. Most of these you likely know.

Table 1.1.2 Converting within U.S. Customary

Measuring	Unit 1	Unit 2
Length	1 mi 1 yd	5280 ft 3 ft
	1 ft	12 in
Volume	1 g 1 qt 1 pt 1 c	4 qts 2 pts 2 c 8 oz
Weight	1 ton 1 lb	2000 lbs 16 oz
Time	1 year 1 day 1 hr 1 min	365 days 24 hrs 60 mins 60 secs

All of these numbers are defined this way. They are not measurments. Of course a year is not always the same number of days, but for planning purposes we can typically use the common 365 days without injury or loss.

Standalone

How many quarts is 2.3 gallons?

Solution. We know each gallon is 4 quarts, so we multiply by 4

$$2.3 \text{ gallons} \cdot \frac{4 \text{ quarts}}{\text{gallon}} = 9.2 \text{ quarts}$$

□

How many cups is 1.7 gallons?

Solution. Because we don't have a number of cups per gallon we will do this in steps.

$$1.7 \text{ gallons} \cdot \frac{4 \text{ quarts}}{\text{gallon}} \cdot \frac{2 \text{ pints}}{\text{quart}} \cdot \frac{2 \text{ cups}}{\text{pint}} = 27.2 \text{ quarts}$$

□

How many days is 17 hours?

Solution. We know each day is 24 hours. Because this is going to a bigger unit, we divide by 24

$$17 \text{ hours} \cdot \frac{1 \text{ day}}{24 \text{ hours}} = 0.708 \text{ days}$$

□

1.1.3 Metric (SI)

Rather than have different names for different scales, metric uses one name of the unit (e.g., liter) and then uses prefixes to indicate size. These can be converted easily, because each one is a power of ten (uniform).

Table 1.1.6 Metric Prefixes

Multiple	Prefix
10^{12}	tera (T)
10^{9}	giga (G)
10^{6}	mega (M)
10^{3}	kilo (k)
10^{2}	hecto (h)
10	deka (da)
10^{-1}	deci (d)
10^{-2}	centi (c)
10^{-3}	milli (m)
10^{-6}	micro (μ)
10^{-9}	nano (n)
10^{-12}	pico (p)

How many centimeters is 3.8 meters?

Solution. We know one centimeter is 10^{-2} meters which means the decimal shifts two positions. 3.8 meters is 380 centimeters. □

How many kilotons is 2.3 megatons?

Solution. We know one kiloton is 10^2 tons and one megaton is 10^3 tons. This means we shift the decimal $3 - 2 = 1$ position. 2.3 megaton is 23 kilotons. □

How many centiliters is 13.6 milliliters?

Solution. We know one centiliter is 10^{-2} liters and one milliliter is 10^{-3} liters. This means we shift the decimal $-2 - (-3) = 1$ position. 13.6 milliliters is 1.36 centiliters. □

1.1.4 Converting between Systems

Commonly we end up with measurements in both U.S. Standard and SI units. We will need to convert all units to one system before using them together. This process is the same as converting one Standard unit to another (e.g., converting miles to feet).

Table 1.1.10 U.S. Customary to SI

Measuring	Standard	SI
Length	1 mi	1.609344 km
	1 ft	0.3048 m
	1 in	2.54 cm
Volume	1 gal	3.785412 L
	1 oz	29.573532 mL
Weight	1 lb	0.453592 kg
	1 oz	28.349523 g

Table 1.1.11 SI to U.S. Customary

Measuring	SI	Standard
Length	1 km	0.621371 mi
	1 m	3.280840 ft
	1 cm	0.393701 in
Volume	1 L	0.264172 gal
	1 mL	0.033814 oz
Weight	1 kg	2.204623 lb
	1 g	0.035274 oz

How many kilometers is 26.2 miles?

Solution. From Table 1.1.10 we know each mile is 1.609344 km.

$$26.2 \text{ miles} \cdot \frac{1.609344 \text{ km}}{\text{mi}} \approx 42.2 \text{ km}$$

□

1.1.5 Exercises

1. Units. Select the correct units to complete the conversion below.
gallons ∗ ______________ = miles
Answer:

(a) gallons/miles

(b) 1/miles

(c) miles/gallons

(d) gallons

(e) miles

(f) 1/gallons

2. Units. Select the correct units to correctly complete the calculation below.
(people)∗(______________) = (dollars)
Answer:

(a) 1/people

(b) people/dollars

(c) people

(d) dollars

(e) 1/dollars

(f) dollars/people

3. Units. Convert the measurement. You may find it useful to use this table[1].
48 c = ____ gal

4. Units. Aurora mixed 3 gallons of lemonade and poured it into two 5-quart jugs. How many cups of lemonade were left over after she filled the jugs?
This table[2] may be useful for this problem.

[1] mital.uaa.alaska.edu/section-units.html#table-customary-convert

_ cups

5. Units. Convert $4\frac{1}{4}$ hours to minutes. Enter your answer as an integer or a reduced fraction.

$4\frac{1}{4}$ hours =______Preview Question 1 minutes

6. Units. Convert the measurement.
3 days = ________ sec

7. Units. A corn stalk grew 4 inches in the first month after it planted, since then it grow another 7 feet.
What is the total height of the corn in feet and inches? __________________ ft __________________ in
What is the to total height of the corn stalk in inches? ____________________Preview Question 1 Part 3 of 4
What is the total height of the corn stalk in feet? ____________________Preview Question 1 Part 4 of 4 Round your answer to 2 decimal places.

8. Units. Select the unit that best fits the scenario
The soda can holds 12

(a) fluid ounce(s)

(b) cup(s)

(c) gallon(s)

of soda

9. Units. Add the following weights:
8 lb 13 oz + 23 lb 13 oz + 28 lb 3 oz

_______________ pounds ____________________ ounces

10. Units. You are in charge of drinks for a community barbecue. You need to supply at least 120 cups of beverage to provide enough for the projected number of people that will attend. So far, you have received the following donations:

- Enough mix to make 3 gallons of lemonade
- 7 bottles of fruit juice that each contain 64 fl. oz.

How many cups of beverage do you have?
____Preview Question 1 Part 1 of 2
Will you have enough for the barbecue?

(a) yes

(b) no

11. Units. Convert 37 miles per hour to feet per second. You may wish to use this table[3].
37 miles per hour = _____ feet per second. Round your answer to the nearest tenth.

12. Units. Convert 878 square inches to square feet.
Round your answer to the nearest hundredth.
_____ square feet

13. Units. Jean's bedroom is 12 feet by 13 feet. She has chosen a carpet which costs \$32.55 per square yard. This includes installation. Determine her cost to carpet her room.
\$ ____________________Preview Question 1 Part 1 of 2

[2] `mital.uaa.alaska.edu/section-units.html#table-customary-convert`
[3] `mital.uaa.alaska.edu/section-units.html#table-customary-convert`

How much would she have saved if she went with the carpet that costs $28.65 per square yard instead?
$ ____________________Preview Question 1 Part 2 of 2

14. Units. Colan is making a quilt and he has determined he needs 1415 square inches of gray fabric and 877 square inches of blue. How many square yards of each material will he need to purchase from the fabric store?
The store only sells fabric by the by the quarter yard.
The gray fabric: ____________________ square yards
The blue fabric: ____________________ square yards
How many total yards of fabric will she have to buy?
____________________ square yards

15. Units. A unit of measure sometimes used in surveying is the *link;* 1 link is about 8 inches. About how many links are there in 9 feet? Do not round your answer.
There are ___ links in 9 feet.

16. Units. 196 in. to yards, feet, and inches
__ yds __ ft __ in

17. Units. David has 14 yd. of material that he will cut into strips 15 in. wide to make mats. How many mats can David make?___

18. Units. Part 1 of 2I saw a job opening for an Audio Engineer that pays $51,000 per year salary. Assume that a regular work week for this job is 36 hours, and that you will work 50 weeks in a year.What is the hourly pay for this job? We will answer the question by converting $51,000 per year into dollars per hour.

If we begin with the fraction $\frac{\$51000}{\text{year}}$, we can multiply by two unit fractions to complete the conversion. What are these fractions? Choose the correct fractions in the calculation below:
$\frac{\$51000}{\text{year}} \times$

(a) 1 week / 50 years

(b) 50 weeks/1 year

(c) 50 years/1 week

(d) 1 year/50 weeks

$\times$

(a) 36 hours/1 week

(b) 1 week/36 hours

(c) 36 weeks/1 hour

(d) 1 hour/36 weeks

Part 2 of 2Finally, what is the hourly pay for this job?
Hourly pay = ____________________ dollars/hour

19. Units. Part 1 of 2
John Mark was driving at 98 feet per second on the freeway the other day. If the speed limit is 65 miles per hour, was he driving too fast? Answer the question by converting 98 feet per second into miles per hour.
To answer this question, we will convert the numerator into miles and the denominator into hours.
If we begin with the fraction $\frac{98 \text{ feet}}{1 \text{ second}}$, we can multiply by three unit fractions to complete the conversion. What are these fractions? Choose the correct fractions in the calculation below:

$\frac{98\text{ feet}}{1\text{ second}} \times$

(a) 60 min/1 sec

(b) 1 min/60 sec

(c) 1 sec/60 min

(d) 60 sec/1 min

$\times$

(a) 60 min/1 hr

(b) 1 min/60 hrs

(c) 60 hrs/1 min

(d) 1 hr/60 min

$\times$

(a) 1 mi/5280 ft

(b) 5280 mi/1 ft

(c) 5280 ft/1 mi

(d) 1 ft/5280 mi

Part 2 of 2The conversion calculation looks like:
$\frac{98\text{ feet}}{1\text{ second}} \times \frac{60\text{ sec}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{1\text{ mi}}{5280\text{ ft}}$
Was John Mark driving too fast?
Answer:

(a) No

(b) Yes

...because he was driving ______ mi/hr (round using significant figures; all speeds are accurate to two sig figs) when the limit was 65 mi/hr

20. Units. Part 1 of 3Darius wants to buy some specialty fabric to make his kindergardner Marquis a halloween costume. The fabric store tells him that the fabric costs 8.35 $\frac{\text{dollars}}{\text{yard}^2}$. Note about this unit[4][Note: Typically, when a fabric store says "yard", they actually mean "square yards". In this problem, I have written it out in the correct format: yards^2]Darius is sure he saw this advertised online for $0.77\frac{\text{cents}}{\text{inch}^2}$, and he wants to know if it is a better deal in the store or online. He will take a minute to convert the price from $\frac{\text{dollars}}{\text{yard}^2}$ to $\frac{\text{cents}}{\text{inch}^2}$ to see which is the better deal.

Darius knows the following facts:

- 1 yard = 3 feet
- 1 ft = 12 inches
- 1 dollar = 100 cents

Darius will perform the conversion by starting with the measured price as a fraction. Choose the correct unit fraction to multiply by in order to complete the conversion:

$\frac{8.35 \text{ dollars}}{\text{yard}^2} *$

(a) 3ft / 1yd

(b) 3yd / 1ft

(c) 1ft / 3yd

(d) 1yd / 3ft

*

(a) 3yd / 1ft

(b) 3ft / 1yd

(c) 1yd / 3ft

(d) 1ft / 3yd

*

(a) 12in / 1ft

(b) 12ft / 1in

(c) 1in / 12ft

(d) 1ft / 12in

*

(a) 12ft / 1in

(b) 1ft / 12in

(c) 12in / 1ft

(d) 1in / 12ft

*

(a) 1dollar / 100cents

(b) 1cent / 100dollars

(c) 100cents / 1dollar

(d) 100dollars / 1cent

Part 2 of 3The correct expression is:

$$\frac{8.35 \text{ dollars}}{\text{yard}^2} * \frac{1 \text{ yd}}{3 \text{ ft}} * \frac{1 \text{ yd}}{3 \text{ ft}} * \frac{1 \text{ ft}}{12 \text{ in}} * \frac{1 \text{ ft}}{12 \text{ in}} * \frac{100 \text{ cents}}{1 \text{ dollar}}$$

What is the final result, rounded to two decimal places?

Answer: ____________________ $\frac{\text{cents}}{\text{inch}^2}$

Part 3 of 3If Darius wants to save the most money, should he shop at the store or online?

Answer:

(a) The Store

(b) Online

[4] #

21. Units. Select the unit that best fits the scenario
The soda can holds 250

(a) milliliter(s)

(b) liter(s)

(c) centimeter(s)

(d) kilogram(s)

(e) meter(s)

(f) kilometer(s)

of soda

22. Units. Select the unit that best fits the scenario
The book weighs 1

(a) milligram(s)

(b) gram(s)

(c) kilogram(s)

(d) centimeter(s)

(e) meter(s)

(f) liter(s)

(g) milliliter(s)

23. Units. Select the unit that best fits the scenario
The bucket of sand weighs 4

(a) milligram(s)

(b) gram(s)

(c) kilogram(s)

24. Units. How many millimeters are there in a meter? ____________________
How many liters are in a decaliter? ____________________
How many centigrams are in there in a gram? ____________________
Which prefix indicates a bigger quantity? kilo
hecto
Which prefix indicates a bigger quantity? deci
deca
Which prefix indicates a bigger quantity? kilo
mega
Which prefix indicates a bigger quantity? centi
milli

25. Units. Convert the measurement
4130 mL = ______ L

26. Units. Convert the measurement
3.1 g = ______ mg

27. Units. Convert the measurement
7 kg = ______ g

28. Units. Convert the measurement
2 m = ______ cm

29. Units. A bottle of Vitamin E contains 100 soft gels, each containing 15 mg of vitamin E. How many total grams of vitamin E are in this bottle?
There are __ grams of Vitamin E in the bottle.

30. Units. Convert the measurement
466 cm = ______ m

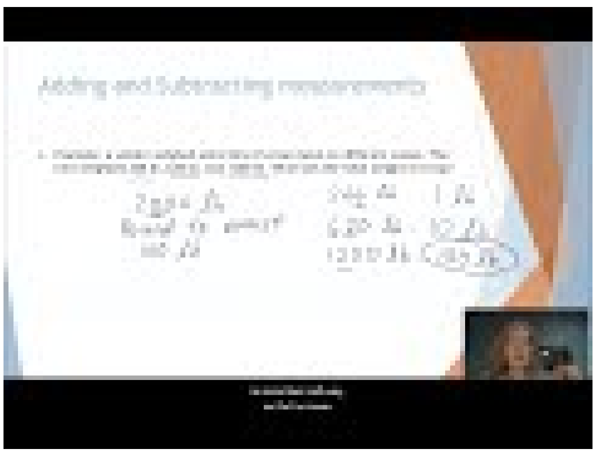
31. Units. Convert the measurement using the rules of SI prefixes.
4 terabyte = __________ gigabyte

32. Units. Convert 6.7 square meters to square centimeters.
_____ square centimeters

33. Units. A small rectangular panel measures 0.7 cm by 0.6 cm. What is its area in square millimeters?
_____ square millimeters

1.2 Accuracy and Precision

1.2.1 Explanation



Standalone

Figure 1.2.1 Introduction to Precision and Accuracy

When working with measurements, we care about the reasonableness of the results. This leads us to two concepts.

Definition 1.2.2 Accuracy. The **accuracy** of a measurement is how close the measurement is to the actual value. ◊

Note, if we are measuring something, it is because we don't know the actual value. Thus we can't determine the exact value of many kinds of data. Instead we will settle for repeatability. If we get the same result often enough, we can convince ourselves that it is reasonably accurate.

Definition 1.2.3 Precision. The **precision** of a measurement is the size of the smallest unit in it. ◊

Note we can have high precision with low accuracy. That is, just because we write a lot of decimal places does not mean they are close to the actual value.

1.2.2 Significant Figures

When writing down measurements we need a way to indicate how precise the measurement is. **Significant digits** also called **significant figures** or simply "sig figs" are a way to do this.

The rules for writing numbers with significant digits have two parts: non-zero digits, and zero digits.

1. All non-zero digits are signficant.
2. Zeros between non-zeros are significant.
3. Any zeros written to the right of the decimal point are significant.
4. If zeros between non-zero digits (on left) and the decimal point (on right) are supposed to be significant, a line is drawn over top of the last significant digit.
5. For numbers less than 1, zeros between the decimal point (on left) and non-zero digits (on right) are not significant.

We can summarize these rules as: write only digits that you mean, and if it is ambiguous clarify.

Each of these numbers is written with five (5) significant digits.

- 10267
- 1.2400
- $7201\bar{0}$
- 2834100
- 0.0010527

□

Checkpoint 1.2.5 How many significant digits does 203 have? ___
How many significant digits does $20\bar{0}0$ have? ___

Answer 1. 3

Answer 2. 3

Solution. Because 203 ends with a non-zero digit all three digits are significant.
Because the 0 in the tens position is marked as significant (the bar) there are 3 significant digits

We also need rules for arithmetic with significant digits. These are based on two principles

- A result of arithmetic cannot be more precise than the least precise measurement.
- The number of significant digits cannot increase.

For addition and subtraction the result (sum or difference) has the same precision as the least precise number added or subtracted. After adding or subtracting we round to the farthest left, last significant digit.

$11050 - 723 = 10330$. This is because the last significant digit of 11050 is the 10's position (with the 5 in it) whereas the last significant digit of 723 is the 1's position (with the 3 in it). We do not know the 1's position of 11050, so we cannot know the 1's position in the result. □

$311 + 8310 + 202200 = 210800$. This is because the farthest left, last significant digit is in the 100's position in 202200. The extra precision of the other two numbers is not useful. □

The significant digits addition/subtraction rule basically says that adding precise data to imprecise data does not increase the precision of the imprecise data. A detailed explanation is in this video.

Standalone

Checkpoint 1.2.8 Calculate $646 + 21.12 + 120$: _____
Calculate $63.97 - 21$: _____

Answer 1. 790

Answer 2. 43

Solution. $646 + 21.12 + 120 = 787.12 \approx 790$ because 120 is significant to only the 10's position (the others are more precise).

$63.97 - 21 = 42.97 \approx 43$ because 21 is significant to only the 1's position (the other is more precise).

For multiplication and division the result (product or quotient) has the same number of significant digits as the least number of the input numbers.

$11050/722 = 15.3$. This is because 722 has only 3 significant digits. □

$17 \times 14\bar{0} \times 3.178 = 7600$. This is because 17 has only two significant digits. □

The significant digit multiplication/division rule basically says that digits that were multiplied by imprecise data cannot be precise. An explanation of why this rule works is in this video.

Standalone

Checkpoint 1.2.11 Calculate 646×21.12: _______
Calculate $63.97/21$: _____

Answer 1. 13600

Answer 2. 3

Solution. $646 \times 21.12 = 13643.52 \approx 13600$ because 646 has only 3 significant digits (the other has more).

$63.97/21 = 3.046190476 \approx 3.0$ because 21 has only 2 significant digits (the other has more).

Significant digit rules must be applied at each step. That is if we have a mix of addition, subtraction, multiplication, and division then we do one operation at a time and apply the appropriate significant digits rule before performing the next arithmetic step.

Consider

$$11,728 + 39(17.9 + 1.23).$$

By order of operations we first calculate $17.9 + 1.23 \approx 19.1$. Second by order of operations we calculate $39 \times 19.1 = 740$. Finally we calculate $11,728 + 740 = 12,470$. □

1.2.3 Rounding

For a variety of reasons in applications we need to round a number, that is ignore some level of precision.

Table 1.2.13 Reasons for Rounding

Reality Constraints	For example we cannot buy partial packages or have fractional people
Remove Detail	For example when describing the population of a nation
Control Error	When used in significant digits

The reason for rounding determines how we do it. Consider the following reality constraints requiring rounding. For example if we need 21 eggs and eggs are sold in cartons of one dozen (12) eggs, we need $21/12 = 1.75$ cartons. Since we cannot purchase part of a carton, we must round 1.75 to 2, and purchase 2 cartons.

Note in this example reality requires us to round up to the nearest integer. We round to an integer because we cannot purchase fractional cartons of eggs. We had to round up, because rounding down would leave us with insufficient eggs (and we are hungry).

Suppose you have a bank account containing \$11410 that accrues 1.65% interest. The bank calculates the payment should be $\$11410 \cdot 0.0165 = \$188.265$. The bank will pay you \$188.26. They round to the nearest one hundredth because cents is a unit which can be paid. They round down, because they like paying less.

For removal of detail consider reporting the population of a country. We might report the population as 9 million rather than 9,904,607. We do this because we don't need the exact number which likely is changing every day. When reporting on salary ranges we might report a range between \$60,000 and \$80,000. That the range is actually \$61,233.57 and \$80,290.11 is unlikely to change a decision. A common usage of removing detail is when we care about the scale of things rather than the count.

Rounding to control error is the use of significant digits.

We can round to any digit. We can round up, down, or simply "round". Context or instructions will specify which digit and which type of rounding.

(a) Round 72481 down to the nearest hundred.

Solution. 72400 is rounding down: we leave the 4 (hundred position) alone and "truncate" (turn to 0) all digits to the right. Note $72400 \leq 72481$.

(b) Round 72481 up to the nearest hundred.

Solution. 72500 is rounding up: we increase the 4 to a 5 and "truncate" (turn to 0) all digits to the right. Note $72500 \geq 72481$.

(c) Round 72481 the nearest hundred.

Solution. Because 72481 is closer to 72500 than it is to 72400, we round to 72500. We can recognize that we should round up because the tens position is $8 \geq 5$ which makes it closer to go up. We could also recognize the need to round up by calculating $500 - 481 = 19$ and $481 - 400 = 81$ and noticing that $81 \geq 19$.

□

(a) Round 72481 down to the nearest thousand.

Solution. 72000 is rounding down: we leave the 2 (thousands position) alone and "truncate" (turn to 0) all digits to the right. Note $72000 \leq 72481$.

(b) Round 72481 up to the nearest thousand.

Solution. 73000 is rounding up: we increase the 2 to a 3 and "truncate" (turn to 0) all digits to the right. Note $73000 \geq 72481$.

(c) Round 72481 to the nearest thousand.

Solution. Because 72481 is closer to 72000 than it is to 73000, we round to 72000. We can recognize that we should round up because the hundreds position is $4 < 5$ which makes it closer to go down. We could also recognize the need to round down by calculating $3000 - 2481 = 519$ and $2481 - 2000 = 481$ and noticing that $481 < 519$.

□

Round 72321.83 to the specified precision.

- Thousands: 72000
- Ones:72322
- Tenths: 72321.8

□

Checkpoint 1.2.17 Round 812,247 to the nearest ten: ___

Round 812,247 to the nearest hundred: ___

Answer 1. 812250

Answer 2. 812200

1.2.4 Greatest Possible Error

When we write a number with significant digits notation or we round a number we write a number that has some error in it. For example if it rained 0.86 inches in a day and we write 0.9 inches, there is an error of 0.4. The important question is "how big can the error be?".

Because our rule for rounding is digits 0-4 round down round up and digits 5-9, rounding will always have a greatest possible error of 5. Consider Example 1.2.18.

What is the greatest possible error if 130 was rounded to the nearest 10?

Solution. One possibility is that 130 was rounded down. Then the original number was one of 130, 131, 132, 133, or 134. 134 is the farthest away from 130 at $134 - 130 = 4$.

The other possibility is that 130 was rounded up. Then the original number was one of 125, 126, 127, 128, or 129. 125 is the farthest away at $130 - 125 = 5$.

Thus the greatest possible error was 5 from the case that 125 was rounded up.

Note in this solution we assumed the number rounded was an integer. However, if we allowed for 134.927 and 125.01 the result would be the same. the extra digits don't change the rounding. □

What is the greatest possible error if 9.31 was rounded to the nearest hundredth?

Solution. The largest possible error is if 9.31 was rounded up from 9.305. Thus the greatest possible error is 5 one thousandths. □

What is the greatest possible error if 223 was rounded up to the nearest one?

Solution. 223 could have been rounded up from 222.1. But it could also have been rounded up from 222.01 or anything else. Thus the greatest possible error is less than 1 $(223 - 222 = 1)$. □

Notice we have to know what type of rounding was used. In most measurements (i.e., significant digits) standard rounding will be used. For example think about measuring on a ruler: if the object isn't exactly on one of the lines, you will choose the closest one. The closest one requires rounding.

Checkpoint 1.2.21 What is the greatest possible error if 8120 was rounded to the nearest ten? ___

Answer. 5

Solution. This could be any number from 8115 to 8124. Thus the greatest possible error is 5.

Checkpoint 1.2.22 What is the greatest possible error in the result $934\bar{0}00$? ___

Answer. 50

Solution. The smallest accurate digit is the hundreds position as indicated by the bar. Thus the number could be anything from 933950-934049. Thus the greatest possible error is 50.

1.2.5 Purposes of Rounding

1.2.6 Exercises

1. Significant Digits. How many significant figures does 2,120,000,000 kg have?_______________

2. Count Significant Digits. Tell how many significant digits there are in each measurement.

(a) 23,000.0 km _____

(b) 2041 m _____

(c) 0.002 mm _____

(d) 8,490,000 yd _____

3. Count Significant Digits. How many significant digits does 0.0020 have?

1
2
3
4
5
6
7
8

4. Count Significant Digits. Determine how many significant figures are in each measurement. If the measurement is exact, select ”exact”. Exact means there is no error in measurement.

(a)
  i. 1
  ii. 2
  iii. 3
  iv. 4
  v. 5
  vi. 6
  vii. 7
  viii. 8
  ix. 9
  x. 10
  xi. exact

(b)
  i. 1
  ii. 2
  iii. 3
  iv. 4

v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(c) i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(d) i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

(e) i. 1
ii. 2
iii. 3
iv. 4
v. 5
vi. 6
vii. 7
viii. 8
ix. 9
x. 10
xi. exact

5. Count Significant Digits. Determine the accuracy (i.e., the number of significant digits) of this number: 0.03081

2
3
4
5

6. Count Significant Digits. Determine the accuracy (i.e., the number of significant digits) of this number: $39\overline{0},0001$

2
3
4
5

7. Significant Digits Arithmetic. Calculate the product below, and express the result with the correct number of significant figures.

$6.13 \times 5.940 =$ _____

8. Significant Digits Arithmetic. Calculate the quotient below, and express the result with the correct number of significant figures.

53.6 &;div; 7.8 = _____

9. Significant Digits Arithmetic. Calculate the sum below, and express the result with the correct number of significant figures.

$44.4 + 0.60 + 320 =$ _____

10. Significant Digits Rounding. Round off the approximate number as indicated.

15.79; 2 significant digits

____

11. Greatest Error. Determine the GPE (i.e., the greatest possible measurement error) of this number: 0.107

(a) ± 0.5

(b) ± 0.05

(c) ± 0.005

(d) ± 0.0005

(e) ± 0.00005

12. Greatest Error. Given the measurement 4.4 gal, find the following.

Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ gal

13. Greatest Error. Given the measurement 0.006 ft, find the following.

Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ ft

14. Greatest Error. Given the measurement 667 psi, find the following.

Precision to nearest ____________________ (thousand, hundred, ten, whole, tenth, hundreth, thousandth)
Accuracy ____________________ (number of significant digits)
Greatest possible error ____________________ psi

1.3 Formulas/Models

Many principles of sciences and other fields can be expressed using mathematical notation. These are often called **formulas** or **mathematical models**. In this section we will practice reading formulas and calculating values from them.

When we have a formula and we know enough of the values we can calculate others. Note in the following examples how the provided values are inserted into the formula before performing the arithmetic.

Ohm's Law relates three properties of electricity: voltage, current, and resistance. **Voltage**, measured in volts (V), is analagous to the amount of pressure to move the electrons. **Current**, measured in amperes (amps), is how much electricity is moving. **Resistance** measured in Ohms (Ω), is, as it sounds, the resitance of a material to letting electricity flow.

The relationship is

$$V = IR$$

where V is voltage, I is current, and R is the resistance.

If we know that the current is 3 amps and the resistance is 8 ohms then we can calculate

$$V = 3 \cdot 8 = 24$$

Thus in this system there must be 24 volts.

Similarly if we know that the current is 1.7 amps and the resistance is 6 ohms, then we can calculate

$$V = 1.7 \cdot 6 = 10.2$$

Thus in this system there must be 10.2 volts. □

The lift equation relates lift (L), air density (ρ), surface area of wing (s), something called the coefficient of lift (C_L), and velocity (v). Lift is the force that keeps aircraft in the air. Air density is the amount of air per volume; you may see this as highs and lows on a weather map and is related to pressurizing high flying aircraft. The lift equation is

$$L = \frac{1}{2}\rho s C_L v^2.$$

If we know that air density is 0.002378 slugs per cubic feet, surface area is $125\,\mathrm{ft}^2$, $C_L = 1.5617$, and velocity is $84.4\,\frac{\mathrm{ft}}{\mathrm{s}}$, we can calculate the lift.

$$L = \frac{1}{2} \cdot 0.002378 \cdot 125 \cdot 1.5617 \cdot 84.4^2 \approx 1653.$$

□

The ideal gas law is relationship between volume, pressure, and temperature.

$$\frac{P_1 V_1}{T_1 + 273} = \frac{P_2 V_2}{T_2 + 273}.$$

P_1, V_1, and T_1 are the initial pressure, volume, and temperature. P_2, V_2, and T_2 are pressure, volume, and temperature at another time. Temperature is in degrees Celsius. Pressure and volume can be in any SI units but must be the same on both sides of the equation.

Suppose the initial conditions are $P_1 = 101.3$ Pa, $V_1 = 0.125$ m^3, and $T_1 = 10.2°$ C. Also $V_2 = 0.125$ m^3 and $T_2 = 50.7°$ C. We can calculate the new pressure.

$$\begin{aligned}
\frac{101.3 \cdot 0.125}{10.2 + 273} &= \frac{P_2 \cdot 0.125}{50.7 + 273} \\
0.0447 &\approx 0.000386 P_2 \\
\frac{0.0447}{0.000386} &\approx \frac{0.000386 P_2}{0.000386} \\
115.8 &\approx P_2
\end{aligned}$$

□

Checkpoint 1.3.4 Calculate the voltage if the current is $I = 3$ amps, $R = 4\Omega$? ____

Answer. 12

Solution. Using $V = IR$: $V = 3 \cdot 4 = 12$ volts.

Checkpoint 1.3.5 Using the ideal gas law to calculate the new pressure (P_2) if $P_1 = 101.3$, $V_1 = 0.250$, $T_1 = 11.3$, and $V_2 = 0.250$, $T_2 = 14.5$? ______

Answer. 102.4

Solution.

$$\begin{aligned} \frac{101.3 \cdot 0.250}{11.3 + 273} &= \frac{P_2 \cdot 0.250}{14.5 + 273} \\ 0.0891 &\approx 0.000870 P_2 \\ \frac{0.0891}{0.000870}; &\approx \frac{0.000870 P_2}{0.000870} \\ 102.4 &\approx P_2 \end{aligned}$$

Checkpoint 1.3.6 Using the ideal gas law calculate the new volume (V_2) if $P_1 = 101.3$, $V_1 = 0.250$, $T_1 = 11.3$, and $P_2 = 98.7$, $T_2 = 14.5$? ______

Answer. 0.0259

Solution.

$$\begin{aligned} \frac{101.3 \cdot 0.250}{11.3 + 273} &= \frac{98.7 \cdot V_2}{14.5 + 273} \\ 0.0891 &\approx 0.343 P_2 \\ \frac{0.0891}{0.343} &\approx \frac{0.343 P_2}{0.343} \\ 0.0259 &\approx P_2 \end{aligned}$$

Chapter 2

Ratios

2.1 Percents

Definition 2.1.1 Percent. A **percent** is a ratio of part of something to the whole of that thing that is written as parts per hundred. ◊

In a class there are 34 students. Of them 21 are female. The percent is calculated as

$$\frac{21}{34} = 0.6176.$$

This number says there are 61 hundreths (remembering our numbering system), so the percent is written as

$$61.76\%$$

□

Generally this means we calculate

$$100 \times \frac{\text{part}}{\text{whole}}.$$

2.1.1 Calculating Percents

In the class there are 34 students. Of them 13 are male. The percent is calculated as

$$100 \times \frac{13}{34} = 100 \times 0.3824 = 38.24\%.$$

□

Checkpoint 2.1.4 In another class there are 78 students and 44 are female. What percent of the students are female? ____

Answer. 56

Solution. A percent is the ratio of part (44 female) to whole (78 total). So this is $\frac{44}{78} \approx 0.56$. This is 56 hundreths, so it is 56%

Note in the first pair of examples we had a whole class of 34 students with 21 female and 13 male. Of course 21+13 = 34, that is the two parts add up to the whole. Because of this 61.76% + 38.24% = 100% as well.

Sometimes we are given the size of the whole and a percent. We must calculate how many in the part.

In a class of 22 students, 18% are Alaska Native. How many students are Alaska Native?

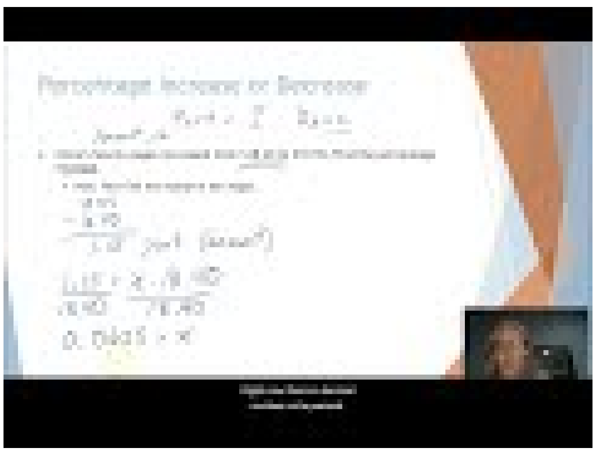
Solution 1. We use the same setup as before, but we don't know the part yet.

$$\begin{aligned} 100 \cdot \frac{P}{22} &= 18\% \\ \frac{P}{22} &= \frac{18}{100} \\ P &= 22 \cdot \frac{18}{100} \\ &\approx 3.96. \end{aligned}$$

Solution 2. We know that a percent is a number out of 100, so we can skip a step from the previous example.

$$\begin{aligned} \frac{P}{22} &= 0.18 \\ P &= 22 \cdot 0.18 \\ P &= 3.96 \end{aligned}$$

□



Standalone

Sometimes we know the size of a part and what percent it is. We can calculate the size of the whole.

In a class 2 Alaska Native students make up 6.25% of the class. How many students are in the class?

Solution. Again we use the same setup, but we don't yet know the whole.

$$\begin{aligned} \frac{2}{W} &= 0.0625. \\ 2 &= 0.0625 \cdot W. \\ \frac{2}{0.0625} &= W. \\ 32 &= W. \end{aligned}$$

□

Checkpoint 2.1.7 Find the number of millilitres of alcohol needed to prepare 150 mL of solution that is 5% alcohol. ____

Answer. 7.5

Solution. We need to know what 5% of 150 mL is: $0.05 \cdot 150 = 7.5$.

Checkpoint 2.1.8 There are 32 students in a class. Below are percents for each racial group tracked. Calculate the number of students in each group.

Solution. In this class there are $0.0625 \cdot 32 = 2$ Alaska Native students; $0.125 \cdot 32 = 4$ Asian students; 2 Black students (same percent as Alaska Native); $0.7188 \cdot 23 \approx 23.0016$ or 23 White students; and $0.0938 \cdot 32 \approx 3.0016$ or 3 students who declared as other.

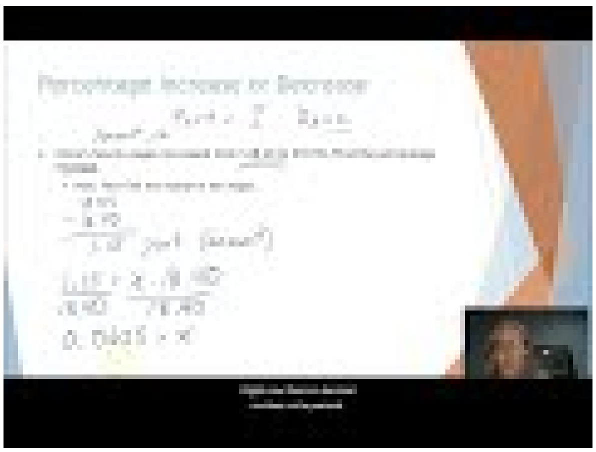
2.1.2 Percent Increase/Decrease

A common use of percents is to indicate how much something has increased (or decreased) from one time to the next.

