Homework 5

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1. Analytic Expressions. Let

$$l_{hinge}(y,t) = max(0, 1 - yt)$$

This function is continuous everywhere, but it is not smooth because it is not differentiable at the point $yt = 1$.

An improvement to this is the smoothed hinge function:

$$l_{huber-hinge}(y,t) = \begin{cases} 0 & yt > 1 + h \\ \frac{(1+h-yt)^2}{4h} & -h \leqslant 1 - yt \leqslant h \\ 1 - yt & yt < 1 - h \end{cases}$$

Huber-hinge loss function is defined as a piecewise function with three "pieces." Each of the functions on these three intervals is continuously differentiable on its domain. Therefore the function is continuously differentiable on those intervals. It is only at the points where the function changes.

$$\nabla l_{huber-hinge}(y,t) = \begin{cases} < 0, 0 > & yt > 1 + h \\ < \frac{-t(1+h-yt)}{4h}, \frac{-y(1+h-yt)}{2h} > & -h \leqslant 1 - yt \leqslant h \\ < -t, -y > & yt < 1 - h \end{cases}$$

Since:

$$\lim_{yt \to 1.5^+} l_{huber-hinge} = \lim_{yt \to 1.5^-} l_{huber-hinge} = \lim_{yt \to 1.5} l_{huber-hinge}$$

and

$$\lim_{yt \to 0.5^+} l_{huber-hinge} = \lim_{yt \to 0.5^-} l_{huber-hinge} = \lim_{yt \to 0.5} l_{huber-hinge}$$

Therefore, the huber-hinge loss function is differentiable on its entire domain, and it is smooth.

Note that: lipschitz continuity $\subseteq absolutecontinuity \subseteq uniformcontinuity$.

Let:

$$dl =< dy, dt > \mid dy = l_{h-h}(y + dy, t) - l_{h-h}(y, t);\ l_{h-h}(, t + dt) - l_{h-h}(y, t)$$

$$\therefore \nabla l = < 0, 0 >$$

is 1-lipschitz

$$\therefore \nabla l = < \frac{-t(1+h-yt)}{4h}, \frac{-y(1+h-yt)}{2h} >$$

is 1.5-lipschitz

$$\therefore \nabla l = < -t, -y >$$

is 0.5-lipschitz