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CML - HW 5

let $l_{hinge}(y,t) = \max(\phi, 1-yt)$

↳ not smooth b/c function is not diff @ $yt = 1$

<u>def:</u>

$$l_{huber\text{-}hinge}(y,t) = \begin{cases} \phi & ; \; yt > 1+h \\ \dfrac{(1+h-yt)^2}{4h} & ; \; |1-yt| \le h \\ 1-yt & ; \; yt < 1-h \end{cases}$$

$l_{h\text{-}h}$ is defined as a piece-wise function w/ 3 "pieces",
all 3 pieces are continuous for $(y,t) \in \mathbb{R}^2$.

$\therefore$ $l_{h\text{-}h}(y,t)$ is differentiable on each separate domain, but it must also be differentiable on the change in intervals (just continuity is NOT enough).

$$↳ \; \nabla l_{h\text{-}h} = \begin{cases} \vec{\phi} & ; \; yt > 1+h \\ \left\langle \dfrac{-(1+h-yt)t}{2h}, \dfrac{-(1+h-yt)y}{2h} \right\rangle & ; \; \overbrace{|1-yt| \le h}^{\text{interval } I} \\ \langle -t, -y \rangle & ; \; yt < 1-h \end{cases}$$

$\therefore$ $\lim\limits_{yt \to 1.5^+} l_{h\text{-}h}(y,t) = \lim\limits_{yt \to 1.5^-} l_{h\text{-}h}(y,t)$ and.

$\lim\limits_{yt \to 0.5^-} l_{h\text{-}h}(y,t) = \lim\limits_{yt \to 0.5^+} l_{h\text{-}h}(y,t)$

$\therefore l_{h-h}(y,t)$ is differentiable on it's entire domain

$\therefore l_{h-h}(y,t)$ is smooth

Note: lipschitz cont. $\subseteq$ absolutely cont. $\subseteq$ uniformly cont.

for $l_{h-h}(y,t) = \emptyset$, the function is 1-lipschitz

let $dl = \langle dy, dt \rangle \mid dy = l_{h-h}(y+dy, t) - l_{h-h}(y,t)$

and $dt = l_{h-h}(y, t+dt) - l_{h-h}(y,t)$

$\therefore |\langle l_{h-h}(y+dy,t) - l_{h-h}(y,t), l_{h-h}(y,t+dt) - l_{h-h}(y,t)\rangle| < L \cdot |dl|$

$\therefore \nabla l_{h-h} = \vec{\emptyset}$ is 1-lipschitz

$\therefore \nabla l_{h-h} = \left\langle \frac{-(1+h-yt)t}{2h}, \frac{-(1+h-yt)y}{2h} \right\rangle$

is 1.5-lipschitz

$\therefore \nabla l_{h-h} = \langle -t, -y \rangle$ is 0.5-lipschitz.