In spring there were 22 students in a class. In fall there were 34 students in the same class. This was an increase of 34-22=12 students. We can calculate what percent 12 is of 22.

$$100 \times \frac{12}{22} = 55\%$$

□



Standalone

We say that the class size had a **percent increase** of 55%. Note this says the **increase** was 55% of the previous **whole**.

We can think of this in another way.

In spring there were 22 students in a class. In fall there were 34 students in the same class.
We calculate the percentage the fall class size is of the spring class size.

$$100 \times \frac{34}{22} = 155\%$$

Because the percent is greater than 100% we know this was an increase. Specifically it was an increase of 55% over the previous semester. □

Checkpoint 2.1.11 What is the percent increase or decrease if enrollment in a class was 57 in spring and 81 in fall? ____

Answer. 42

Solution. The ratio is $\frac{81}{57} \approx 1.42$. Because 81 is greater than 57 this is a percent increase. The increase is 42%.

Checkpoint 2.1.12 What is the percent increase or decrease if enrollment in a class was 78 in fall and 38 in spring? ____

Answer. 51

Solution. The ratio is $\frac{38}{78} \approx 0.49$. Because 38 is greater than 78 this is a percent decrease. Because the new *is* 49% of the previous the decrease is 100%-49%=51%.

2.1.3 Limitations

When presenting data a percent by itself can be deceptive.

Which of the following do you suppose represents a greater reduction in students?

Percent reduction	Total
18%	495
1.85%	54
60%	5

Solution.

Percent reduction	Total	Number reduced
18%	495	90
1.85%	54	1
60%	5	3

The 18% of 495 represents the largest number of students. The 60% is a higher percent, but because the total is so small it represents very few students. A percent is more useful if we also know the total number.

Did you calculate 89 for 18% of 495? Compare the following to see why both are reasonable responses. 89 is what percent of 495? 90 is what percent of 495? □

2.1.4 Exercises

1. Decimal to Percentage. The decimal 0.41 is equivalent to what percent?
____ %
(Do not enter the % sign)

2. Decimal to Percentage. The decimal 0.25 is equivalent to what percent?
____ %
(Do not enter the % sign)

3. Fraction to Percentage. The fraction $\frac{9}{10}$ is equivalent to what percent? ____________________ %
(Do not enter the % sign)

4. Fraction to Percentage. The fraction $\frac{2}{5}$ is equivalent to what percent?
____________________ %
(Do not enter the % sign)

5. Percent of Whole. 75 is what percent of 60.
____%

6. Percent of Whole. There are $15,000$ students attending the community college. Find the percent of students that attend classes in the evening if there are $3,750$ evening students.
____________________Preview Question 1 %

7. Percent of Whole. There are $15,000$ students attending a private college. There are $5,250$ evening students. What percent of all students are the evening students?
Evening students are _% of all the college's students.

8. Application of Percent. There are 12000 students attending the community college. Find the percent of students that attend classes in the evening if there are 2553 evening students.
____________________Preview Question 1 %
Round to units. Do not type the %

9. Application of Percent. There are $15,000$ students attending a private college. There are $5,250$ evening students. What percent of all students are the evening students?
Evening students are _% of all the college's students.

10. Use Percent to Calculate Total. If the first chapter of a certain book is 30 pages long and makes up 10% of the book, how many pages does the entire book have?
____________________ pages

11. Calculate Original from Percent. This week Shania got a promotion at work that came with a 1 % pay increase. If now her monthly salary is $ 2777.5 , how much was she making before the raise? Enter your answer as a decimal. If needed, round to the nearest penny. Do not use commas.
$ ________

12. Calculate Whole from Part and Percent. Anakin is running a race. He has completed 10 km, which happens to be 50% of the total race. How long is the race?
____________________Preview Question 1 km

13. Calculate Whole from Part and Percent. Carson is running a race. He has completed 8 km, which happens to be 80% of the total race. How long is the race? ____________________Preview Question 1 km

14. Percent Decrease. Cai's car insurance bill this year has decreased by 7%. He will be paying $54.11 less this year than last. How much did he pay last year?
____________________Preview Question 1

15. Percents as Fractions. Out of the last 43 days, it rained 16 days. What percent of these days did it rain? ____________________ %
Round to 2 decimal places.

16. Percents as Fractions. In 29.55 % of the last 44 days it rained. How many days did it not rain? ____________________ days
Round to the nearest whole.

17. Percents as Fractions. In 38.46 % of the last 26 days it rained. How many days did it not rain? ____________________ days
Round to the nearest whole.

18. Percents as Fractions. An e-book regularly costs $36.99. How much does it cost if it's on sale for 74% of the regular cost?
____________________ dollars
Round to the nearest cent.

19. Percents as Fractions. Laura invested $6200 in stocks which were later sold for $7500. What percent of the initial investment were they sold for?
____________________ %
Round to 2 decimal places.

20. Percents as Fractions. A city has a population 160,000 in a state with population 5,400,000. What percent of the state live in this city?
____________________ %
Round to 2 decimal places.

2.2 Mixtures

There are many situations where we desire to mix two or more substances together in precise ratio. These include mixing medicines in diluents (like water) and mixing ingredients in a recipe. Here we will look at how to calculate some of those using percents.

2.2.1 Mixtures

Mixture problems involve having solutions in which typically a desired substance is mixed into another. For example an active ingredient (medicine) may be mixed into a diluent (often water).

We can calculate a resulting percent when we combine things with differing percents.

Suppose we have a container with a solution that is 22% sugar and the rest water and another container with a solution that is 16% sugar and the rest water. If we combine 150 g of the first solution and 250 g of the second solution, what is the percent sugar?

Solution. To calculate a percent we need the total amount and amount of the part. We can calculate the total directly: $T = 150 + 250 = 400$.

To calculate the part we need to know how much (rather than what percent) sugar.

$$\begin{aligned} 150 \cdot 0.22 &= 33. \\ 250 \cdot 0.16 &= 40. \\ P &= 33 + 40 \\ &= 73. \end{aligned}$$

Thus the percent of the mixture is $P/T = 73/400 = 18.25\%$. □

Checkpoint 2.2.2 Suppose one container has a solution that is 11% alcohol and another container has a solution that is $4\bar{0}\%$. Both are percent by volume. If 4.0 oz of the first is mixed with 1.0 oz of the second what is the percent alcohol by volume? _____

Answer. 17

Solution. We know we will end up with 5.0 oz total. We calculate the volume of alcohol in the first to be $0.11 \cdot 4.0 = 0.44$ and the volume alcohol in the second to be $0.40 \cdot 1.0 = 0.40$. Thus the alcohol part is $0.44 + 0.40 = 0.84$. The percent alcohol by volume of the result is $P/T = 0.84/5.0 = 0.17$ or 17%.

The next problem is producing a isopropyl alcohol with a lower concentration of alcohol than the original solution. We begin with 16.0 oz of a 91.0% isopropyl alcohol solution. The other ingredient is water.

Suppose we add 4.00 oz of water to this mixture. What will the percent alcohol be?

Solution. The percent alcohol is the amount of alcohol divided by the total volume. The volume of alchol is $0.910(16.0) = 14.6$. The volume of water from the original solution is $0.0900(16.0) = 1.44$. We are adding 4.00 oz, thus the total volume is $16.0 + 4.00 = 20.0$ oz. The final percent alcohol is

$$\frac{14.6}{20.0} = 0.730$$

or 73.0%. □

If we added more water would the percent alcohol be greater or less? Note that if we are using all of the alcohol solution, the amount of water we add determines the percent alcohol.

If we start with 16.0 oz of 91.0% alcohol solution, how much water do we add to get 25.0 oz of a 55.0% alcohol solution?

How much solution total does this produce? Remember it is not necessarily 25.0 oz.

Solution. This is a percent problem with the total alcohol unchanged ($(0.91)16 = 14.56$) and adding only some amount w of water. Thus we setup

$$\begin{aligned} \frac{(0.910)16.0}{16.0 + w} &= 0.550 \\ (0.910)16.0 &= 0.550(16.0 + w) \\ (0.910)16.0 &= (0.550)16.0 + (0.550)w \\ 14.6 &= 8.80 + (0.550)w \\ 5.8 &= (0.550)w \\ \frac{5.8}{0.550} &= w \\ 11 &\approx w \end{aligned}$$

Note this means we end up with $16 + 11 = 27$ oz of new solution. Also the percent alcohol is $\frac{0.910(16.0)}{16.0+11} = 0.54$ This is not exactly 55% because of the rounding in the calculations. □

Standalone

2.2.2 Exercises

1. **Mixture.** You need a 60% alcohol solution. On hand, you have a 420 mL of a 70% alcohol mixture. How much pure water will you need to add to obtain the desired solution?

A) Write an equation using the information as it is given above that can be used to solve this problem. Use x as your variable to represent the amount of pure water you need to use. Equation: ______________________________Preview Question 1 Part 1 of 3

B) You will need
_____ mL of pure water
to obtain
_____ mL of the desired 60% solution.

2. **Mixture.** 8.00 liters of fuel containing 2.7% oil is available for a certain two-cycle engine. This fuel is to be used for another engine requiring a 4.3% oil mixture. How many liters of oil must be added?

Give your answer to 3 significant digits.

____

3. **Medical Proportion.** Quinidine gluconate is a liquid mixture, part medicine and part water, which is administered intravenously. There are 120.0 mg of quinidine gluconate in each cubic centimeter (cc) of the liquid mixture. Dr. Alverez orders 1560 mg of quinidine gluconate to be administered daily to a patient with malaria.

How much of the solution would have to be administered to achieve the recommended daily dosage?
____________________ cc

4. **Medical Proportion.** Albuterol is a medicine used for treating asthma. It comes in an inhaler that contains 15 mg of albuterol mixed with a liquid. One actuation (inhalation) from the mouthpieces delivers a 90 μg dose of albuterol. (Reminder: 1 mg = 1000 μg .)

a.) Dr. Olson orders 2 inhalations 3 times per day. How many micrograms of albuterol does the patient inhale per day?
____________________ μg

b.) How many actuations are contained in one inhaler?
____________________ actuations

c.) Shelby is going away for 5 months and wants to take enough albuterol to last for that time. Her physician has prescribed 2 inhalations 3 times per day. How many inhalers will Shelby need to take with her for the 5 period? Assume 30 month days.

Hint: she can't bring a fraction of an inhaler, and she does not want to run out of medicine while away.

____

5. **Medical Ratio.** Amoxicillin is a common antibiotic prescribed for children. It is a liquid suspension composed of part amoxicillin and part water.

In one formulation there are 150 mg of amoxicillin in 5 cubic centimeters (cc's) of the liquid suspension. Dr. Scarlotti prescribes 225 mg per day for a 2-yr old child with an ear infection.

How much of the amoxicillin liquid suspension would the child's parent need to administer in order to achieve the recommended daily dosage?

____

6. **Medical Proportion.** Diphenhydramine HCL is an antihistamine available in liquid form, part medication and part water. One formulation contains 14 mg of medication in 5 mL of liquid. An allergist orders 56-mg doses for a high school student. How many milliliters should be in each dose?

____________________ mL

7. **Percent Concentration.** How many mL of sodium hydroxide are required to prepare 400 mL of a 17% solution? Assume the sodium hydroxide dissolves in the solution and does not contribute to the overall volume.

____________________ mL

8. **Dilution Ratio.** You are asked to make a 1/7 dilution using 1 mL of serum. How much diluent do you need to use?

____________________ mL

9. **Dilution Ratio.** A clinical lab technician determines that a minimum of 40 mL of working reagent is needed for a procedure. To prepare a $\frac{1}{9}$ dilution ratio of the reagent from a stock solution, one should measure 40 mL of the reagent and ____________________ mL of the diluent.

10. **Dilution Ratio.** A patient's glucose result is suspected to be outside the range of the analyzer, so the techs decide to dilute the sample before running it. 45 microliters of serum is added to 90 microliters of diluent and the diluted sample is analyzed. The analyzer reads that the glucose value of the diluted sample is 70 $\frac{mg}{dL}$.

What was the ratio the sample was diluted to?

____________________Preview Question 1 Part 1 of 2

What is the glucose value of the original sample?

____________________ $\frac{mg}{dL}$

11. **Dilution Ratio.** A thyroid peroxidase antibody test was performed on a 45 year old man. The dilution sequence was 20 μL serum added to 80 μL of diluent in tube 1. Then 70 μL from tube 1 was added to 490 μL of diluent in tube 2. Finally 45 μL from tube 2 was added to 135 μL of diluent in tube 3.

All dilution ratios should be given as fractions.

a.) What is the dilution ratio in tube 1?

____________________Preview Question 1 Part 1 of 4

b.) What is the dilution ratio in tube 2?

____________________Preview Question 1 Part 2 of 4

c.) What is the dilution ratio in tube 3?

____________________Preview Question 1 Part 3 of 4

d.) What is the overall (serial) dilution ratio?

____________________Preview Question 1 Part 4 of 4

2.3 Ratios

With percents we are comparing the size of a part to the whole or perhaps comparing the before and after sizes of the same thing. With ratios we can compare many different quantities.

In a class there are 21 female students and 13 male students. We can write the ratio

$$\frac{21 \text{ female}}{13 \text{ male}}$$

Note that this is simply a statement. There is no question to answer, and nothing more to do with the ratio.

Consider that $\frac{21}{13} \approx 1.62 > 1$. Because it is greater than one the ratio tells us that there are more female students than male students in the class.

The ratio

$$\frac{13 \text{ male}}{21 \text{ female}}$$

gives the exact same information. That is, the order of a ratio does not change the information though it does change the number thought of as a fraction. □

In a neighborhood there are 7 dogs and 12 cats. We can write the ratio

$$\frac{7 \text{ dogs}}{12 \text{ cats}}$$

Note that $\frac{7}{12} \approx 0.58 < 1$. Because it is less than one the ratio tells us that there are fewer dogs than cats in the neighborhood. □

2.3.1 Using Ratios

Ratios imply amounts at fixed periods. They can also be used to calculate amounts of parts from totals.

2.3.1.1 Listing Amounts

Often we use ratios to indicate a fixed (unchanging) relationship. For example a faucet fully open may have a ratio of 2 gallons per minute. This is true in the first minute and in the second minute. This is in contrast to when we turn on a faucent: it goes from a rate of 0 g/min to 2 g/min over a few seconds. When a ratio is fixed we can use it to calculate amounts.

The Diamond DA-20 cruises at the rate of

$$\frac{110 \text{ nautical miles}}{1 \text{ hour}}.$$

With a rate we can calculate amounts. See Table 2.3.4 for results. Note the first row is the given ratio.

For the second row, note that the ratio implies that the rate does not change. If in the first hour the plane travelled 110 nm (nautical miles), then it travelled 110 nm in the second hour as well. So all total it travelled 220 nm.

In general we can add another 110 nm for each hour.

For 0.5 hours we note the following mathematics.

$$\begin{aligned}\frac{110 \text{ nautical miles}}{1 \text{ hour}} &= \frac{110 \text{ nautical miles}}{1 \text{ hour}} \cdot \frac{1/2}{1/2} \\ &= \frac{55 \text{ nautical miles}}{0.5 \text{ hour}}.\end{aligned}$$

We scaled the number of hours. What we do to the bottom (number of hours) we must also do to the top (number of miles).

For 2.5 hours we note again that because the rate does not change (110 nm for every hour) we can add 220 nm for two hours and 55 nm for the half hour giving 275 nm total. □

Table 2.3.4 Airspeed and Distance

Time	Distance
1 hour	110 nm
2 hour	220 nm
0.5 hour	55 nm
2.5 hour	275 nm

Water is flowing out of a hose at a rate of 11 gallons per minute. How many gallons have come out after 2.7 minutes?

Solution. We can set this up like the half hour calculation in Example 2.3.3. That is we scale the ratio to give us 2.7 minutes.

$$\begin{aligned} \frac{11 \text{ gallons}}{1 \text{ minute}} &= \frac{11 \text{ gallons}}{1 \text{ minute}} \cdot \frac{2.7}{2.7} && \text{Scale the ratio.} \\ &= \frac{29.7 \text{ gallons}}{2.7 \text{ minutes}} \end{aligned}$$

The answer is approximately 30 gallons. □

Checkpoint 2.3.6 Calculate how far a plane flying at a rate of 110 nm/hour would travel in 3.1 hours ____

Answer. 341

Solution.

$$\begin{aligned} \frac{110 \text{ nautical miles}}{1 \text{ hour}} &= \frac{110 \text{ nautical miles}}{1 \text{ hour}} \cdot \frac{3.1}{3.1} && \text{Scale the ratio.} \\ &= \frac{341 \text{ nautical miles}}{3.1 \text{ hours}}. \end{aligned}$$

2.3.1.2 Calculating Parts

Like a percent, ratios indicate a relative amount, but do not directly specify an amount. A ratio can be used in calculations the same way a percent is.

Based on data from the FDA the average amount of mercury found in fresh or frozen salmon is 0.022 ppm (parts per million). This means there are 0.022 mg of mercury in one liter of salmon. If a meal portion of salmon is 0.002 liters how much mercury is consumed?

Solution. Note we cannot use a percent because we are comparing a weight to a volume: dissimilar measurements. We can use the ratio $\frac{0.022 \text{ mg}}{1\ \ell}$. We apply this ratio to the given volume of 0.002 liters.

$$\begin{gathered} \frac{0.022 \text{ mg}}{1\ \ell} \cdot 0.002\ell = \\ \frac{0.022 \text{ mg}}{1\ \cancel{\ell}} \cdot 0.002\ \cancel{\ell} = 0.000044 \text{ mg}. \end{gathered}$$

□

A saline solution intended for nasal rinsing has a ratio of 2.5 g of salt (sodium chloride) per 240 mL of pure water. How much salt is needed to make a half liter of saline solution?

Solution. We can apply the given ratio (2.5 g/240 mL) to the given amount (0.5 L). First it will be convenient to conver a half liter to milliliters. This is also a ratio problem. See Table 1.1.6 for the conversion ratio.

$$\begin{gathered} \frac{1000 \text{ mL}}{1\text{L}} \cdot 0.5 \text{ L} = \\ \frac{1000 \text{ mL}}{1\ \cancel{\text{L}}} \cdot 0.5\ \cancel{\text{L}} = 500 \text{ mL} \end{gathered}$$

Next we can apply the ratio to the volume.

$$\begin{gathered} \frac{2.5 \text{ g}}{240 \text{ mL}} \cdot 500 \text{ mL} = \\ \frac{2.5 \text{ g}}{240\ \cancel{\text{mL}}} \cdot 500\ \cancel{\text{mL}} = 5.2 \text{ g} \end{gathered}$$

Note we use two significant digits because the 2.5 and 240 both have two significant digits and 500 is not a measurement. □

Checkpoint 2.3.9 One formulation of amoxicillin, a drug used to treat infections in infants, contains 125 mg of amoxicillin per 5.00 mL of liquid. How much amoxicillin is in 12.0 mL of liquid? ____

Answer. 300

Solution.

$$\frac{125\text{ mg}}{5.00\text{ mL}} \cdot 12.0\text{ mL} =$$
$$\frac{125\text{ g}}{5.00\,\cancel{\text{mL}}} \cdot 12.0\,\cancel{\text{mL}} = 300\text{ mg}$$

2.3.2 Conversion of Units

One common type of ratio is a rate. These typically have units. Examples you probably know are miles per hour, kilometers per hour, and gallons per minute. When we know a rate we can calculate amounts (e.g., number of miles, number of kilometers, or number of gallons). Sometimes we convert one unit to another. This is accomplished by applying ratios as we did in Example 2.3.7 .

Find the ratio of 5 cm to 40 mm.

Solution. We can write simply

$$\frac{5\text{ cm}}{40\text{ mm}}$$

but the mismatched units make this less informative than it could be. We know 5 cm is 50 mm, so the ratio could also be written

$$\frac{50\text{ mm}}{40\text{ mm}} = \frac{5\text{ mm}}{4\text{ mm}}.$$

Note it is equally valid to note that 40 mm is 4 cm and write the ratio

$$\frac{5\text{ cm}}{4\text{ cm}}.$$

Note that the units do not affect the ratio when it is reduced. □

Standalone

We will convert 2.1 hours to a number of minutes. Note we know there are 60 minutes per hour. Notice that the conversion is like a rate. Specifically it is a fraction. If we multiply minutes/hour by hours we will end up with just minutes.

Solution.

$$2.1\text{ hours} \cdot \frac{60\text{ minutes}}{\text{hour}} =$$
$$2.1\,\cancel{\text{hours}} \cdot \frac{60\text{ minutes}}{\cancel{\text{hour}}} =$$
$$126\text{ minutes}.$$

□

We will convert 450 feet per minute to units of inches per second.

Solution.

Standalone

$$\begin{aligned}\frac{450\text{ feet}}{1\text{ minute}}\cdot\frac{12\text{ inches}}{1\text{ foot}}\cdot\frac{1\text{ minute}}{60\text{ seconds}}&=\\ \frac{450\cdot 12}{60}\frac{\cancel{\text{feet}}}{\cancel{\text{foot}}}\cdot\frac{\cancel{\text{minute}}}{\cancel{\text{minute}}}\cdot\frac{\text{inches}}{\text{second}}&=\\ \frac{90\text{ inches}}{1\text{ second}}&.\end{aligned}$$

□

We will convert 450 feet per minute to units of meters per minute. Note when converting between dissimilar units the conversions are approximations: select a sufficiently precise approximation for your application. For this problem we will use the ratio of one foot per 0.3048 meters. We will need to divide by feet and multiply by meters so we need to reciprocate this ratio.

Solution.

$$\begin{aligned}\frac{450\text{ feet}}{\text{minute}}\cdot\frac{0.3048\text{ meters}}{\text{foot}}&=\\ 450\cdot 0.3048\frac{\cancel{\text{feet}}}{\cancel{\text{foot}}}\cdot\frac{\text{meters}}{\text{minute}}&\approx\\ \frac{140\text{ meters}}{\text{minute}}&.\end{aligned}$$

□

Checkpoint 2.3.14 Look up the conversion from kilometers to miles. How many miles per hour is 12 kilometers per hour? ____

Answer. 7.5

Solution.

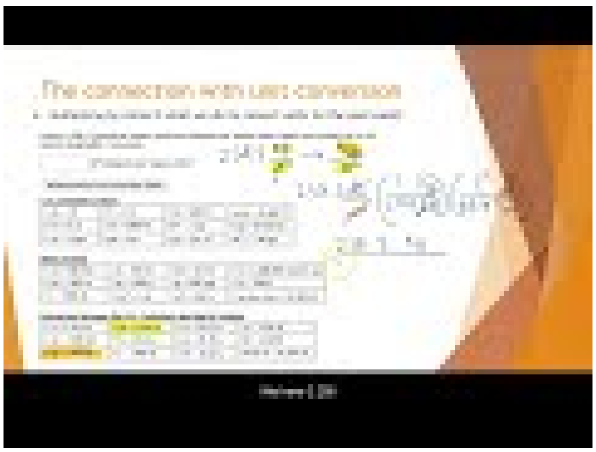
$$\begin{aligned}\frac{12\text{ km}}{\text{hour}}&\approx\\ \frac{12\text{ km}}{\text{hour}}\cdot\frac{0.621371\text{ miles}}{\text{km}}&\approx\\ \frac{7.5\text{ miles}}{\text{hour}}\cdot\frac{\cancel{\text{km}}}{\cancel{\text{km}}}&\approx\\ \frac{7.5\text{ miles}}{\text{hour}}&.\end{aligned}$$

We will convert 153 square inches to square centimeters. Note that 1 inch is approximately 2.54 centimeters. Note first that **square** here refers to the inches not to the 153. Notice also how we adjust the conversion process for this exponent.

Solution.

$$\begin{aligned} 153 \text{ inches}^2 &= \\ 153 \text{ inches}^2 \cdot \left(\frac{2.54 \text{ cm}}{\text{inch}}\right)^2 &\approx \\ 153 \text{ inches}^2 \cdot \frac{6.45 \text{ cm}^2}{\text{inch}^2} &\approx \\ 987 \text{ cm}^2 \cdot \frac{\cancel{\text{inches}^2}}{\cancel{\text{inch}^2}}. &\approx \\ 987 \text{ cm}^2. & \end{aligned}$$

□



Standalone

Checkpoint 2.3.16 Note that a pound (lb) is a unit of force. It is approximately equal to 4.45 Newtons which is another unit of force. How many Newtons per square meter is 51.2 lbs per square inch? Look up other conversions as needed. _______

Answer. 353000

Solution.

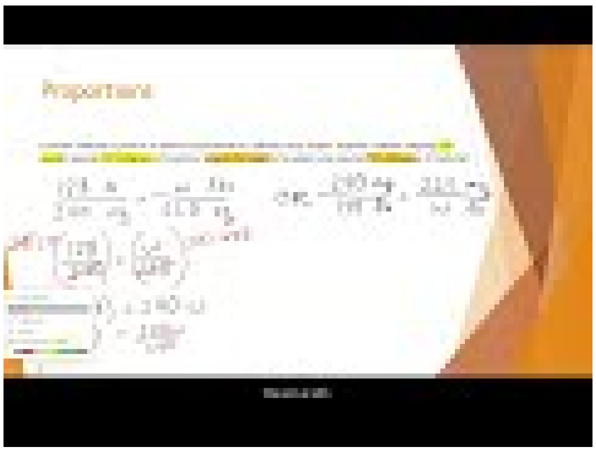
$$\begin{aligned} \frac{51.2 \text{ lbs}}{\text{inch}^2} &\approx \\ \frac{51.2 \text{ lbs}}{\text{inch}^2} \cdot \frac{4.45 \text{ N}}{1.00 \text{ lbs}} \cdot \left(\frac{1.00 \text{ inch}}{0.0254 \text{ m}}\right)^2 &\approx \\ \frac{51.2 \cdot 4.45}{(0.0254)^2} \frac{\text{lbs}}{\text{inch}^2} \cdot \frac{\text{N}}{\text{lbs}} \cdot \frac{\text{inch}^2}{\text{m}^2} &\approx \\ \frac{228}{0.000645} \frac{\text{lbs}}{\text{inch}^2} \cdot \frac{\text{N}}{\text{lbs}} \cdot \frac{\text{inch}^2}{\text{m}^2} &\approx \\ 353000 \frac{\cancel{\text{lbs}}}{\cancel{\text{inch}^2}} \cdot \frac{\text{N}}{\cancel{\text{lbs}}} \cdot \frac{\cancel{\text{inch}^2}}{\text{m}^2} &\approx \\ \frac{353000 \text{ N}}{\text{m}^2}. & \end{aligned}$$

2.4 Proportions

Ratio problems presume that the ratio does not change. This makes sense in examples like conversion of units. For example 1 inch is always approximately 2.54 cm. In contrast rates often change: your average speed may be 25 mph, but you must have driven slower and faster during that drive. For ratios that do not change we can write equations and solve for properties.

2.4.1 Proportion Examples

We can solve for values in ratio problems by setting up the equation (ratio equals ratio) and then multiplying and dividing as needed to solve. Often people remember this with the mnemonic device: cross multiplication which is also known as clearing the denominators.



Standalone

The Diamond DA-20 cruises at the rate of

$$\frac{110 \text{ nautical miles}}{1 \text{ hour}}.$$

How long will it take to travel 236 nm?
Because this is a fixed ratio we can write

$$\begin{aligned}
\frac{110 \text{ nm}}{\text{hour}} &= \frac{236 \text{ nm}}{t \text{ hours}}. \\
t \text{ hours} \cdot \frac{110 \text{ nm}}{1\text{hour}} &= t \text{ hours} \cdot \frac{236 \text{ nm}}{t \text{ hours}}. \\
t \cdot 110 \text{ nm} &= 236 \text{ nm}. && \text{Clearing the denominators.} \\
\frac{t \cdot 110 \text{ nm}}{110 \text{ nm}} &= \frac{236 \text{ nm}}{110 \text{ nm}}. \\
t &= \frac{236}{110}. \\
t &\approx 2.1 \text{ hours}.
\end{aligned}$$

□

Checkpoint 2.4.2 If the Diamond DA-20 climbs at the rate of

$$\frac{450 \text{ feet}}{1.0 \text{ minute}}$$

how long will it take it to climb 2500 ft? ____

Answer. 5.5

Solution.

$$\begin{aligned}\frac{450\text{ ft}}{1.0\text{ minute}} &= \frac{2500\text{ ft}}{t\text{ minutes}}.\\ t\text{ minutes}\cdot\frac{450\text{ ft}}{1.0\text{ minutes}} &= t\text{ minutes}\cdot\frac{2500\text{ ft}}{t\text{ minutes}}.\\ t\cdot 450\text{ ft} &= 2500\text{ ft}.\\ \frac{t\cdot 450\text{ ft}}{450\text{ ft}} &= \frac{2500\text{ ft}}{450\text{ ft}}.\\ t &= \frac{2500}{450}\\ t &\approx 5.5\text{ minutes}.\end{aligned}$$

For a particular cheesecake recipe there is $15\bar{0}$ g of eggs and $150\bar{0}$ g of cream cheese. We will determine how many grams of eggs we need if we double the recipe. This means everything will be in ratio of 2/1. Note the 2 and 1 have infinite significant figures (they are exact numbers rather than approximated measurements).

$$\begin{aligned}\frac{2}{1} &= \frac{E\text{ g}}{15\bar{0}\text{ g}}.\\ \frac{2}{1}\cdot 150\text{ g} &= \frac{E\text{ g}}{15\bar{0}\text{ g}}\cdot 150\text{ g}.\\ 2\cdot 15\bar{0}\text{ g} &= E\cdot 1. && \text{Eliminating the denominator.}\\ 30\bar{0}\text{ g} &= E\text{ of eggs.}\end{aligned}$$

□

For a particular cheesecake recipe there is $15\bar{0}$ g of eggs and $150\bar{0}$ g of cream cheese. If we have $35\bar{0}$ g of egg how much cream cheese do we need? We know that the egg to cream cheese ratio must be 150/1500. We also notice this is 1/10.

$$\begin{aligned}\frac{1}{10} &= \frac{35\bar{0}\text{ g}}{C\text{ g}}.\\ \frac{10}{1} &= \frac{C\text{ g}}{35\bar{0}\text{ g}}.\\ \frac{10}{1}\cdot 350\text{ g} &= \frac{C\text{ g}}{350\text{ g}}\cdot 350\text{ g}.\\ 35\bar{0}0 &= C\text{ g of cream cheese.}\end{aligned}$$

Notice we flipped the ratio in the second step to make the arithmetic easier to follow. You can think of this as the addage "what you do to one side, you must also do to the other". □

Checkpoint 2.4.5 For a particular cheesecake recipe there is $15\bar{0}$ g of eggs and $150\bar{0}$ g of cream cheese. Suppose you have $35\bar{0}$ g of eggs and $340\bar{0}$ g of cream cheese. How much of the egg and cream cheese can you use?

Egg: ____

Cream cheese: _____

Answer 1. 340

Answer 2. 3400

Solution. First we can figure out whether the egg or the cream cheese is the limiting ingredient. The egg to cream cheese must be in a

$$\frac{15\bar{0}}{150\bar{0}} = \frac{1.00}{10.0}$$

ratio. We can calculate the ratio of ingredients we have

$$\frac{35\bar{0}\text{ g}}{340\bar{0}\text{ g}} \approx 0.103$$

Because this is more than $0.103 > 0.1 = 1/10$, we have more egg than we can use, because the numerator (egg) is bigger. Thus we will set up the proportion using all of the cream cheese.

We can setup the ratio multiple ways. The first is using the egg to cheese ratio.

$$\begin{aligned}
\frac{15\bar{0}}{150\bar{0}} &= \frac{E\text{ g}}{340\bar{0}\text{ g}}.\\
\frac{1.00}{10.0} &= \frac{E\text{ g}}{340\bar{0}\text{ g}}.\\
\frac{1.00}{10.0}\cdot 340\bar{0}\text{ g} &= \frac{E\text{ g}}{340\bar{0}\ \text{ g}}\cdot 340\bar{0}\text{ g}.\\
34\bar{0}.0 &= E\text{ g}.
\end{aligned}$$

We can use 340 of the 350 grams of egg.

Another way is using the recipe to scaled up ratio.

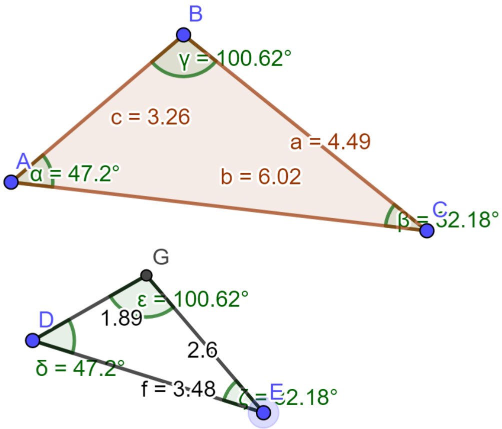
$$\begin{aligned}
\frac{15\bar{0}}{E} &= \frac{150\bar{0}}{340\bar{0}}.\\
\frac{15\bar{0}}{E}E &= \frac{150\bar{0}}{340\bar{0}}E.\\
15\bar{0} &= \frac{150\bar{0}}{340\bar{0}}E.\\
\frac{340\bar{0}}{150\bar{0}}\cdot 15\bar{0} &= \frac{340\bar{0}}{150\bar{0}}\cdot\frac{150\bar{0}}{340\bar{0}}E.\\
\frac{340\bar{0}\cdot 15\bar{0}}{150\bar{0}} &= E.\\
34\bar{0} &= E.
\end{aligned}$$

We can use 340 of the 350 grams of egg.

2.4.2 Similar polygons

Definition 2.4.6 Similar Triangles. Two triangles are **similar** if and only if corresponding angles are congruent. ◊

Congruent in this case means the same length. Because triangles are similar corresponding side lengths are proprotional.



Standalone
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Figure 2.4.7 Similar Triangles

We can use the proportionaliy of similar triangle sides lengths to calculate the lengths using the same technique as Example 2.4.1.

Suppose triangle A has angles $30°, 60°, 90°$ with side lengths $1.000, 1.732, 2.000$. If triangle B also has angles $30°, 60°, 90°$ it is similar. Suppose the smallest side length is 2.000. Then we know.

$$\begin{aligned}
\frac{1.000}{2.000} &= \frac{1.732}{s}. \\
\frac{1.000}{2.000} \cdot 2.000s &= \frac{1.732}{s} \cdot 2.000s. && \text{Clearing the denominators.} \\
s &= 1.732(2.000). \\
s &= 3.464.
\end{aligned}$$

We can calculate the length of the third side in the same way.

$$\begin{aligned}
\frac{1.000}{2.000} &= \frac{2.000}{s}. \\
\frac{1.000}{2.000} \cdot 2.000s &= \frac{2.000}{s} \cdot 2.000s. \\
s &= 4.000
\end{aligned}$$

□

Checkpoint 2.4.9 Suppose triangle A has angles $40°, 60°, 80°$ and side lengths 1.000, 1.347, 2.000. If triangle B has the same angle measures and the shortest side is length 2.500, what are the other two side lengths?

Corresponds to 1.347: ______

Corresponds to 2.000: ______

Answer 1. 3.368

Answer 2. 5

Solution. We setup the proportions for the other two sides.

$$\frac{1.000}{2.500} = \frac{1.347}{s_2}.$$
$$s_2 = 3.368.$$
$$\frac{1.000}{2.500} = \frac{2.000}{s_3}.$$
$$s_3 = 5.000.$$

Shapes other than triangles can be similar. For example there are similar rectangles and similar pentagons. To be similar they must have the same number of sides, corresponding angles must be the same, and corresponding sides must be in the same ratio. Note that just having the same angles is insufficient: any two rectangles have all the same angles (right angles) but not every pair is similar.

Similar Polygons. One way to define similar polygons that avoids the trap illustrated by rectangles is to require every triangle made from vertices to have the same angles. Take two, non-similar rectangles and find the triangles that don't match.

One place where similar shapes (beyond triangles) is used is scale drawing and scale models. If you ever built a model of a car or a plane or some such there was most likely a scale given. For example they may be 1/32 scale. This means that one inch on the model is 32 inches on the actual object.

2.4.3 Exercises

1. Fractions. Compare the two fractions by choosing the correct symbol to place between them.

$\frac{7}{12}$

(a) <</label>

(b) >

(c) =

$\frac{1}{2}$

2. Fractions. Compare the two fractions by choosing the correct symbol to place between them.

$\frac{5}{6}$

(a) <</label>

(b) >

$\frac{4}{7}$

3. Proportion (no context). Find the unknown number in the proportion

$\frac{6}{36} = \frac{8}{x}$____________________

4. Proportion (no context). Find the unknown number in the proportion

$\frac{x}{80} = \frac{27}{72}$____________________

5. Proportion (no context). Cellular phone service that charges per-minute will charge $20 for 100 minutes. How much would 938 minutes cost?

Round your answer to the nearest cent.

$_____

6. Proportion (no context). Solve for the variable in $\frac{x}{7.3} = \frac{5.28}{8.8}$

$x =$__________

7. **Proportion Application.** Ben goes to the grocery store at a rate of 5 times a week. How many times would he be expected to go to the grocery store in 13 weeks? Use x as the variable.

Table 2.4.10

Translate to a proportion: ____________________ Preview Question 1 Part 1 of 2
$x =$___ times in 13 weeks

8. **Proportion Application.** Sue goes to the fabric store at a rate of 4 times a month. How many times would she be expected to go to the fabric store in 6 months?

Table 2.4.11

Translate to a proportion: ____________________ Preview Question 1 Part 1 of 2
$x =$___ times in 6 months

9. **Proportion Application.** A carpet store charges \$416.00 to install 64 square yards of carpet. Assuming they charge the same rate per square yard regardless of the amount of carpet installed, how much would they charge to install 200 square yards of carpet? What is the unit price for installation per square yard of carpet?

Table 2.4.12

Translate to a proportion: ____________________ Preview Question 1 Part 1 o
They would charge \$___ to install 200 square yards of carpet.

\$___ per square yard

10. **Proportion Application.** Gwen's Gravel Company supplied a homeowner with 12 cubic yards of gravel for his driveway at a cost of \$930.00. Assuming they charge the same rate per cubic yard regardless of the amount of gravel supplied, what would they charge for 34 cubic yards of gravel?

Table 2.4.13

Translate to a proportion: ____________________ Preview Question 1 Part 1 of 2
\$_____ for 34 cubic yards of gravel

11. **Proportion Application.** If a 32-acre alfalfa field produces 224 tons of hay, how many acres would be needed for a field to produce 336 tons of hay?

Table 2.4.14

Translate to a proportion: ____________________ Preview Question 1 Part
A field would need to be _____ acres to produce 336 tons of hay.

12. **Proportion Application.** A recipe for lemon tea cookies calls for $1\frac{1}{3}$ cups of flour for every $\frac{3}{3}$ cup of sugar. How many cups of sugar are needed if $2\frac{2}{3}$ cups of flour are used?

For $2\frac{2}{3}$ cups of flour you need __________Preview Question 1 cups of sugar.

13. **Solution.** A label reads: "2.5 mL of solution for injection contains 1,000 mg of streptomycin sulfate." How many millilitres are needed to give 600 mg of streptomycin?

____________________Preview Question 1 Part 1 of 2
A matching example in the textbook is Example

(a) 2.5.1

(b) 2.5.2

(c) 2.5.3

(d) 2.5.4

(e) 2.5.5

(f) 2.5.6

(g) 2.5.7

(h) 2.5.8

(i) 2.5.9

(j) 2.5.10

(k) 2.5.11

(l) 2.5.12

(m) 2.5.13

(n) 2.5.14

(o) other

(p) none

14. Similar Triangles. The side lengths of ΔABC are given below.
$AB = 2$ $BC = 10$ $AC = 9$
The side lengths of ΔRST are given below.
$RS = 10$ $ST = 50$ $RT = 45$
Simplify the given corresponding side ratios:
$\frac{RS}{AB} =$ _____ $\frac{ST}{BC} =$ _____ $\frac{RT}{AC} =$ _____
Is $\Delta ABC \sim \Delta RST$?

(a) No

(b) Yes

15. Similar Triangles. Suppose you are standing such that a 30-foot tree is directly between you and the sun. If you are 6 feet tall and the tree casts a 125-foot shadow, how far away from the tree can you stand and still be completely in the shadow of the tree?

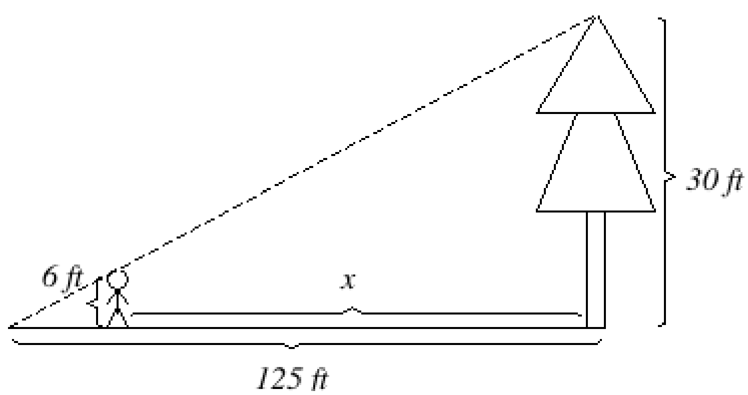


Figure 2.4.15

The distance between you and the tree is ____________________Preview Question 1 ft (If needed, round to 1 decimal place.)

16. Similar Triangles. A stick 1 meter long casts a shadow 1.4 meters long. A building casts a shadow 22 meters long. How tall is the building (to 2 decimal places)
____________________Preview Question 1 meters

17. Similar Triangles. In the following figure, the two triangles are similar. The lengths of their sides are shown. What is the value of y in terms of x?

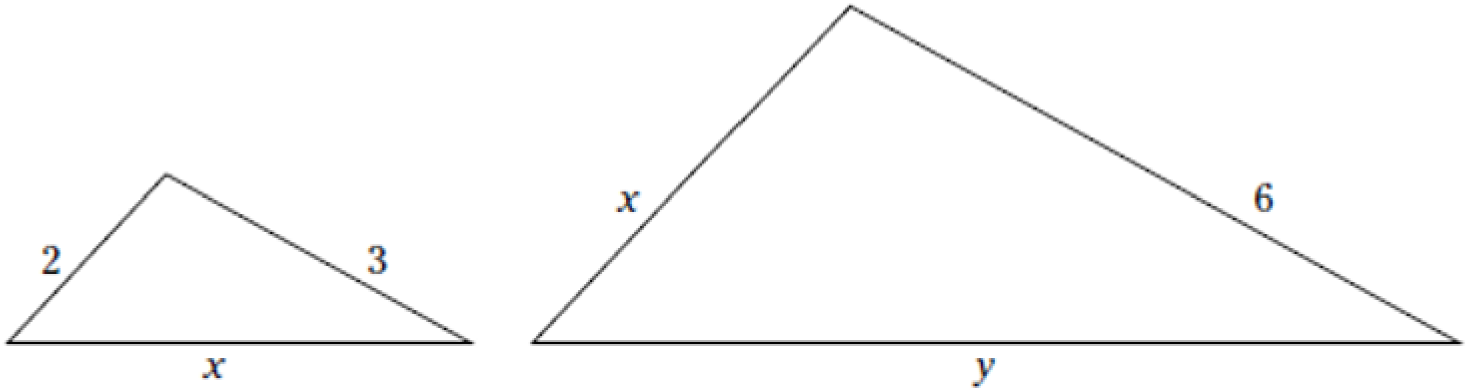


Figure 2.4.16

(a) $x + 2$

(b) $x + 3$

(c) $x + 5$

(d) $x + 4$

(e) $x + 1$

You have 2 attempts to correctly answer this question.

18. Similar Triangles. Victoria holds a fishing pole with fishing line extended according to the picture below. How far is the fish from her hook? (Solve for x)

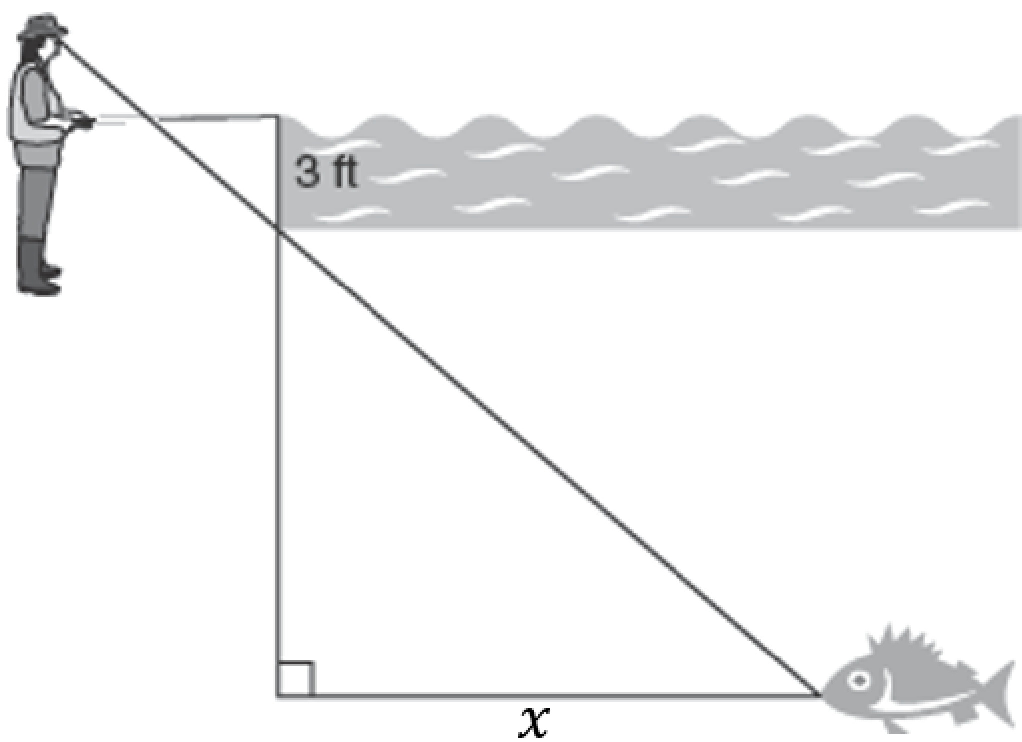


Figure 2.4.17

$10ft$ $28.5ft$
$x =$ _____

(a) cm

(b) m

(c) ft

(d) in

(e) °

19. Similar Triangles. Name the polynomial expression below.
$11h^4 + 12h - 7$

(a) Constant

(b) Linear

(c) Quadratic

(d) Cubic

(e) Quartic

(f) Quintic

(g) 6th degree or higher

(a) Monomial

(b) Binomial

(c) Trinomial

(d) Polynomial

20. Similar Triangles. A floor plan has a 32 : 1 scale. On the drawing, one of the rooms measures $1\frac{1}{4}$" by $3\frac{3}{4}$". Show answers to the nearest .01 The actual dimensions would be: ___________________Preview Question 1 Part 1 of 3 feet by: ___________________Preview Question 1 Part 2 of 3 feet. The area of the room would be ____________________Preview Question 1 Part 3 of 3 square feet.

2.5 Medical Ratios

A common application of ratios in medicine is creating drugs of a desired strength. For example some drugs need to be administered based on the body weight of he patient. This requires the medical personnel to mix the drug they have on hand to the needed strength.

We will work the following three types of medical problems.

- Measure drug concentration
- Dilute a drug to a lower concentration
- Determine how much drug to use

2.5.1 Terminology

The active ingredient in a drug is often added to a inactive ingredient (often liquid) to administer it. This liquid is known as a **diluent**. The diluent might be water, saline solution, or other substances.

Even if the drug can be administered directly (e.g., is already liquid) we sometimes need to dilute the **stock solution** (undiluted drug) for ease of use.

In some problems the drug mixture will be divided into parts. These parts are sometimes called **aliquots**. For example when testing substances (like blood samples) we may divide the total amount into aliquots for each test to be run.

The most important concept is measuring how concentrated a solution is. There are three ways this information can be written.

- Ratio of concentrated drug to diluent, e.g., 1:4
- Ratio of concentrated drug to total, e.g., 1:5
- **Dilution Factor**, e.g., 5
- **Percent concentration**, e.g., 20%

Suppose we add 500 mL of water to 100 mL of a concentrated solution of hydrochloric acid.

- The ratio of solution to diluent is $\frac{100}{500}$
- The ratio of solution to total is $\frac{100}{600}$
- The dilution factor is $\frac{600}{100} = 6$
- The percent concentration is $\frac{100}{600} \cdot 100 = 16.7\%$

Note this is about diluting the solution, not about determining the concentration of acid in diluent. We need more information for that. □

Standalone

2.5.2 Making Solutions

A solution of a desired concentration can be produced to have a desired percent concentration. Note be careful in each problem to determine if this is by weight or by volume. The result can be significantly different.

A standard concentration of HCl (hydrochloric acid) is 37% by weight. The solvent is water. How much water in volume do we need to add to 37 g of HCl to make this solution?

Solution. Because the concentration is given as a percent we can calculate this like Example 2.1.5.

$$37 \text{ g} = 0.37S \text{ g}.$$

$$\frac{37 \text{ g}}{0.37} = \frac{0.37S \text{ g}}{0.37}.$$

$$100 \text{ g} = S.$$

We have a total of 100 g and 37 g is HCl, thus the water is

$$100 \text{ g} - 37 \text{ g} = 63 \text{ g}$$

of water. 1 g of water at standard conditions has a volume of 1 mL. This is now a unit conversion problem.

$$63g \cdot \frac{1 \text{ mL}}{1 \text{ g}} = 63 \text{ mL}.$$

□

Solutions are sometimes diluted by a specified amount to produce a less concentrated version. First, we will practice calculating dilution factors.

A solution is produced from 3 mL of concentrated chloroform and 37 mL of water. What is the dilution factor?

Solution. The dilution factor is the ratio of the total to the substance. The total is substance plus diluent. Here that is $3 + 37 = 4\bar{0}$. The ratio then is $\frac{4\bar{0}}{3} \approx 13$. □

To produce a solution 3.0 mL of concentrated chloroform that will have a dilution factor of $5\bar{0}$, how much diluent do we add?

Solution. The dilution factor is the ratio of the total to the substance. The total is substance plus diluent. In this case we have

$$\begin{aligned}
\frac{3.0 + D}{3.0} &= \frac{5\bar{0}}{1.0}. \\
3.0 \cdot \frac{3.0 + D}{3.0} &= 3.0 \cdot \frac{5\bar{0}}{1.0}. \\
3.0 + D &= 150. \\
-3.0 + 3.0 + D &= -3.0 + 150. \\
D &= 147.
\end{aligned}$$

So we need 147 mL of diluent. □

2.5.3 Dilution

A dilution ratio tells us how much an existing solution has been diluted. Dilution ratios are used like percents or applying ratios.

If we have a solution of HCl and water that has a ratio of solution to total of 37/100, and we apply a dilution ratio of 1/2, what is the new ratio of solution to total?

$$\frac{37}{100} \cdot \frac{1}{2} = \frac{37}{200}$$

Note this means there are 37 g of HCl and $200 - 37 = 163$ grams of water. □

If we know the original ratio of solution to diluent and the resulting ratio of solution to diluent, then we can calculate the dilution ratio.

Suppose the original solution of HCl and water had a ratio of solution to total of 37/100 and the resulting solution had a ratio of 74/300. What was the dilution ratio? We setup the same calculation as in Example 2.5.5 but instead of 1/2 we have the unknown ratio.

$$\begin{aligned}
\frac{37}{100} r &= \frac{74}{300} \\
\frac{100}{37} \cdot \frac{37}{100} r &= \frac{100}{37} \cdot \frac{74}{300}
\end{aligned}$$

$$r = \frac{2}{3}.$$

The dilution ratio was 2/3. □

One usage of dilution is to reduce the concentration so that instruments can accurately measure it. Consider trying to measure an acid without disolving the tools used to measure it.

A sample of a suspected high blood glucose value was obtained. According to the manufacturer of the instrument used to read blood glucose values, the highest glucose result which can be obtained on this particular instrument is 500 mg/dL. When the sample was run, the machine gave an error message (concentration too high).

The serum was diluted to 1/10 and retested. The machine gave a result of 70 mg/dL. What was the initial concentration?

Note that the ratio is milligrams to decilitres (weight to volume). In these types of problems the amount of substance is so small that it does not affect the volume.

Solution. Before we jump into an equation, let's try an experiment. Suppose we take 1 dL of the original serum. Because the blood sample is so small, we can calculate as if all the volume is the diluent. To dilute to a ratio of 1/10 we need to add $10 - 1 = 9$ dL of diluent. No blood glucose was added thus the concentration is changed only by the diluent. Thus the concentration of the diluted serum would be

$$\begin{aligned}
\frac{C + 0 \text{ mg}}{1 + 9 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} \\
\frac{C \text{ mg}}{10 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} && \text{clearing the denominators} \\
(C \text{ mg})(\text{dL}) &= (70 \text{ mg})(10 \text{ dL}) \\
\frac{(C \text{ mg})(\text{dL})}{\text{dL}} &= \frac{(70 \text{ mg})(10 \text{ dL})}{\text{dL}} \\
C &= 700 \text{ mg}
\end{aligned}$$

Did this result depend on our selecting 1 dL of the original serum? If we are uncertain we can try the problem again and select 2 dL of the original serum. To figure out the total amount of which 2 is 1/10, we can treat this like Example 2.3.5

$$\begin{aligned}
\frac{1}{10} &= \frac{1}{10}\frac{2}{2} \\
&= \frac{2}{20}.
\end{aligned}$$

This means we need $20 - 2 = 18$ dL of diluent to have the desired dilution ratio. Also we will have twice as much of the blood glucose.

$$\begin{aligned}
\frac{2C + 0 \text{ mg}}{2 + 18 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} \\
\frac{2C \text{ mg}}{20 \text{ dL}} &= \frac{70 \text{ mg}}{\text{dL}} && \text{clearing the denominators} \\
(2C \text{ mg})(\text{dL}) &= (70 \text{ mg})(20 \text{ dL}) \\
\frac{(2C \text{ mg})(\text{dL})}{2\text{dL}} &= \frac{(70 \text{ mg})(20 \text{ dL})}{2\text{dL}} \\
C &= 700 \text{ mg}
\end{aligned}$$

Notice the result is the same. This makes sense, because we are setting up a proportion, and ratios do not depend on the amount. □

Sometimes we dilute more than one time. Here we experiment to determine what the effect of **serial dilution** is upon the dilution factor.

Suppose you have a solution consisting of 10 mL of acyl chloride and 90 mL of water. If this is diluted to a dilution ratio of 1/2 and then diluted again to a dilution ratio of 1/3, what is the final dilution ratio?

Solution. We can do the calculations one at a time. First we calculate the original concentration.

$$\begin{aligned}\frac{10 \text{ mL}}{10 + 90 \text{ mL}} &= \frac{10}{100}\\ &= \frac{1}{10}.\end{aligned}$$

To dilute to a ratio of 1/2 we can calculate the amount of diluent to add as a proportion problem like in Example 2.4.3.

$$\begin{aligned}\frac{1}{2} &= \frac{100 \text{ mL}}{T \text{ mL}}\\ 1 \cdot (T \text{ mL}) &= 2 \cdot (100 \text{ mL})\\ T &= 200 \text{ mL}.\end{aligned}$$

The total will be 200 mL so we need to add 200 mL − 100 mL = 100 mL of additional diluent. Note at this point the concentration is

$$\frac{10 \text{ mL acyl chloride}}{200 \text{ mL diluent}} = \frac{1}{20}.$$

To dilute again to a ratio of 1/3 we can calculate the amount of diluent to add

$$\begin{aligned}\frac{1}{3} &= \frac{200 \text{ mL}}{T \text{ mL}}\\ 1 \cdot (T \text{ mL}) &= 3 \cdot (200 \text{ mL})\\ T &= 600 \text{ mL}.\end{aligned}$$

The total will be 600 mL so we need to add 600 mL − 200 mL = 400 mL of additional diluent. Note at this point the concentration is

$$\frac{10 \text{ mL acyl chloride}}{600 \text{ mL diluent}} = \frac{1}{60}.$$

Now we can determine what the resulting dilution ratio after diluting twice (1/2 and then 1/3). We set this up like Example 2.5.6

$$\begin{aligned}\frac{1}{10} \cdot F &= \frac{1}{60} && \text{clear the denominator on the left}\\ 10 \cdot \frac{1}{10} \cdot F &= 10 \cdot \frac{1}{60}.\\ F &= \frac{1}{6}.\end{aligned}$$

Notice that $\frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$. This relationship is always true for serial dilution. □

2.5.4 Dosage

If we know the concentration of a drug, we can determine how much is needed for a given dose.

In medicine some substances are measured in **International Unit** or IU. For each substance this is defined by the effect of that amount of the drug.

One IU of insulin is 0.0347 mg. A common concentration of insulin is U-100 which is 100 IU/mL. This is produced by combining 100 units of insulin in one mL of diluent.

If a person needs 2 units of insulin, how many mL of solution will that be?

Solution. This is a unit conversion problem. We need to convert from IU to mL. We know the ratio is 100

IU/mL. Because we want to end up with mL, we will write the ratio as

$$\begin{aligned}
\frac{\text{mL}}{100\text{ IU}} &= \frac{v\text{ mL}}{2\text{ IU}} && \\
(1\text{ mL})(2\text{ IU}) &= (100\text{ IU})(v\text{ mL}) && \text{clear denominators} \\
\frac{(1\text{ mL})(2\text{ IU})}{(100\text{ IU})} &= \frac{(100\text{ IU})(v\text{ mL})}{(100\text{ IU})} && \text{divide to isolate the variable} \\
\frac{1}{50}\text{ mL} &= v. && \\
0.02\text{ mL} &= v. &&
\end{aligned}$$

□

A label reads "2.5 mL of solution for injection contains 1000 mg of streptomycin sulfate." How many millilitres are needed to contain 800 mg of streptomycin?

Solution. The label gives us a ratio of 1000 mg per 2.5 mL. We can find the desired volume by setting up the following proportion.

$$\begin{aligned}
\frac{1000\text{ mg}}{2.5\text{ mL}} &= \frac{800\text{ mg}}{v\text{ mL}}. && \text{clear the denominators} \\
1000\text{ mg}\cdot v\text{ mL} &= 800\text{ mg}\cdot 2.5\text{ mL}. && \text{divide to isolate the variable} \\
v\text{ mL} &= \frac{800\text{ mg}\cdot 2.5\text{ mL}}{1000\text{ mg}}. && \\
v &= 2\text{ mL}. &&
\end{aligned}$$

□

Standalone

Give 1500 mL of saline solution IV with a drop factor of 10 drops per mL as a rate of 50 drops per minute to an adult patient. Determine how long in hours the IV should be administered.

Solution. This is also a unit conversion problem like Example 2.3.12. We need to convert from drops per minute to drops per mL.

$$\frac{50\text{ drops}}{\text{minute}}\cdot\frac{\text{mL}}{10\text{ drops}} = \frac{5\text{ mL}}{\text{minute}}$$

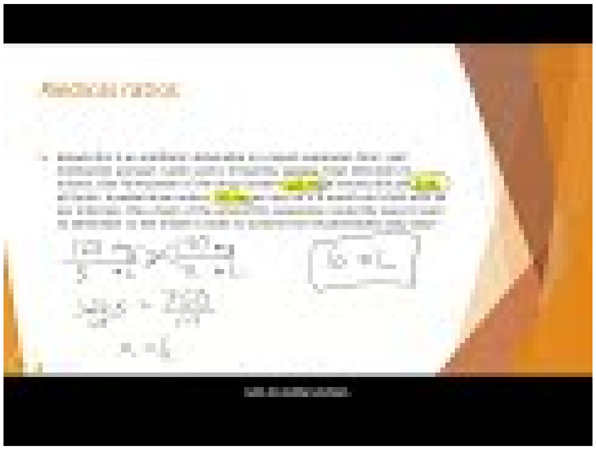
Now that we know the rate, we can determine how long it will take to give the whole IV solution. Notice how we invert the rate to make the mL units match.

$$1500\text{ mL}\cdot\frac{\text{minute}}{5\text{ mL}} = 300\text{ minutes}$$

The final step is to convert minutes to hours. This is another unit conversion problem using a conversion from Table 1.1.2

$$300 \text{ minutes} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 5 \text{ hours}$$

□



Standalone

Amoxicillin is an antibiotic obtainable in a liquid suspension form, part medication and part water, and is frequently used to treat infections in infants. One formulation of the drug contains 125 mg of amoxicillin per 5 mL of liquid. A pediatrician orders $15\bar{0}$ mg per day for a 4-month-old child with an ear infection. How much of the amoxicillin suspension would the parent need to administer to the infant in order to achieve the recommended daily dose?

Solution. Here we need to scale the amount (from 125 mg to $15\bar{0}$ mg). This is a proportion problem, that is, the ratio of medicine to volume is the same so we can setup an equation based on the drug concentration.

$$\begin{aligned}
\frac{125 \text{ mg}}{5 \text{ mL}} &= \frac{15\bar{0} \text{ mg}}{A \text{ mL}} && \text{clear the denominators}\\
(125 \text{ mg})(A \text{ mL}) &= (5 \text{ mL})(15\bar{0} \text{ mg}) && \text{divide to isolate the variable}\\
\frac{(125 \text{ mg})(A \text{ mL})}{125 \text{ mg}} &= \frac{(5 \text{ mL})(15\bar{0} \text{ mg})}{(125 \text{ mg})}\\
A \text{ mL} &= \frac{(5 \text{ mL})(15\bar{0} \text{ mg})}{(125 \text{ mg})}\\
A \text{ mL} &= 6.
\end{aligned}$$

□

Checkpoint 2.5.13 A 5% dextrose solution (D5W) contains 5 g of pure dextrose per 100 mL of solution. A doctor orders 500 mL of D5W IV for a patient. How much dextrose does the patient receive from that IV?

Solution. Once again we need to scale the amount (from 100 mL to 500 mL). This is also a proportion problem, that is, the ratio of medicine to volume is the same so we can setup an equation based on the drug concentration.

$$\begin{aligned}
\frac{5 \text{ g}}{100 \text{ mL}} &= \frac{D \text{ g}}{500 \text{ mL}} && \text{clear the denominators}\\
(5 \text{ g})(500 \text{ mL}) &= (D \text{ g})(100 \text{ mL}) && \text{divide to isolate the variable}\\
\frac{(5 \text{ g})(500 \text{ mL})}{(100 \text{ mL})} &= \frac{(D \text{ g})(100 \text{ mL})}{(100 \text{ mL})}\\
25 \text{ g} &= D
\end{aligned}$$

A sample of chloroform water has a dilution factor of 40. If 2 mL of chloroform are needed how many milliliters total are needed?

Solution. A dilution factor of 40 indicates that 1 mL of chloroform is in 40 mL total of solution. We can setup a proportion to answer this.

$$\begin{aligned} \frac{1 \text{ mL}}{40 \text{ mL}} &= \frac{2 \text{ mL}}{T \text{ mL}} && \text{clear the denominators} \\ (1 \text{ mL})(T \text{ mL}) &= (2 \text{ mL})(40 \text{ mL}) \\ \frac{(1 \text{ mL})(T \text{ mL})}{1 \text{ mL}} &= \frac{(2 \text{ mL})(40 \text{ mL})}{1 \text{ mL}} \\ T &= 80 \text{ mL}. \end{aligned}$$

□

2.5.5 Exercises

1. **Medical Ratio.** A 1 litre (1,000 mL) IV bag of dextrose solution contains 40 g of dextrose. Find the ratio of grams per millilitre of dextrose. (Enter your answer in fraction form.) ________________Preview Question 1

2. **Medical Ratio.** Find the flow rate (in drops/min) for the given IV (assume a drop factor of 15 drops/mL).

1400 mL in 7.0 h

____________________Preview Question 1 drops/min

3. **Medical Ratio.** Find the length of time (in h) the IV should be administered (assume a drop factor of 14 drops/mL).

1,000 mL at a rate of 70 drops/min

____________________Preview Question 1 h

4. **Medical Proportion.** A label reads: "2.5 mL of solution for injection contains 1,000 mg of streptomycin sulfate." How many millilitres are needed to give 600 mg of streptomycin?

____________________Preview Question 1

5. **Medicine to Solution.** Quinidine gluconate is a liquid mixture, part medicine and part water, which is administered intravenously. There are 120.0 mg of quinidine gluconate in each cubic centimeter (cc) of the liquid mixture. Dr. Alverez orders 1560 mg of quinidine gluconate to be administered daily to a patient with malaria.

How much of the solution would have to be administered to achieve the recommended daily dosage?

____________________ cc

6. **Medical Ratio with Rounding.** Albuterol is a medicine used for treating asthma. It comes in an inhaler that contains 15 mg of albuterol mixed with a liquid. One actuation (inhalation) from the mouthpieces delivers a 90 μg dose of albuterol. (Reminder: 1 mg = 1000 μg .)

a.) Dr. Olson orders 2 inhalations 3 times per day. How many micrograms of albuterol does the patient inhale per day?

____________________ μg

b.) How many actuations are contained in one inhaler?

____________________ actuations

c.) Shelby is going away for 5 months and wants to take enough albuterol to last for that time. Her physician has prescribed 2 inhalations 3 times per day. How many inhalers will Shelby need to take with her for the 5 period? Assume 30 month days.

Hint: she can't bring a fraction of an inhaler, and she does not want to run out of medicine while away.

____

7. **Concentration.** Amoxicillin is a common antibiotic prescribed for children. It is a liquid suspension composed of part amoxicillin and part water.

In one formulation there are 150 mg of amoxicillin in 5 cubic centimeters (cc's) of the liquid suspension. Dr. Scarlotti prescribes 225 mg per day for a 2-yr old child with an ear infection.

How much of the amoxicillin liquid suspension would the child's parent need to administer in order to achieve the recommended daily dosage?

____

8. **Concentration.** Diphenhydramine HCL is an antihistamine available in liquid form, part medication and part water. One formulation contains 14 mg of medication in 5 mL of liquid. An allergist orders 56-mg doses for a high school student. How many milliliters should be in each dose?

____________________ mL

9. **Concentration.** How many mL of sodium hydroxide are required to prepare 400 mL of a 17% solution? Assume the sodium hydroxide dissolves in the solution and does not contribute to the overall volume.

____________________ mL

10. **Dilution Ratio.** You are asked to make a 1/7 dilution using 1 mL of serum. How much diluent do you need to use?

____________________ mL

11. **Dilution Ratio.** A clinical lab technician determines that a minimum of 40 mL of working reagent is needed for a procedure. To prepare a $\frac{1}{9}$ dilution ratio of the reagent from a stock solution, one should measure 40 mL of the reagent and ____________________ mL of the diluent.

12. **Dilution Ratio.** A patient's glucose result is suspected to be outside the range of the analyzer, so the techs decide to dilute the sample before running it. 45 microliters of serum is added to 90 microliters of diluent and the diluted sample is analyzed. The analyzer reads that the glucose value of the diluted sample is 70 $\frac{mg}{dL}$.

What was the ratio the sample was diluted to?

____________________Preview Question 1 Part 1 of 2

What is the glucose value of the original sample?

____________________ $\frac{mg}{dL}$

13. **Serial Dilution.** A thyroid peroxidase antibody test was performed on a 45 year old man. The dilution sequence was 20 μL serum added to 80 μL of diluent in tube 1. Then 70 μL from tube 1 was added to 490 μL of diluent in tube 2. Finally 45 μL from tube 2 was added to 135 μL of diluent in tube 3.

All dilution ratios should be given as fractions.

a.) What is the dilution ratio in tube 1?

____________________Preview Question 1 Part 1 of 4

b.) What is the dilution ratio in tube 2?

____________________Preview Question 1 Part 2 of 4

c.) What is the dilution ratio in tube 3?

____________________Preview Question 1 Part 3 of 4

d.) What is the overall (serial) dilution ratio?

____________________Preview Question 1 Part 4 of 4

2.6 Linear Expressions

We have been solving problems involving ratios, also called proportions. These model situations where the ratio/proportion between items never changes. We also call these linear equations.

2.6.1 Solving Linear Equations

Before reading farther solve the equation $5x - 7 = 12$. What steps did you use? Why do they work? Example 2.6.1 is an example of solving another linear equation.

Solve $-8x - 3 = 5$.

Solution.

$$-8x - 3 = 5.$$

$$\begin{aligned}-8x-3+3&=5+3.\\-8x&=8.\\\frac{-8x}{-8}&=\frac{8}{-8}.\\x&=-1.\end{aligned}$$

Note we added three because it eliminates the -3 (undoes subtraction of 3). We divided by negative eight because it eliminates the -8 (undoes the multiplication by -8). □

Checkpoint 2.6.2 What is the solution to $4x+2=14$. $x=$___

Answer. 3

Solution.

$$\begin{aligned}4x+2&=14\\4x&=14-2\\x&=\frac{14-2}{4}\\x&=3.\end{aligned}$$

Some linear equations need one more technique. What would you need to solve $17-4y=14-y$? Example 2.6.3 is an example of solving a similar linear equation.

Solve $17-4y=5+2y$.

Solution.

$$\begin{aligned}17-4y&=5+2y.\\-5+17-4y&=-5+5+2y.\\12-4y&=2y.\\12-4y+4y&=2y+4y.\\12&=(2+4)y.\\12&=6y.\\\frac{12}{6}&=\frac{6y}{6}.\\2&=y.\end{aligned}$$

Notice we had to combine like terms (factor and add). □

Checkpoint 2.6.4 What is the solution to $10x+8=5x+18$. $x=$___

Answer. 2

Solution.

$$\begin{aligned}10x+8&=5x+18\\10x-5x&=18-8\\(10-5)x&=18-8\\x&=\frac{18-8}{10-5}\\x&=2.\end{aligned}$$

Another linear equation is $\frac{x}{3}+\frac{x}{4}=\frac{7}{12}$. How would you solve it?

We can solve this the same as in Example 2.6.3 but there is another technique as well. It is shown in Example 2.6.5.

Solve $\frac{x}{5}+\frac{2x}{7}=\frac{34}{35}$

Solution.

$$\begin{aligned}
\frac{x}{5}+\frac{2x}{7} &= \frac{34}{35}.\\
5\cdot\left(\frac{x}{5}+\frac{2x}{7}\right) &= 5\cdot\frac{34}{35}.\\
\frac{5x}{5}+\frac{10x}{7} &= 5\cdot\frac{34}{35}.\\
x+\frac{10x}{7} &= \frac{34}{7}.\\
7\cdot\left(x+\frac{10x}{7}\right) &= 7\cdot\frac{34}{7}.\\
7x+\frac{7\cdot 10x}{7} &= 7\cdot\frac{34}{7}.\\
7x+10x &= 34.\\
(7+10)x &= 34.\\
17x &= 34.\\
\frac{17x}{17} &= \frac{34}{17}.\\
x &= 2.
\end{aligned}$$

This is referred to as clearing denominators. We are once again eliminating division by multiplying. Always remember to distribute. Note, we could multiply once if we figured out the correct number, but there are no prizes for doing this fast, so you can do this either way. □

2.6.2 Solving with Multiple Variables

When we are using models (also called formulas) they often include more than one variable (also known as named constants). The process for solving such models is the same.

Solve the equation $V = IR$ for R. Note, this formula is explained in Example 1.3.1.

Solution.

$$\begin{aligned}
V &= IR. &&\\
\frac{V}{I} &= \frac{IR}{I} && \text{divide to undo multiplication.}\\
\frac{V}{I} &= R. &&
\end{aligned}$$

□

Solve the lift equation $L = \frac{1}{2}\rho S C_L v^2$ for S.

Solution.

$$\begin{aligned}
L &= \frac{1}{2}\rho S C_L v^2.\\
2L &= 2\frac{1}{2}\rho S C_L v^2.\\
2L &= \rho S C_L v^2.\\
\frac{2L}{\rho} &= \frac{\rho S C_L v^2}{\rho}.\\
\frac{2L}{\rho} &= S C_L v^2.
\end{aligned}$$

$$\frac{2L}{\rho C_L} = \frac{SC_L v^2}{C_L}.$$
$$\frac{2L}{\rho C_L} = Sv^2.$$
$$\frac{2L}{\rho C_L v^2} = \frac{Sv^2}{v^2}.$$
$$\frac{2L}{\rho C_L v^2} = S.$$

□

Solve the lift equation $L = \frac{1}{2}\rho SC_L v^2$ for S when $L = 255\bar{0}$, $\rho = 0.002378$, and $C_L = 1.430$.

Solution. Because we know some of the values, we can first insert them before solving.

$$255\bar{0} = \frac{1}{2}(0.002378)S(1.430)v^2.$$
$$255\bar{0} = 0.001700Sv^2.$$
$$\frac{255\bar{0}}{0.001700} = \frac{0.001700Sv^2}{0.001700}.$$
$$150\bar{0}000 = Sv^2.$$
$$\frac{150\bar{0}000}{v^2} = \frac{Sv^2}{v^2}.$$
$$\frac{150\bar{0}000}{v^2} = S.$$

□

Checkpoint 2.6.9 Solve the lift equation $L = \frac{1}{2}\rho SC_L v^2$ for ρ.

2.6.3 Identifying Linear Expressions

All of the equations with which we just worked are linear. What do you use to identify linear expressions or linear equations? Table 2.6.10 shows examples of linear expressions and non-linear expressions.

Table 2.6.10 Linear and Non-linear

Linear	Non-linear
$5x + 3$	$5x^2 - x + 3$
$y = 11 - \frac{7}{13}x$	$y = \frac{17}{x}$
$7x - 9y = 8$	$3 - 2xy = 12$

Some equations that may not appear to be linear can be solved using the same methods.

Solve $\frac{11}{x} + 2 = \frac{18}{x} - 5$.

Solution.

$$\frac{11}{x} + 2 = \frac{18}{x} - 5.$$
$$x \cdot \left(\frac{11}{x} + 2\right) = x \cdot \left(\frac{18}{x} - 5\right).$$
$$11 + 2x = 18 - 5x.$$
$$-11 + 11 + 2x = -11 + 18 - 5x.$$
$$2x = 7 - 5x.$$
$$2x + 5x = 7 - 5x + 5x.$$
$$7x = 7.$$

$$\frac{7x}{7} = \frac{7}{7}.$$
$$x = 1.$$

□

2.6.4 Exercises

1. **Solve.** Solve the equation below.
$2(x-3)-7=14x-85$
Answer: $x =$ ____________________

2. **Solve.** Solve $-7(x+7)+8=-3(x-5)$ for x algebraically. If your answer is a fraction, write it in reduced, fractional form. Do NOT convert the answer to a decimal.$x =$____________________Preview Question 1

3. **Solve.** Solve the equation for the given variable:
$\frac{-4n-7}{-5} = -2$
If your answer is a fraction, write it in fraction from and reduce it completely. Do NOT convert to decimals.
$n =$ ________Preview Question 1

4. **Solve.** Solve the equation $\frac{1}{3}y+1=\frac{1}{5}y$.
$y =$ ____________________Preview Question 1

5. **Solve.** Solve the equation for the given variable. If your answer is a fraction, write it in reduced, fractional form. Do NOT convert the answer to a decimal.
$\frac{y}{2}+\frac{y}{4}=\frac{7}{6}$
Answer: $y =$ ____________________Preview Question 1

6. **Solve.** In certain deep parts of oceans, the pressure of sea water, P, in pounds per square foot, at a depth of d feet below the surface, is given by the following equation:
$P = 16+\frac{5d}{11}$
If a scientific team uses special equipment to measures the pressure under water and finds it to be 216 pounds per square foot, at what depth is the team making their measurements?
Answer: The team is measuring at ______ feet below the surface.

7. **Solve.** Solve for k in the equation: $\frac{2}{3}k-\frac{9}{7}=-8-\frac{1}{9}k$.
Round your answer to three decimal places. *Note: round only on the last step!*
$k =$ __________

8. **Solve.** Solve the following formula for x
$y = 2mx-4b$
$x =$ ____________________Preview Question 1
Enter your answer as an expression.
But be careful...to enter an expression like $\frac{a+b}{3+m}$ you need to type *(a+b)/(3+m)*. You need parentheses for both the numerator and denominator.

9. **Solve.** Solve the following formula for m
$c = amt$
$m =$ ____________________Preview Question 1
Enter your answer as an expression.
But be careful...to enter an expression like $\frac{a+b}{3+m}$ you need to type *(a+b)/(3+m)*. You need parentheses for both the numerator and denominator.

10. Solve. Solve the formula $d = rt$ for t.
$t =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

11. Solve. Solve the formula $V = \pi r^2 h$ for h. HINT: type π as pi.
$h =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

12. Solve. Solve the formula $A = \frac{1}{2}bh$ for h.
$h =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

13. Solve. Solve the formula $A = \frac{1}{2}h(a+b)$ for a.
$a =$ _______Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

14. Solve. Solve the formula $S = P(1 + rt)$ for P.
$P =$ __________Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

15. Solve. Solve the formula $A = P + Prt$ for t.
$t =$ __________Preview Question 1
Enter your answer as an expression.
Be sure to PREVIEW your answer before submitting!

2.7 Working with Applications

We are emphasizing using math to address questions that arise from various circumstances. This means the problems do not come with a model (think equation or function) and instructions on using it. In this section we will consider how we can go about using a description (typically verbal) to identify and use a model.

2.7.1 Process Overview

Our first task is to read the problem to understand it.

- Read the problem description a few times.
  - If you can paraphrase it, you understand it enough.
  - Drawing a picture and labeling parts may help.
- Identify what we are asked to do.
- Identify the information we are given. Note distinctions like measurements and rates.
- Identify any units. These often help us set up a model.
- Write everything! We do not model in our heads.

Next we write the mathematical model (equation or function).

- Use the description to determine which application type (e.g., percent, proportion, linear model, etc.). Note units can suggest this (e.g., meters and meters squared indicate something was squared).

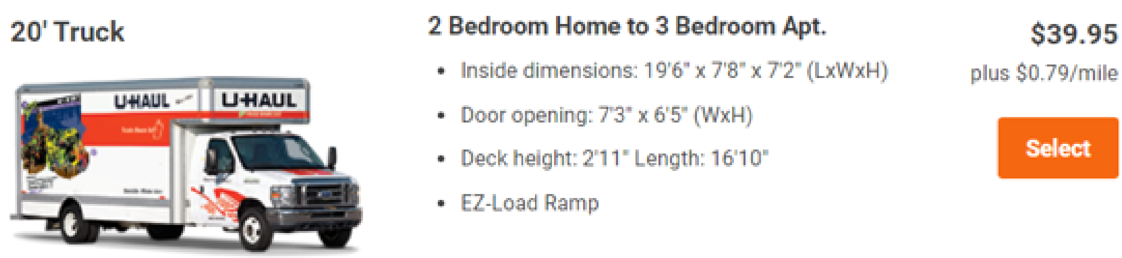
- Do not insert any numbers yet.
- Do not do any calculations yet.

Now we will have a model that matches our situation and possibly some numbers to insert.

- Insert numbers into the model. You may have to calculate some of these (e.g., you are given two points but not the slope you need).
- Solve for the desired value. Note it may help to do some calculations with the numbers first.
- State your answer and use units appropriately.

Finally we should check that our answer makes sense. We should not have negative prices (usually) or distances larger than the earth (when working with terrestrial problems).

You moved across town and rented a 20 foot moving truck for the day. You want to make sure the bill you received is correct. If you paid \$81.03, for how many miles were you charged? Assume there were no extra fees.



Solution. We want to compare the bill we received to the price listed in the add. The question is about how many miles (not how much money).

We are given the price per mile (\$0.79 per mile). There is also a fixed cost for the rental (\$39.95). Adding the fixed cost and the milage cost will give us the total.

Our model is $C = \$39.95 + \$0.79m$ where C is the total cost and m is the number of miles.

We know the total cost, which will leaves m in the equation, the number of miles, which is what we want to calculate. We can use the solving technique in Section 2.6

$$\begin{aligned} \$81.03 &= \$39.95 + \$0.79m \\ -\$39.95 + \$81.03 &= -\$39.95 + \$39.95 + \$0.79m \\ \$41.08 &= \$0.79m \\ \frac{\$41.08}{\$0.79} &= \frac{\$0.79}{\$0.79}m \\ 52 &= m. \end{aligned}$$

The charge is for 52 miles. □

Standalone

Figure 2.7.2 Using math modeling for rental truck

2.7.2 Exercises

1. Minimum Grade. A student in a Pre-Calculus class has test scores of 77, 76, 64, and 72. The final exam is worth 4 test grades.

Write a linear equation that models this problem, where x is the grade on the final exam and y is the student's grade in the course. The whole grade is based on these tests and the final.

____________________Preview Question 1 Part 1 of 2

What grade is needed on the final to earn a B (average score of 80%)?

____

2. Triangle Properties.

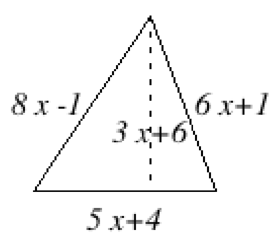


Figure 2.7.3

Consider the triangle shown on the picture. Find the value of x, given that the perimeter of the triangle is 80 unit.

$x =$ _______ unit

Preview Question 1

3. Bike Rental. Amanda rented a bike from Ted's Bikes.

It costs \$11 for the helmet plus \$5.25 per hour.

If Amanda paid about \$34.63, how many hours did she rent the bike?

a) Let $h =$ the number of hours she rented the bike. Write the equation you would use to solve this problem.

____________________Preview Question 1 Part 1 of 2

b) Now solve your equation

Amanda rented the bike for ____________________ hours. (Round your answer to the nearest tenth of an hour.)

4. Pilot Training. Part 1 of 5

The cost of a private pilot course is \$1443. The flight portion costs \$431 more than the ground school portion. What is the cost of the flight portion alone?

a) Let x represent the cost of the ground school portion. Write a variable expression to represent the cost of the flight portion.

____________________Preview Question 1 Part 1 of 7

Part 2 of 5b) The total cost of the private pilot course can be represented by: Cost of ground school portion + Cost of flight portion = Total Cost

Fill in the boxes using the information from the problem and your expressions from part a:

Table 2.7.4

Cost of Ground School	+	Cost of flight portion
____________________Preview Question 1 Part 2 of 7	+	____________________Preview Question 1 Part 3 of 7

Part 3 of 5

c) Solve the equation $x + (x + 431) = 1443$ to answer the question.

$x =$ ____________________

Part 4 of 5

d) Since $x = 506$, this tells us:

(a) Flight school portion costs \$506

(b) Ground school portion costs \$506

Part 5 of 5

e) We used the expression $x+431$ to represent the cost of the flight portion. Knowing that $x = 506$, what is the cost of the flight portion alone?

Flight portion costs \$____________________

5. Thunder. In a thunderstorm, the formula:

$M = \dfrac{t}{5}$

gives the approximate distance, M, in miles, from a lightning strike if it takes t seconds to hear the thunder after seeing the lightning. If you are 6.3 miles away from the lightning flash, how long will it take the sound of the thunder to reach you.

Answer: It will take ______ seconds for the sound to reach you.

6. Speeding. In a Northwest Washington County, speeding fines are determined by the formula:

$F = 7(s - 50) + 90$

where F is the cost, in dollars, of the fine if a person is caught driving at a speed of s miles per hour.

If a fine comes to \$195, how fast was the person speeding?

Answer: The person's speed was ______ miles per hour.

7. Area of Triangle. The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where A is the area of the triangle, b is its base and h is its height.

Solve the formula, $A = \frac{1}{2}bh$, for h.

$h =$ ____________Preview Question 1 Part 1 of 2

Find the height of the triangle with base of 8 meters and area of 111 square meters. Write your answer as a decimal, rounded to the nearest hundredth, when necessary.

height = ______Preview Question 1 Part 2 of 2 meters

8. Unit cost. You and your classmates create 15 abstract paintings to sell at the PTA auction to raise money for your school. In order to pay for the materials you bought to make the abstract paintings, you need to sell each of the abstract paintings for \$3.75. When transporting the abstract paintings to the auction, one of your parents accidentally drops three of the abstract paintings in the street and they are run over by a school bus. What is the new minimum price you need to set for the remaining abstract paintings in order to pay for the materials?

Keep in mind that you do not want to collect less money than you paid for the materials. This may affect how you round your answer.

$____________________

9. Paychecks. Your weekly paycheck is 25 percent less than your coworker's. Your two paychecks total 640. Find the amount of each paycheck.

Your coworker's is : $ __________ and yours is $ __________ .

Given your answers to the nearest cent

10. Mixture. The radiator in a car is filled with a solution of 70 per cent antifreeze and 30 per cent water. The manufacturer of the antifreeze suggests that for summer driving, optimal cooling of the engine is obtained with only 50 per cent antifreeze.

If the capacity of the radiator is 3.9 liters, how much coolant (in liters) must be drained and replaced with pure water to reduce the antifreeze concentration to 50 per cent?

Round your answer to two significant figures.

____________________Preview Question 1 L

11. Area of Rectangle. A rectangular garden is 40 ft wide. If its area is 3400ft^2, what is the length of the garden?

Your answer is : ____________________Preview Question 1 ft

12. Shelving. A bookshelf containing 5 bookshelves is to be constructed. The floor-to-ceiling clearance is 7 ft 2.0 in. Each shelf is 1.0 in thick. An equal space is to be left between the shelves, the top shelf and the ceiling, and the bottom shelf and the floor. (There is no shelf on the ceiling or floor.)

What space should be between each shelf and the next? Round your answer to the nearest tenth of an inch.

____________________ in

13. Fax Cost. An online fax company, EFaxIt.com, has a customer plan where a subscriber pays a monthly subscription fee of $5.75 dollars and can send/receive 110 fax pages at no additional cost. For each page sent or received past the 110 page limit, the customer must pay an overage fee of $0.19 per page. The following expression gives the total cost, in dollars, to send p pages beyond the plan's monthly limit.

$C = 0.19p + 5.75$

If the monthly bill under this plan comes out to be $20.19, what was the total number of pages that were sent or received?

Answer: The total number of pages sent/received was ____________________.

14. Population Decrease. The current population of a small city is 37500 people. Due to a loss of jobs, the population is decreasing by an average of 325 people per year. How many years (from now) will it take for the population to decrease to 35875 people?

A) Write an equation you can use to answer this question. Be sure all the numbers given above appear in your equation. Use x as your variable and use no commas in your equation.

The equation is ____________________Preview Question 1 Part 1 of 2

B) Solve your equation in part [A] for x.

Answer: $x =$ ____________________

15. Sales. After a 40% reduction, you purchase a new digital player for $276. What was the price of the digital player before the reduction?

A) First write an equation you can use to answer this question. Use x as your variable and express any percents in decimal form in the equation.

The equation is ____________________Preview Question 1 Part 1 of 2

B) Solve your equation in part [A] to find the original price of the digital player.

Answer: The original price of the digital player was ____________________ dollars.

16. Bookcase. A bookcase is to have 4 shelves including the top as pictured below.

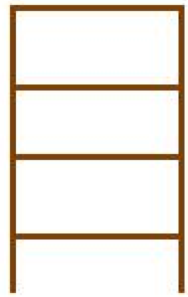


Figure 2.7.5

The width is to be 4 feet less than 2 times the height. Find the width and the height if the carpenter expects to use 24 feet of lumber to make it.

Width: _____ feet

Height: _____ feet

17. Knitting. It takes Rylla 18 hours to knit a scarf. She can only knit for 1.5 hours per day. How many days will it take her to knit the scarf?

Part 1: Let x be the number of days it will take her to knit the scarf. Choose the correct translation of this problem into an equation:

(a) 1.5 x=18

(b) x=(1.5)(18)

(c) 18-1.5=x

Part 2: Solve for x.

____________________Preview Question 1 Part 2 of 2

18. Rental Cost. A rental car company charges $40 plus 25 cents per each mile driven.

Part1. Which of the following could be used to model the total cost of the rental where m represents the miles driven.

(a) $C = 25m + 40$

(b) $C = 2.5m + 40$

(c) $C = 0.25m + 40$

Part 2. The total cost of driving 300 miles is;

$____________________Preview Question 1 Part 2 of 2

19. Mixture. You need a 60% alcohol solution. On hand, you have a 420 mL of a 70% alcohol mixture. How much pure water will you need to add to obtain the desired solution?

A) Write an equation using the information as it is given above that can be used to solve this problem. Use x as your variable to represent the amount of pure water you need to use. Equation: ______________________________Preview Question 1 Part 1 of 3

B) You will need

_____ mL of pure water

to obtain

_____ mL of the desired 60% solution.

20. Mixture. 8.00 liters of fuel containing 2.7% oil is available for a certain two-cycle engine. This fuel is to be used for another engine requiring a 4.3% oil mixture. How many liters of oil must be added?

Give your answer to 3 significant digits.

____

21. Wire Cutting. A wire 23 cm long is cut into two pieces. The longer piece is 3 cm longer than the shorter piece.

Find the length of the shorter piece of wire

____________________ cm

2.8 Project: False Position

Project 1 Method of False Position. In this project, we are going to learn about an ancient algebraic technique that is built around correcting guesses. We may gain greater appreciation for the value of *wrong* guesses and what we can gain from them.

(a) Solve the following equation any way you would like.

$$x\left(1+\frac{1}{3}+\frac{1}{4}\right)=14.$$

Check your answer using technology.

(b) Notice that 12 is the least common multiple of 3 and 4: the denominators. Distribute 12 in the following expression.

$$12\left(1+\frac{1}{3}+\frac{1}{4}\right).$$

Is this bigger, equal to, or smaller than 14?

(c) We multiplied by a convenient number, which is not quite right. Because it is multiplication we can scale (multiply) our not quite right guess to make it right. Consider

$$y\cdot 12\left(1+\frac{1}{3}+\frac{1}{4}\right)=14.$$

Replace 12 times the sum with your result from the previous step.

Solve the resulting equation for y.

(d) Note that y is the correction to our guess of 12. Calculate $y \cdot 12$.

This will match your original solution. If not, check your calculations.

(e) This method is called *false position* because it guesses a convenient number which is typically false then corrects it. One of the original motivations for this method was the lack of a useful notation for fractions (it dates to the Sumerians and ancient Egyptians).

Many people today use similar methods when dealing with fractions. What is a reason people might distribute a convenient number before doing the solving?

2.9 Project: Arclength Estimation

Project 2 Estimating Arc Lengths. In aviation it is sometimes useful to estimate a distance between points as the length of a circular arc. This results from navigation methods (search for VOR and DME arc if curious). To estimate on the fly they use what is known as the 60:1:1 approximation. It means that 60 miles from a point a one degree arc is approximately one mile in length. Note in aviation the distances would be in nautical miles (nm), but the ratio does not change if we use statute miles (the usual type).

Here we will practice using the method to approximate then check why it works.

(a) *Using the Ratio.*

(i) View Example 2.9.1 to Example 2.9.3.

(ii) What is the arclength of 2 degrees at a distance of 30 miles?

(iii) What is the arclength of 5 degrees at a distance of 30 miles?

(iv) What is the arclength of 10 degrees at a distance of 20 miles?

(b) *Explaining the Ratio.*

(i) Calculate the perimeter of a circle with radius 60 miles using the formula $P = 2\pi r$ where P is the perimeter and r is the radius.

(ii) Calculate the perimeter of a semi-circle (half circle) with radius 60 miles.

(iii) Calculate the perimeter of a quarter of a circle with radius 60 miles.

(iv) Calculate the perimeter of 1/360 of a circle with radius 60 miles.

(v) Note that the previous task is the 60:1:1 ratio (1 degree is 1/360th of a circle). Does your result match (i.e., is the result approximately 1 mile)?

Calculate the arclength of 3 degrees at 60 miles.

Solution. If each degree is one mile then 3 degrees is $A = 3 \cdot 1 = 3$ miles. □

Calculate the arclength of 1 degree at 30 miles.

Solution. At 30 miles we are only half way $(30/60 = 1/2)$, so the length is $A = \frac{1}{2} \cdot 1 = \frac{1}{2}$ miles. □

Calculate the arclength of 4 degree at 18 miles.

Solution. The radius is $(18/60 = 3/10)$ of the usual. Thus each degree is $\frac{3}{10}$ of a mile. This arc is $4°$ so the length is $A = \frac{3}{10} \cdot 4 = 1.2$ miles. □

Chapter 3

Variation

3.1 Representing Data

We often represent numerical data using tables, diagrams, and graphs. These include various kinds of charts like bar graphs and pie charts, and graphing of functions. We do this to make certain traits of the data easier to notice. Here we will look at how some of these are produced and begin to learn to recognize differences due to rates.

3.1.1 Reading Tables of Data

Sometimes we simply write down the numbers in tables. This works well if we have a limited number of entries and only two aspects to consider. The two aspects become headers for the rows and columns.

Table 3.1.1 Stall speed at 2550 lbs, most rearward center of gravity, speeds KIAS

	Angle of Bank			
Flap Setting	0°	30°	45°	60°
Up	48	52	57	68
10°	43	46	51	61
Full	40	43	48	57

Note that stall speed refers to the speed at which a wing will produce insufficient lift to keep a plane flying. It results in the plane lowering its nose to regain speed. Angle of bank refers to how steeply the plane is tipped (left or right) in order to turn. KIAS stands for knots indicated air speed. Indicated airspeed is a speed pilots can see (think speedometer). Flaps are a structure extended for landing and sometimes take-off. Up means they are not in use. Others refer to varying degrees of extension.

What is the stall speed in a 30° bank angle with flaps up?

We can determine this by looking for the column labeled "30°" and the row labeled "Up". In that cell is the number 52. Thus the stall speed at that bank angle with flaps up is 52 KIAS. □

What is the stall speed in a 15° bank angle with flaps at 10°?

First, we note that there is no column for 15° bank angle. However we have 0° and 30°. 15° is half way between these two. For this chart and some others it is reasonable to estimate our desired number by calculating the number between those given.

The two stall speeds are 43 and 46. The number in between (the average) is $(43 + 46)/2 = 44.5$. When considering stall speeds, it is safest to assume a higher stall speed, so we will round to 45 KIAS. □

Checkpoint 3.1.4 What is the stall speed at 60° bank angle and 10° flap setting? ____ KIAS

Answer. 61

Checkpoint 3.1.5 What is the stall speed at 37.5° bank angle and full flap setting? ____ KIAS

Answer. 46

Solution. Note this bank angle is half way between 30° and 45°. We can average the entries for those bank angles.

$$\frac{43+48}{2}=45.5$$

We round up to 46 KIAS for safety.

3.1.2 Reading Graphs

Graphs that are curves (like lines) are read by finding a vertical heading that matches our question (think row) and read the corresponding horizontal heading (think column). Note this could be reversed, that is, find a horizontal heading that matches and read the corresponding vertical one.

If the plane with maximum engine out glide represented in Figure 3.1.7 is 2400 ft above the ground how many nautical miles can it glide forward?

Note each horizontal major line is 2000 ft. Counting we find 10 minor lines between each major line, so we know they represent $2000/10 = 200$ ft. each vertical major line is 2 nm. Again there are 10 minor lines between each major line, so we know they represent $2/10 = 0.2$ nm.

2400 is two minor lines above 2000. We follow that to the blue line, then we follow the gray (minor) line down to the bottom. It is two minor lines before 4. This is $4 - 2(0.2) = 3.8$ nm. □

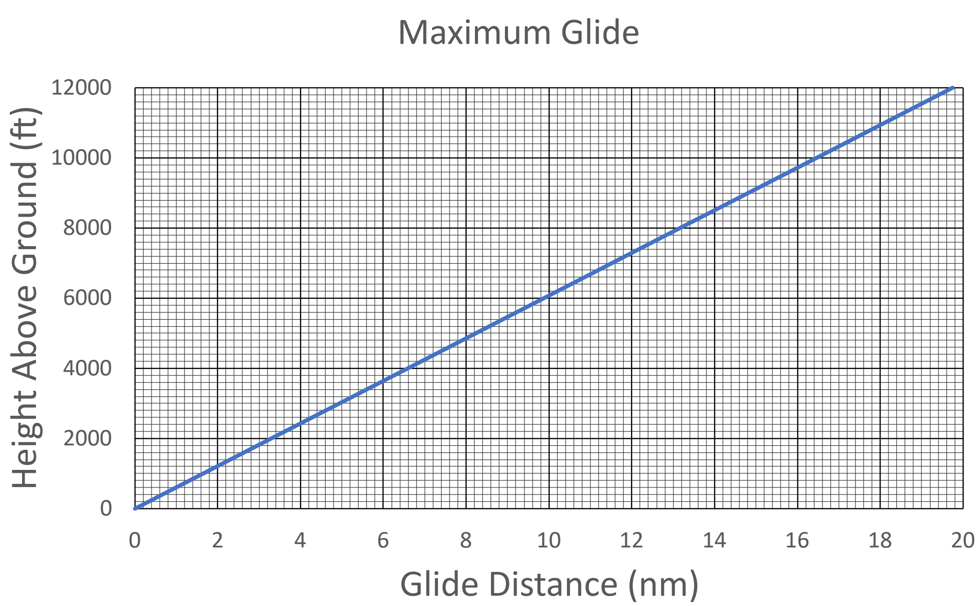


Figure 3.1.7 Graph Representing Maximum Engine Out Glide

Checkpoint 3.1.8 What is the maximum glide for an aircraft 5000 above the ground? ____ nm

Answer. 8.2

Graphs can be from raw data which can seem random. We still read these the same way.

Figure 3.1.10 has the temperature and dewpoint read by a radiosonde (instruments on weather balloon) as it rose in the atmosphere. Note the vertical axis is the pressure reading. This is not the same as altitude, but it does correspond mostly to altitude. Dew point is the temperature at which water will condense, so it is also a temperature.

What are the temperature and dewpoint at the 700 millibar level?

We follow the 700 mb line over to the dewpoint (green, dashed) line. It is just above $-20°$ C. We estimate $-18°$ C. Continuing across the 700 mb line to the temperature (red, solid) line we find it about half way between 0 and 10. We estimate the temperature is $5°$ C. □

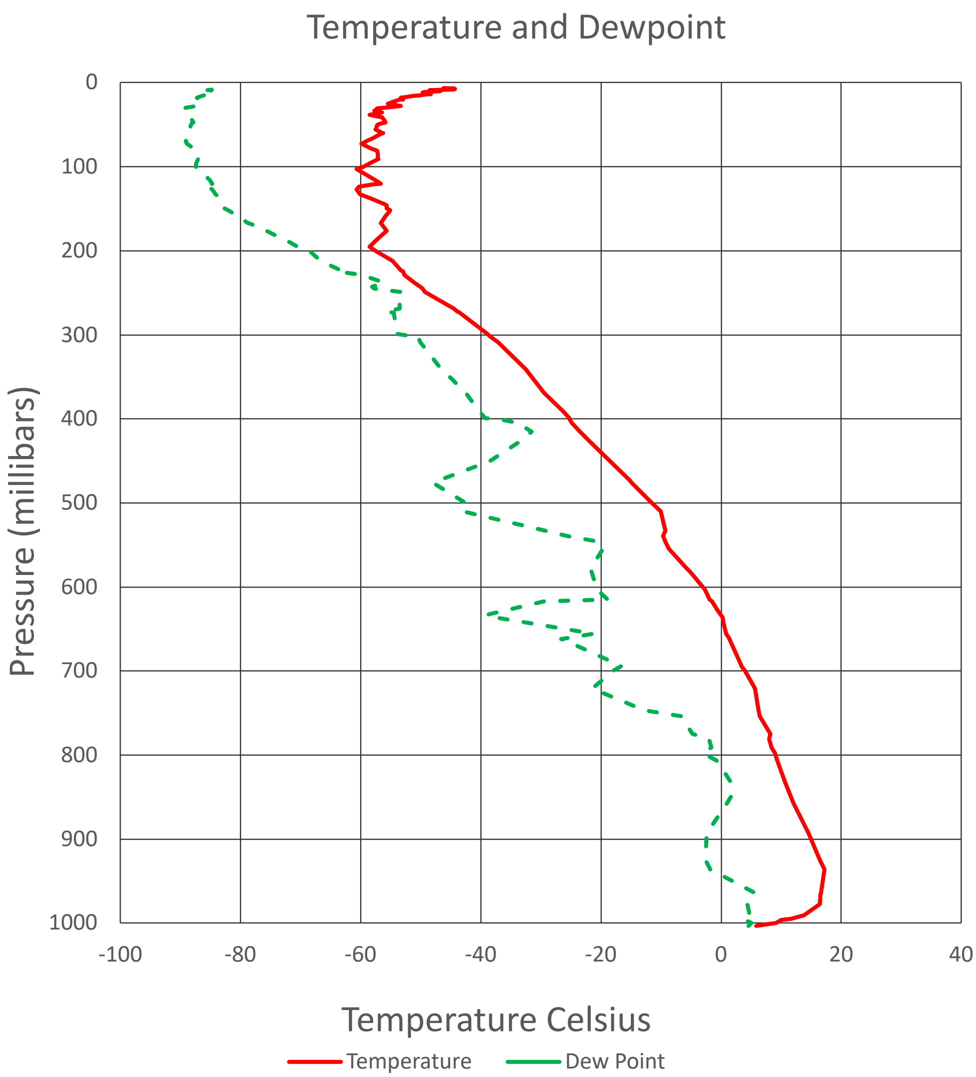


Figure 3.1.10 Graph of Temperature and Dewpoint

Note some charts like Figure 3.1.10 are not meant to convey specific numbers but rather to show trends.

Clouds form when the temperature reaches the dewpoint. We see in Figure 3.1.10 that the temperature is always higher than the dewpoint, so no clouds are expected where this sounding (weather balloon reading) was taken. □

3.1.3 Graphing Discrete Data

When Vasya was hired in 2017 she was paid an annual salary of $62,347.23. Her work has been good, so each year she has received raises of $5000.00.

To represent this data we first need to calculate her salary each year. We calculate this by iteratively adding the $5000 for each year. This is found in Table 3.1.13

We will represent her salary over time using the bar graph in Figure 3.1.14. Notice the horizontal axis is labeled with years and the vertical axis is labeled in dollars. There is one bar for each year, because her salary was changed only once each year.

- Does this graph tell us when Vasya received her raises each year?
- If we made the horizontal axes dollars and the vertical axis years, would this change the information presented? Would it be easier or harder to read?

□

Table 3.1.13 Vasya's Salary

2017	$62,347.23
2018	$67,347.23
2019	$72,347.23
2020	$77,347.23
2021	$82,347.23

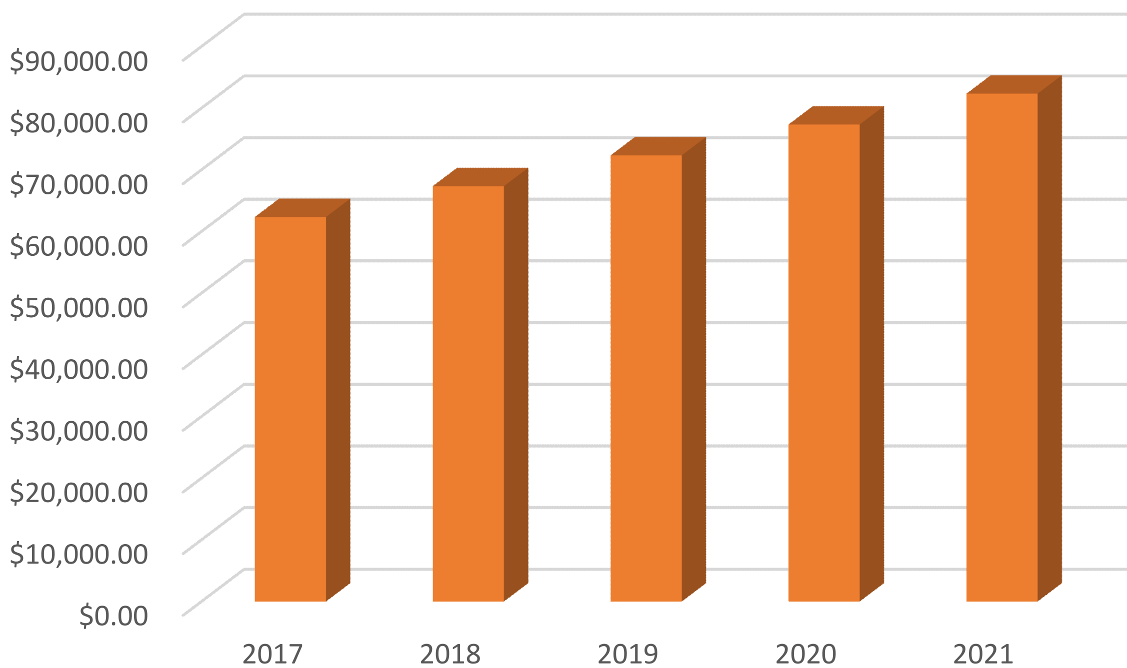


Figure 3.1.14 Vasya's Salary

When Tien was hired he was pair an annual salary of $52,429.33. His work has been good so each year he has received raises. His salaries are found in Table 3.1.16. A graph representing his salaries is found in Figure 3.1.17.

- How frequently did Tien receive raises?
- Can we tell the size of the raises from the graph?

□

Table 3.1.16 Tien's Salary

2017	\$52,429.33
2018	\$55,050.80
2019	\$57,803.34
2020	\$60,693.50
2021	\$63,728.18

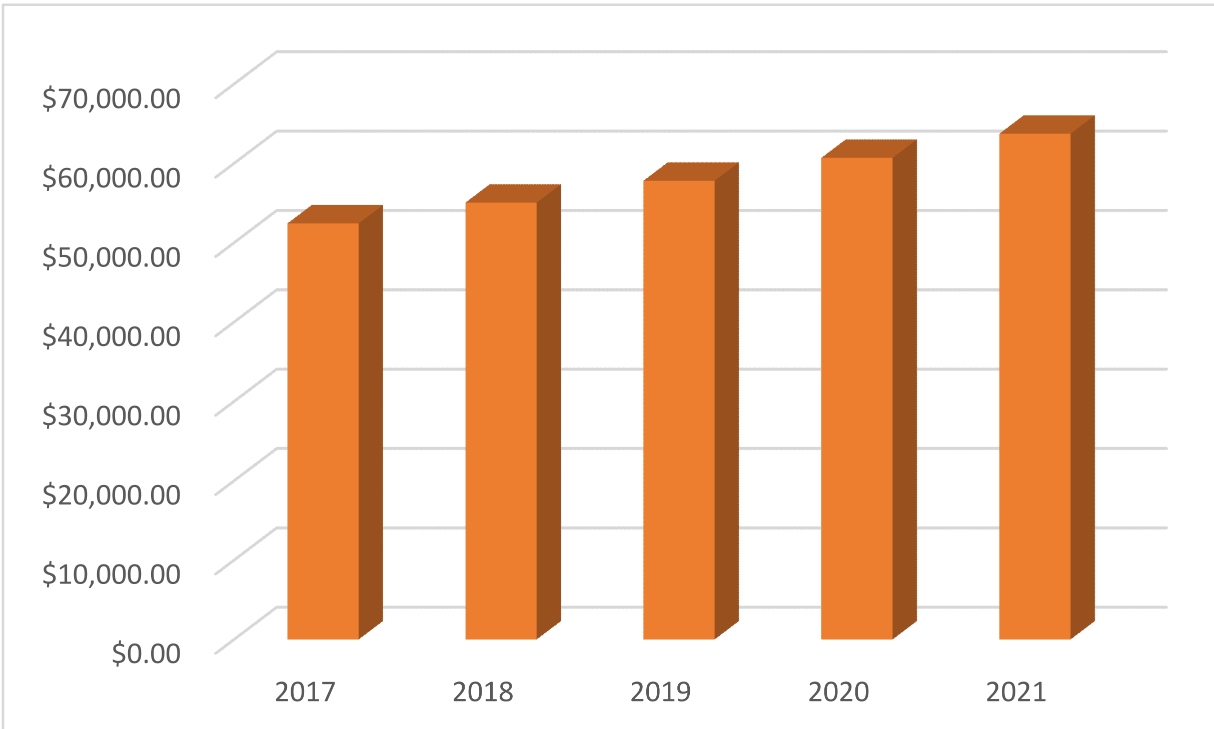


Figure 3.1.17 Tien's Salary

3.1.4 Graphing Continuous Data

Because the salaries changed only once per year we could use a single entry each year. For other data the changes occur constantly.

A model of a space shuttle is 1:144. This means one inch on the model represents 144 inches on the actual shuttle. Similarly 2 inches on the model represents 288 inches on the actual shuttle. Lengths are not just integers, we could have a part that is 1.72 inches. This would be $1.72 \cdot 144 = 247.68$ inches on the actual shuttle.

To represent the conversion from inches on the model to inches on the actual object we need to plot points and connect them continuously. A graph of this in Figure 3.1.19. Notice the two points and the extension of those.

Why does the graph start at 0? □

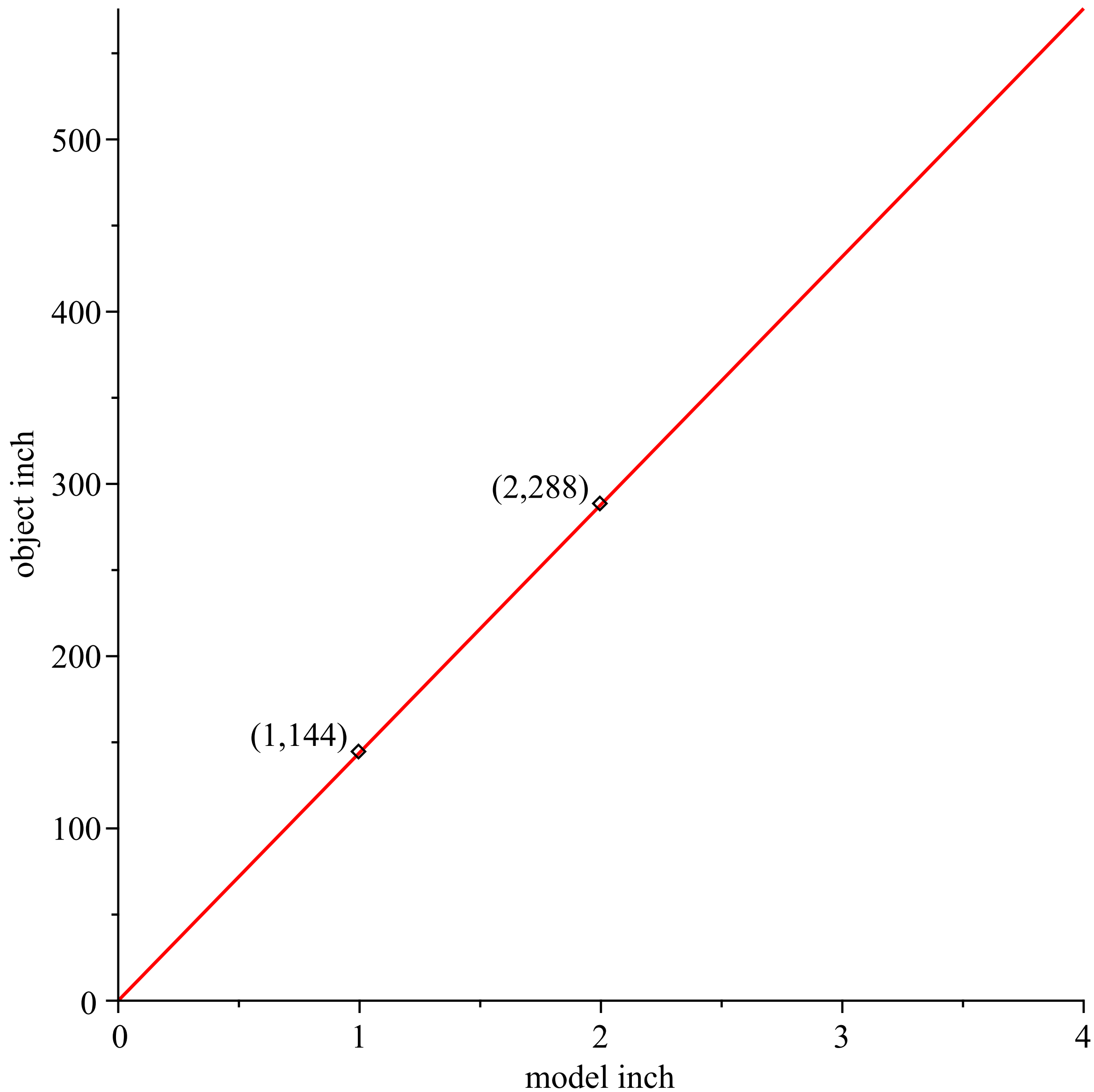


Figure 3.1.19 Graph of Scale

Recall Ohm's Law $V = IR$ from Example 1.3.1.
Consider a 12 V system with an 8 Ohm resistor.

$$12 = I \cdot 8.$$
$$\frac{12}{8} = I.$$
$$1.5 = I.$$

If we reduce the resistance to 4 Ohms we get

$$12 = I \cdot 4.$$
$$\frac{12}{4} = I.$$

$$3 = I.$$

If instead we increase the resistance to 16 Ohms we get

$$\begin{aligned} 12 &= I \cdot 16. \\ \frac{12}{16} &= I. \\ 0.75 &= I. \end{aligned}$$

We can plot these three points and draw a curve through them. This is found in Figure 3.1.21.

- The graph starts with 1 Ohm. Why does it not start at 0?
- As the resistance increases what happens to the current?

□

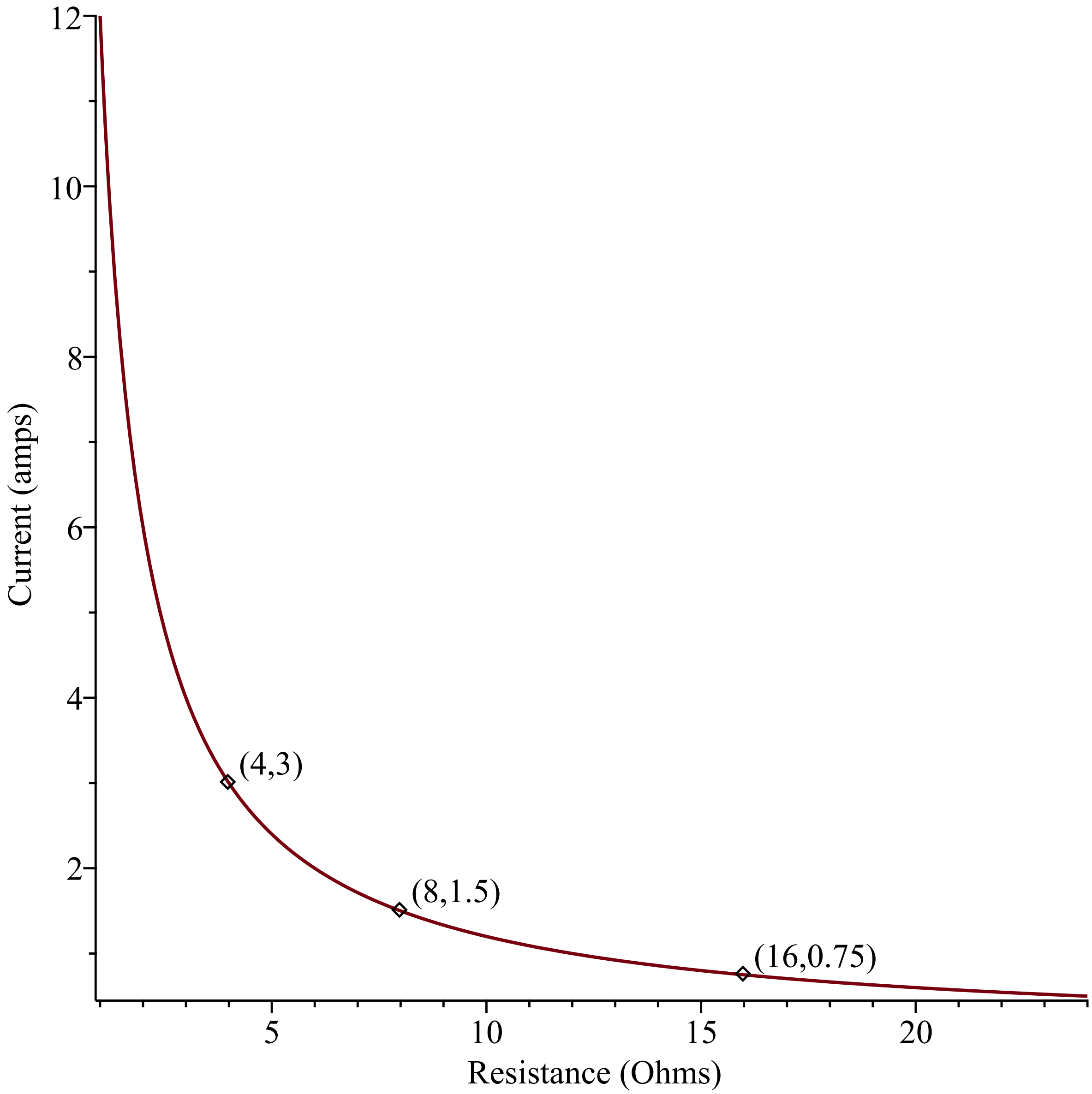


Figure 3.1.21 Graph of Ohms Law

Checkpoint 3.1.22 The ideal gas law expresses a relationship between pressure, volume, and temperature of a gas. It is given by

$$P \cdot V = k \cdot T$$

where P is the pressure, V is the volume, T is the temperature, and k is a constant dependent on the specific gas.

(a) Draw a graph for the equation $P = \frac{8.3145T}{2.0000}$. Note the units are Kelvin (Celsius + 273.15) for temperature and Jules/litre for pressure. These do not need to be labeled here.

(b) Draw a graph for the equation $P = \frac{8.3145 \cdot 293.15}{V}$.

3.1.5 Representing Linear Relations

We have seen what linear data looks like in data tables, discrete graphs (e.g., bar graph), and continuous graphs. Here we will add a representation in mathematical symbols that are useful for calculation.

Remember from Example 3.1.12 that Vasya's annual salary follows a linear pattern. Specifically it increases \$5000.00 per year. That means in over one year it has increased \$5000.00. Over two years it has increased

$$\begin{aligned}\$5000.00 + \$5000.00 = \\ 2 \cdot \$5000.00 = \$10,000.00.\end{aligned}$$

Over three years it has increased

$$\begin{aligned}\$5000.00 + \$5000.00 + \$5000.00 = \\ 3 \cdot \$5000.00 = \$15,000.00.\end{aligned}$$

Generally then we can say that after n years, Vasya's salary has increased $5000n$ dollars. To know her actual salary we need to add her initial salary. An equation for Vasya's salary then is

$$S = \$62,347.23 + \$5000.00n.$$

□

Checkpoint 3.1.24 If Dimitri's salary is given by the equation

$$S = \$73,240.00 + \$6000.00n$$

where n is the year of employement, what was his initial salary? _______
What was the size of his annual raises? _______

Hint. His initial salary is his salary in year 0.

Answer 1. 73240

Answer 2. 6000

In Example 3.1.18 we graphed a line. In terms of the graph of a line, the rate of change is known as the **slope**. Because the rate of change is constant, the slope can be calculated from any two points.

Note a rate is a ratio of changes. In terms of points expressed in Cartesian coordinates the calculation is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

This is read, "change in y divided by change in x."

The graph in Figure 3.1.26 is linear. We will calculate the slope twice.

Solution 1. We will use the points (4000,33.94) and (2000,31.94).

$$\begin{aligned}\frac{33.94 - 31.94}{4000 - 2000} &= \frac{2}{2000} \\ &= \frac{1}{1000}.\end{aligned}$$

This means the slope is 1 inHg (inch of mercury) per 1000 feet above mean sea level.

Solution 2. We will use the points (8000,37.94) and (4000,33.94).

$$\begin{aligned}\frac{37.94 - 33.94}{8000 - 4000} &= \frac{4}{4000} \\ &= \frac{1}{1000}.\end{aligned}$$

As expected this is the same slope, because on a line the rate of change (slope) is constant. □

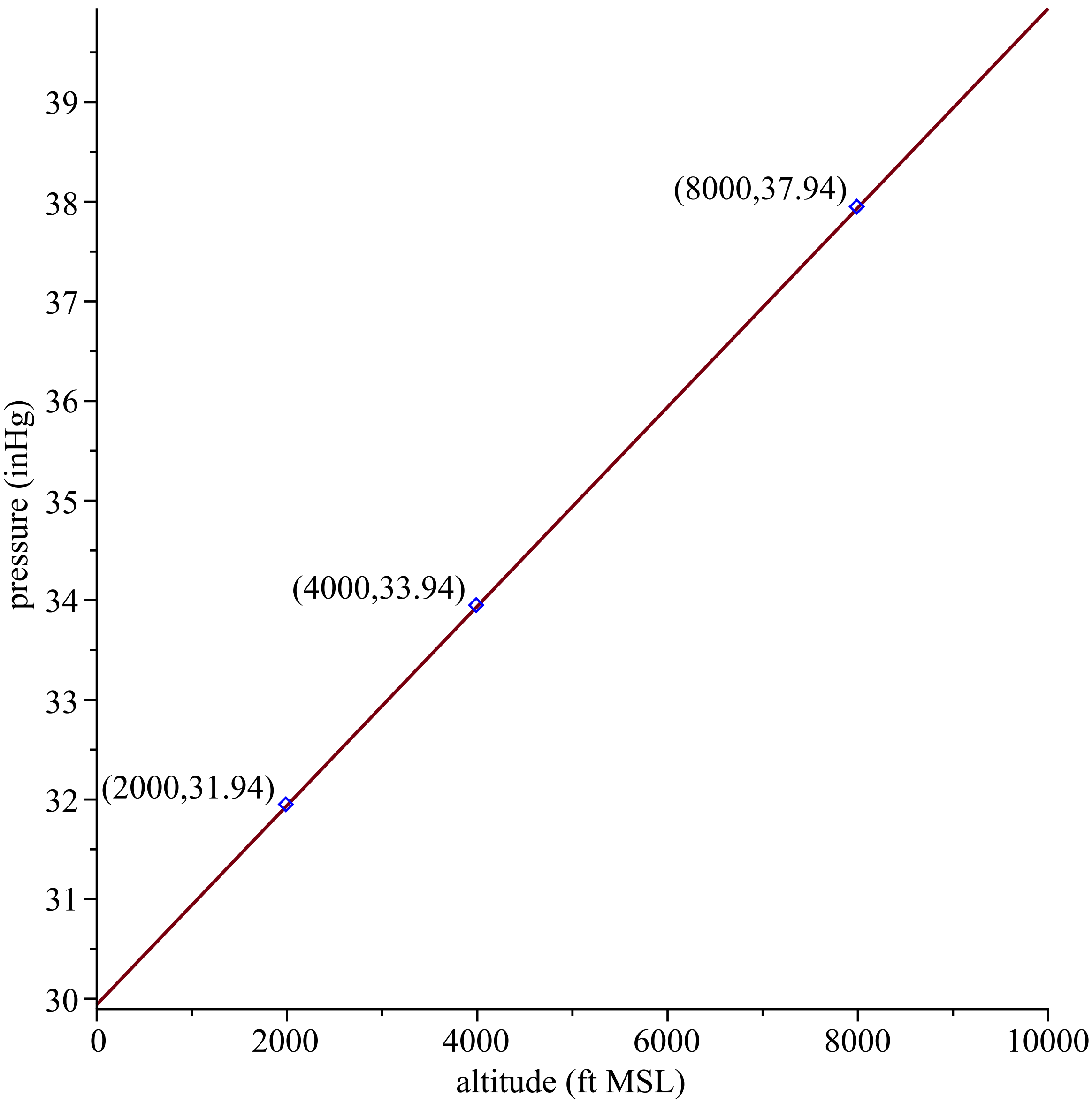


Figure 3.1.26 Calculating Slope

Checkpoint 3.1.27 Suppose that as dry air rose it dropped in temperature in a linear fashion. If the temperature was measured at 1000 ft MSL as 21° C and at 3000 ft MSL as 17° C, what is the rate of change of temperature with respect to altitude? ____

What does this imply the temperature is at 0 ft MSL? ____

Answer 1. $\frac{-1}{500}$

Answer 2. 23

Solution. The slope can be calculated as

$$\frac{17-21}{3000-1000} = \frac{-4}{2000} = -\frac{2}{1000}.$$

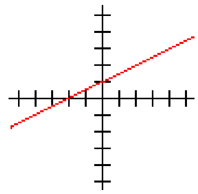
Alternately because the temperature drops as the air rises, the temperature at a lower altitude (0 is lower than 1000) will be higher by 2 degrees. So the temperature is $21+2=23$.

3.1.6 Exercises

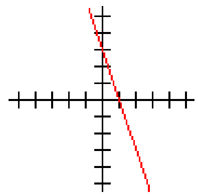
Exercise Group. Answer these questions about interpreting data.

1. **Determine Linear.** Identify what each equation below represents?

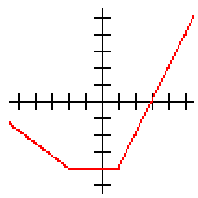
(a)



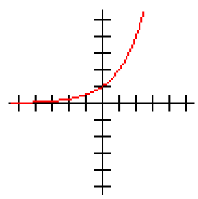
(b)



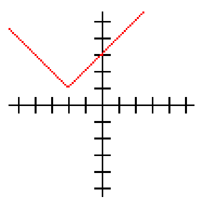
(c)



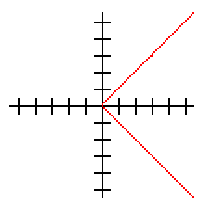
(d)



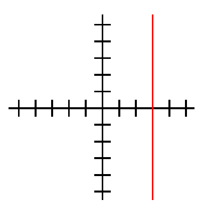
(e)



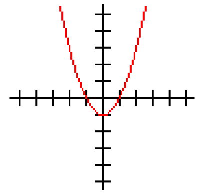
(f)



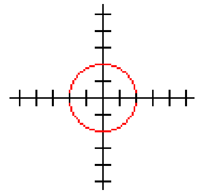
(g)



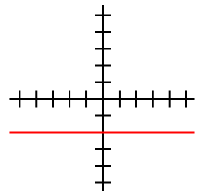
(h)



(i)



(j)



(a) Linear function

(b) Nonlinear function

(c) Not a function

2. **Graph and Table.** Determine if the following table represents a linear relation.

Table 3.1.28

x	0	1	2	3	4	5
y	0.3	-0.9	-2.1	-3.3	-4.5	-5.7

(a) Yes

(b) No

3. **Graph and Table.** Determine if the following table represents a linear relation.

Table 3.1.29

x	0	1	2	3	4	5
y	2.6	4.3	9.4	17.9	29.8	45.1

(a) Yes

(b) No

4. **Graph and Table.** Determine if the following table represents a linear relation.

Table 3.1.30

x	0	1	2	3	4	5
y	2.4	4.4	10.4	28.4	82.4	244.4

(a) Yes

(b) No

5. **Graph and Table.** Determine if the following table represents a linear relation.

Table 3.1.31

x	1	2	3	4	5	6
y	900	225	100	56.25	36	25

(a) Yes

(b) No

6. Graph and Table. Calculator[1]

Examine the linear function below.

$y = 3x - 5$

Which table represents the same function?

(a) **Table 3.1.32**

x	y
-2	-11
0	-5
2	1
4	7

(b) **Table 3.1.33**

x	y
-4	-7
-2	-1
0	5
2	11

(c) **Table 3.1.34**

x	y
0	-5
5	25
10	55
15	85

(d) **Table 3.1.35**

x	y
0	3
3	12
6	21
9	30

7. Compare Linear Functions. Put the the people in order from lowest pay rate to greatest pay rate.

(Note variables: x = time in hours, y = dollars earned)

- *Person A*$y = 12.5x$
- *Person B*Draeden earned \$42 after 4 hours of work.
- *Person C*

[1] #

Table 3.1.36

Time (hours)	Total (dollars)
0.5	6.75
2	27
6	81

- *Person D*

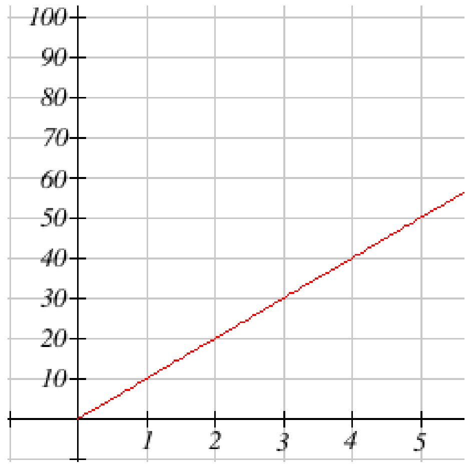


Figure 3.1.37

Lowest to Greatest

(a) A

(b) B

(c) C

(d) D

(a) A

(b) B

(c) C

(d) D

(a) A

(b) B

(c) C

(d) D

(a) A

(b) B

(c) C

(d) D

8. Interpret Graph. Becky takes out a 30-year mortagage for which her monthly payment is \$1100. During the early years of the mortgage, most of each payment is for interest and the rather small remainder for principal. As time goes on, the portion of each payment that goes for interest decreases while the portion for principal increases, as shown in the following graph:

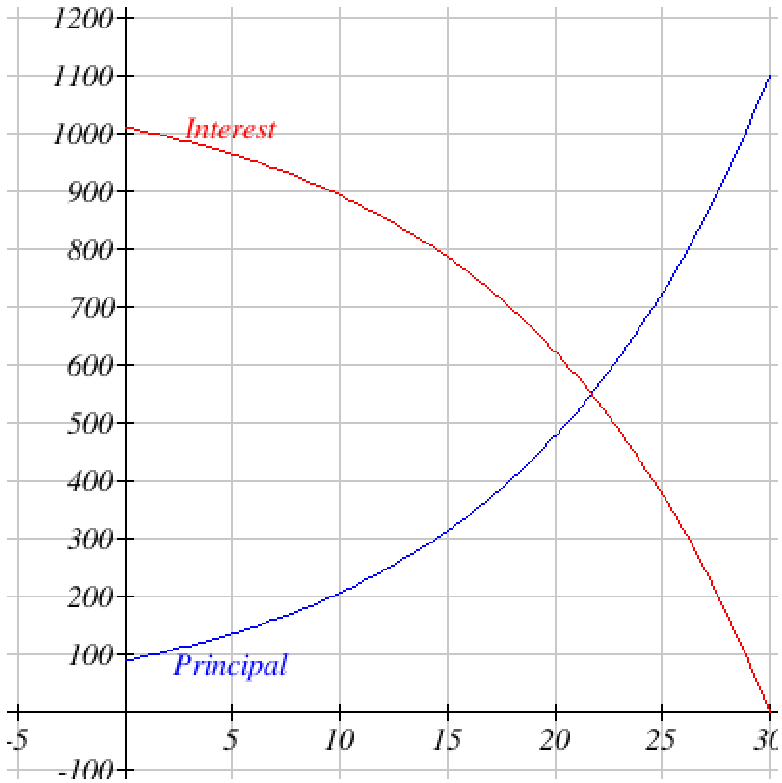


Figure 3.1.38

a) Approximately how much of the \$1100 monthly payment goes for interest in year 15?
\$____________________

b) In what year will the monthly payment be equally divided between interest and principal?
Year ____________________

9. Interpret Graph. Our company's new widget has been growing in sales. The histogram below shows sales in millions for the years shown.

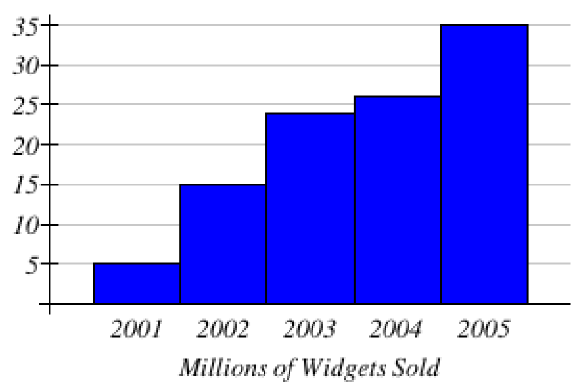


Figure 3.1.39

Approximately how many millions of widgets were sold in year 2001?____________________

10. Interpret Graph. Our company's new widget has been growing in sales. The histogram below shows sales in millions for the years shown.

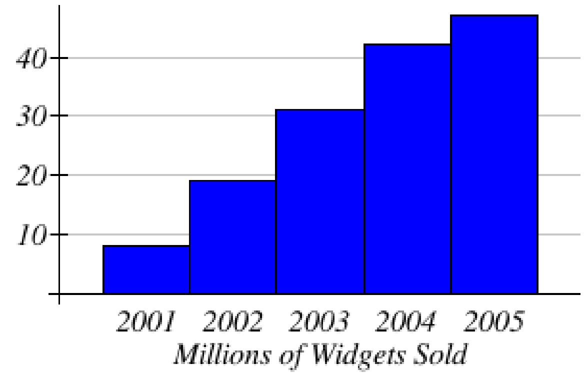


Figure 3.1.40

In which year did sales first exceed 36 million widgets?____________________

11. Interpret Graph. An animal shelter once tracked the length of the whiskers of three cats every week starting from the time they received the cats. The three graphs are shown below.

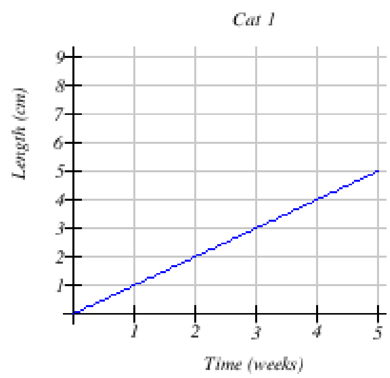


Figure 3.1.41

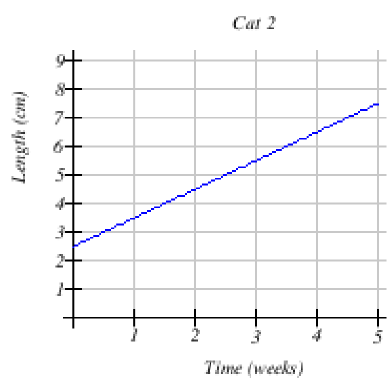


Figure 3.1.42

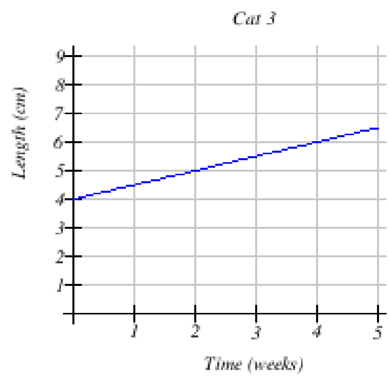


Figure 3.1.43

Which cat was born in the animal shelter?

(a) Cat 1

(b) Cat 2

(c) Cat 3

At what rate are the whiskers of Cat 1 growing? ____

(a) cm per week

(b) weeks per cm

(c) cm

(d) weeks

Which cat's whiskers are growing more slowly than the other two?

(a) Cat 1

(b) Cat 2

(c) Cat 3

How long were the whiskers of Cat 2 when it was received by the animal shelter? ____Preview Question 1 Part 5 of 6

(a) cm per week

(b) cm

(c) weeks per cm

(d) weeks

12. Interpret Graph. In the graph below the input x is the number of units produced by a machine in a factory. The output y is the profit made by the sale of these units when they are produced. Determine the intercepts and interpret the meaning of each.

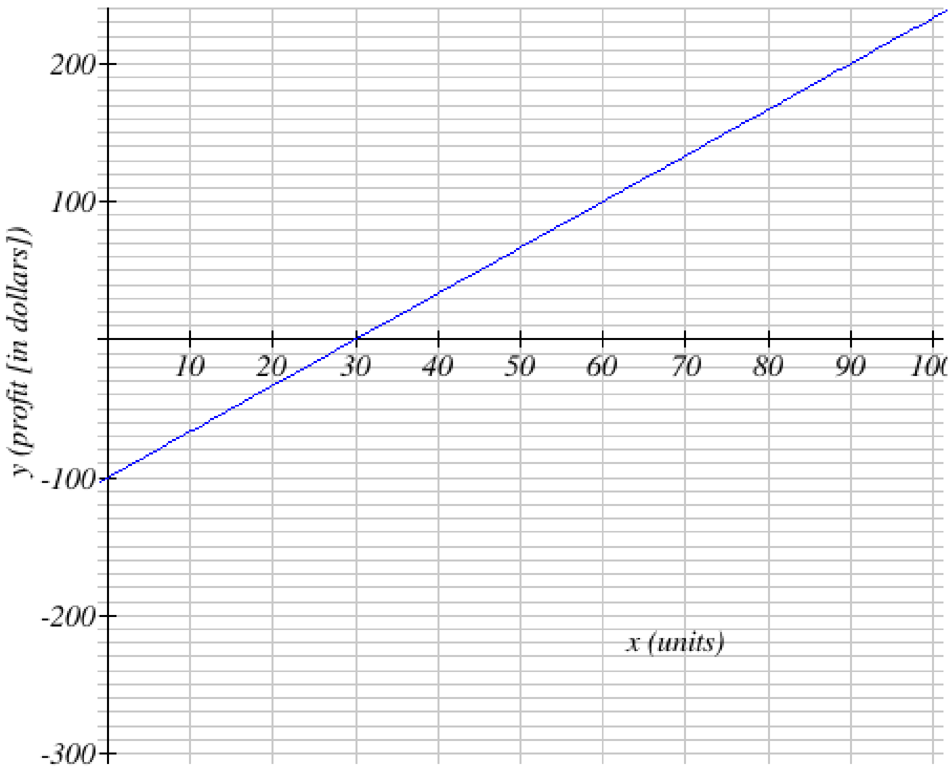


Figure 3.1.44

A. Give the coordinates of the x-intercept: ____________________
B. Interpret the meaning of the x-intercept:

(a) The profit is zero when -100 units are produced and sold.

(b) There are zero units produced and sold when the profit is $30.

(c) The profit is $30 when -100 display units are produced and sold.

(d) There are zero units produced and sold when the profit is $-100

(e) The profit is zero when 30 units are produced and sold.

(f) The profit is $-100 when 30 display units are produced and sold

C. Give the coordinates of the y-intercept: ____________________
D. Interpret the meaning of the y-intercept:

(a) There are zero units produced and sold when the profit is $30.

(b) The profit is zero when 30 units are produced and sold.

(c) The profit is zero when -100 units are produced and sold.

(d) The profit is $-100 when zero units are produced.

(e) The profit is $30 when -100 display units are produced and sold.

(f) The profit is $-100 when 30 display units are produced and sold

E. Determine the slope of the line: ____________________Preview Question 1 Part 5 of 9
[Make sure your slope is written as a reduced fraction.]
F. Interpret the meaning of the slope. For each 7-unit increase in units sold there is a(n)

(a) decrease

(b) increase

in profit of \$__________________. Round your answer to the nearest cent. *G.* Write the equation of the line in $y = mx + b$ form: __________________Preview Question 1 Part 8 of 9

H. Use your equation to determine what quantity of units will yield a profit of \$60: __________________

13. Interpret Graph. The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?

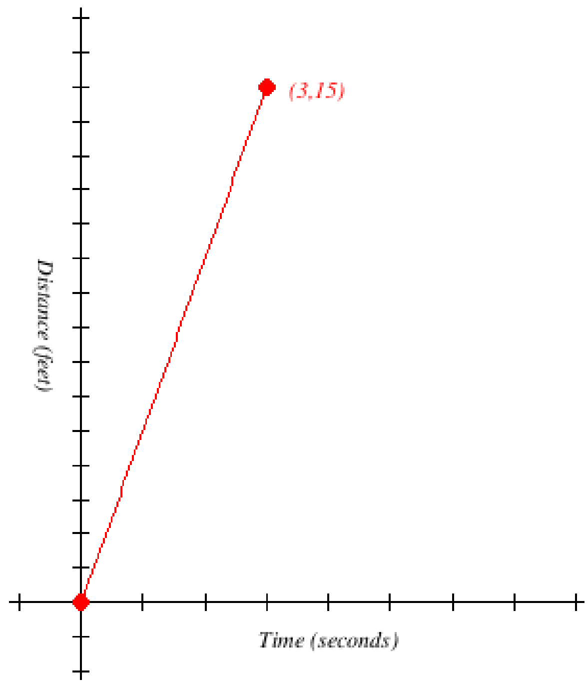


Figure 3.1.45

(a) The cat was 15 feet away from the milk and ran away from it at a rate of 3 feet per second.

(b) The cat was 15 feet away from the milk and ran toward it reaching it after 3 seconds.

(c) The cat was 3 feet away from the milk and ran toward it reaching it after 15 seconds.

(d) The cat ran away from the milk at a rate of 3 feet per second.

(e) The cat ran away from the milk at a rate of 5 feet per second.

14. Interpret Graph. The graph below shows the value of Bob's Beanie Baby collection over several years.

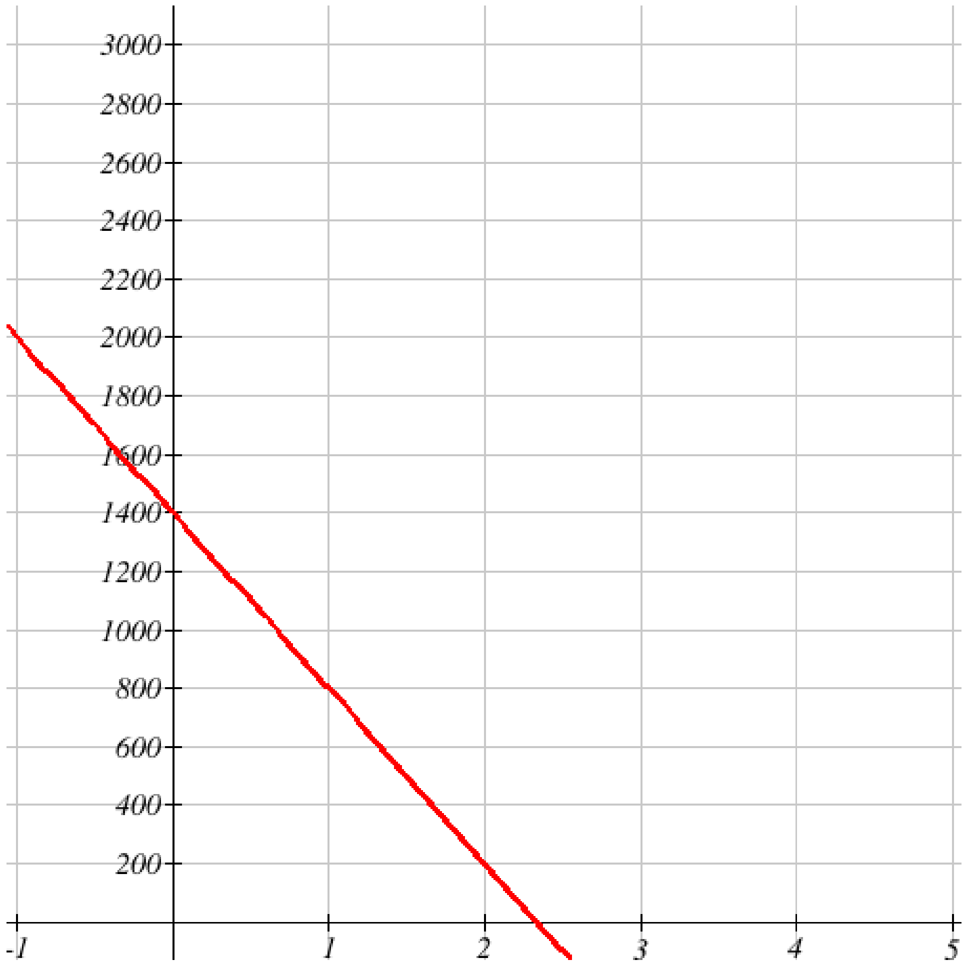


Figure 3.1.46

The collection's value decreased at a rate of ____________________...

(a) dollars per year

(b) Beanie Babies per dollar

(c) years per dollar

(d) dollars per Beanie Baby

15. Interpret Graph. The graph shows the velocity of a ball that is thrown upwards and remains in the air for 4.69 seconds.

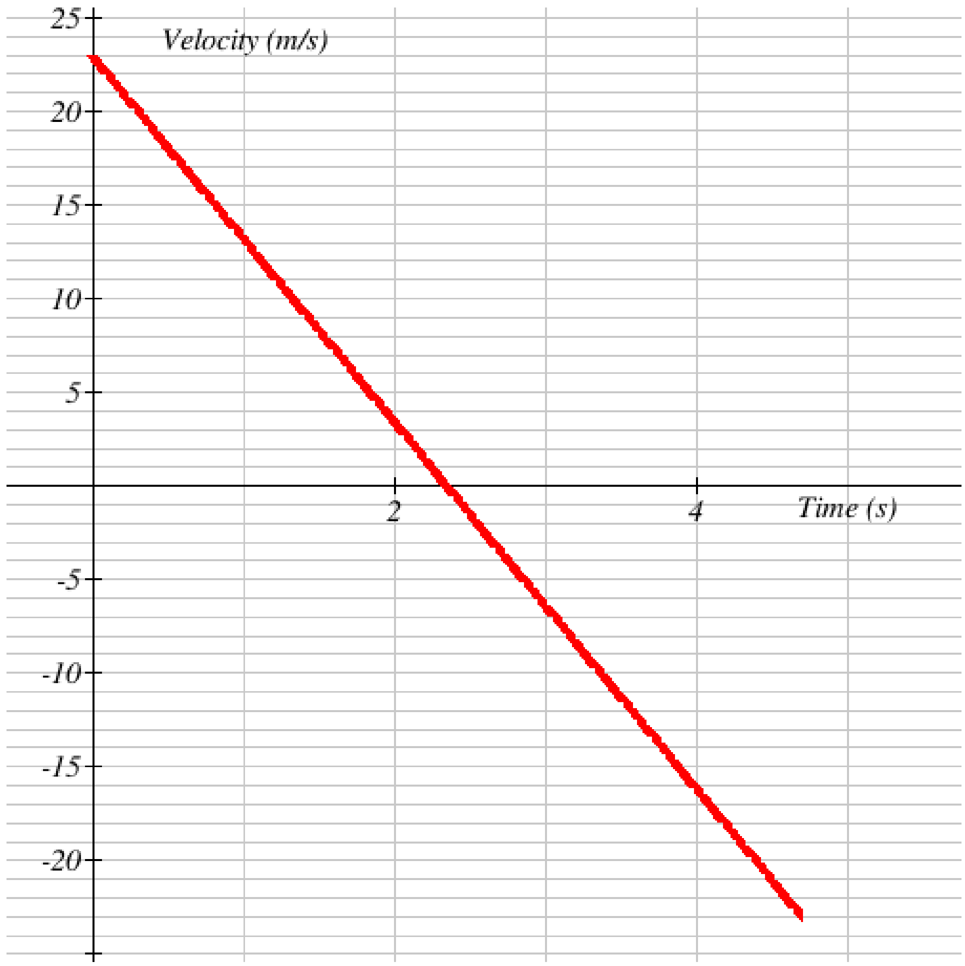


Figure 3.1.47

What is the final velocity of the ball?

v_f = ____________________ unit____________________

At approximately what time does the ball change direction?

t_f = ____________________ unit____________________

16. Interpret Graph. The graph shows the distance traveled by a vehicle over time. Describe the change in distance.

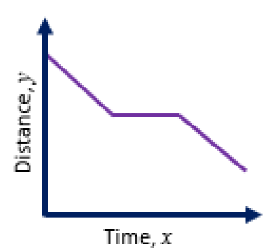


Figure 3.1.48

(a) First the distance *increases* at a constant rate. Then the distance *decreases* at a constant rate before staying the *same* for a while.

(b) First the distance *decreases* at a constant rate. Then the distance *increases* at a constant rate before staying the *same* for a while.

(c) First the distance *decreases* at a constant rate. Then the distance *stays the same* for a while before *decreasing* at a constant rate.

(d) First the distance *increases* at a constant rate. Then the distance *stays the same* for a while before *decreasing* at a constant rate.

17. Interpret Graph. The park service records the number of reported rattle snake sightings on Bald Mountain Trail (light blue) and Camel Back Trail (gold). The data gives rise to the following stacked bar graph.

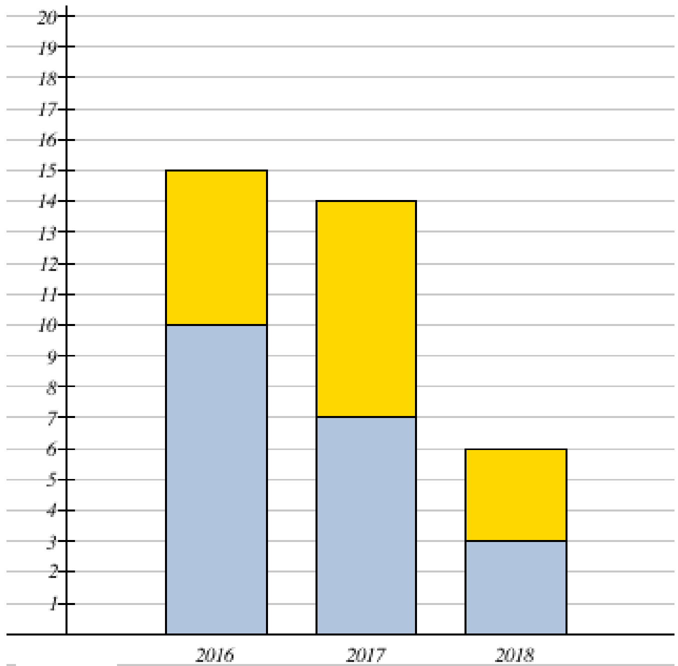


Figure 3.1.49

(a) How many rattle snake sightings were there on Bald Mountain Trail in 2016? _____
(b) How many rattle snake sightings were there on Camel Back Trail in 2016? _____
(c) How many total rattle snake sightings were there in 2017? _____
(d) What percentage of rattlesnake sightings in 2018 were from Bald Mountain trail? _____%
Report your answer to at least two decimal places.

18. Interpret Graph. In 1994, the moose population in a park was measured to be 3280. By 1999, the population was measured again to be 4530. If the population continues to change linearly:
Find a formula for the moose population, P, in terms of t, the years since 1990.
$P(t) =$ ____________________Preview Question 1 Part 1 of 2
What does your model predict the moose population to be in 2006?
____________________Preview Question 1 Part 2 of 2

19. Determine if Linear. Select all of the following tables which could represent a linear function.

Table 3.1.50

(a)

x	$f(x)$
0	-4
5	11
10	26
15	41

Table 3.1.51

x	$g(x)$
0	6
5	-19
10	-44
15	-69

(b)

Table 3.1.52

x	$h(x)$
0	6
5	31
10	106
15	231

(c)

Table 3.1.53

x	$k(x)$
5	24
10	44
20	84
25	104

(d)

20. Find Equation. Is the function $k(x) = 5x - 6$ a linear function?

(a) Cannot Be Determined

(b) Yes

(c) No

21. Interpret Graph. Below, one description, one graph, and one equation are equivalent. Choose the proper set.

(a) You start with 3 Xbox games and each month you buy 3 new games.

(b) You start with 2 Xbox games and each month you buy 4 new games.

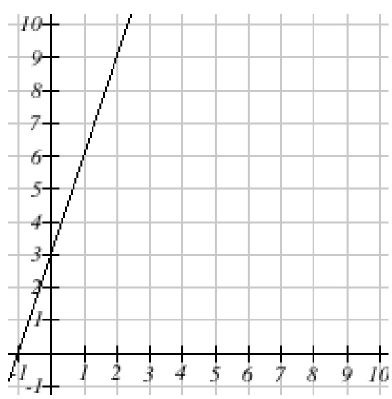


Figure 3.1.54

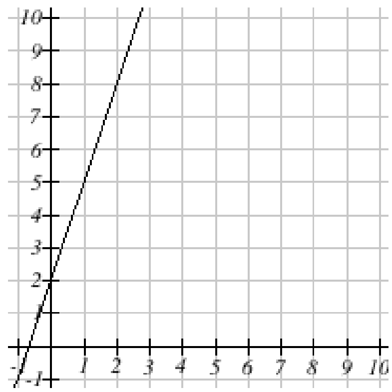


Figure 3.1.55

(a) $y = 3x + 3$

(b) $y = 4x + 2$

Exercise Group. Answer these questions about lines.

22. Rate from Data. Superman needs to save Lois from the clutches of Lex Luthor. It takes Superman 15 seconds to get to Lois who is 855 feet away. What is Superman's rate?

_____

(a) s/ft or seconds per foot

(b) ft/s or feet per second

23. Rate from Data. You are on an oceanographic research expedition that began in San Juan, Puerto Rico on September 14.

The ship left port at 0630 hr on 14 September and covered a distance of 1679 km to the first drill location (Site 1) where you are going retrieve a drill core of seafloor sediments. The ship arrived at the first drill site at 1800 hr on 16 September.

Calculate the rate of travel (i.e., speed) of the ship during its transit to the first drill site. *Round your answer to the nearest tenth.*

_____Preview Question 1 km/hr

24. Find Slope from Points. Find the slope of the line that goes through the points (8,-8) and (9,7).

Slope, $m =$ ____________________Preview Question 1

Enter your answer as an integer or a reduced fraction in the form A/B

25. Find Slope from Graph.

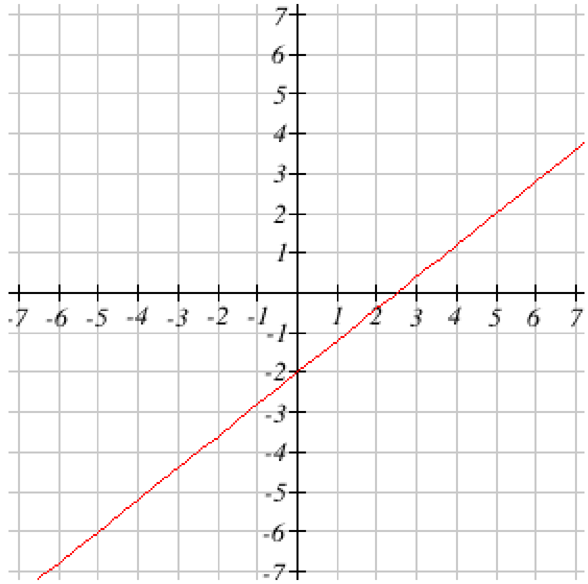


Figure 3.1.56

Find the slope of the line.

Slope $= m =$ __________Preview Question 1

Enter your answer as an integer or as a reduced fraction in the form A/B.

26. Population. A city's population in the year x =1986 was y =3,377,000. In 2000 the population was 3,380,500.

Compute the slope of the population growth (or decline) and choose the most accurate statement from the following:

(a) The population is increasing at a rate of 450 people per year.

(b) The population is decreasing at a rate of 450 people per year.

(c) The population is increasing at a rate of 50 people per year.

(d) The population is increasing at a rate of 250 people per year.

(e) The population is decreasing at a rate of 250 people per year.

(f) The population is decreasing at a rate of 50 people per year.

27. Identify Slope.

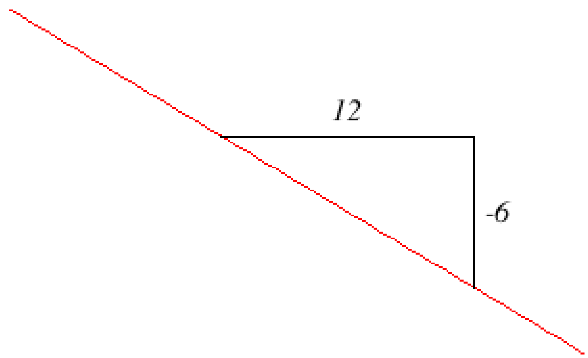


Figure 3.1.57

State the run, rise, and slope of the line above.
run = _
rise = _
m = _

28. Equation from Table. Given the table of Celsius and corresponding Fahrenheit temperatures, find the linear relationship where Celsius, c, is the input and Fahrenheit, f, is the output.

Table 3.1.58

Celsius	Fahrenheit
-10	14
0	32
10	50
20	68
30	86
40	104
50	122
60	140

____________________Preview Question 1

29. Equation from Table. Find the constant rate of change described from the table.

Table 3.1.59

hours	dollars
7	102
12	82
17	62
22	42

Rate/Slope: _____

(a) hours

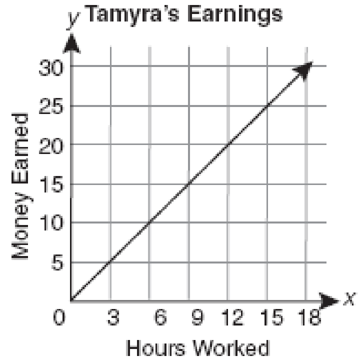
(b) hours per dollar

(c) dollars

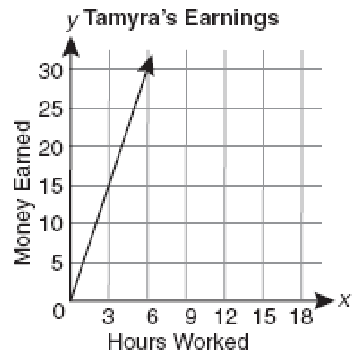
(d) dollars per hour

30. Find Slope from Graph. Tamrya is babysitting to earn money to visit her aunt. She earns \$3.00 for each hour of babysitting. Which graph represents her earnings from babysitting?

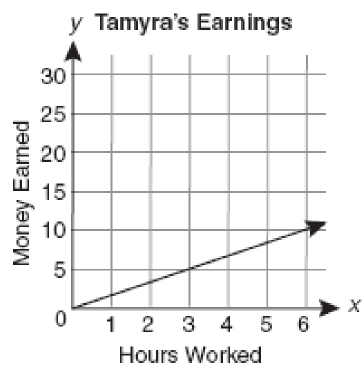
(a)



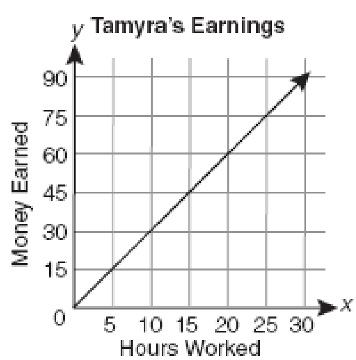
(b)



(c)



(d)



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31. Tuition. The graph below shows the total cost of attending a particular college based on the number of classes taken. (x = number of classes, y = total cost)

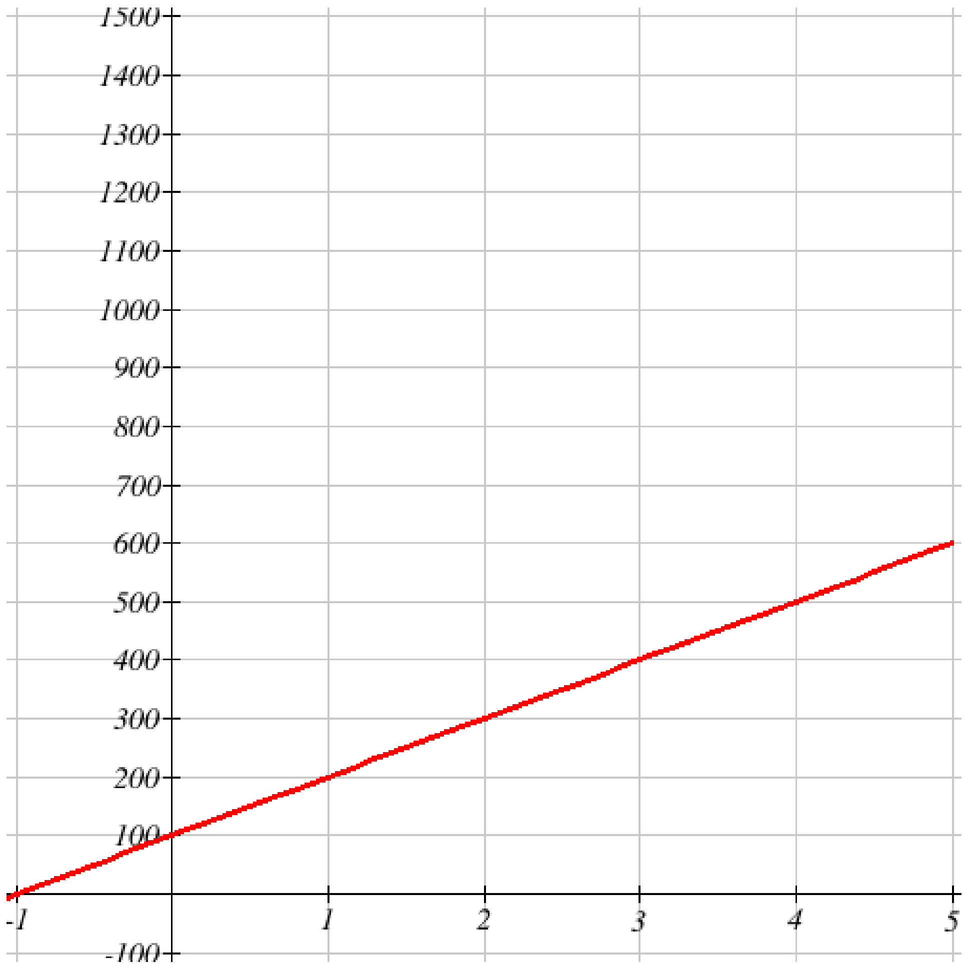

Figure 3.1.60

The slope of this line tells us that the cost of attending this college increases by ________________

(a) dollars

(b) dollars per year

(c) classes

(d) dollars per class

Use the graph to find the cost of attending the college if 2 classes are taken.
$ ____________________

32. Check if Linear. Identify the rate of change and initial value for the linear situation modeled below.

Table 3.1.61

x	y
0	8
2	20
4	32
6	44
8	56

Rate of change: _______
Initial Value: _______

33. Check if Linear. Identify the rate of change and initial value for the linear situation modeled below.

$y = 3x + 7$
Rate of change: _______
Initial Value: _______

34. Write Equation. Find the equation (in terms of x) of the line through the points (-5,-2) and (1,2)

$y =$ ____________________Preview Question 1

35. Write Eqation. Write the equation in slope-intercept form of the line that has slope -3 and y-intercept $(0, 5)$.

$y =$ ____________________Preview Question 1

36. Write Eqation. Suppose you start with a full tank of gas (15 gallons) in your truck. After driving 3 hours, you now have 8 gallons left.

If x is the number of hours you have been driving, then y is the number of gallons left in the tank.

At what rate is the gas left in the tank changing? State your answer as a reduced fraction.

_ gallons per hour

Find an equation of a line in the form y = mx + b that describes the amount of gas in your tank.

$y =$ _____Preview Question 1 Part 2 of 2

37. Graph Line.

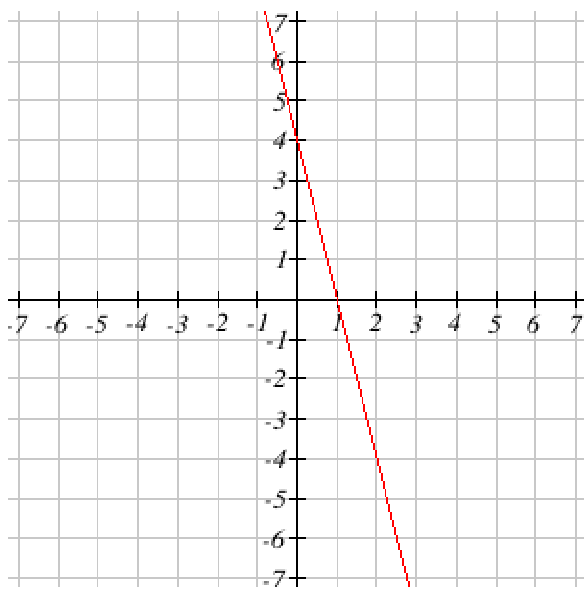

Figure 3.1.62

What is the slope of the graph? Leave your answer in simplest form.
Slope = ___Preview Question 1 Part 1 of 3
Identify the y-intercept.
y-intercept = (0, __)
Write an equation in slope-intercept form for the graph above.
$y =$ ______Preview Question 1 Part 3 of 3

3.2 Identifying Rates

A very important mathematical concept is rates. Often the rate at which something is happening is more important than the current scale or other measures. Here we will learn to identify rates from data.

3.2.1 Differences

One way to measure rates is to look at the differences between data point.

One of the purposes of graphing is to compare how data is changing. Here we will compare the way the raises for Vasya and Tien were calculated.

We know that Vasya's salary rose by \$5000 each year. We need to calculate how much Tien's salary was raised each year. These calculations are in Table 3.2.2.

Notice that Tien's raises were not the same amount each year. How did the raises change? Vasya's raises were the same each year, so her salary changed by a constant amount. When the rate of change is constant we call the pattern **linear**. We will learn other patterns as well. □

Table 3.2.2 Tien's Raises

2018	\$55,050.80-\$52,429.33	=\$2,621.47
2019	\$57,803.34-\$55,050.80	=\$2,752.54
2020	\$60,693.50-\$57,803.34	=\$2,890.17
2021	\$63,728.18-\$60,693.50	=\$3,034.68

Definition 3.2.3 Linear Relation. A relation is **linear** if and only if the rate of change is constant. ◊

Note we used this constant addition property when working with ratio problems like Example 2.3.3. Relations defined by fixed ratios like these are linear.

Checkpoint 3.2.4 Which of these employees' raises were linear?

Moses	Freya	Jamal
\$57,233.00	\$61,199.00	\$58,769.00
\$61,239.31	\$63,499.00	\$61,924.00
\$65,526.06	\$65,799.00	\$65,079.00
\$70,112.89	\$68,099.00	\$68,234.00
\$75,020.79	\$70,399.00	\$71,389.00
\$80,272.24	\$72,699.00	\$74,544.00

Hint. Subtract consecutive pairs of salaries. If all the differences are the same, then that person's raises were linear.

For linear data we noted that consecutive differences are always the same. In Example 3.2.5 to Example 3.2.8 we will see known data and how the differences look.

Note in Table 3.2.6 that the first differences are not the same. However, they increase in a suspiciously simple pattern. Checking the second differences (the differences of the 1st differences) we see a linear pattern. This turns out to be true for all quadratic data. □

Table 3.2.6 Quadratic Data

n	n^2	1st difference	2nd difference
1	1		
2	4	4-1=3	
3	9	9-4=5	5-3=2
4	16	16-9=7	7-5=2
5	25	25-16=9	9-7=2
6	36	36-25=11	11-9=2

Definition 3.2.7 Quadratic Relation. A relation is **quadratic** if and only if the second differences are constant. ◊

In Table 3.2.9 the differences are not the same nor do they show the pattern of quadratics. However, there is a pattern in the differences. Notice that the differences are exactly equal to the original data. This measn the rate is determined by the current scale. This is the pattern of data that varies exponentially. □

Table 3.2.9 Exponential Data

n	2^n	Difference
1	2	
2	4	4-2=2
3	8	8-4=4
4	16	16-8=8
5	32	32-16=16
6	64	64-32=32

Note that the differences for exponential data are not always exactly equal to the data.

In Table 3.2.11 note that the differences are not exactly equal to the original numbers. However, note that $6 = 2 \cdot 3$ $18 = 2 \cdot 9$, and $54 = 2 \cdot 27$. The differences are double the original numbers. In general for exponential data the differences will be the original data scaled by some number. □

Table 3.2.11 Exponential Data with Scale

n	3^n	Difference
1	3	
2	9	9-3=6
3	27	27-9=18
4	81	81-27=54
5	243	243-81=162
6	729	729-243=486

Definition 3.2.12 Exponential Relation. A relation is **exponential** if and only if the differences are a multiple of the original values, that is the rate is proportional to the value. ◊

3.2.2 Quotients

Rather than thinking about the change as the difference (subtraction) of consecutive numbers (salaries in these examples), we can consider the percent increase for each pair of consecutive numbers.

We will first calculate the percent increase of salary each year for Tien and Vasya. Because salary numbers are exact, we will not use significant digits. Rather we will round to the nearest percent. This is in Table 3.2.14 and Table 3.2.15.

Notice that for Tien the percent increase is the same each year. It is 5%. For Vasya, the percent increase is not the same each year. How does the percent increase change for her? □

Table 3.2.14 Percent Increase for Tien

2018	\$55,050.80/\$52,429.33	=1.05
2019	\$57,803.34/\$55,050.80	=1.05
2020	\$60,693.50/\$57,803.34	=1.05
2021	\$63,728.18/\$60,693.50	=1.05

Table 3.2.15 Percent Increase for Vasya

2018	\$67,347.23/\$62,347.23	=1.08
2019	\$72,347.23/\$67,347.23	=1.07
2020	\$77,347.23/\$72,347.23	=1.07
2021	\$82,347.23/\$77,347.23	=1.06

Definition 3.2.16 Exponential. A relation is **exponential** if and only if the percent increase is constant. ◊

Althought Definition 3.2.12 and Definition 3.2.16 are phrased differently they both accurately describe exponential relations. Generally it is easier to test if data is exponential by testing the ratios of terms rather than the differences. Table 3.2.17 shows an example of both.

Table 3.2.17 Exponential Data 2 Ways

n	$5\left(\frac{4}{3}\right)^n$	Difference	Ratio
1	$\frac{20}{3}$		
2	$\frac{80}{9}$	$\frac{20}{9}$	$\frac{4}{3}$
3	$\frac{320}{27}$	$\frac{80}{27}$	$\frac{4}{3}$
4	$\frac{1280}{81}$	$\frac{320}{81}$	$\frac{4}{3}$
5	$\frac{5120}{243}$	$\frac{1280}{243}$	$\frac{4}{3}$
6	$\frac{20480}{729}$	$\frac{5120}{729}$	$\frac{4}{3}$

3.2.3 Exercises

1. Determine Rate. For each table below, could the table represent a function that is linear, exponential, or neither?

Table 3.2.18

x	1	2	3	4
$f(x)$	80	64	51.2	40.96

f(x) is

(a) Exponential

(b) Linear

(c) Neither

Table 3.2.19

x	1	2	3	4
$g(x)$	60	24	-8.8	-39.04

g(x) is

(a) Exponential

(b) Linear

(c) Neither

Table 3.2.20

x	1	2	3	4
$h(x)$	80	60	40	20

h(x) is

(a) Exponential

(b) Linear

(c) Neither

2. **Check if Linear.** Is the following equation linear and consequently produce a graph that is a straight line?

$y = x^2 + 4x + 13$

(a) Yes

(b) No

3. **Check if Linear.** Is the following equation linear and consequently produce a graph that is a straight line?

$3x + 11y = 20$

(a) Yes

(b) No

4. **Check if Linear.** Is the following equation linear and consequently produce a graph that is a straight line?

$20x = 21 - 14y$

(a) Yes

(b) No

5. **Check if Linear.** Is the following equation linear and consequently produce a graph that is a straight line?

$2y = \dfrac{14}{x - 17}$

(a) Yes

(b) No

6. **Find Slope for Linear.** Determine Linearity and Slope from a Table

For each of the following functions, determine if the function is linear.

If it is linear, give the slope. If it is not linear, enter "DNE" for the slope.

Table 3.2.21

x	5	6	7	12	18
$f(x)$	-45	-52	-59	-94	-136

Behavior:

(a) Not Linear

(b) Linear

Slope: ____________________

Table 3.2.22

x	1	2	6	10	15
$f(x)$	3.5	10	36	62	94.5

Behavior:

(a) Linear

(b) Not Linear

Slope: ____________________

Table 3.2.23

x	2	9	10	14	18
$f(x)$	-17	-41.5	-45	-57.5	-73

Behavior:

(a) Not Linear

(b) Linear

Slope: ____________________

Table 3.2.24

x	3	4	5	14	20
$f(x)$	40	43	54	117	159

Behavior:

(a) Not Linear

(b) Linear

Slope: ____________________

7. Determine Rate. One of the patterns in these tables is a quadratic relationship. Which table is quadratic?

(a) **Table 3.2.25**

x	y
0	4
1	13
2	22
3	31
4	40
5	49

(b) **Table 3.2.26**

x	y
0	4
1	5
2	7
3	10
4	14
5	19

(c) **Table 3.2.27**

x	y
0	7
1	21
2	63
3	189
4	567
5	1701

8. Determine Rate.

Table 3.2.28

x	y
0	3
1	7
2	15
3	27
4	43
5	63

is:

(a) Linear

(b) Exponential

(c) Quadratic

Table 3.2.29

x	y
0	4
1	8
2	16
3	32
4	64
5	128

is:

(a) Linear

(b) Exponential

(c) Quadratic

Table 3.2.30

x	y
0	6
1	13
2	20
3	27
4	34
5	41

is:

(a) Linear

(b) Exponential

(c) Quadratic

9. Determine Rate. Identify which type of pattern each table is. Continue each pattern, filling in the missing rows.

Table 3.2.31

x	y
0	9
1	18
2	36
3	72
4	__
5	__
6	576

is:

(a) Linear

(b) Exponential

(c) Quadratic

Table 3.2.32

x	y
0	2
1	4
2	8
3	14
4	__
5	__
6	44

is:

(a) Linear

(b) Exponential

(c) Quadratic

Table 3.2.33

x	y
0	6
1	9
2	12
3	15
4	__
5	__
6	24

is:

(a) Linear

(b) Exponential

(c) Quadratic

10. Determine Rate. Directions: Graph each set of values and determine whether the function is linear, quadratic, or exponential.

Table 3.2.34

x	y
-2	-2
-1	-2.5
0	-2
1	-1
2	1

The function is:

(a) Quadratic

(b) Exponential

(c) Linear

I know because the graph/table have a rate of change that

(a) the rate of change is slow at first, but then goes very steep

(b) the rate of change is constant

(c) the function decreases, then increases

HINT: Just graph the 5 points on this graph.

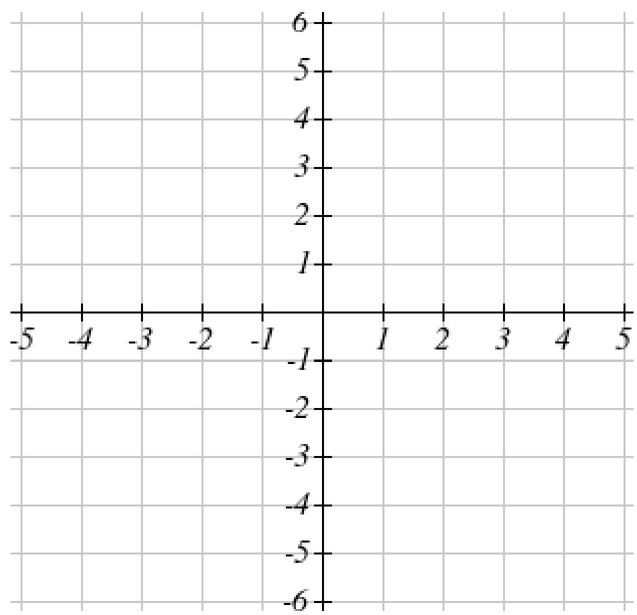

Figure 3.2.35

11. Determine Rate. For each scenario, identify the appropriate growth model that describes how it's changing.

(a) The number of new polio cases has been cut in half each year due to vaccination efforts

(b) The amount of pollutants in the lake has been increasing by 4 milligrams per Liter each year

(c) The number of arrests grew for several years, but now has been decreasing

(d) Tuition is currently $2,000 a quarter has been growing by 7% a year

(a) Linear

(b) Exponential

(c) Neither

12. Validity of Model. Your friend Pat says to you, "I'm on a new diet plan and I've been able to lose about a pound a week." Come up with a linear equation that models this situation and use it to answer the following questions:
If Pat currently weighs 220 pounds, how much would Pat weigh in a year (52 weeks)?
____________________ pounds
Assuming Pat stays on this plan and the equation is still valid, how much would Pat weigh in 4 years?
____________________ pounds
Does this equation still seem like it would be valid after 4 years? Why or why not?

13. Write next terms. For the following sequence, state the next three terms.
6, 10, 14, 18, ___, ___, ___
State a formula for the nth term: ______Preview Question 1 Part 4 of 5
What type of sequence is this?

(a) arithmetic (linear)

(b) quadratic

(c) geometric (exponential)

(d) other

14. Write next terms. For the following sequence, state the next three terms.
18, 54, 162, 486, ___, ___, ___
What type of sequence is this?

(a) arithmetic (linear)

(b) quadratic

(c) geometric (exponential)

(d) other

15. Write next terms. For the following sequence, state the next three terms.
2, 4, 7, 11, ___, ___, ___
What type of sequence is this?

(a) arithmetic (linear)

(b) quadratic

(c) geometric (exponential)

(d) other

3.3 Variation

In Section 3.2 we compared data based on the rate. Here we will emphasize how one variable affects another.

3.3.1 Direct Relations

In Section 3.2 we saw examples where the data increased. For example the salaries went up each year. The quadratic (Example 3.2.5) and exponential (Example 3.2.8) also are increasing data. When an increase in one variable causes another to increase, we call the relationship between them **direct**.

For some problems it makes sense to look at these another way. Consider the situation in Figure 3.3.1. The pressure exerted on a fluid by a piston is the ratio of the force exerted and the area of the piston. On the left that is $P = \frac{F_1}{A_1}$. The same is true on the right $P = \frac{F_2}{A_2}$. Because the hydraulic fluid is contiguous the pressure is the same on both sides. Thus

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

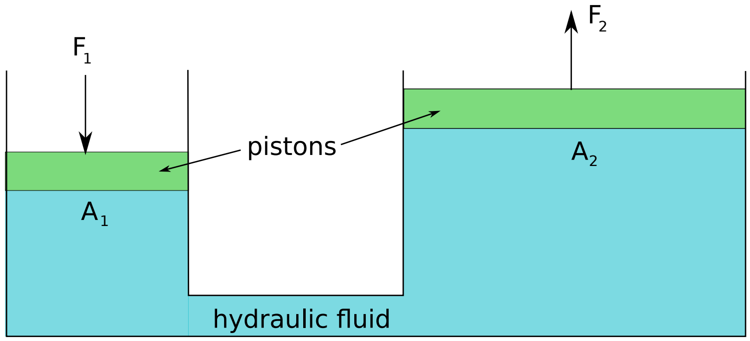

Figure 3.3.1 Hydraulic Press

If the left piston has area 16 cm^2, and 5.0 N of force is exerted, what is the force exerted on the second piston if it has area 25 cm^2?

Solution. Notice this is a proportion problem. We can use the relationship

$$\begin{aligned}\frac{16\text{ cm}^2}{5.0\text{ N}} &= \frac{25\text{ cm}^2}{F_2}.\\ (16\text{ cm}^2)F_2 &= (25\text{ cm}^2)(5.0\text{ N}). && \text{Eliminating the denominators.}\\ \frac{(16\text{ cm}^2)F_2}{(16\text{ cm}^2)} &= \frac{(25\text{ cm}^2)(5.0\text{ N})}{(16\text{ cm}^2)}.\\ F_2 &\approx 7.8\text{ N}.\end{aligned}$$

□

Next lets consider how changing the size of the second piston affects the force.

If the second piston's area is change from 25 cm^2 to 36 cm^2, what is the increase or decrease in force?

Solution. First we setup the same problem as in Example 3.3.2

$$\begin{aligned}\frac{16\text{ cm}^2}{5.0\text{ N}} &= \frac{36\text{ cm}^2}{F_2}.\\ (16\text{ cm}^2)F_2 &= (36\text{ cm}^2)(5.0\text{ N}). && \text{Eliminating the denominators.}\\ \frac{(16\text{ cm}^2)F_2}{(16\text{ cm}^2)} &= \frac{(36\text{ cm}^2)(5.0\text{ N})}{(16\text{ cm}^2)}.\\ F_2 &\approx 11\text{ N}.\end{aligned}$$

The piston is larger and the force is also larger □

Voltage and resistance vary directly (see further Example 3.1.20) . If voltage is 24 V and resistance is 8Ω, how would the voltage have to change if the resistance changes to 6Ω? to 10Ω?

Solution. Because the relationship is direct we can write

$$\begin{aligned} \frac{V_1}{R_1} &= \frac{V_2}{R_2}. \\ \frac{24\text{ V}}{8\Omega} &= \frac{V_2}{6\Omega}. \\ \frac{24\text{ V}}{8\Omega}(6\Omega) &= V_2. && \text{Eliminating the denominator on the right} \\ 18\text{ V} &= V_2. \end{aligned}$$

We do the same for the second case.

$$\begin{aligned} \frac{V_1}{R_1} &= \frac{V_2}{R_2}. \\ \frac{24\text{ V}}{8\Omega} &= \frac{V_2}{10\Omega}. \\ \frac{24\text{ V}}{8\Omega}(10\Omega) &= V_2. && \text{Eliminating the denominator on the right} \\ 30\text{ V} &= V_2. \end{aligned}$$

Notice that the resistances we chose change in a linear fashion: 6, 8, 10. Also note that the voltages changed in a linear fashion: 18, 24, 30. □

3.3.2 Inverse Relations

In Example 3.1.20 we saw data where the increase in one variable caused a decrease in the other. That example is known as an **inverse** relation.

Whereas a direct relation can be expressed as $\frac{V_1}{R_1} = \frac{V_2}{R_2}$ inverse relation can be expressed as $I_1R_1 = I_2R_2$.

If the current is 3 amps when the resistance is 8 Ohms, what will the current be when the resistance is 6 Ohms? 4 Ohms?

Solution. Because this is an inverse relation, we can write

$$\begin{aligned} I_1R_1 &= I_2R_2 \\ (3\text{ amps})(8\Omega)&I_2(6\Omega) \\ \frac{(3\text{ amps})(8\Omega)}{6\Omega}&I_2 && \text{Eliminating the denominator on the right} \\ 2.25\text{ amps} &= I_2. \end{aligned}$$

The second case can be written

$$\begin{aligned} (3\text{ amps})(8\Omega)&I_2(4\Omega) \\ \frac{(3\text{ amps})(8\Omega)}{4\Omega}&I_2 && \text{Eliminating the denominator on the right} \\ 1.5\text{ amps} &= I_2. \end{aligned}$$

□

Note that the relation in Example 3.3.5 the resistances we chose change in a linear fashion: 4, 6, 8. The current also change in a linear fashion: 3, 2.25, 1.5. Thus the relation is inverse linear.

In Table 3.3.7 we see data for an exponential with a negative exponent. Note that this decreases. The differences display the same pattern of matching the data except they are negative. The negative should make sense because the data is decreasing. This is inverse exponential. □

Table 3.3.7 Negative Exponential

n	2^{-n}	Difference
1	1/2	
2	1/4	-1/4
3	1/8	-1/8
4	1/16	-1/16
5	1/32	-1/32
6	1/64	-1/64

3.3.3 Constant of Variation

We defined direct and indirect variation in terms of how change in one variable affects another. Here we consider this from a different perspective.

For direct variation the ratio of variables is always equal. In Example 3.3.4 we have $\frac{V_1}{R_1} = \frac{V_2}{R_2}$. The specific results were $\frac{30}{10} = \frac{24}{8} = \frac{18}{6} = 3$. The number 3 is known as the **constant of variation**. In some cases it has meaning. In this case we recognize it to be the current (i.e., 3 amps).

Lift (L, the force that keeps aircraft in the air) increases with air density (ρ), surface area of wing (s), and the square of velocity (v^2). This can be expressed as

$$\frac{L_1}{\rho_1 s_1 v_1^2} = \frac{L_1}{\rho_2 s_2 v_2^2}$$

Thus if air density decreases ($\rho_2 < \rho_1$) either surface area or velocity must increase to maintain lift.

We can also express this as

$$\frac{L}{\rho s v^2} = k$$

or

$$L = k\rho s v^2.$$

For lift it is known that $k = \frac{1}{2}C_L$ where C_L is known as the coefficient of lift. It depends on a variety of factors such as the shape of the wing and the angle the wing makes with the air. □

3.3.4 Another Mixture Problem

In Subsection 2.2.1 we learned to calculate percents for mixtures and how to dilute a mixture to a specified percent.

By adding water we can of course not increase the percent alcohol, so 91% is the highest we can achieve. If we add enough water we can dilute it to as little as we want. That requires not restricting the final volume. This should make us wonder about a relationship between the desired volume and the minimum/maximum amount of alchol.

This first question is the same as Example 2.2.4. Use it to check your understanding of the process.

Checkpoint 3.3.9 Suppose we want 20.0 oz of 70.0% alcohol solution. How much water should we add? ____ How much solution will we have? ____

Answer 1. 4.9

Answer 2. 20.9

Solution. This is a percent problem with the total alcohol unchanged ($(0.91)16 = 14.56$) and adding only some amount w of water. Thus we setup

$$
\begin{aligned}
\frac{(0.910)16.0}{16.0+w} &= 0.700 \\
(0.910)16.0 &= 0.700(16.0+w) \\
(0.910)16.0 &= (0.700)16.0 + (0.700)w \\
14.6 &= 11.2 + (0.700)w \\
3.4 &= (0.700)w \\
\frac{3.4}{0.700} &= w \\
4.9 &\approx w
\end{aligned}
$$

Thus we end up with $16 + 4.9 = 20.9$ oz of the new solution.

We can confirm this work by calculating the percent water. The percent alcohol is

$$\frac{0.910(16.0)}{16.0+4.9} = 0.70$$

which is 70%.

Next we will illustrate that for a desired volume, we cannot obtain every percent.

Checkpoint 3.3.10 If we start with 16.0 oz of 91.0% alcohol solution, how much water do we add if we want 25.0 oz of a 60.0% alcohol solution? ____

How much solution total does this produce? ____

Answer 1. 8.56

Answer 2. 24.6

Notice that we did not produce 25 oz but a little less. One question we can ask then is "what is the percent alcohol solution we can obtain for a given volume?"

If we start with 16.0 oz of 91.0% alcohol solution and we want exactly 20.0 oz of solution, what is the percent alcohol we can obtain?

Solution. If we call the percent alcohol P then the percent water is $(1-P)$. Thus the final solution will have

$$(1-P)20.0 \text{ oz}$$

of water. We already have

$$0.0900 \cdot 16.0 = 1.44 \text{ oz}$$

of water. Thus we need

$$(1-P)20.0 - 1.44$$

oz of water. The total volume then will be

$$(1-P)20.0 - 1.44 + 16.0 = 20.0 \text{ oz}$$

Solving this tells us the percentage possible is

$$
\begin{aligned}
(1-P)20.0 - 1.44 + 16.0 &= 20.0 \\
(1-P)20.0 + 14.6 &= 20.0 \\
(1-P)20.0 &= 5.4 \\
1-P &= \frac{5.4}{20.0} \\
-P &= \frac{5.4}{20.0} - 1 \\
P &= 1 - \frac{5.4}{20.0} \\
P &\approx 0.73
\end{aligned}
$$

Thus the percent we obtain for exactly 20.0 oz is 73%. □

Checkpoint 3.3.12 If we start with 16.0 oz of 91.0% alcohol solution, what is the percent alcohol if we produce exactly 24.0 oz of solution? ____

Answer. 61

Checkpoint 3.3.13 Does the percent alcohol vary linear, quadratic, exponential, or the inverse of one of these with respect to the volume? You will need to determine the percent alcohol for 28.0 oz of solution and perhas 32.0 oz of solution. Use these data points to determine the relation.

3.3.5 Exercises

1. Distinguish Direct and Indirect Variation.

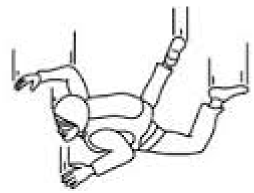

Figure 3.3.14

The terminal velocity of a skydiver can be determined by the equation $v = \left(\dfrac{2D}{\rho CA}\right)^{1/2}$ where

- D is the skydiver's weight
- ρ is the density of the air
- C is the skydiver's coefficient of drag
- A is the skydiver's ground-facing surface area

Use the concepts of direct and inverse variation to describe how changing each of these parameters impacts terminal velocity.

(a) Increasing ρ will

(a) decrease velocity

(b) increase velocity

(b) Increasing A will

(a) decrease velocity

(b) increase velocity

(c) Decreasing C will

(a) decrease velocity

(b) increase velocity

(d) Decreasing D will

(a) increase velocity

(b) decrease velocity

2. Describe Relation. For the following exercise, assume the constant k is positive.

V varies directly with t. Describe what happens to the value of V as t increases.

(a) no change

(b) increases

(c) undeterminable

(d) decreases

3. Describe Relation. For the following exercise, assume the constant k is positive.
W varies inversely with n. Describe what happens to the value of W as n increases.

(a) increases

(b) decreases

(c) undeterminable

(d) no change

4. Application. The force F (in pounds) needed on a wrench handle to loosen a certain bolt varies inversely with the length L (in inches) of the handle. A force of 40. pounds is needed when the handle is 8.0 inches long. If a person needs 25 pounds of force to loosen the bolt, estimate the length of the wrench handle. Calculate using significant digits.
____________________ inches

5. Application. The electrical current, in amperes, in a circuit varies directly as the voltage. When 18 volts are applied, the current is 6 amperes. What is the current when 51 volts are applied?
____________________Preview Question 1 amperes

6. Application. The number of hours required to build a fence is inversely proportional to the number of people working on the fence. If it takes 3 people 57 hours to complete the fence, then how long will it take 11 people to build the fence?
(Round the answer to 2 decimal places if needed)
____ hours.

7. Application. The capacitive reactance, X, in a circuit varies inversely as the frequency, f, of the applied voltage. If the reactance is 617 ohms when the frequency is 68.4 hertz, find the reactance when the frequency is 49.4.
_____ Ω

8. Application. The loudness, L, of a sound (measured in decibels, dB) is inversely proportional to the square of the distance, d, from the source of the sound.
When a person 6.0 feet from a jetski, it is 85.0 decibels loud. How loud is the jetski when the person is 40 feet away?
Round using the rules of significant figures.
____________________Preview Question 1 dB

9. Contextless Practice. S varies directly as p and q. If $p = 7$ and $q = 8$ then $S = 140$. Find the constant of proportionality.
$k =$ ____________________Preview Question 1

10. Contextless Practice. Write the equation representing the relationship, use k for the constant of variation.
f varies directly as y

(a) f / y = k

(b) f y = k

11. Contextless Practice. Write the equation representing the relationship, use k for the constant of variation.
b is inversely proportional to z

(a) b / z = k

(b) b z = k

12. Application. Hooke's law states that the distance that a spring is stretched by hanging object varies directly as the mass of the object. If the distance is 100.0 cm when the mass is 15.0 kg, what is the distance when the mass is 10.0 kg?

Round using the rules of significant figures.

____________________ cm

13. Application. The volume of a gas varies inversely as the pressure upon it. The volume of a gas is 800 cm^3 under a pressure of 8 kg/^2. What will be its volume under a pressure of 160 kg/cm^2?

Round your answer to two significant figures.

____________________ cm^3

14. Application. The wavelength of a radio wave varies inversely as its frequency. A wave with a frequency of 1800 kilohertz has a length of 200 meters. What is the length of a wave with a frequency of 400 kilohertz?

____________________ meters

15. Contextless Practice. Write a function describing the relationship of the given variables.

V varies directly with the square of t and when $t = 5$, $V = 350$

$V =$____________________Preview Question 1

16. Contextless Practice. Write a function describing the relationship of the given variables.

V varies inversely with the square of t and when $t = 5$, $V = 11$

$V =$____________________Preview Question 1

17. Contextless Practice. Write a function describing the relationship of the given variables.

C varies directly with the square root of L and when $L = 16$, $C = 12$

$C =$____________________Preview Question 1

18. Application. The velocity v of a falling object varies directly with the time t of the fall. If after 5.00 seconds, the velocity of an object is 160 feet per second, what is the velocity after 7.0 seconds?

Your answer should have 3 significant figures.

_____ feet per second Preview Question 1

19. Application. The weight of an object above the surface of Earth varies inversely with the square of distance from the center of Earth. If an object weighs 50.00 pounds when it is 3960 miles from Earth's center, what would the same object weigh when it is 4,020.0 miles from Earth's center?

Your answer should have 4 significant figures.

______ pounds

20. Application. Newton's Law of Gravitation says that two objects with masses m_1 and m_2 attract each other with a force F that is jointly proportional to their masses and inversly proportional to the square of the distance r between the objects. Newton discovered the constant of proportionality is 6.67×10^{-11}.

In a small laboratory experiment, two 600 kg masses are separated by 0.9 meters. What would the gravitational force between the objects be?

Force = ____________________Preview Question 1 Newtons

3.4 Rational Expressions

In Section 3.3 we worked with some relations that involved one dividing by a variable. Here we will work with more applications that involve rational (fractional) expressions.

3.4.1 Understand a Model

Activity 3 When one gear directly drives another the speeds at which they rotate (in rotations per minute or rpm) is given by

$$r_1 g_1 = r_2 g_2$$

where r_1 and r_2 are the rotation speeds of the two gears and g_1 and g_2 are the number of teeth on each gear.

(a) Consider a pair of gears with the larger have 32 teeth and the smaller having 16 teeth. If the larger gear does one, full rotation, 32 teeth move around. This means the smaller gear also has 32 teeth move around. How many rotations is this?

(b) Next consider a pair of gears with the larger having 36 teeth and the smaller having 20 teeth. If the larger gear rotates 2 times, how many times has the smaller gear rotated?

(c) Next we consider the effect of increasing the disparity in the number of teeth. Suppose the smaller gear has 10 teeth. How many times does it rotate for a single rotation if the larger gear has 30 teeth? 36 teeth? 42 teeth? 48 teeth? Is this linear, quadratic, exponential, inverse of one of these?

(d) The speed of the larger gear will always be what compared to the speed of the lower gear?

3.4.2 First Example

We worked with Ohm's Law relating voltage, current, and resistance. That applies to electricity flowing along one path (e.g., wire). In the next example we look at what effect connecting multiple paths has on resistance.

When two resistors are in parallel as shown in Figure 3.4.1 then the resulting resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

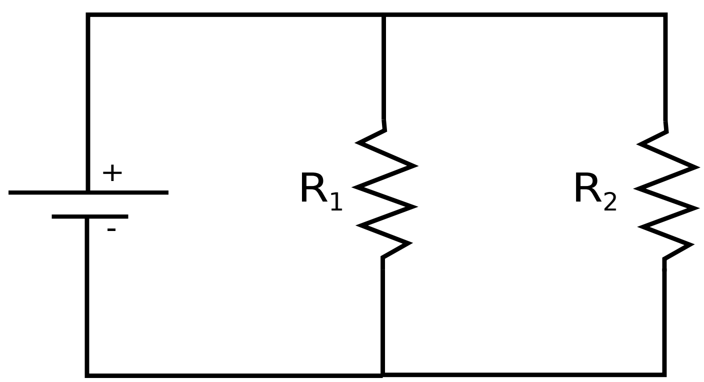

Figure 3.4.1 Resistors in Parallel

Calculate the resulting resistance when one resistor is 4 Ohms ($R_1 = 4$) and the other is 12 Ohms ($R_2 = 12$).

Solution.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$
$$\frac{1}{R} = \frac{1}{4} + \frac{1}{12}.$$

$$\begin{aligned}
\frac{1}{R} &= \frac{3}{3} \cdot \frac{1}{4} + \frac{1}{12}. \\
\frac{1}{R} &= \frac{3}{12} + \frac{1}{12}. \\
\frac{1}{R} &= \frac{4}{12} \\
&= \frac{1}{3}. \\
\frac{1}{R} \cdot 3R &= \frac{1}{3} \cdot 3R \\
3 &= R.
\end{aligned}$$

Note the need for a common denominator in the third line. The final step is our now frequently used clearing of denominators (i.e., 'cross multiplication'). □

In the previous example we knew the two resistors and calculated the resulting resistance. In other cases we know how much resistance we need and one of the resistors. We must calculate the resistance for the other resistor.

If we need a five Ohm resistance and have an eight Ohm resistor already, what do we add as the second resistor? Due to common accuracy of resistor measurement we can use two significant digits.

Solution.

$$\begin{aligned}
\frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2}. \\
\frac{1}{5.0} &= \frac{1}{8.0} + \frac{1}{R_2}. \\
\frac{1}{5.0} - \frac{1}{8.0} &= \frac{1}{R_2}. \\
\frac{8.0}{8.0} \cdot \frac{1}{5.0} - \frac{5.0}{5.0} \cdot \frac{1}{8.0} &= \frac{1}{R_2}. \\
\frac{8.0}{4\bar{0}} - \frac{5}{4\bar{0}} &= \frac{1}{R_2}. \\
\frac{3.0}{4\bar{0}} &= \frac{1}{R_2}. \\
R_2 \cdot \frac{4\bar{0}}{3.0} \cdot \frac{3.0}{4\bar{0}} &= R_2 \cdot \frac{4\bar{0}}{3.0} \cdot \frac{1}{R_2}. \\
R_2 &= \frac{4\bar{0}}{3.0} \\
&\approx 13.
\end{aligned}$$

Note the need for a common denominator in the fourth line. The final step is once again clearing of denominators (i.e., 'cross multiplication'). □

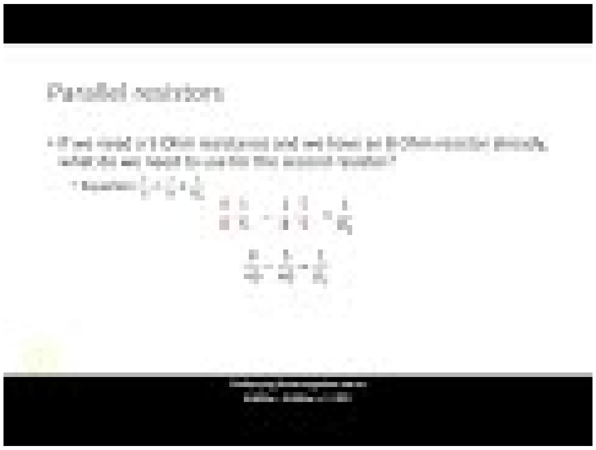

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Figure 3.4.4 Parallel Resistance Solving

Checkpoint 3.4.5 If two resistors are connected in parallel and they are 4 and 16 ohms, what is the resulting resistance? ______

Answer. $\frac{16}{5}$

Checkpoint 3.4.6 If we need 5 Ohm resistance and one of our resistors is a 4 Ohm resistor, can we find a second resistor to make this work? Explain.

3.4.3 Re-arranging Rational Expressions

Equations show relationships between parameters. Sometimes we want to re-arrange the equation for a specific parameter (variable).

There is a relation between the number of teeth (N) of a gear, the outside diameter (D_o), and the pitch diameter (D_p). It is

$$D_p = \frac{D_o N}{N+2}.$$

If we want to find the number of teeth given the two diameters we will need to solve for N.

Solution.

$$\begin{aligned}
D_p &= \frac{D_o N}{N+2}.\\
D_p \cdot (N+2) &= \frac{D_o N}{N+2} \cdot (N+2).\\
D_p(N+2) &= D_o N.\\
D_p N + 2D_p &= D_o N.\\
D_p N - D_o N &= -2D_p.\\
(D_p - D_o)N &= -2D_p.\\
\frac{(D_p - D_o)N}{D_p - D_o} &= \frac{-2D_p}{D_p - D_o}.\\
N &= \frac{-2D_p}{D_p - D_o}.
\end{aligned}$$

Notice we needed to collect the terms with N. This required distributing (third line), collecting on one side, then factoring. □

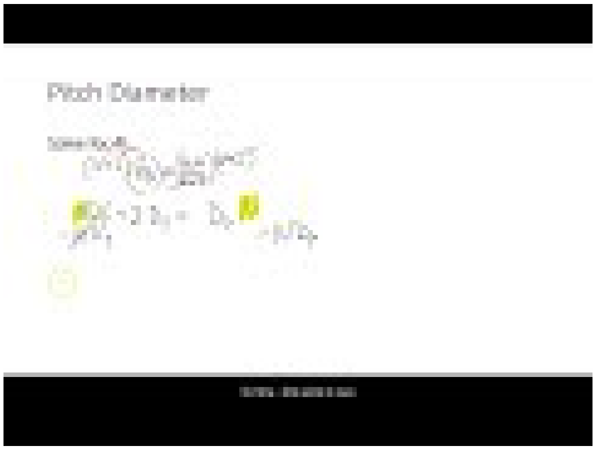

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Figure 3.4.8 Pitch Diamter

There is a relationship between volume, pressure, and temperature.

$$\frac{P_1V_1}{T_1+460}=\frac{P_2V_2}{T_2+460}.$$

P_1, V_1, and T_1 are the initial pressure, volume, and temperature. P_2, V_2, and T_2 are pressure, volume, and temperature at another time. One or more might change.

Checkpoint 3.4.9 Consider a situation where the initial measurements are $P_1 = 29.97$ inHg, $V_1 = 28.88$ in^3, and $T_1 = 55.00°$ F.

(a) Calculate P_2 when $T_2 = 55°$ F and $V_2 = 28.88$ in^3.

(b) Suppose $T_2 = 55.00°$ F (i.e., temperature is unchanged). Calculate P_2 when $V_2 = 25.88$, $V_2 = 22.88$, and $V_2 = 19.88$. Is the relationship between volume and pressure linear?

(c) Suppose $V_2 = 28.88$ in^3 (i.e., the object is not changing size). Calculate P_2 when $T_2 = 110.00$, $T_2 = 165.00$, and $T_2 = 220.00$. Is the relationship between pressure and temperature linear? How does the equation indicate this?

(d) Suppose $V_2 = 28.88$ in^3. What temperature, T_2, will produce a pressure of 31.23 inHg?

(e) We used inHg as the unit of pressure. Would this process still work if we used lbs/in^2 (psi)?

- **(i)** To figure this out copy your calculation for $V_2 = 28.88$ and $T_2 = 55$. Note that the conversion from inHg to psi is 1.000 inHg to 0.4912 psi. Convert $P_1 = 29.97$ inHg to psi.
- **(ii)** Use this to recalculate P_2. This is in psi.
- **(iii)** Convert it back to inHg.
- **(iv)** Does this match your first answer?
- **(v)** Does this equation work when units are changed?

3.4.4 Rates

There are many times when we need to calculate the rate at which something can be accomplished when more than one person/thing is working on it.

A company has two pumps available for draining flooded basements. One pump can drain a basement in 4.0 hours, whereas the other pump can do the job in only 3.0 hours. How long would it take to drain the basement if both pumps are used simulataneously?

Solution. If both pumps are running it will take less than 3 hours which one alone could do. The question is how to find the combined speed. Note speed is a rate, so this suggests we first write down the rates. The first pump operates at a rate of $\frac{1\text{ basement}}{4.0\text{ hours}}$ and the second pump operates at a rate of $\frac{1\text{ basement}}{3.0\text{ hours}}$. When the pumps are working together their rates would combine; we should end up with a faster rate (less than 3 hours per basement). The combined rate is

$$\begin{aligned} \frac{1\text{ basement}}{4.0\text{ hours}} + \frac{1\text{ basement}}{3.0\text{ hours}} &= \\ \frac{3}{3}\cdot\frac{1\text{ basement}}{4.0\text{ hours}} + \frac{4}{4}\cdot\frac{1\text{ basement}}{3.0\text{ hours}} &= \\ \frac{3\text{ basement}}{12.0\text{ hours}} + \frac{4\text{ basement}}{12.0\text{ hours}} &= \frac{7\text{ basement}}{12.0\text{ hours}}. \end{aligned}$$

Now we need to scale this rate from 7 basements per 12 hours to 1 basement per N hours.

$$\frac{1/7}{1/7}\cdot\frac{7\text{ basement}}{12.0\text{ hours}} = \frac{1\text{ basement}}{1.7\text{ hours}}.$$

Thus if both pumps are working it will take 1.7 hours drain the basement. □

Checkpoint 3.4.11 If one shop can do seven float changes in two days, and a second shop can do thirteen float changes in three days, how long will it take the pair of shops to do 65 float changes? ____

If they start on Monday and work only weekdays, on what day of the week will they finish? (☐ Monday ☐ Tuesday ☐ Wednesday ☐ Thursday ☐ Friday)

Answer 1. 8.3

Answer 2. Wednesday

3.5 Linear Systems

In Example 3.3.11 we wanted to solve a problem with two constraints. We could meet one exactly (percent alcohol) and the other in inequality (at least that volume). Many problems have more than one constraint (condition we want to meet). Here we will learn to solve one type of them.

3.5.1 Motivation

In Example 3.3.11 we diluted a mixture using just diluent (water in that case). In other situations we will have two mixtures and want to combine them.

Suppose we have 16 oz of 91% isopropyl alcohol and 12 oz of 75% isopropyl alcohol. How much of each do we need to mix to produce 10.0 oz of 85% alcohol?

Solution. A common technique in mathematics is to start by writing the answer. We will declare that we will use A oz of 91% alcohol and B oz of 75% alcohol. Next we will express our dual constraints using these answers (variables).

The first constraint is that we end up with 10 oz of solution. Thus

$$A + B = 10.0.$$

The second contraint is the percent alcohol. As in Example 3.3.11 we will start by figuring out how much alcohol total will be in the resulting solution. Because it will be 85% alcohol there will be

$$(0.85)10.0 = 8.5\text{ oz}.$$

Because A oz of the first solution will be added and it is 91% alcohol, it will contribute $(0.91)A$ oz of alcohol. Similarly the second solution will contribute $(0.75)B$ oz of alcohol. Combined we will obtain

$$(0.91)A + (0.75)B = 8.5.$$

Now we just need a way to solve this pair of equations. □

3.5.2 Crossing Lines

Our goal here is to consider what causes lines to cross. We will do this by looking at a pair of lines and seeing where they cross.

Recall that a line is a relation (set of points) such that the change between any two, equally space points is the same. Often you have heard this described as rise over run or slope. Slope is a geometric interpretation referring to how **steep** the line is.

Suppose we start at a point $(1,3)$. If we know that the line increases 2 units for each step then the points $(2,5)$ and $(3,7)$ are also on the line. This shows us that for each x value we add 2. This gives us $2x$ because that adds two for each x. But we must account for starting at $(1,3)$. $2(1)=2$ so we need to add one to shift it up. This is why the line is $2x+1$. This is the slope-intercept form of a line. The coefficient of x, 2 in this case, tells us how steep (how much it goes up for each incease of x). The constant, 1 in this case, shifts the line up or down. □

Checkpoint 3.5.3 In Figure 3.5.4 there are two lines. One goes through point $A=(1,3)$. It rises at a slope of 2/1 (two up for each one over). The other line goes through the point $B=(1,2)$ which is below A.

(a) Use the slider to set the slope of the second line to 3. Does the line cross the one through A? Where (left or right of point B)?

(b) Use the slider to set the slope to something bigger than 3. Does the line cross the first one? Where (left or right of point B)?

(c) Use the slider to set the slope to 1. Does the line cross the first one? Where (left or right of point B)?

(d) In general if one slope is steeper than the other will they cross?

(e) Can you select a slope so that they don't cross?

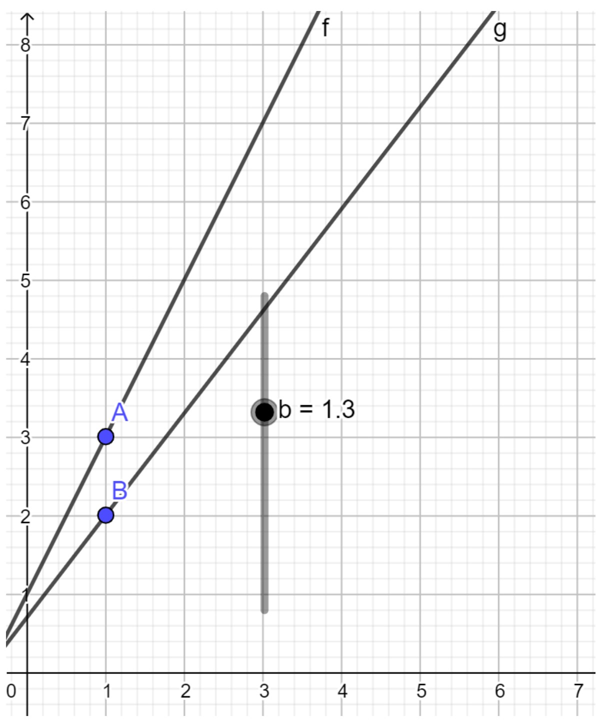

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Figure 3.5.4 Crossing Lines

Checkpoint 3.5.5 Vasya's initial pay was \$62,347.23. She received \$5,000 raises each year. Pyotr's initial pay was \$67,242.33. He receives \$3,500 raises each year. If they were both hired in 2012 in what year does Vasya first have a higher salary?

Solution 1. We could make a table.

Year	Vasya	Pyotr
2012	\$62,347.23	\$67,242.33
2013	\$67,347.23	\$70,742.33
2014	\$72,347.23	\$74,242.33
2015	\$77,347.23	\$77,742.33
2016	\$82,347.23	\$81,242.33

We see that is in 2016 that Vasya is first paid more.

Solution 2. We could note that Vasya's raises are \$1,500 more each year than Pyotr's raises. This means she closes the gap by \$1,500 each year. The difference in their initial salaries is $67242.33 - 62347.23 = 4895.10$. Because she gains by 1500 each year it will take $4895.10/1500 = 3.2634$ years. Because they receive raises once a year it will take 4 years. Thus she is first paid more in 2016.

3.5.3 Solving Linear Systems

We will consider two ways of solving linear systems of this type. The second method is very important for larger systems.

We will solve the system from Example 3.5.1. The two equations are

$$\begin{aligned} A + B &= 10.0. \\ (0.91)A + (0.75)B &= 8.5. \end{aligned}$$

Solution. Notice we can solve the first equation for B, then substitute it into the second.

$$\begin{aligned} B &= 10.0 - A. \\ (0.91)A + (0.75)(10.0 - A) &= 8.5. \\ (0.91)A + 7.5 - (0.75)A &= 8.5. \\ (0.16)A &= 1.0. \\ A &= \frac{1.0}{0.16} \\ &= 6.3. \end{aligned}$$

Now that we know that $A = 6.3$ we can substitute that into $A + B = 10.0$. This gives us

$$\begin{aligned} A + B &= 10.0. \\ 6.3 + B &= 10.0. \\ B &= 3.7. \end{aligned}$$

We can check that this works in the other equation (about percent alcohol).

$$\begin{aligned} (0.91)A + (0.75)B &= \\ (0.91)(6.3) + (0.75)(3.7) &= \\ 5.7 + 2.8 &= 8.5. \end{aligned}$$

□

If we had 7 variables instead of two, substituting would take a while. Instead we can use the following method which is more like solving as we know it, that is isolating a variable. We call it **elimination**.

We will solve the system

$$A + B = 10.0.$$

$$(0.91)A + (0.75)B = 8.5.$$

Solution. Note how we modify the first equation to partially match the second one.

$$\begin{aligned} A + B &= 10.0. \\ -(0.91)(A+B) &= -(0.91)10.0. \\ -(0.91)A - (0.91)B &= -9.1. \\ (0.91)A + (0.75)B &= 8.5. \\ -(0.16)B &= -0.6. \\ B &= \frac{-0.6}{-0.16} \\ &= 3.7. \end{aligned}$$

In the fifth line we added the two equations. Because they had opposite coefficients for A, that was eliminated leaving us with just B. This can always be done with systems of linear equations. ☐

Checkpoint 3.5.8 Solve the linear system below using substitution.

$$\begin{aligned} 2x + 3y &= 17. \\ 4x + 2y &= 14. \end{aligned}$$

$x =$ ____
$y =$ ____

Answer 1. 1

Answer 2. 5

Solution. We can solve the first equation for x and substitute.

$$\begin{aligned} 2x + 3y &= 17. \\ 2x &= 17 - 3y. \\ x &= \frac{1}{2}(17 - 3y). \end{aligned}$$

$$\begin{aligned} 4x + 2y &= 14. \\ 4\left(\frac{1}{2}(17 - 3y)\right) + 2y &= 14. \\ 34 - 6y + 2y &= 14. \\ -4y &= -20. \\ y &= 5. \\ 2x + 3(5) &= 17. \\ 2x + 15 &= 17. \\ 2x &= 2. \\ x &= 1.. \end{aligned}$$

Checkpoint 3.5.9 Solve the linear system below using elimination.

$$\begin{aligned} 2x + 3y &= 17. \\ 4x + 2y &= 14. \end{aligned}$$

$x =$ ____
$y =$ ____

Answer 1. 1

Answer 2. 5

Solution. We note that $4/2 = 2$ so we should multiply the first equation by -2.

$$\begin{aligned} -2(2x+3y) &= -2(17). \\ -4x-6y &= -34. \\ 4x+2y &= 14. \\ -4y &= -20. \\ y &= 5. \\ 2x+3(5) &= 17. \\ 2x+15 &= 17. \\ 2x &= 2. \\ x &= 1. \end{aligned}$$

Checkpoint 3.5.10 Find the solution to this system.

$$\begin{aligned} 6x+8y &= 22. \\ 9x+13y &= 35. \end{aligned}$$

$x =$ ___
$y =$ ___

Answer 1. 1

Answer 2. 2

Solution. To eliminate the $9x$ term we need to multiply the $6x$ equation by $-9/6 = -3/2$

$$\begin{aligned} -3/2(6x+8y) &= -3/222 \\ -9x-12y &= -33 \\ 9x+13y &= 35 \\ y &= 2 \\ 6x+8(2) &= 22 \\ 6x &= 6 \\ x &= 1 \end{aligned}$$

3.5.4 Other Cases

In Figure 3.5.4 we found a slope that caused no intersection. If we were solving a pair of linear equations that represented lines like this we would find no solution. These are known as **inconsistent** systems.

Find all solutions to the system

$$\begin{aligned} 2x+3y &= 5. \\ 4x+6y &= 7. \end{aligned}$$

Solution. We will use elimination. If we multiply -2 by the first equation we will obtain -4 (opposite of x in the second equation).

$$\begin{aligned} 2x+3y &= 5. \\ -2(2x+3y) &= -2(5). \\ -4x-6y &= -10. \\ 4x+6y &= 7. \\ 0 &= -3. \end{aligned}$$

Our work is correct, but the conclusion is clearly false. This means there are no solutions. You can think of this as saying, for a solution to exist 0 must equal -3. ☐

There is a third case.

Find all solutions to the system

$$2x + 3y = 5.$$
$$4x + 6y = 10.$$

Solution. We will use elimination. If we multiply -2 by the first equation we will obtain -4 (opposite of x in the second equation).

$$2x + 3y = 5.$$
$$-2(2x + 3y) = -2(5).$$
$$-4x - 6y = -10.$$
$$4x + 6y = 10.$$
$$0 = 0.$$

This time we have a true, but rather uninformative statement. We notice that after scaling (multiplying by -2) the two equations were identical. Essentially we had only one equation. Because one can be obtained from the other we call them **dependent**. ☐

Checkpoint 3.5.13 Determine whether this system is inconsistent or dependent.

$$6x + 8y = 52.$$
$$9x + 12y = 78.$$

The system is

⊙ Inconsistent

⊙ Dependent

Answer. Dependent

3.5.5 Exercises

1. Two Equations. Use substitution to solve this system of linear equations. $-2x + 2y = -6$ $x = 5y - 9$ Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One or more solutions: ____________________
- No solution
- Infinite number of solutions

2. Two Equations. Use substitution to solve this system of linear equations. $2x - 5y = 4$ $6x + y = -20$ Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

3. Two Equations. Use substitution to solve this system of linear equations. $x + 2y = -6$ $6y = -18 - 3x$ Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

4. **Two Equations.** Use substitution to solve this system of linear equations. $y = -4x + 11$ $-2y = 8x - 21$ Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

5. **Two Equations.** Use elimination to solve this system of linear equations. $\begin{array}{rcrcr} -3x & + & 2y & = & 4 \\ 3x & + & 3y & = & -9 \end{array}$
Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

6. **Two Equations.** Use elimination to solve this system of linear equations. $\begin{array}{rcrcr} 7x & + & y & = & -38 \\ 3x & - & 4y & = & -3 \end{array}$
Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

7. **Two Equations.** Use elimination to solve this system of linear equations. $\begin{array}{rcrcr} 3x & + & 3y & = & 3 \\ -9x & + & 2y & = & -20 \end{array}$
Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

8. **Two Equations.** Use elimination to solve this system of linear equations. $\begin{array}{rcrcr} 2x & - & 3y & = & -7 \\ -6x & + & 9y & = & 21 \end{array}$
Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

9. **Two Equations.** Use elimination to solve this system of linear equations. $\begin{array}{rcrcr} 8x & + & 4y & = & -20 \\ 12x & + & 6y & = & -31 \end{array}$
Select the correct choice below and, if necessary, enter an ordered pair (x, y) to complete your answer.

- One solution: ____________________
- No solution
- Infinite number of solutions

10. **Linear System Application.** We have a jar of coins, all quarters and dimes. All together, we have 265 coins, and the total value of all coins in the jar is $ 52.90. How many quarters are there in the jar?
Answer: ____ quarters

11. Linear System Application. A hoverboard manufacturer has just announced the *Glide 5* hoverboard. The accounting department has determined that the cost to manufacturer the *Glide 5* hoverboard is $y = 33.58x + 39150$. The revenue equation is $y = 85.78x$. What is the break even point for the *Glide 5* hoverboard?

The break even point for the *Glide 5* hoverboard is __________

12. Linear System Application. A store owner wants to mix chocolate and nuts to make a new candy. How many pounds of chocolate costing \$8.30 per pound should be mixed with 20 pounds of nuts that cost \$3.70 per pound to create a mixture worth \$5.67 per pound?

The owner needs to mix __________ pounds of chocolate.
(round to the nearest whole pound)

13. Linear System Application.

Figure 3.5.14

A coffee distributor plans to mix some House coffee that sells for \$8.70 per pound with some Queen City coffee that sells for \$14.00 per pound to create 30 pounds of a new coffee blend that will sell for \$10.82 per pound.

How many pounds of each kind of coffee should they mix? *Round to the nearest pound.* ____ pounds of House coffee. ____ pounds of Queen City coffee.

14. Linear System Application.

Figure 3.5.15

An airplane flying with the wind takes *5* hours to travel a distance of *900* miles.
The return trip takes *6* hours flying against the wind.
What is the speed of the airplane in still air and how fast is the wind blowing?
Answer:
The speed of the airplane in still air is _____ miles per hour.
The wind speed is _____ miles per hour.
Round your values to the nearest whole number.

15. Linear System Application.

Figure 3.5.16

A certain bread recipe asks you to combine yeast and flour with *2* cups of warm *120* °F water. If the water is hotter or colder than that, then the bread won't rise.

All you have available are hot tap water that is *140* °F and ice that is *32* °F.

How much hot tap water and ice should you mix together to get *2* cups of *120* °F water?

Answer:

____ cups of hot tap water.

____ cups of ice.

Round your answers to 2 decimal places.

16. Linear System Application. A 3.00 % solution of pesticide and a 5.00 % solution of pesticide must be combined to produce 146 mL of a 3.96 % solution. How much of each type should be mixed? Round using the rules of working with significant figures.

____________________ mL of 3 %

____________________ mL of 5 %

17. Linear System Application. At a farmers' market, Frederick buys 3 pounds of apples and 3 pounds of cherries for \$12.00. At the same farmers' market, Wilhelmina buys 6 pounds of apples and 15 pounds of cherries for \$50.82. Determine the price per pound of apples and cherries at the farmers' market.

Apples cost \$_______ per pound.

Cherries cost \$_______ per pound.

3.6 Project: Literal Formula

Project 4 Literal Formula. Most math books define the area of a circle as follows: $A = \pi r^2$, where A is the area of the circle and r is the radius of a circle. A text used in UAA's Automotive Diesel program defines the area of a circle as $A = 0.7854d^2$, where A is the area of the circle and d is the diameter of the circle.

The purpose of this project is to determine when each formula is most useful.

(a) What is the mathematical relationship between radius and diameter? Your answer can be a sentence or an equation.

(b) Show mathematically how to get from the formula $A = \pi r^2$ to the formula $A = 0.7854d^2$. This should take you multiple steps.

(c) Explain in words what you did in each step to change the first formula into the second. What assumptions did you have to make? Anyone reading this answer should be able to replicate the math by just reading your answer. That is, talk me through all the steps.

(d) Did you have any false starts or did you see how to change the formula right away? There is no wrong answer here; I just want you to think about your process.

(e) For this problem, you will need a tape measure or a ruler. If doing this on a device it must be a computer and ensure you are at 100% magnification. Your phone or a scaled version will distort the results. First *measure* the radius of the circle in Figure 3.6.1. Then *measure* the diameter of the circle below and record your answer. ***Do not calculate the diameter!*** This must be measured, not calculated. Try to be as precise as is reasonably possible. Include units.

Was it easier to measure the radius or the diameter?

(f) What is one reason why it might be more practical on a job to use the formula $A = 0.7854d^2$ instead of $A = \pi r^2$? If it helps, you may wish to ask yourself why the auto diesel students in particular use this less traditional formula.

(g) Determine how many significant figures are in each measurement. If the number is not a measurement or the measurement has no error then it is called 'exact'.

- π
- 0.7854

(h) Which of the two formulas is more accurate? Which is more precise? Give a reason to back up your answer.

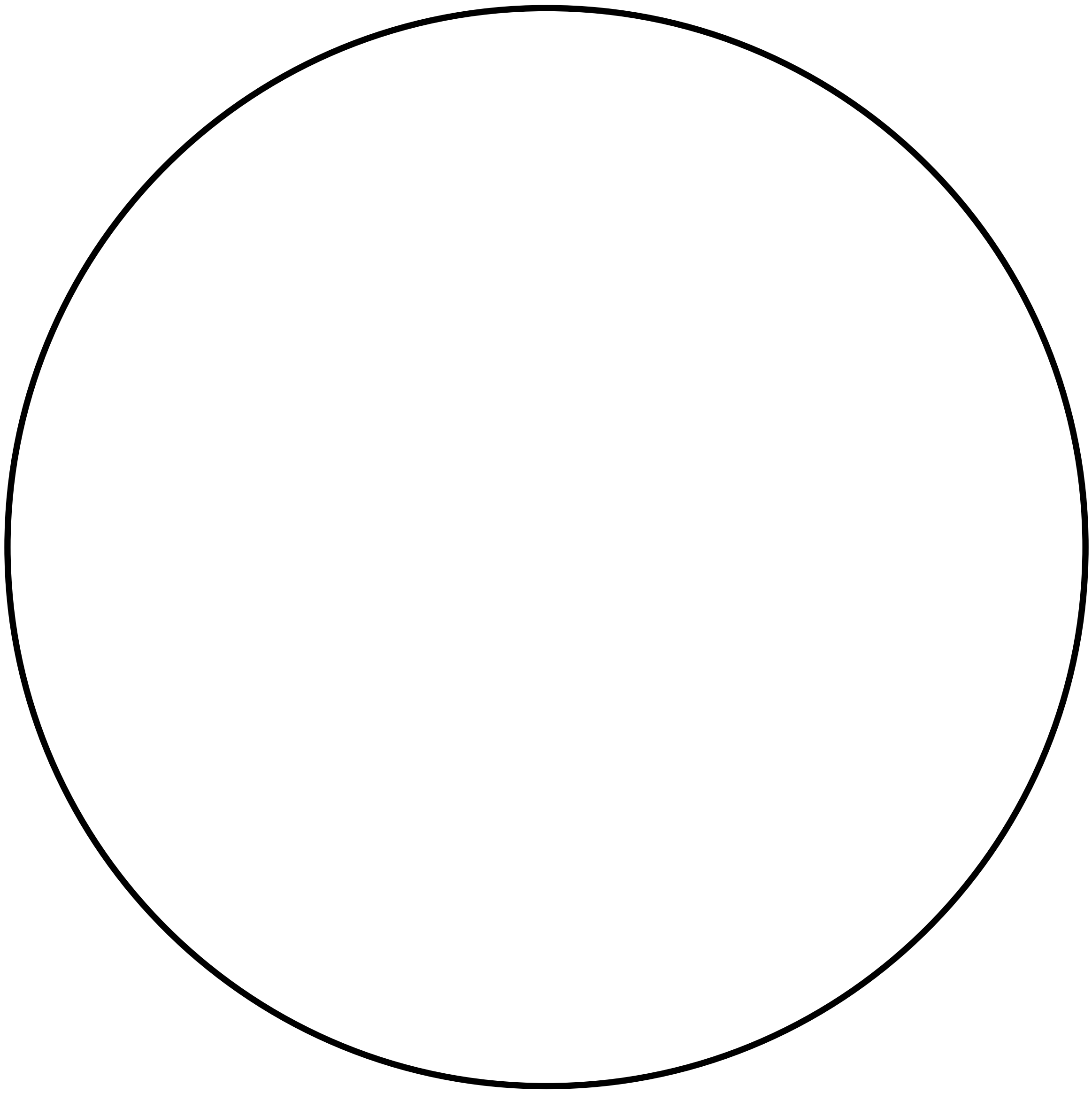


Figure 3.6.1 Circle

3.7 Project: Biking in Kansas and Alaska

Project 5 Project: Biking in Kansas and Alaska. In this project, we're going to think about what makes a relationship linear or not linear. Each question is worth two points.

(a) This is a graph of a linear relationship. Looking at it, what about it tells you that it is linear?

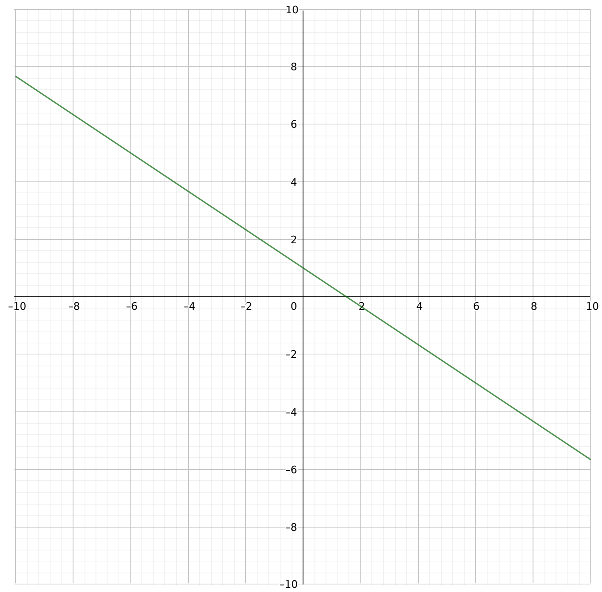


Figure 3.7.1 Graph of Line

(b) Here is a table of some of the points represented on the above graph. This data also represents a linear relationship. Without graphing, how can you tell that this relationship is linear?

Table 3.7.2 Table of Points

x	y
-6	5
-3	3
0	1
3	-1
6	-3

(c) Friends Jacob and Mike like to bike. For a math conference, the two traveled to Kansas and decided to go on a bike ride one evening. Mike enjoys tracking his data and so took note of his distance traveled at regular intervals. Here is a table of Mike's time and mileage:

Table 3.7.3 Time and Distance

Time biked (in minutes)	Distance traveled (in miles)
10	2
20	4
30	6
40	8
50	10

Does this table represent a linear relationship? Give some supporting computations OR write a sentence to support your answer.

(d) Jacob is more absent minded in tracking his mileage over time, and so took note of his distance traveled sporadically. Here is a table of Jacob's time and mileage:

Table 3.7.4 Time and Distance

Time biked (in minutes)	Distance traveled (in miles)
7	1.4
12	2.4
20	4
35	7
54	10.8

Does this table represent a linear relationship? Give some supporting computations OR write a sentence to support your answer.

(e) After returning home to Alaska, the friends decide to go on another ride. This bike ride was on a trail in the foothills of the Chugach Mountains. Again, Mike took note of his distance traveled at regular intervals Here is a table of Mike's time and mileage:

Table 3.7.5 Time and Distance

Time biked (in minutes)	Distance traveled (in miles)
10	2.3
20	4
30	5.2
40	6
50	8

Does this table represent a linear relationship? Give some supporting computations OR write a sentence to support your answer.

(f) Again, Jacob is absent minded in tracking his mileage over time, and so took note of his distance traveled sporadically. Here is a table of Jacob's time and mileage:

Table 3.7.6 Time and Distance

Time biked (in minutes)	Distance traveled (in miles)
11	2.5
20	4
25	4.8
43	7
65	10.5

Does this table represent a linear relationship? Give some supporting computations OR write a sentence to support your answer.

(g) Slope is $\frac{\text{change in y values}}{\text{change in x values}}$. If the first columns of the four tables above represent x values and the second columns represent y values, find the unit of the slope. Your answer should be a unit, like $\frac{\text{ft}}{\text{s}}$ or in^2, *not a number*.

(h) (1 point extra credit): Consider your answer to the previous question. What does this unit represent? Your answer can be one word.

3.8 Project: Radiation Dosage

Project 6 Calculating Effects of Radiation. The purpose of this project is to build a mathematical model for a situation. We will focus on the structure of the equations, and what they tell us about the mathematical relationships of the data. The emphasis is not on actual numbers.

For an unrelated example of a model, consider dropping a ball off a cliff. Ignoring air resistance, the ball's position can be modeled by the equation $h = kt^2$, where t is the time and h is the height of the ball.

There are some other numbers that go in there, but what is changing is the time and position (height). That equation framework is the mathematical model.

You may find this to be a challenging project. Do the best you can and use your own common sense. Math should make sense! After you finish this, please read back over your work and make sure your answers are logically consistent.

Remember that you can ask questions and meet up with a tutor, but you should *not* be looking up answers or just writing down what someone else says. Do not let someone else copy your answers. That is academic dishonesty and you should not allow it. Your work should be your own.

For this project, imagine that you are working with radioactive material. Since we do not want you harmed by radiation, we should understand how time and distance impacts the radioactive dose. (You should also be behind material that shields you from the radiation, but that math is more complicated, so we'll focus on time and distance.)

(a) As your time near a radioactive source increases, does the radiation level in your body increase, decrease, or stay the same?

(b) Do you think the relationship between time and dosage is linear or not linear?

(c) Let E stand for radiation exposure, t stand for length of time of exposure, and k be the constant of variation. Write an equation representing the relationship between E, t, and k.

(d) Examine the above equation you just wrote.

(a) Is it linear or non-linear? This should match your answer to Task 6.b.

(b) What in the equation indicates it is linear or non-linear? That is, how did you know the answer to part Item 1?

(e) We have determined the relation between time and radiation exposure. Next we determine the relation between distance to radiation source and radiation exposure. These can be different.

Radiation radiates outwards from a source evenly in all directions (like light radiates out evenly from a lightbulb) unless it is obstructed by something (like a lead shield). Imagine a radiation source floating in the center of a sphere. All parts of the sphere would be getting hit with an equal amount of radiation. We are going to figure out the radiation for a given patch of area on this sphere.

It will be helpful to know the following formula: $A = 4\pi r^2$, where A is the surface area of the sphere and r is the radius of the sphere.

If a sphere had a radius of 2 m, what is the surface area of the sphere? Remember to include units. Leave your answer in terms of π (meaning it should look like __________π m^2).

(f) A very rough approximation of the surface area of the front of a person is $1\,\text{m}^2$. We will consider what percent of the surface area of a sphere this is.

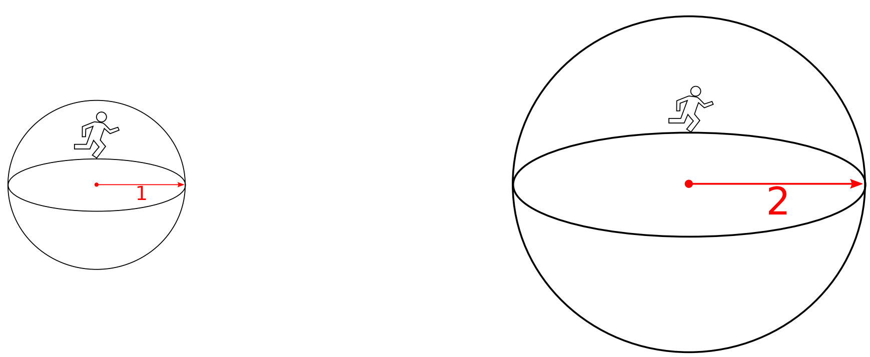


Figure 3.8.1 Surface Area Ratio

(i) What percent of the surface area of a sphere of radius 1 is $1\,\text{m}^2$?

(ii) What percent of the surface area of a sphere of radius 2 is $1\,\text{m}^2$?

(iii) Why does the change in percent make sense? See Figure 3.8.1.

(iv) In general as the radius increases what will the percent of the surface that is $1\,\text{m}^2$ do? Increase/remain the same/decrease?

(g) Note that the amount of radiation (energy) remains the same regardless of the radius of the sphere. That is a sphere with radius 1 and surface area 4π has the same energy as a sphere with radius 2 and surface area 16π.

(i) If the radiation is being emitted at an intensity of 5 Sieverts per hour ($\frac{\text{Sv}}{\text{h}}$), what amount of radiation will be hitting our $1\,\text{m}^2$ person who is at a distance of $1\,\text{m}$ from the source?

(ii) If the radiation is being emitted at an intensity of 5 Sieverts per hour ($\frac{\text{Sv}}{\text{h}}$), what amount of radiation will be hitting our $1\,\text{m}^2$ person who is at a distance of $2\,\text{m}$ from the source?

(iii) In general as the distance (radius) increases what happens to the amount of radiation absorbed by the person do? Increase/remain the same/decrease?

(h) If the radiation is being emitted at an intensity of x Sieverts per hour ($\frac{\text{Sv}}{\text{h}}$), what amount of radiation will be hitting our human-sized cutout on the surface of the sphere? Your answer should be in terms of x.

(i) Complete the following table. Notice that you already found the values for the first row.

Table 3.8.2 Ratio of Surface Areas

Radius of circle	Ratio of $1\,\text{m}^2$ to surface area
$2\,\text{m}$	
$3\,\text{m}$	
$4\,\text{m}$	
$5\,\text{m}$	
r m	

(j) Graph the points in Table 3.8.2.

Hint. Your horizontal axis (radius) should be from 0 to 5. Because the output numbers are small, we need a scale that matches. Make the units on the vertical scale 0.002.

(k) Does the data represented in the table above represent a linear relationship or a non-linear relationship? Give a reason to justify your answer.

(l) Is the relationship between distance and the potential amount of radiation hitting the person better modeled by direct variation or inverse variation?

(m) Consider the numbers 16, 36, 64, 100. These numbers are significant in mathematics. What is the pattern or significance of these numbers? (Note: you only see these numbers if you completed the table in terms of π (do not multiply and round). Go back and fix your table if you do not see these numbers.)

(n) As the radius of the sphere increases, does the level of radiation hitting our person-sized cutout increase or decrease? Does this increase or decrease change linearly (at a constant rate) or non-linearly (at a changing rate)? Circle the appropriate answer.

As the radius of the sphere increases, the radiation intensity is *increasing / decreasing* (circle one) in a *linear / non-linear* (circle one) fashion.

(o) The sphere with a floating radiation source is a good model for us to use when thinking through how distance impacts radiation levels because we can disregard complicating factors like the walls, floor, and ceiling of the room as long as a person is still directly exposed to the radiation source. Still, the relationship you found between radius and radiation holds true. With that in mind, answer the following questions.

Let I stand for radiation experienced by the person, r stand for distance, and k be the constant of variation. Write an equation of variation representing the relationship between I, r, and k.

Hint. Look back at your recent answers and the table you built. Is the formula you wrote logically consistent with these answers? That is, if you plugged in the r with some constant value k, would you get the right answer for I?

(p) Let R stand for radiation exposure, t stand for length of time of exposure, r stand for distance, and k be the constant of variation (this k may be different from your k in Task 6.c or Task 6.n). Write an equation of joint variation representing the relationship between R, r, t, and k.

Hint. Is your answer consistent with your answers to #3 and #13?

(q) Use the equation you just built. If you are 5 meters away for 3 hours, how long would you stay at 2 meters away to receive the same radiation dose?

(r) At the end of it all, if you find yourself next to a radioactive source, what do you do? Full points will be awarded for all reasonable answers that address both time and distance.

Chapter 4

Geometry

4.1 Geometric Reasoning

Here we will use a variety of formulas for geometric properties on basic shapes to analyze objects in context and more complicated shapes. These formulas are provided (largely) without explanation. Our goal is to break down complex problems into simpler problems we can solve using these formulas on basic shapes.

4.1.1 Properties

Two of the properties of shapes we will consider are **perimeter** and **area**. The **perimeter** of a shape is a measure of the size of its border (edges). The **area** of a shape is a measure of what it takes to fill the shape.

4.1.2 2D Shapes

Definition 4.1.1 Parallelogram. A **parallelogram** is a four sided shape for which opposing pairs of sides are parallel. ◊

Note this includes **rectangles**, which are parallelograms with four right angles, and **rhombi** which are parallelograms with four equal length sides. Notice that a square is a rectangle and a rhombus.

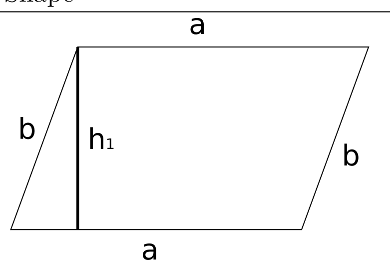
Shape	Perimeter	Area
	$2(a+b)$	$h_1 a$

Figure 4.1.2 Parallelograms

What are the perimeter and area of the parallelogram in Figure 4.1.4?

Solution. The perimeter is the sum of the sides which in this case is

$$2(4.05 + 5.00) = 18.1.$$

The area of a parallelogram, given in Figure 4.1.2 is $h_1 a$. For this parallelogram that is

$$\text{Area} = 3.00 \cdot 5.00 = 15.0.$$

□

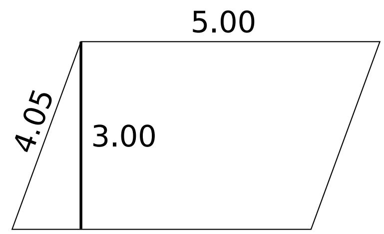


Figure 4.1.4 Calculate the area

Checkpoint 4.1.5 In Figure 4.1.2 the height is labeled h_1. Where is another height that could be used?

Checkpoint 4.1.6 What does h_1 equal in a rectangle?

Definition 4.1.7 Trapezoid. A **trapezoid** is a four sided shape for which one pair of opposing sides are parallel. ◊

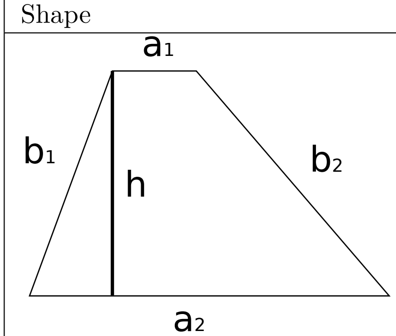
Shape	Perimeter	Area
	$a_1 + b_1 + a_2 + b_2$	$\frac{h}{2}(a_1 + a_2)$

Figure 4.1.8 Trapezoid

What are the perimeter and area of the trapezoid in Figure 4.1.10?

Solution. The perimeter of this trapezoid is the sum of the four side lengths

$$4.05 + 1.76 + 3.04 + 5.00 = 13.85.$$

The area of a trapezoid, given in Figure 4.1.8 is $\frac{h}{2}(a + b)$. For this trapezoid that is

$$\text{Area} = \frac{3.00}{2}(1.76 + 5.00) \approx 10.1.$$

□

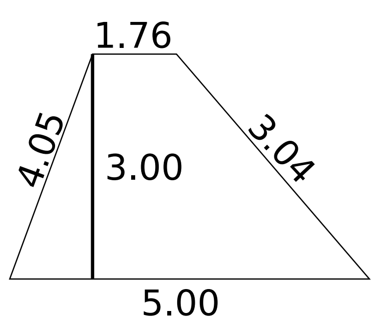


Figure 4.1.10 Calculate the area

Definition 4.1.11 Triangle. A **triangle** is a three sided shape. ◊

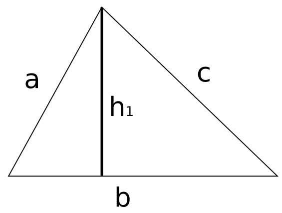
Shape	Perimeter	Area
	$a + b + c$	$\frac{1}{2}bh$

Figure 4.1.12 Triangle

Note that the value h in the area formula is called the **height** of the triangle. It is the length of a line segment perpendicularly down from a vertex to the opposing side (or extension of it). The vertical, dashed line segments in Figure 4.1.14 are heights for those two triangles. The one on the left is from the top vertex down to the bottom side. The one on the right is from the top vertex down to the extension (to the left) of the bottom side.

What are the perimeter and area of the triangles in Figure 4.1.14?

Solution. The perimeter of the triangle on the left is

$$5.98 + 10.9 + 8.19 \approx 25.1.$$

The perimeter of the triangle on the right is

$$5.98 + 2.87 + 8.19 \approx 17.0.$$

The area of a triangle, given in Figure 4.1.12 is $\frac{1}{2}bh$. For the triangle on the left that is

$$\text{Area} = \frac{1}{2}10.90 \cdot 4.43 \approx 24.1.$$

For the triangle on the right the area is

$$\text{Area} = \frac{1}{2}2.87 \cdot 4.43 \approx 6.36$$

□

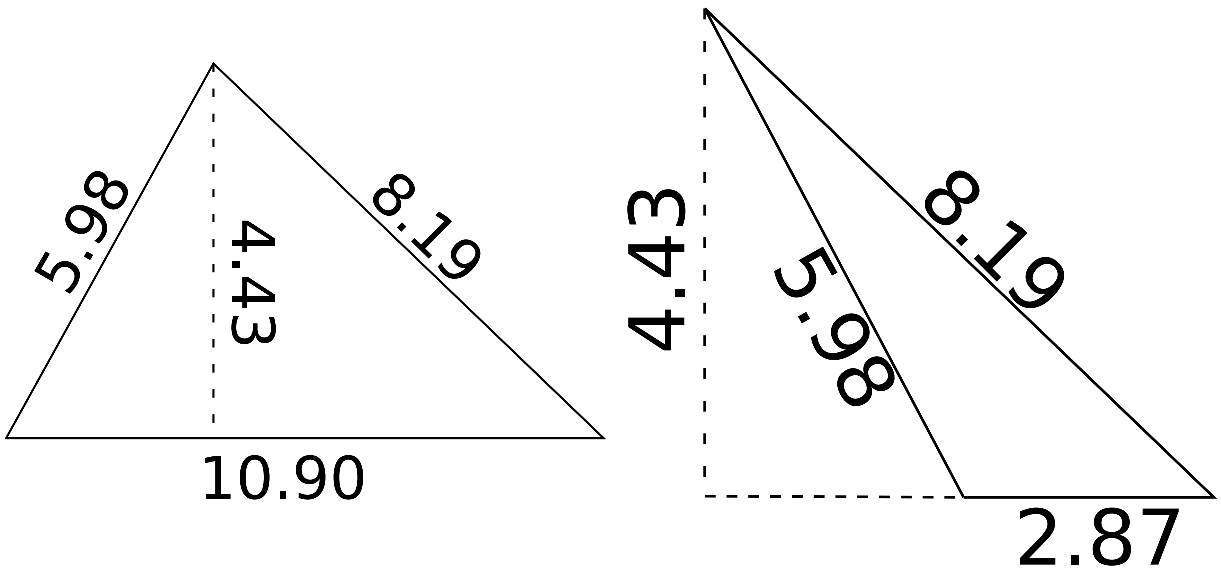


Figure 4.1.14 Calculate the area

Checkpoint 4.1.15 In Figure 4.1.12 the height is labeled h_1. Where are other possible heights and where are their bases?

Theorem 4.1.16 Pythagorean Theorem. *For a triangle containing a right angle*

$$a^2 + b^2 = c^2$$

where a and b are the lengths of the sides adjacent to the right angle and c is the third side.

Consider the triangle on the right in Figure 4.1.14. Consider the segments of length 4.43, 5.98, and the horizontal dashed segment. 5.98 is the length of the side not adjacent to the right angle (c in the formula). We can calculate the length of the horizontal, dashed segment using the formula.

$$\begin{aligned} 4.43^2 + b^2 &= 5.98^2. \\ 19.6 + b^2 &= 35.8. \\ b^2 &= 16.2. \\ \sqrt{b^2} &= \sqrt{16.2}. \\ b &\approx 4.06. \end{aligned}$$

□

In Section **??** we will develop a version of this statement for triangles without a right angle.

Theorem 4.1.18 Heron's Formula. *The area of a triangle can be calculated using the three sides.*

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$.

Calculate the area of the triangles in Figure 4.1.14.

Solution. According to Heron's formula for the triangle on the left

$$\begin{aligned} s &= \frac{1}{2}(5.98 + 8.19 + 10.90) \\ &\approx 12.54. \\ \text{Area} &= \sqrt{12.54(12.54 - 5.98)(12.54 - 8.19)(12.54 - 10.90)} \end{aligned}$$

$$= 24.2.$$

For the triangle on the right

$$\begin{aligned} s &= \frac{1}{2}(5.98 + 8.19 + 2.87) \\ &\approx 8.52. \\ \text{Area} &= \sqrt{8.52(8.52 - 5.98)(8.52 - 8.19)(8.52 - 2.87)} \\ &\approx 6.35. \end{aligned}$$

$\square$

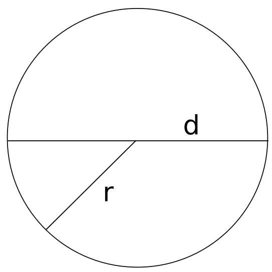
Shape	Perimeter	Area
	$2\pi r$ πd	πr^2 $\pi \frac{d^2}{4}$

Figure 4.1.20 Circle

For a circle with radius 7.31 what are the perimeter and area?
The perimeter, given in Figure 4.1.20, is $2\pi r$. For radius 7.31 the perimeter is

$$2\pi(7.31) \approx 45.9.$$

The area, given in Figure 4.1.20, is πr^2. For radius 7.31 the area is

$$\pi(7.31)^2 \approx 168.$$

$\square$

What are the perimeter and area of a semi-circle with diameter 11.7?
The perimeter includes half the usual perimeter plus the length of the diameter.

$$\frac{1}{2}\pi(11.7) + 11.7 \approx 30.1.$$

The area is simply half of the usual area.

$$\frac{1}{2}\pi(11.7)^2 \approx 215.$$

$\square$

4.1.3 Applying Geometry

Our first task in using geometry properties is to break down a problem into the kinds of shapes we already know. Then we can use the properties to calculate results.

(a) Find the area of this wall given the dimensions given in feet.

Solution. First we note that we can describe the wall as a rectangle with a triangle on top of it.

The sides of the rectangle area are 7 ft (height) and 24 ft (width). This means that area is 7 ft $\cdot$ 24 ft = 168 ft^2.

The top is a triangle with two sides of length 13 and one of length 24. We don't know the height of the triangle so it will be easier to use Heron's formula for area.

$$\begin{aligned} s &= \frac{1}{2}(13+13+24) \\ &= 25. \\ \text{area} &= \sqrt{25(25-13)(25-13)(25-24)} \\ &= 60. \end{aligned}$$

The total area then is $168 + 60 = 228$ square feet.

(b) Find the perimeter of this wall given the dimensions given in feet.

Solution. There are five (5) edges. There sum is $7 + 24 + 7 + 13 + 13 = 64$ feet.

(c) What is the (tallest) height of the wall?

Solution. We need the height of the triangular portion of the wall to find the height at the peak. Height is part of the area formula, and from Heron's formula we already know the height.

$$\begin{aligned} 60 &= \frac{1}{2}24h. \\ 5 &= h. \end{aligned}$$

The total height is then $7 + 5 = 12$.

□

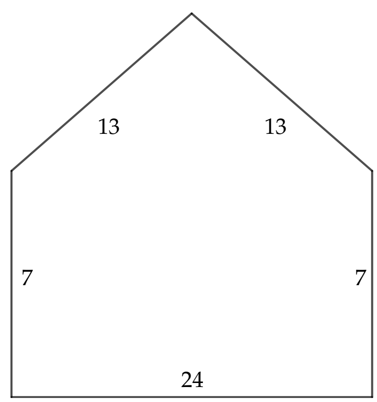


Figure 4.1.24 Wall

Katie is building a large scale abacus for a park. Her plan is to build it from treated 2x4 lumber. Her plan is shown in Figure 4.1.26 Note the depth of each piece of wood is 3.5". If you are wondering why a 2x4 is 1.5 in x 3.5 in, note that the nominal size (2x4 in this case) is based on the initial cut. The lumber shrinks as it cures and again when it is planed smooth.

Because we must have enough wood, we will round up all approximations.

(a) What is the total number of feet of lumber (2x4) needed?

Solution. There are two boards of length 60.0 inches and three boards of length 27.0 inches. The total length is

$$2 \cdot 60.0 \text{ in} + 3 \cdot 27.0 \text{ in} = 12\bar{0} \text{ in}.$$

We convert this to feet using a ratio.

$$120.\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}} = 10.0\text{ ft}$$

(b) If a standard 2x4 is 96.0 inches long, what is the smallest number of boards Katie can purchase to have enough lumber?

Solution. If a 60.0 inch section is cut from a 96.0 inch board, we have 96.0 in − 60.0 in = 36.0 in left. This is long enough for one of the 27.0 inch segments but not more. Thus two board will cover all but the last 27.0 inch segment. We need 3, 96.0 inch boards.

(c) If the boards are painted before they are assembled, what is the total surface area of the boards to be painted?

Solution. Each board has six surface. Each surface size appears twice (e.g., top and bottom). For the long segments these areas are

$$\begin{aligned}60.0\text{ in}\cdot 3.5\text{ in} &= 210\text{ in}^2,\\ 60.0\text{ in}\cdot 1.5\text{ in} &= 9\bar{0}\text{ in}^2,\\ 1.5\text{ in}\cdot 3.5\text{ in} &\approx 5.3\text{ in}^2.\end{aligned}$$

For the short segments these are

$$\begin{aligned}27.0\text{ in}\cdot 3.5\text{ in} &\approx 95\text{ in}^2,\\ 27.0\text{ in}\cdot 1.5\text{ in} &\approx 41\text{ in}^2,\\ 1.5\text{ in}\cdot 3.5\text{ in} &= 5.3\text{ in}^2.\end{aligned}$$

Thus the total area is

$$\begin{aligned}&2(2)(210\text{ in}^2)+2(2)(9\bar{0}\text{ in}^2)+2(2)(5.3\text{ in}^2)\\ &+3(2)(95\text{ in}^2)+3(2)(41\text{ in}^2)+3(2)(5.3\text{ in}^2)=\\ &810+360+21.2+570+246+31.8\text{ in}^2 = 2069\text{ in}^2\end{aligned}$$

This is 2069 in^2.

(d) What is the area that is hidden, that is, cannot be seen after assembling?

Solution. This would be where the three short boards touch the long boards. There are six places where this happens which are all the same shape $3.5\text{ in}\cdot 1.5\text{ in}\approx 5.3\text{ in}^2$. The covered surface is on both the short boards and the matching spot on the long boards, so there are 12 of these surfaces for $12\cdot 5.3\text{ in}^2\approx 64\text{ in}^2$.

□

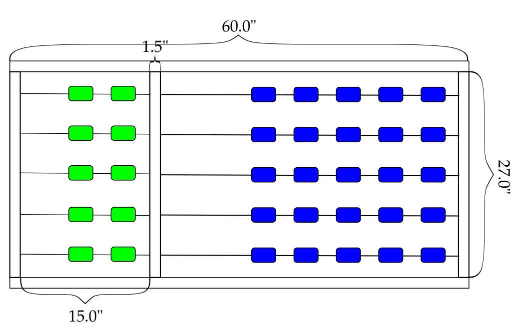


Figure 4.1.26 Abacus